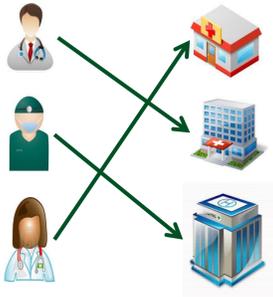


# Socially stable matchings in the Hospitals/Residents problem

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## 1. Matching problems generally involve...



- ✓ assigning agents to each other
  - medical graduates to hospitals
  - kidney patients to donors
  - students to colleges
- ✓ some agents may have **preferences**
  - 1<sup>st</sup> choice, 2<sup>nd</sup> choice, etc.
- ✓ agents have a limit on the number of assignments they can have
- ✓ an example is the *Hospitals/Residents problem (HR)*

## 2. We seek to...

- ✓ generate **stable** matchings
  - one in which no two agents prefer each other to their current partners
  - else dissatisfied agents may abandon the scheme and form private arrangements
  - we call these agent pairs **blocking pairs**
- ✓ centralised matching schemes that produce unstable matchings generally don't last

### 3a. A sample instance

Residents	Hospitals
$r_1: h_2 h_1$	$h_1(2): r_1 r_3 r_2 r_5 r_6$
$r_2: h_1 h_2$	$h_2(2): r_2 r_6 r_1 r_4 r_5$
$r_3: h_1 h_3$	$h_3(2): r_4 r_3$
$r_4: h_2 h_3$	
$r_5: h_2 h_1$	
$r_6: h_1 h_2$	

- ✓ preference lists are strictly ordered
- ✓  $r_1$  prefers  $h_2$  to  $h_1$  etc.
- ✓ each hospital has a capacity of 2

### 3b. An unstable matching

Residents	Hospitals
$r_1: (h_2) h_1$	$h_1(2): r_1 r_3 (r_2) (r_5) r_6$
$r_2: (h_1) h_2$	$h_2(2): r_2 (r_6) (r_1) r_4 r_5$
$r_3: (h_1) (h_3)$	$h_3(2): (r_4) (r_3)$
$r_4: h_2 (h_3)$	
$r_5: h_2 (h_1)$	
$r_6: h_1 (h_2)$	

- ✓  $(r_3, h_1)$  forms a blocking pair

### 3c. A stable matching

Residents	Hospitals
$r_1: (h_2) h_1$	$h_1(2): r_1 (r_3) (r_2) r_5 r_6$
$r_2: (h_1) h_2$	$h_2(2): r_2 (r_6) (r_1) r_4 r_5$
$r_3: (h_1) h_3$	$h_3(2): (r_4) r_3$
$r_4: h_2 (h_3)$	
$r_5: h_2 h_1$	
$r_6: h_1 (h_2)$	

- ✓ stability requirement causes one resident to be unmatched

## But will all blocking pairs undermine a matching in practice?

### 4. Answer: No! (Social Stability)

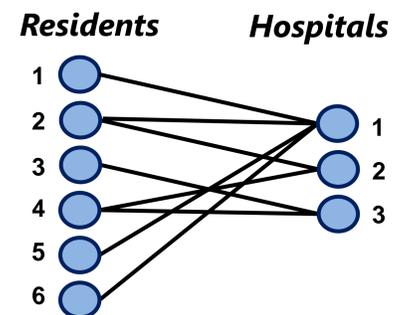
- ✓ Agents may not know about each other's preferences
  - Hospitals' preference lists may be unknown to residents
- ✓ Agents may not know about each other's partners
  - matchings may not be made public
- ✓ Only agents with a social relationship may be aware that they form blocking pairs
- ✓ This can increase the size of stable matchings

### 5a. A new model

Residents	Hospitals
$r_1: h_2 h_1$	$h_1(2): r_1 r_3 r_2 r_5 r_6$
$r_2: h_1 h_2$	$h_2(2): r_2 r_6 r_1 r_4 r_5$
$r_3: h_1 h_3$	$h_3(2): r_4 r_3$
$r_4: h_2 h_3$	
$r_5: h_2 h_1$	
$r_6: h_1 h_2$	

A HR instance

+



A social network

### 5b. HR under Social Stability

- ✓ The Hospitals/Residents problem under social stability (**HRSS**) consists of:
  - A HR instance  $I$  and
  - A Social network graph  $G$
- ✓ Agent pairs represented by edges in  $G$  are said to be **acquainted** and can form blocking pairs.
- ✓ Agent pairs **not** represented by edges in  $G$  are said to be **unacquainted** and **cannot** form blocking pairs.
- ✓ Socially stable matchings may be of different sizes
- ✓ We define **MAX HRSS** as the problem of finding a maximum socially stable matching

### 5c. A socially stable matching

Residents	Hospitals
$r_1: (h_2) h_1$	$h_1(2): r_1 r_3 (r_2) (r_5) r_6$
$r_2: (h_1) h_2$	$h_2(2): r_2 (r_6) (r_1) r_4 r_5$
$r_3: h_1 (h_3)$	$h_3(2): (r_4) (r_3)$
$r_4: h_2 (h_3)$	
$r_5: h_2 (h_1)$	
$r_6: h_1 (h_2)$	

- ✓  $(r_3, h_1)$  are unaware they form a blocking pair
- ✓ Socially stable matching has all residents assigned

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### 6. Some results

- ✓ MAX HRSS is NP hard
- ✓ MAX HRSS is approximable to within  $3/2$
- ✓ MAX HRSS is not approximable to within  $3/2 - \epsilon$  for any  $\epsilon > 0$  (assuming UGC)
- ✓ MAX HRSS is solvable in polynomial time when:
  - number of acquainted pairs is constant
  - number of unacquainted pairs is constant
  - hospitals have capacity 1 and residents have at most 2 hospitals on their list