# An Introduction to the Universe of Biform Games

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## What is a biform game?

A biform game is a straightforward combination of non-cooperative and cooperative games, introduced by Brandenburger and Stuart (2007). It is a two-stage game: in the first stage, decisive players choose their strategies in a non-cooperative way, thus forming the second stage of the game, in which all the players cooperate.

### Non-cooperative game[1]

An $n$-player non-cooperative game is a collection $\Gamma^N_{norm} = (S^1, \ldots, S^n; u^1(s^1, \ldots, s^n), \ldots, u^n(s^1, \ldots, s^n))$ where

*a)* $S^i$ denotes the set of player $i$'s strategies (with $s^i \in S^i$); and

*b)* $u^i : S^1 \times \cdots \times S^n \to \mathbb{R}$ denotes player $i$'s pay-off function.

The subscript $norm$ refers to the normal form representation of the non-cooperative game and the superscript $N$ indicates the set of players $N$.

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### Cooperative game[2]

An $n$-player transferable utility (TU) game is a function $v \in \mathcal{G}^N$, where $v : 2^N \to \mathbb{R}$ and $\mathcal{G}^N$ denotes the class of $N$-player games. By convention, $v(\emptyset) = 0$.

We call function $v \in \mathcal{G}^N$ the characteristic function of the game.
In the literature, another definition is accepted, i.e. a collection $(N, v)$. We leave the set of players of the definition of the cooperative game while we constrain $v$ to be in the class of $N$-player games.

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### Linkage between the stages

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### Biform game

An $n$-player *biform game* is a collection $(S^1, \ldots, S^n; v; \psi)$, where

*a)* $S^i$ is a finite set, for all $i = 1, \ldots, n$;

*b)* $v : S^1 \times \ldots \times S^n \to M$, where $M : 2^N \to \mathbb{R}$, $v(s^1, \ldots, s^n)(\emptyset) = 0$; and

*c)* $\psi : 2^N \to \mathbb{R}^N$ such that $\psi(v) \in C(v)^3$ whenever $C(v) \neq \emptyset$.

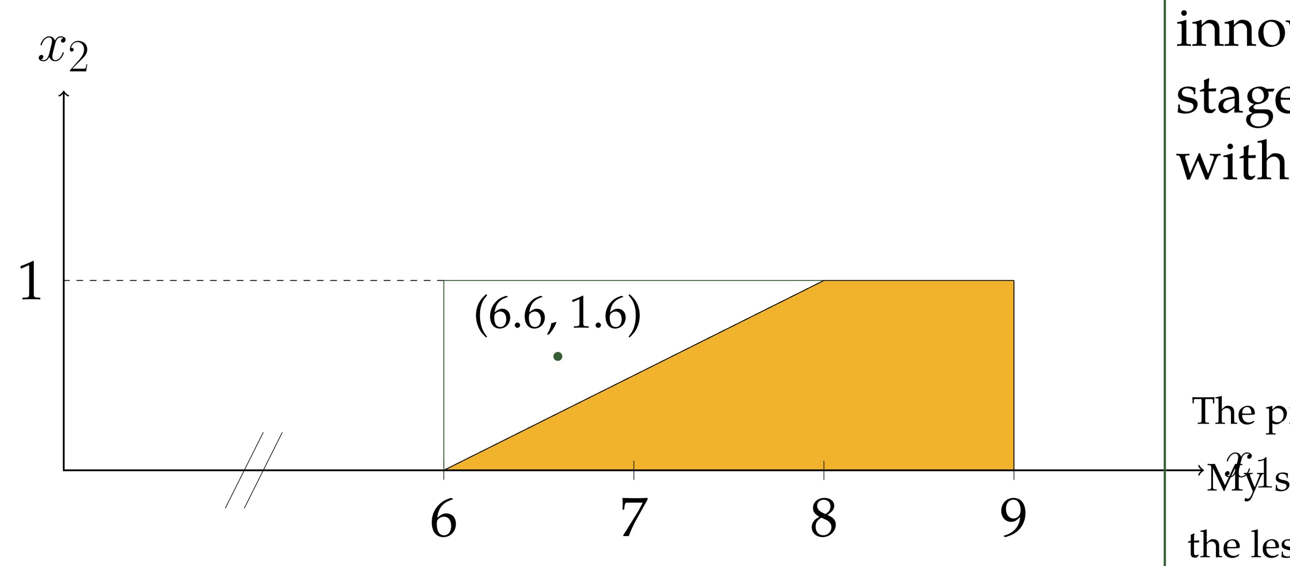
## What is new?

