

IMPROVING ON THE NDC*

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ABSTRACT

The NDC (Notional Defined Contribution) is widely considered as the optimal combination of the defined contribution principle of certain funded private systems and the lack of funds of unfunded public systems. While this paper appreciates the virtues of the NDC, it argues that the NDC incentives are too strong and lead to excessive redistribution from the ex-ante shorter lived to the ex-ante longer lived. Using the tools of optimal mechanism design can improve on the existing NDCs.

Keywords: flexible retirement, asymmetric information, actuarial fairness, mechanism design

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1. INTRODUCTION

Since the publication of World Bank (1994), the reform of public pension systems has come to the fore. At the beginning, the World Bank suggested the replacement of the dominant unfunded defined benefit (DB) public systems with dominantly funded defined contribution (DC) systems. Indeed, some post socialist-countries (e.g. Hungary and Poland) have introduced a multi-pillar pension system, but the dominance of the unfunded public pillar was retained and the entry for the mixed system remained voluntary for significant if not all the working cohorts (Fultz, eds. 2002 and Simonovits, 2003b). In other countries, notably in Germany a similar reform was introduced in 2002. Although the US has already a multi-pillar system, favored by the World Bank, President Bush has recently attempted to extend the funded pillar to the poorer workers.

These privatization plans have several aims but here we only mention two of them: (i) to prefund old-age consumption in a quickly aging world and (ii) to replace DB pensions with DC pensions, creating better incentives. (It is of interest that originally the bulk of the funded private pension systems were also DB, but they are now being quickly replaced by DC for new workers. Nevertheless, the underfundedness of these private DB plans causes a lot of problems to the business communities, especially in the US car industry and airlines.) Here we single out two related dimensions of pension incentives: (a) creating stronger links between monthly contributions and monthly benefits, i.e. weakening the progressivity and (b) creating stronger links between lifetime contributions and lifetime benefits, i.e. improving the actuarial adjustment. (The two dimensions are not the same. For example, in Germany, there has always been a very tight connection in dimension (a), but it was quite loose in dimension (b), notably the actuarial adjustment for working longer or shorter than normal was very weak until 1992 (cf. Börsch-Supan, 2004). In the US, there has always been a strong redistribution from the rich to the poor in dimension (a), but actuarial adjustment has always been quite strong.) In general, there is a widespread conviction that low retirement ages are connected to insufficient incentives (Gruber and Wise, eds. 1999).

During the reform process more and more experts have come to the conclusion that aim (i) is more important and easier to achieve than aim (ii). For example, two outspoken critics of unfunded public systems, Axel Börsch-Supan and Robert Holzmann have recently shifted their emphasis from privatization (cf. their comments on Orszag and Stiglitz, 2001) to introducing NDC (Börsch-Supan and Wilkies, 2004?) and Holzmann, 2004?).

Indeed, there are four countries, Sweden, Italy, Poland and Latvia, which have recently introduced NDC. Other countries, like France and Germany have shifted in this direction (Legros, 2006).

What is NDC? It is an unfunded public pension system which mimics a funded DC private system. Each participant has his own account, where a virtual capital accumulation takes place. Using a virtual interest rate, for example, the average (real) wage rate, the individual annual contributions are accumulated. At retirement, dividing the accumulated pension capital by an annuity divisor, the capital is turned into a life annuity.

What are the advantages of the NDC? Transforming a public DB pension system with weak incentives into an NDC, a country obtains a strong link between lifetime contributions and lifetime benefits, without prefunding the system. Those who contribute

more to the system, receive proportionally more from the system. A remarkable feature of the NDC is the so-called *actuarial fairness*: if somebody works 1 year longer, his life annuity increases hyperbolically: not only the capital increases by his additional contribution in the numerator, but the annuity divisor decreases in the nominator.

There are some problems with the NDC, too. Valdés-Prieto (2000), Legros (2006) and Marin (2006) discussed some of them in details, here we only mention the arbitrary choice of virtual interest rate. According to the theory, the interest rate should be equal to the growth rate of the total covered wage-fund; in practice, it is different. For example, in Sweden, it is equal to the average real wage growth. Another problem is the sensitivity of the annuity to the actual financial situation. A good system needs a huge buffer fund, and only the Swedish NDC system has one.

