Maxmin Mechanism in Common Value Auction

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Consider a common value auction setting. Each agent has a private signal $s_i \in S_i = \mathbb{R}_+$ about the true value of the object. No probability distribution of the signal space is specified; we focus on worst case analysis (Goldberg, etc., 2006; Moulin, 2008) Our goal is to achieve the largest portion of the true value in the worst scenario for the seller.

Alternative way to model common value

Beysian approach (Wilson 1972)	Classical approach (this paper)
$True\,value:\ V \sim F(\cdot)$	$True\,value:\ parameter\,v$
$S \sim G_i(\cdot \mid V = v)\,with\,E(S_i) = v$	$S_i \sim G_i(\cdot; v)\,with\,E(S_i) = v$
$S_i\,i.i.d.\,cond.\,on\,V$	$S_i\,not\,necessarily\,i.i.d.$
$\rightarrow v = E(V \mid s_1, \cdots, s_n)$	$\rightarrow v = \frac{1}{n}\sum_{i=1}^{n} s_i\,(robust\,estimation)$

Axioms

Let $\mathcal{M}$ be the class of direct mechanisms that satisfy feasibility, ex-post implementability, individual rationality and no free lunch.

A direct mechanism (a, t) is ex-post implementable if $a(s) \cdot v(s) - t_i(s) \geq a_i(\tilde{s}_i; s_{-i}) \cdot v(s) - t_i(\tilde{s}_i; s_{-i})$, $\forall s, \forall \tilde{s}_i, \forall i$.

Main result

Definition. A mechanism in $\mathcal{M}$ is called the Maxmin mechanism if $a_i(s) = \frac{1}{n}$,$t_i = \frac{1}{n^2}\sum_{j \neq i} s_j$,$\forall s$, $\forall i$.

The Maxmin mechanism gives each agent the same probability to win the object, and when some agent wins, he or she pays the average of the signal vector, with his or her own signal replaced by 0.

Theorem. *For any $\mu \in \mathcal{M}$, $Q(\mu) = \frac{n-1}{n}$ if and only if μ is the Maxmin mechanism, where $Q(\mu) = inf_{s \in S \setminus \{0\}} \sum_{i=1}^{n} t_i / v(s)$.*

Tradeoff within order mechanisms

Definition. (Order mechanisms) Let μ^m be the direct mechanism that has the following form:

$\forall s$, $\forall i$, $a_i(s) = \frac{1}{\bar{m}}$, $t_i(s) = \frac{1}{\bar{m}} \cdot \frac{1}{n}(s_{-i}^{(m)} + \sum_{j \neq i} s_j)$ if $s_i \geq s_{-i}^{(m)}$; $a_i(s) = 0$, $t_i(s) = 0$ if $s_i < s_{(m)}$, where $s_{-i}^{(m)}$ is the m^{th} largest signal in $(0; s_{-i})$ and $\bar{m} = |\{i \in I : s_i \geq s_{-i}^{(m)}\}|$, for $1 \leq m \leq n$.

Two extreme cases: "second price" auction ($m = 1$) and Maxmin mechanism ($m = n$). Guaranteed portion: $0 \rightarrow \frac{n-1}{n}$; Incentive: strong to week