Matching Search with Noisy Types

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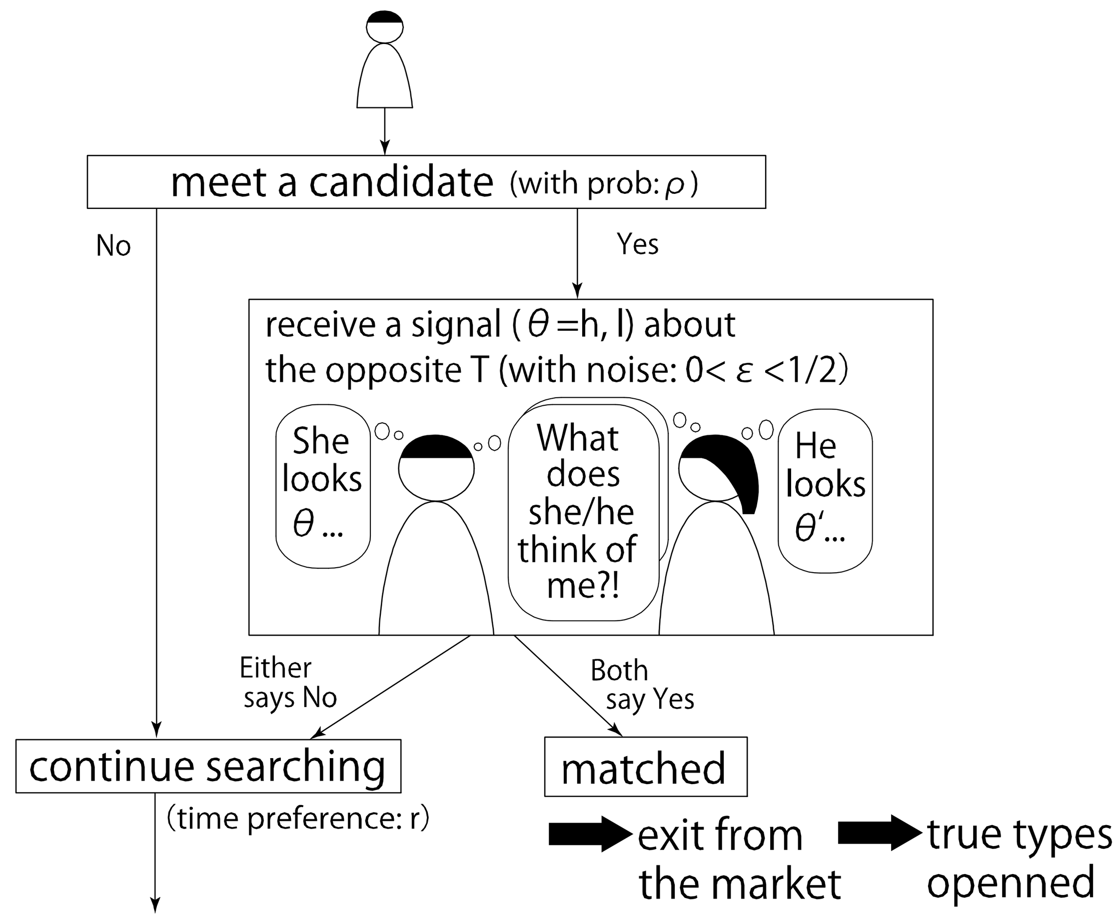
1 The aim of the research

- We study the dynamics of the marriage market or the labor market under imcomplete information.
- We consider that the constant noise causes that the signal concerned with the others' true type goes wrong.

2 Setting

- We referred to Burdett and Coles (1997) and Shimer and Smith (2000) for modelling.
  - continuous time, infinite horizon, random encountering, no divorce.
  - After matched couple exit from the market, their 'clones' enter the market.
- Two types of productivity: $T = H$ (high), L(low)
  →These types exist by the same amount.
- The timing at each instant:



- Payoffs:
  - $g_T > 0$: 'good result'. the (flow) payoff of type T matched with H.
  - $b_T > 0$: 'bad result'. the (flow) payoff of type T matched with L.
  - $g_T > b_T > 0$ for each $T = H, L$.
- Strategies: all of the agents with the same type take the same strategy.
  → $\delta_T(\theta) \in [0, 1]$: the probility with which type T chooses a candidates when receiving θ.
  → $\delta_T := (\delta_T(h), \delta_T(l))$: the type T's strategy.
  → $\delta := (\delta_H, \delta_L)$: a strategy profile.
- For ease, $\beta := \rho/2r$.