A third type of pitfalls of the NDC will be discussed in the present paper: NDC neglects the correlation between individual life expectancies and the retirement ages in calculating the annuity. If people retiring later live statistically longer, then the NDC principle not only overrewards them, at the cost of the other participants but the total balance of the system can also be upset.

And we have good reasons to assume that such a correlation exists. First, there is some evidence on such correlation in the US (Waldron, 2001) and people have quite reliable ideas on their own life expectations (Smith et al., 2001). Second, in a world of asymmetric information, one can prove that for a large class of benefit–retirement age functions, the optimal retirement ages—maximizing his lifetime utility—leads to such a correlation (i.e. Simonovits, 2003a). Third, there may be a reverse causation: just because somebody receives higher benefits, he can lengthen his life span (Philipson and Becker, 1998).

What is the solution, or better, what are the solutions to the improvement on the NDC?

(i) The simplest approach is to confine our attention to *linear* benefit–retirement age schedules (Simonovits, 2003a). We need a social welfare function and maximize its value by looking for a socially optimal benefit–retirement age function. Then a classical constrained maximization problem arises. It turns out that the optimal linear schedule relies on weaker incentives than the NDC. Moreover, these two solutions are the only designs when we can relax the assumption of one-dimensional heterogeneity and are able to consider multi-dimensional heterogeneity as well (Simonovits, 2003a). A further improvement is possible if we introduce a second factor, reflecting not only the actuarial adjustment but also the increase in the lifetime contribution: the *bilinear* schedule.

(ii) But why should we restrict the analysis to linear or bilinear schedules? Since Mirrlees (1971) we have a powerful tool to consider general schedules as well: *mechanism design*. Because of asymmetric information, one must take into account the so-called *incentive compatibility constraints*, i.e. the pairs of benefit,retirement age, depending on the life expectancy, should be such that each type is interested in choosing that pair which are designed for it. Then the *second-best* benefits and retirement ages as functions of life expectancies can be calculated by optimal control theory.

There are two possibilities with general schedules. (a) If we want to retain the *neutrality* (called also fairness) of our mechanism, then the type-specific retirement age is determined by the benefit, i.e. only the benefit–life expectancy series need be determined (Simonovits, 2004). (b) If we are ready to sacrifice neutrality and introduce

redistribution, then a series of pairs of benefit–life expectancy and retirement–life expectancy need be calculated (Esó and Simonovits, 2002). It is of interest that in a large part of the parameter space, the redistributive optimum Pareto-dominates the neutral optimum.

(iii) There is a third dimension of heterogeneity, strongly emphasized by the analysts of progressivity in DB public systems, namely the average lifetime earnings. Indeed, the critics (e.g. World Bank 1994, p. 131 and Liebman, 2001/2002) and the defenders (e.g. Diamond and Orszag 2004) of such systems equally emphasize that “there [is] little [lifetime] redistribution from the rich to the poor, despite progressive benefit formulas,...because [they are] earnings-related ...[and]... upper-income people enter the labor force later in life and live longer after retirement.” We shall neglect this dimension, however, because the earning is common knowledge, thus pension design does not directly apply to this situation.

This paper is a simplified survey of previous papers written by the author (and of a joint paper by Péter Esó). It does not contain full proofs but it relies on a common framework, missing from the original papers.

At this point we make a short discourse on the relevant literature. Rather than listing all the relevant papers on the subject, I only mention Diamond (2003) on the application of optimal mechanism design to pension economics. I also call attention to Spiezia (2002), as a rare source that emphasizes that not every worker is allowed to decide on his retirement.

To my best knowledge, only Simonovits (2004) and (2006) analyzed neutral (i.e. nonredistributive) *second-best* pension rules, though in some other fields of information economics (e.g. the insurance model of Rothschild and Stiglitz, 1976), neutrality has been a natural requirement for decades. Moreover, the idea of Pareto-dominant redistribution also appears with Rothschild and Stiglitz (1976, p. 638): “The [neutral] separating equilibrium ... may not be Pareto optimal even relative to the information that is available.” On the other hand, the original problem of optimal income taxation by definition requires redistribution.

