Partially Strategyproof Mechanisms for the Assignment Problem

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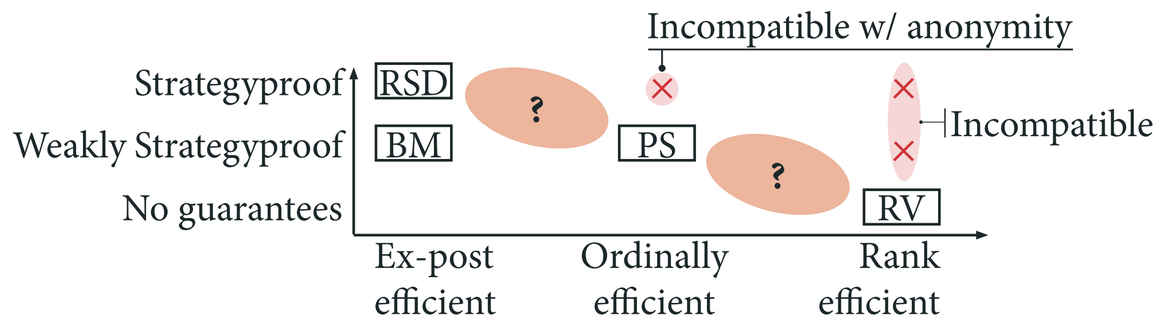
Abstract

We propose a new way of relaxing strategyproofness by only requiring mechanisms to be non-manipulable for a subset of all possible utility functions, namely those bounded away from indifference. We construct hybrid mechanisms that make desirable and scalable trade-offs between efficiency and incentive properties, and we use this technique to design hybrids of RDS and PS.

Problem Statement

2 Consider the problem of assigning students to schools ...

3 ... which requires incentive/efficiency trade-offs.



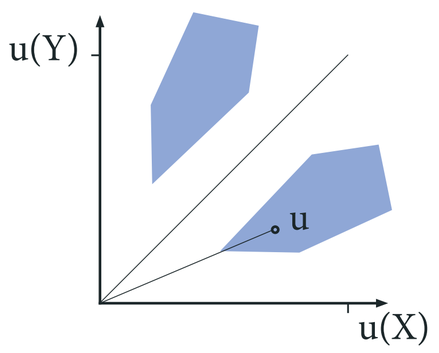
Research Question

4 How can we design assignment mechanisms that make desirable trade-offs between efficiency and incentives?

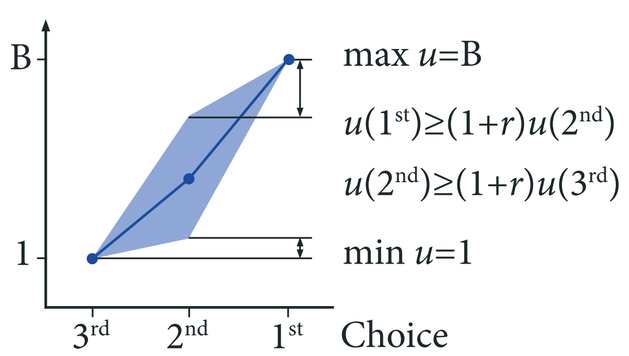
Research Idea

5 Introduce intermediate incentive concepts...

- *Partial Strategyproofness (PSP):* require SP constraints to hold for a subset of the possible utility functions.



- *Uniformly Relatively Bounded Utility (URB):* SP constraints must hold for utilities that are bounded away from indifference.



6 ... and introduce intermediate efficiency concepts.

- *g dominates f:* allocations resulting under *g* dominate those resulting under *f*.

relax to

- *g imperfectly dominates f:* allocations resulting under *g* dominate those resulting under *f* *whenever they are comparable.*

Theory Results

7 Introduce hybrid mechanisms, ...

- *Prop. 1.* Hybrid mechanisms $h_\beta(f,g)=(1-\beta)f+\beta g$ are well-defined.
- *Prop. 2.* PSP on URB characterized by finitely many constraints.
- *Prop. 8.* If *g* imperfectly dominates *f*, then $h_\beta(f,g)$ imperfectly dominates *f*.

8 ... which preserve PSP, are computable, ...

- *Thm. 1./Cor. 1.* Given *f* SP and *g* weakly less varying than *f*, we have $\forall (r,B)\ \exists \beta > 0$: $h_\beta(f,g)$ is PSP on URB(*r*,*B*,*m*).
- *Prop. 3./Cor. 2.* For *f* SP, any mechanism *g*, bounds (*r*, *B*), and *m* objects, there exists a maximal value $\beta_{max}>0$ for which $h_\beta(f,g)$ is PSP on URB(*r*,*B*,*m*), and this β_{max} is computable.

9 ... and yield a hierarchy of manipulability and efficiency.

- *Prop. 9./10.* Given *f* SP, *g* manipulable and weakly less varying than f, g imperfectly dominates f, $0\leq\beta<\beta'\leq 1$, then
  1. $h_{\beta'}(f,g)$ is intensely and strongly more manipulable than $h_\beta(f,g)$
  2. $h_{\beta'}(f,g)$ imperfectly dominates $h_\beta(f,g)$.

Instantiations

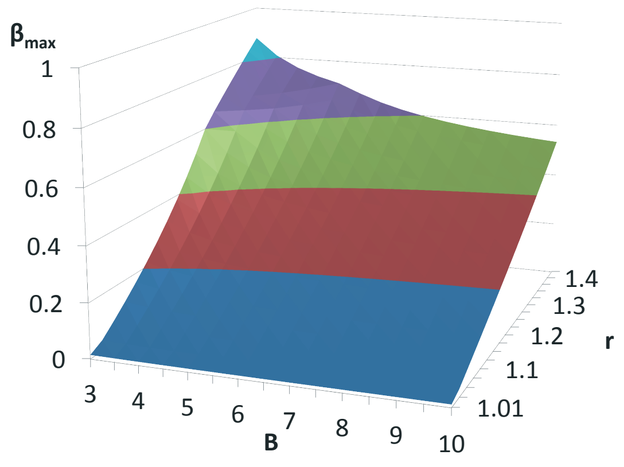
10 PS and RSD can be used to design interesting hybrids...

- *Thm 2.* PS is weakly less varying than RSD, i.e., whenever PS changes the allocation, so does RSD.
- *Cor. 3.* Given bounds (*r*, *B*), and a setting with m objects, there exists β>0 such that
  1. h_β(RSD,PS) is PSP on URB(*r*,*B*,*m*)
  2. h_β(RSD,PS) imperfectly dominates RSD.

12 The construction may fail when mixing RSD and Rank Value mechanisms.

- Prop. 11. There exist Rank Value mechanisms that are not weakly less varying than RSD.

11 ... and the mixing factor can be high.



Maximal mixing factor β_{max} for hybrids of PS and RSD with 4 agents and 4 objects.

13 Future Research

- Derive reduction theorem to characterize PSP in terms of URB utilitiy sets.
- Introduce intermediate efficiency concepts, based on weaker dominance notions.