3 Equilibrium

- V_T: the expected lifetime payoff of an type T agent.
- V_T^θ: the expected net gain of an type T agent by accepting θ.

$$\text{e.g.) } V_H^h = (1-\varepsilon)\overbrace{\left(\underbrace{(1-\varepsilon)\,\delta_H(h)}_{\text{He observes } h.} + \underbrace{\varepsilon\delta_H(l)}_{\text{He observes } l.}\right)}^{\text{The candidate (he) is } H.}\left(\frac{g_H}{r} - V_H\right)$$

$$+\varepsilon\overbrace{\left(\underbrace{(1-\varepsilon)\,\delta_L(h)}_{\text{He observes } h.} + \underbrace{\varepsilon\delta_L(l)}_{\text{He observes } l.}\right)}^{\text{The candidate (he) is } L.}\left(\frac{b_H}{r} - V_H\right)$$

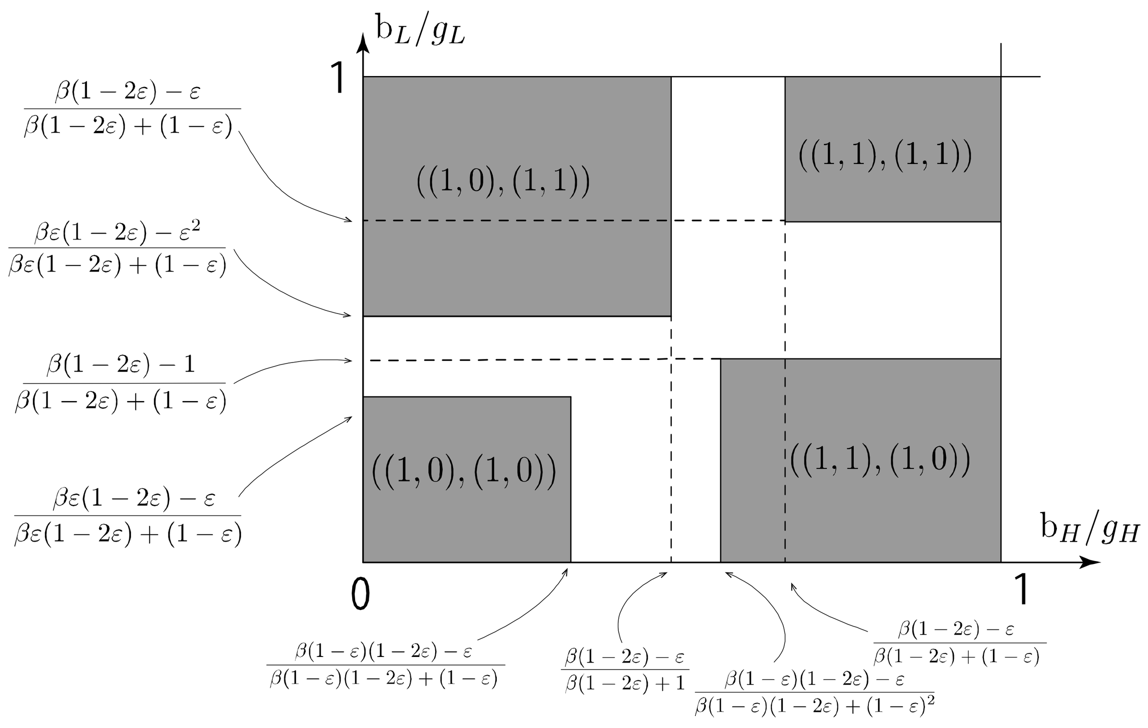
- The Bellman equation for type T:

$$V_T = \frac{\rho}{r}\left[\frac{1}{2}\max\left\{V_T^h, 0\right\} + \frac{1}{2}\max\left\{V_T^l, 0\right\}\right]$$
$$= \beta\left[\max\left\{V_T^h, 0\right\} + \max\left\{V_T^l, 0\right\}\right].$$

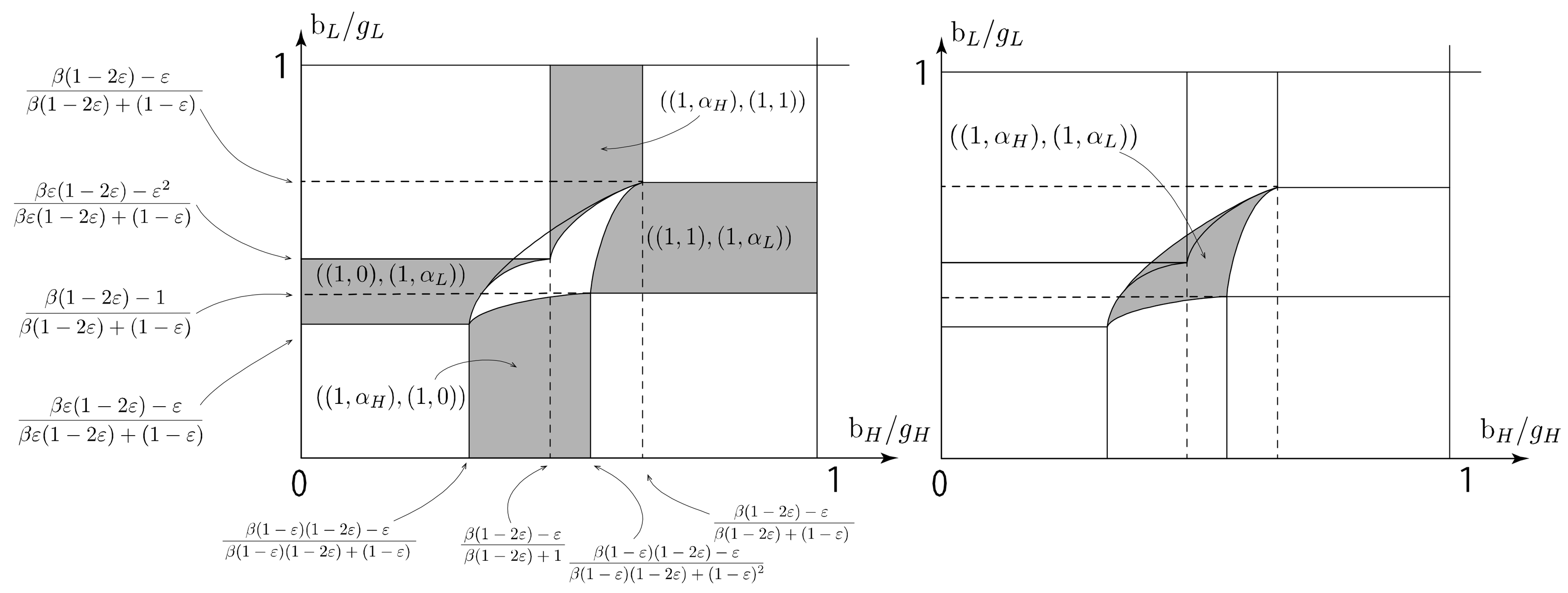
- δ is a search eqm. $\iff \delta_T(\theta)\begin{cases} = 0 & \text{if } V_T^\theta < 0; \\ \in [0,1] & \text{if } V_T^\theta = 0; \\ = 1 & \text{if } V_T^\theta > 0, \end{cases}$ $\forall T, \theta.$