The structure of the chapter is as follows. Section 2 presents the case for the NDC. Section 3 discusses a pitfall in the NDC. Sections 4 and 5 analyze the linear and general improvements, respectively. Section 6 provides the numerical illustrations and Section 7 concludes.

2. THE CASE FOR NDC

We consider a standard model, where there is no growth, no inflation, no interest, and where a worker has a life expectancy D and age-invariant annual wage w . To avoid lengthy notations, assume that he starts to work at age 0. Confining our attention to old-age pensions, we assume that he retires before his death: $R < D$. Assuming that our worker contributes τw to the unfunded public system per year, the NDC-rule is as follows:

$$\bar{b}(R) = \frac{\tau w R}{D - R}. \tag{1}$$

Indeed, during his working period he contributes a total $\tau w R$, and during his retirement

period he obtains a total $b(R)(D - R)$, and their equality implies (1).

But there are a lot of workers. Let us distinguish different types, and denote the type of a worker by integer $i = 1, 2, \dots, n$ and assume that the difference between types is revealed by their different retirement ages R_i . (We could derive different optimal retirement ages from the maximization of lifetime utility functions with different labor disutilities.) Following the tradition of the NDC literature, for a moment we assume that workers have the same life expectancy \bar{D} and then (1) is generalized into

$$\bar{b}_i = \frac{\tau w R_i}{\bar{D} - R_i}. \quad (2)$$

It is easy to see again that individual expected lifetime balances are zero:

$$\bar{z}_i = \tau w R_i - \bar{b}_i \cdot (\bar{D} - R_i) = 0.$$

In summary: *a clear advantage of the NDC is that it allows workers of common life expectancy but of different propensity to retire at different ages without burdening any type by this flexibility.*

3. A PITFALL OF NDC

What happens, however, if the different retirement ages are generated by different life expectancies? Then we shall see that the NDC-rule breaks down.

Let us denote the type-specific life expectancy by D_i and calculate the individual balances again, now by type-specific life expectancies: $z_i = \tau w R_i - b_i \cdot (D_i - R_i)$. Under NDC, z_i simplifies to

$$\bar{z}_i = \tau w R_i - \bar{b}_i \cdot (\bar{D} - R_i) + \bar{b}_i \cdot (\bar{D} - D_i) = \bar{b}_i \cdot (\bar{D} - D_i). \quad (3)$$

Since typically $D_i \neq \bar{D}$, the individual balances differ from zero: $\bar{z}_i \neq 0$. We have shown that *with heterogeneous life expectancies, NDC is not fair.*

Let us define the population share of type i by f_i , $f_i > 0$ and $\sum_i f_i = 1$. Then we can define the expected lifetime balance of an average worker as

$$Z = \sum_i f_i z_i.$$

After substitution of (3),

$$\bar{Z} = \sum_i f_i \bar{b}_i \cdot (\bar{D} - D_i). \quad (4)$$

As a rule, one can assume that workers expectedly living longer also work longer. Let us index our workers according to increasing life expectancy: $D_i < D_{i+1}$ and—as discussed in the Introduction—assume that workers of higher life expectancies retire later: $R_i < R_{i+1}$. By monotonicity, the benefits are also increasing: $\bar{b}_i < \bar{b}_{i+1}$. It can be shown that $\bar{Z} < 0$. *In sum, under NDC with heterogeneous life expectancies, and*

with positively correlated retirement ages, not only individual balances are different from zero but even the average balance is negative.

It is quite easy to eliminate the average loss arising at the NDC. Let us distinguish between two contribution rates: (i) the rate to be paid in, denoted by τ and (ii) the rate to be paid out, to be denoted by $\check{\tau}$. Then

$$\check{b}(R_i) = \frac{\check{\tau}wR_i}{D - R_i} \quad (\check{2})$$

and

$$\check{z}_i = \tau w R_i - \check{b}_i \cdot (D_i - R_i). \quad (\check{3})$$

For any contribution rate $\tau > 0$, it is possible to choose a reduced contribution rate $(\tau >) \check{\tau} > 0$, such that the aggregate balance is restored:

$$\check{Z}(\tau, \check{\tau}) = 0. \quad (\check{4})$$

Until now we have assumed that for any life expectancy D_i , there corresponds a unique retirement age R_i . This is not necessarily the case, however, and it is possible that for any life expectancy D_i there correspond many retirement ages R_{ij} . This more general case can also be examined but we turn to the improvement of the NDC.