My innovation is the generalization of the originally plotted biform game. In their article, Brandenburger and Stuart (2007) use a special form of the above-mentioned linkage, which leads to inconsistency in the model. I illustrate this inconsistency with a simple example.

### The original definition: confidence indices

Brandenburger and Stuart (2007) define the so-called confidence indices. These are simple performance measures that indicate *a priori* how well will the players bargain in the cooperative second stage. I.e. a confidence index of player $i$ of 60% means that player $i$ believes prior to entering the second stage that he will capture the maximal value of the projection of the core to his plane with 60% probability, and the minimal value with 40%.

A small example: Suppose there are two players and the core projections to their planes of payoffs are as follows:




As it is seen, there are constellations of confidence indices which assigns such believed payoffs to players which fall outside the core, like the upper-left point. Furthermore, the confidence indices are exogeneously given parameters of the biform game, and they lack any justification.

### My definition: single-valued solution

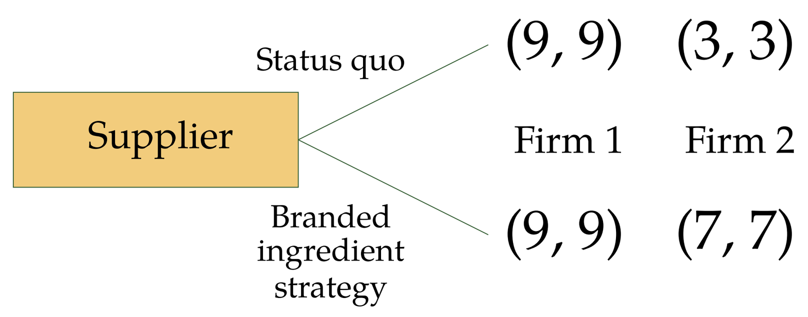
As seen in the bottom of the first column, my contribution is the switch from confidence indices to single-valued solutions of the cooperative second stage. No additional assumptions needed. The only requirement for these solutions is that it should belong to the core whenever the core is non-empty.

## What is the big deal?

Unlike many applications of game theory, the need for biform games is from empirics. Many real world situations can be described by biform games.

### Branded ingredient strategy

This example is from Brandenburger and Stuart (2007).




A supplier decides in the first stage whether or not he should oblige the downstream firms to advertise the ingredient on the end product. Typical examples are Intel stickers on computers or Gore-Tex labels on clothes.

### Innovation

Basically, any type of innovation can be modelled with biform games. Its general framework allows the researcher to feature firms with an innovation option in the non-cooperative first stage, and capture the second-stage bargaining with cooperative games.

### Disclaimer

The present poster is based on my bachelor's thesis (Gyetvai, 2012). My supervisor was Dr. Tamás Solymosi. The thesis was created by the lessons of research seminar 'Cooperation and Allocation' led by Dr. Solymosi. During the seminar, we compiled a paper with co-author and friend Tamás Török for the Conference of Scientific Students' Associations. Our findings won us the **3rd place** on the conference and **distinguishment** in the national round.

### References

Brandenburger, A., Stuart, H. (2007): Biform games. *Management Science*, Vol. 53, No. 4, pp. 537-549.

Gyetvai, A. (2012): *An Analysis of Innovation with Biform Games*. Bachelor's Thesis, Faculty of Economics, Corvinus University of Budapest.

Gyetvai, A., Török, T. (2012): *Biform Games* (in Hungarian). Paper for the Conference of Scientific Students' Associations, Corvinus University of Budapest.

[1]Normal form representation.
[2]Transferable utility assumption.
[3]$C(v)$ denotes the core of a TU game; i.e. $C(v) = \{x \in \mathbb{R}^N : v(N) = x(N), e(v, x)_S \leq 0, S \subseteq N\}$