4 Eqm. condtion and existence

- We characterized the eqm. conditions for all possible strategy profiles.
- $\delta = ((0,0), (0,0))$ is always a search eqm.
  →We call this the 'trivial' search eqm, and the others 'non-trivial' search equilibria.
- A non-trivial search eqm. satisfies that $\delta_H(h) = \delta_L(h) = 1$.
- The eqm. conditions for pure strategy profiles:

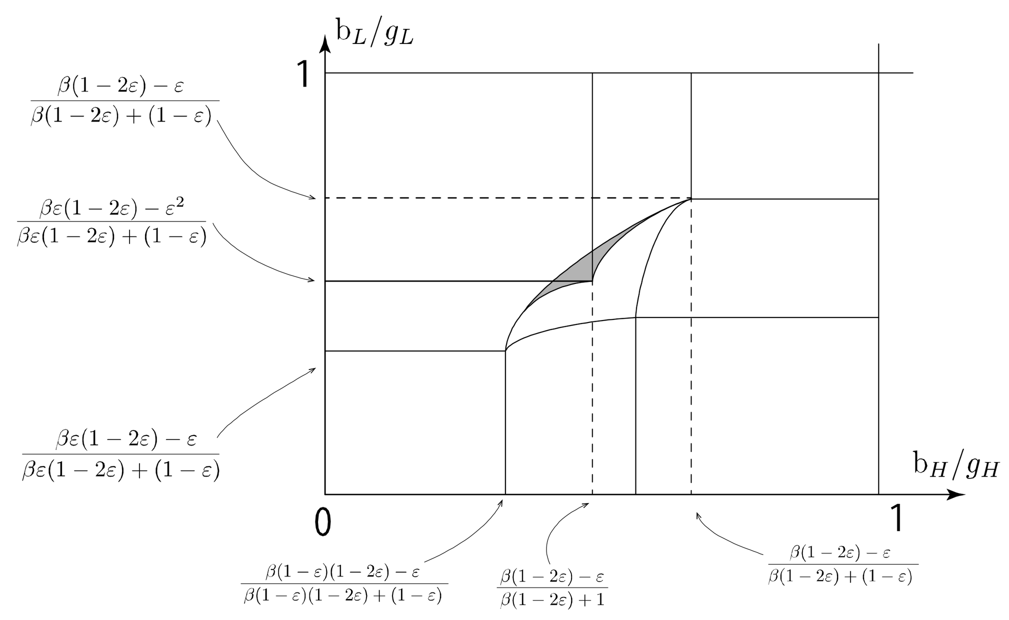


- The eqm. conditions for mixed (not pure) strategy profiles:



→By take mixed strategy into consideration, there always exists a non-trivial search eqm.

- The number of nontrivial search equilibria: There are three equilibria in the shaded area (two on the boundary).



5 The change of V_H, V_L w.r.t β

- $\beta := \rho/2r$: the proportion of the encountering rate over discount rate.
- There is a search eqm. where $\frac{\partial V_H}{\partial \beta} < 0$ or $\frac{\partial V_L}{\partial \beta} < 0$ in the shaded area of each figure below.
- When $\delta_H(l) = 0$ or 1, the increase of β causes the increase of V_H and V_L (the direct effect).
- But, when $0 < \delta_H(l) < 1$, the increase of β causes the decrease of $\delta_H(l)$ as well (the indirect effect).
  →Because type H get more likely to wait for h.
  →The decrease of $\delta_H(l)$ leads to the decrease of V_H and V_L.
  →The indirect effect is bigger than the direct one.