4. LINEAR SCHEDULES

Turning to the improvement on the NDC, in this Section we shall consider linear schedules, retaining redistribution.

Lifetime utility

To find socially optimal schedules, we must introduce individual utility functions which workers maximize by choosing optimal retirement ages (Sheshinski, 1978). In the simulations below we shall also apply this technique to determine retirement behavior with NDC.

Let us assume that the individuals have well-behaved instantaneous *utility functions*, depending on consumption c and leisure l : $\mathbf{u}(c, l)$. We maintain the assumption that the life span is divided into working stage and retirement stage, where the leisure of workers and pensioners are equal to $0 < l_m < l_M$, respectively. Now the instantaneous utility functions are $u(a) = \mathbf{u}(a, l_m)$ and $v(b) = \mathbf{u}(b, l_M)$, where $a = (1 - \tau)w$ and b are consumption of the worker and of the pensioner, respectively. Because a pensioner has more leisure than a worker has, $u(c) < v(c)$ holds for any c . The difference between v and u is the labor disutility. The *lifetime utility function* is time-additive:

$$U(D, R, a, b) = u(a)R + v(b)(D - R). \quad (5)$$

Social welfare

To evaluate the social welfare of a pension system, we must introduce a social welfare function. We shall start with the simplest one, called *utilitarian*:

$$V = \sum_i f_i U_i. \quad (6')$$

If we are interested not only in efficiency but equity as well, then we should use a more general social welfare function. For example, defining an increasing concave scalar-scalar function ψ , one has a *social welfare function*

$$V = \sum_i f_i \psi(U_i). \quad (6)$$

Obviously, $\psi(U_i) = U_i$ corresponds to the utilitarian case.

Linear benefit-retirement age function

In practice, the benefit function is frequently a simple parametric function of the observable characteristics of the population. For example, linear benefit-retirement age functions are very popular in practice:

$$b(R) = \varphi w [1 + \chi(R - R^*)], \quad (7')$$

where parameter R^* is the normal (or full benefit) retirement age, χ is the linear actuarial adjustment coefficient and φ is the constant rate. To simplify the formula, we rewrite (7') as

$$b(R) = w(\gamma + \alpha R), \quad (7)$$

where γ and α are the corresponding parameters. Parameter γ has no direct interpretation but the normal unit-wage benefit $\varphi = \gamma + \alpha R^*$ defines it in terms of other parameters.

Our social optimization is as follows: Choose parameters τ , γ and α that maximize V under the constraint $Z = 0$.

In principle, this is a very simple constrained maximization problem which can be solved by the method of Lagrange multiplier. In practice, this problem is quite involved, because the U 's defining the objective function V in (6) are themselves the maxima of individual optimization problems. Moreover, as usual in such problems, the objective function is quite insensitive to the arguments. Again, one can consider multi-dimensional populations.

Finally, we may improve on the linear benefit schedule by introducing a second factor, reflecting the proportionality between retirement age and lifetime contribution. Such bilinear benefit-retirement age functions are very popular in practice:

$$b(R) = \varphi w R [1 + \chi(R - R^*)], \quad (8')$$

where parameter R^* is the normal (or full benefit) retirement age, χ is the linear actuarial adjustment coefficient and φ is the uniform accrual rate. To simplify the formula, we rewrite (8') as

$$b(R) = w \frac{R}{R^*} (\gamma + \alpha R), \quad (8)$$

where γ and α are the corresponding parameters. Parameter α remains as before. Parameter γ has no direct interpretation but the normal unit-wage benefit $\varphi R^* = \gamma + \alpha R^*$ defines it in terms of other parameters.

5. GENERAL SCHEDULES

In this Section, we turn to the general, nonlinear schedules. First we stick to the neutrality condition, then we relax it. For the sake of simplicity, here we fix the contribution rate τ and use notation $u = u((1 - \tau)w)$.

