Combinatorial Optimization & Graph Algorithms
TU Berlin
www.coga.tu-berlin.de

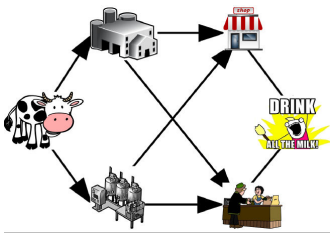
Stability in networks

Ágnes Cseh, Martin Skutella

Technische Universität Berlin
Berlin Mathematical School
Methods for Discrete Structures
Deutsche Telekom Stiftung

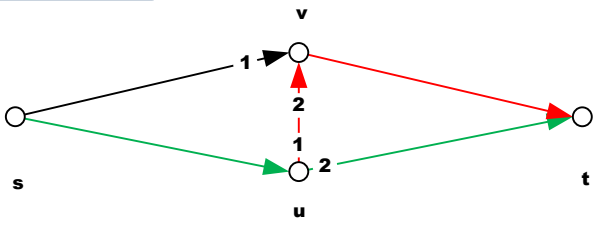
Market modeling

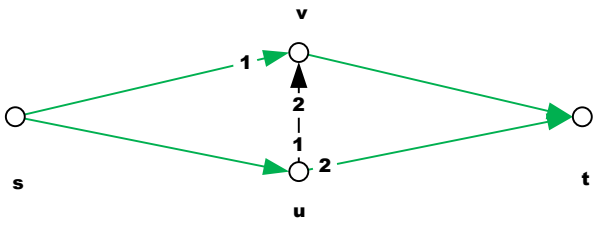
traders → vertices
possible deals → edges



- selfish strategy → max flow
- profit → min cost flow
- time constraints → flow over time
- social welfare → stable flow

Stable flows

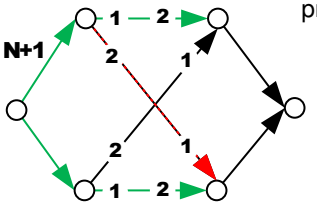




- **Instance**
  - directed network with a set of terminals
  - preference lists on possible deals
- **Definition**
  A walk $W = (v_1, ..., v_k)$ is blocking, if
  - the edges of W are unsaturated and
  - v_1 is terminal or it prefers v_1v_2 to its worst selling and
  - v_k is terminal or it prefers $v_{k-1}v_k$ to its worst purchase.

  A feasible flow is *stable* if there is no blocking walk.
- **Properties [Fleiner, 2010]**
  stable flows
  - always exist
  - can be found in polynomial time
  - all have the same flow value

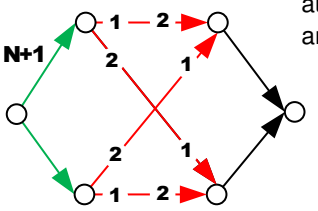
The Gale-Shapley algorithm



proposal and refusal steps

- starts at terminals
- preflow-push algorithm
- runtime: exponential

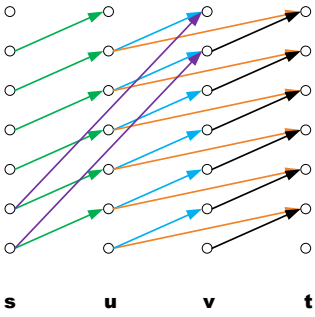
The augmenting path algorithm

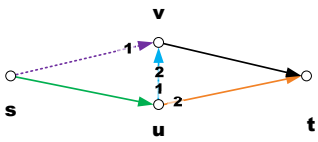


augmenting paths of proposal and refusal pointers

- starts at terminals
- several Gale-Shapley steps at once
- runtime: polynomial

Flows over time





- **Instance**
  - directed network, preference lists
  - transit times on edges, time horizon
- **Definition**
  A walk $W = (v_1, ..., v_k)$ is blocking, if its edges fulfill the blocking properties exactly at the time when they have to forward flow.
- **Properties**
  - Stable flows over time always exist and they can be found in pseudo-polynomial time.
  - If the time is considered to be infinte, periodic stable flows can also be found.
  - The possibility of storage can be handled efficiently.