Neutral second-best

If $z_i = 0$, then we can express the retirement age as a function of benefit:

$$R_i = \frac{b_i D_i}{b_i + \tau w}. \quad (9)$$

Then the two-variable decision problem reduces to a one-variable problem. Substituting (9) into (5) results in

$$\tilde{U}(b_i) = u R_i + v(b_i)(D_i - R_i) = \frac{u b_i + v(b_i) \tau}{\tau + b_i} D_i = \varphi(b_i) D_i. \quad (10)$$

First we maximize $\tilde{U}(b_i)$ regardless of other constraints. Taking the derivative $\varphi(b_i)$ and equating it to 0,

$$u - v(b^*) + v'(b^*)(\tau + b^*) = 0 \quad (11)$$

provides the uniform optimal benefits $b_i = b^*$, thus the optimal proportional retirement ages $R_i = R_i^* = (b^*/(\tau w + b^*)) D_i$.

If the government, however, made this proposal (called *neutral first best*), then every worker would pretend that he belongs to the shortest lived type $i = 1$ and retire at the minimal R_1^* even if his proper retirement age is R_2^*, \dots, R_n^* , respectively. To avoid such an error, we introduce the *incentive-compatibility constraints*: the lifetime utility of type i at $b = b_i$ should be greater than or equal to the lifetime utility of type i at $b = b_j$, $j \neq i$. Denote the lifetime utility of type i taking contract j as $U_{i|j}$:

$$U_{i|j} = u R_j + v(b_j)(D_i - R_j).$$

One can show that at the social optimum, only the adjacent incentive compatibility conditions are effective:

$$U_{i|i} = U_{i|i-1}, \quad i = 1, 2, \dots, n. \quad (12)$$

Because there is no constraint on b_n , $\bar{b}_n = b^*$. (This is generally the case of optimal design!) Because of (9), (12) is a scalar first-order nonlinear difference equation, which can be solved in reversed order, starting from the end-condition $\bar{b}_n = b^*$ and yielding the benefits \bar{b}_i sequentially. Then (9) yields the corresponding retirement ages \bar{R}_i . Of course, *in neutral second-best optimum, the longer the type lives, the larger is his benefit and the later he retires: $\bar{b}_i < \bar{b}_{i+1}$ and $\bar{R}_i < \bar{R}_{i+1}$.*

To simplify the notation, we shall measure life expectancy D_i in such units that $D_i = \delta_0 + \delta_i$, $i = 1, \dots, n$, where $n > 1$ is the number of types. Changing notations, (12) results in a standard difference equation for b_i s.

Redistributive second-best

As we shall see in the simulations below, there are, however, problems with the neutral second-best. Therefore we shall also consider redistributive schemes as well. We follow Esó and Simonovits (2002). The basic difference between the neutral and the redistributive solution is that in the former the benefits determine the retirement ages, while in the latter they do not.

We now combine the constrained social welfare maximization with the incentive-compatibility constraints. Our new problem is as follows: Maximize the social welfare function V under the average balance constraint $Z = 0$, taking into account the incentive compatibility conditions (12).

Since (9) is not required, the determination of the redistributive second-best is much more difficult than that of the neutral one. Nevertheless, the optimal benefit of the longest-lived type is again the absolute optimum $\hat{b}_n = b^*$. The benefits \hat{b}_i and the retirement ages \hat{R}_i are also increasing with the life expectancies and the balances \hat{z}_i are probably decreasing, starting with positive and ending with negative values.

In the simplest, utilitarian case, all the benefits and all the retirements are the same: $\hat{b}_i = b^*$ and $\hat{R}_i = R^*$. Then $\hat{z}_i = (\tau w + b^*)R^* - b^*D_i$ is decreasing.

It is worth comparing the neutral and the redistributive second-best solutions. We shall see in the simulation below that the neutral optimum has rather undesirable properties, which are related: *(i) for longer-lived types, a 1-year increase in the life expectancy implies a greater than a 1-year increase in the retirement age. (ii) The welfare provided by the neutral second-best solution is not only much inferior to that of the redistributive one, but the individual lifetime utilities are also lower for most if not all types.*

6. SIMULATIONS

To have a feeling for the order of magnitudes, it will be helpful to simulate our findings (Esó and Simonovits, 2002). For simplicity, we assume unit wages: $w = 1$. Assume that from the point of view of the government, the individuals' life expectancies (starting from entering the workforce at age 20) are uniformly distributed between $D_1 = 49$ and $D_{11} = 59$ years. Let the pensioner's instantaneous utility function be of CRRA-type, $v(x) = x^\sigma / \sigma + \theta$, where θ is large enough to ensure that $u(x) \equiv v(x) - \varepsilon > 0$, (otherwise $U_{i+1} > U_i$ may not hold); $1 - \sigma$ being the coefficient of relative risk aversion, $\varepsilon > 0$ is the labor disutility. We set $\theta = 4.1$, $\sigma = -.5$ and $\varepsilon = 1.398$.

Let us start with the NDC with the socially optimal contribution rate $\tau = 0.212$. Table 1 presents the resulting optimal retirement ages, benefits, balances and utilities as functions of life expectancies, respectively.

Table 1. *Characteristics of the NDC solution*

Life expectancy D_i	benefit \bar{b}_i	N D C		
		retirement age (yrs) \bar{R}_i	lifetime balance \bar{z}_i	lifetime utility \bar{U}_i
49	0.615	40.14	3.07	31.73
50	0.645	40.64	2.58	33.31
51	0.679	41.14	2.04	34.95
52	0.716	41.65	1.43	36.65
53	0.756	42.16	0.76	38.42
54	0.801	42.69	0.00	40.25
55	0.851	43.22	-0.85	42.15
56	0.908	43.77	-1.82	44.11
57	0.972	44.32	-2.91	46.15
58	1.046	44.89	-4.18	48.25
59	1.133	45.48	-5.66	50.43

Adult retirement ages range from 40.1 to 45.5 years, meaning 60.1 to 65.5 years. Correspondingly, the benefits increase from 61 to 113 percent of the total wage, with full net replacement (80 percent) at the middle. It can be seen that although the lifetime balance of the medium type $D_6 = 54$ is zero, the lifetime balances are asymmetric: for example, while the shortest-lived produces $\bar{z}_{49} = 3.1$, that of the longest-lived is $\bar{z}_{59} = -5.7$. Small wonder that the expected lifetime balance is also negative: $\bar{Z} = -0.5$.

As mentioned at the end of Section 3, this problem of aggregate imbalance can easily be solved by introducing a lower contribution rate for the benefit function than in the contribution function. Indeed, choosing $\check{\tau} = 0.2$, one arrives to $\check{Z} = 0$.

Table 2. *Characteristics of the modified NDC solution*

Life expectancy D_i	benefit \check{b}_i	M o d i f i e d NDC		
		retirement age (yrs) \check{R}_i	lifetime balance \check{z}_i	lifetime utility \check{U}_i
49	0.604	40.56	3.51	31.09
50	0.635	41.06	3.03	32.64
51	0.669	41.57	2.51	34.27
52	0.707	42.08	1.92	35.95
53	0.748	42.60	1.26	37.71
54	0.794	43.13	0.52	39.53
55	0.846	43.67	-0.32	41.42
56	0.905	44.22	-1.28	43.38
57	0.972	44.78	-2.38	45.41
58	1.051	45.36	-3.66	47.52
59	1.146	45.97	-5.18	49.71

Due to the modification, the benefits diminish, the retirement ages increase together with the balances. Note that even the middle type's balance is now positive ($z_{54} = 0.5$) and is close to the negative of the previous average balance \bar{Z} . Note that for the median worker, the benefit does not really change, while the length of employment increases from 42.7 to 43.1 years!

We now consider a linear benefit rule. It is mentioned that originally I had to be satisfied with the grid method but Alács (2004) has developed a very sophisticated algorithm. The grid method assured us that the socially optimal incentive is definitely weaker than suggested by the NDC.

First we obtain the following optimal parameters: $\tau = 0.202$, $\gamma = -2.962$ and $\alpha = 0.0869$. Table 3 displays the resulting characteristics of the optimal linear solution.

Table 3. *Characteristics of the optimal linear solution*

Life expectancy D_i	benefit b_i	L i n e a r		
		retirement age (yrs) R_i	lifetime balance z_i	lifetime utility U_i
49	0.663	41.70	3.59	31.29
50	0.688	41.99	2.98	32.96
51	0.712	42.27	2.33	34.67
52	0.736	42.55	1.64	36.41
53	0.759	42.81	0.92	38.20
54	0.782	43.08	0.17	40.02
55	0.805	43.34	-0.62	41.87
56	0.827	43.59	-1.45	43.76
57	0.848	43.84	-2.30	45.67
58	0.870	44.09	-3.19	47.61
59	0.891	44.33	-4.11	49.58

The linear optimum is again welfare-superior to the optimal NDC like in Simonovits (2003a). All but the longest-lived lifetime utilities are greater than the corrected NDC's counterparts. The expected social welfare is higher than its counterpart: $40.19 > 39.87$. *The partial equalization of the retirement ages in the linear design dominates the sensitivity of retirement ages on life expectancies.*

Skipping the bilinear benefit function, we turn now to the display of the second-best solutions, neutral and redistributive.

Table 4. *Characteristics of the neutral (and the redistributive) second-best*

Life expectancy D_i	benefit \bar{b}_i	N e u t r a l		Redistributive
		retirement age (yrs) \bar{R}_i	lifetime utility \tilde{U}_i	lifetime utility \hat{U}_i
49	0.424	33.3	31.7	30.9
50	0.433	34.2	32.7	32.8
51	0.443	35.1	33.7	34.7
52	0.455	36.1	34.8	36.5
53	0.468	37.1	36.0	38.4
54	0.484	38.2	37.2	40.3
55	0.504	39.4	38.4	42.1
56	0.528	40.6	39.7	44.0
57	0.562	42.0	41.0	45.9
58	0.614	43.7	42.4	47.7
59	0.800	47.2	44.0	49.6

(The last column of Table 4 shows the utilities of the redistributive second-best.)

The most surprising feature of the neutral second-best solution is that the longest-lived type must work 3.5 years longer than the next type (just because he lives one year longer). On the other hand, he receives a much higher benefit than his neighbor does: 80% vs 61.4%. The other types' (retirement age, benefit) pairs seem to be acceptable, although the greater than one-to-one reduction of retirement age and life expectancy is excessive for types $D_i = 52, \dots, 59$.

Finally, we recapitulate the simplest simulation of the redistributive second-best solution (Eső and Simonovits, 2002), namely with utilitarian social welfare function. As was already mentioned, in this case, there exists a second-best solution with uniform first-best retirement ages $\hat{R}_i \equiv \bar{D}b^*/(\tau + b^*)$ and benefits $\hat{b}_i \equiv b^*$, where \bar{D} is the average life expectancy of the cohort. We shall use this *pooling* solution as a benchmark. It is evident that in the neutral second-best solution, the longest-lived must work much longer than in the redistributive one: $47.2 > 43.2$ years. Since the longest-lived's benefits are the same in both systems, the longest-lived's lifetime utility is much higher in the redistributive system than in the neutral one. And this is so for other adjacent types as well. In our case, 10 out of 11 types have higher lifetime utility in the redistributive second-best than in the neutral one. This is almost Pareto-dominance. If we insist on stronger equity in the social welfare function, then we could easily obtain Pareto-dominance.

7. Conclusions

We have calculated various optimal solutions to the flexible retirement system. First, we showed the original NDC does not even assure the total balance between contributions and benefits. Second, the total balance can be reestablished with a trivial modification of the original benefit formula, but neutrality is still missed. Third, considering linear and bilinear benefit–retirement age functions, one can obtain an optimal solutions, where the redistribution is less severe than in the modified NDC. Fourth, introducing incentive compatibility conditions, it is possible to determine the neutral second-best solution, but it leads to excessively low retirement for the shorter lived. Fifth, renouncing neutrality, the redistributive second-best can Pareto-dominate the neutral second-best.

Unfortunately, our general second-best solutions presume a one-dimensional heterogeneity, namely in the life expectancies. We must continue our research to extend the results further. What we can only establish, is the following: NDC needs improvement, namely by reducing the strength of the actuarial adjustment.

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