MAGYAR TUDOMÁNYOS AKADÉMIA
Közgazdaság- és Regionális Tudományi Kutatóközpont

MTA KRTK KTI

Centre for Economic and Regional Studies
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK DISCUSSION PAPERS

MT-DP – 2015/4

Ants and crickets: arbitrary saving rates in an agent-based model with infinitely lived-agents

GERGELY VARGA – JÁNOS VINCZE

INSTITUTE OF ECONOMICS, CENTRE FOR ECONOMIC AND REGIONAL STUDIES,
HUNGARIAN ACADEMY OF SCIENCES BUDAPEST, 2015

Discussion papers
MT-DP – 2015/4

Institute of Economics, Centre for Economic and Regional Studies,
Hungarian Academy of Sciences

Ants and crickets: arbitrary saving rates in an agent-based model with infinitely lived-agents

Authors:

János Vincze
research advisor
Institute of Economics
Centre for Economic and Regional Studies
Hungarian Academy of Sciences
and professor
Department of Mathematical Economics and Economic Analysis
Corvinus University of Budapest
e-mail: vincze.janos@krtk.mta.hu

Gergely Varga
Corvinus University of Budapest
e-mail: gergely.varga@uni-corvinus.hu

January 2015

ISBN 978-615-5447-62-4
ISSN 1785 377X

Ants and crickets: arbitrary saving rates in an agent-based model with infinitely lived-agents

Gergely Varga - János Vincze

Abstract

Savings behaviour seems to exhibit heterogeneity across nations, and within nations, too. Large changes in saving rates have been observed in the last decades that can be viewed as signs of the arbitrariness of saving. There is a long tradition in the savings literature that separates people into two groups: those who behave soberly (ants), and those who act in an extremely short-sighted fashion (crickets). A puzzle remains: why does an apparently inferior behavioural pattern persist? Our aim is to provide a model that exhibits the arbitrariness of savings by exploring the two-types idea, and also makes intelligible why both types can coexist in the long run. Our approach consists in setting up an agent-based model starting from a traditional production and factor market framework. The model features an evolutionary mechanism that promotes the behaviour conducive to the highest satisfaction of the consumption goal. Our main findings include the prevalence of non-ergodicity, and the genericity of non-stationarity. The model becomes stationary when the selection pressure is very high, and crickets are eliminated. Though in general ants have somewhat higher per capita consumption than crickets, and are less indebted, we have found cases where the total average consumption is higher with many crickets than without them.

Keywords: saving-consumption puzzles, bounded-rationality, agent-based macromodel

JEL classification: E03, E14, E27

Acknowledgement:
We are grateful for valuable comments from Andras Simonovits. The reseach was financed by OTKA K 108 658.

Hangyák és tücskök: a megtakarítási ráta esetlegessége egy ágensalapú modellben, végtelen ideig élő szereplőkkel

Varga Gergely – Vincze János

Összefoglaló

A megtakarítási magatartás heterogenitást mutat mind időben, mind országok között, mind pedig az egyes országokon belül. Hatalmas változásokat lehet megfigyelni az utóbbi évtizedekben a megtakarítási rátákban, amit úgy tekinthetünk, mint a megtakarítások „esetlegességét". A megtakarítási irodalomban régóta jelen van az a gondolat, hogy az emberek két csoportra oszthatók: egyrészt vannak hosszú távon gondolkodó, józan „hangyák", másrészt léteznek csak a mának élő „tücskök". Az utóbbiak létezése talány: hogyan maradhat fenn tartósan egy ilyen „irracionális" viselkedés? Célunk ebben a tanulmányban az, hogy felírjunk egy olyan modellt, amelyben a megtakarítások „esetlegesek". Ebben felhasználjuk a két típus gondolatát, és megindokoljuk azt, hogy miért maradhatnak fenn ezek egymás mellett. Kiindulva egy hagyományos termelési struktúrájú és hagyományos tényezőpiacokkal rendelkező heterogén ágenses makromodellből, egy ágensalapú modellt írunk fel. Ebben létezik olyan szelekciós mechanizmus, amely azokat a magatartásokat részesíti előnyben, amelyek nagyobb hosszú távú átlagos fogyasztást realizálnak. A modell nem ergodikus viselkedést mutat, amelyben a stacionaritás nem elfajult tulajdonság. A modell stacionáriussá válik, amikor a szelekciós nyomás nagyon nagy, és a tücskök eliminálódnak. Bár igaz, hogy általában a tücskök átlagosan jobban el vannak adósodva, mint a hangyák és átlagos fogyasztásuk kisebb, találtunk olyan eseteket, amelyekben az egész gazdaság fogyasztása a tücskökkel együtt átlagosan nagyobb, mint tücskök nélkül.

Tárgyszavak: fogyasztási-megtakarítási rejtély, korlátozott racionalitás, ágens-alapú makromodell

JEL kód: E03, E14, E27

Ants and crickets: arbitrary saving rates in an agent-based model with infinitely lived-agents*

Gergely Varga†, János Vincze‡

Abstract

Savings behaviour seems to exhibit heterogeneity across nations, and within nations, too. Large changes in saving rates have been observed in the last decades that can be viewed as signs of the arbitrariness of saving. There is a long tradition in the savings literature that separates people into two groups: those who behave soberly (ants), and those who act in an extremely short-sighted fashion (crickets). A puzzle remains: why does an apparently inferior behavioural pattern persist? Our aim is to provide a model that exhibits the arbitrariness of savings by exploring the two-types idea, and also makes intelligible why both types can coexist in the long run. Our approach consists in setting up an agent-based model starting from a traditional production and factor market framework. The model features an evolutionary mechanism that promotes the behaviour conducive to the highest satisfaction of the consumption goal. Our main findings include the prevalence of non-ergodicity, and the genericity of non-stationarity. The model becomes stationary when the selection pressure is very high, and crickets are eliminated. Though in general ants have somewhat higher per capita consumption than crickets, and are less indebted, we have found cases where the total average consumption is higher with many crickets than without them.

Keywords: saving-consumption puzzles, bounded-rationality, agent-based macromodel

JEL classification: E03, E14, E27

*We are grateful for valuable comments from Andras Simonovits. The reseach was financed by OTKA K 108 658.

†Corvinus University of Budapest, e-mail: gergely.varga@uni-corvinus.hu

‡Corvinus University of Budapest and Institute of Economics (Centre for Economic and Regional Studies of Hungarian Academy of Sciences), e-mail: janos.vincze@uni-corvinus.hu

Hangyák és tücskök: a megtakarítási ráta esetlegessége egy ágensalapú modellben, végtelen ideig élő szereplőkkel

Varga Gergely - Vincze János

Összefoglaló

A megtakarítási magatartás heterogenitást mutat mind időben, mind országok között, mind pedig az egyes országokon belül. Hatalmas változásokat lehet megfigyelni az utóbbi évtizedekben a megtakarítási rátákban, amit úgy tekinthetünk, mint a megtakarítások "esetlegességét". A megtakarítási irodalomban régóta jelen van az a gondolat, hogy az emberek két csoportra oszthatók: egyrészt vannak hosszú távon gondolkodó, józan "hangyák", másrészt léteznek csak a mának élő "tücskök". Az utóbbiak létezése talány: hogyan maradhat fenn tartósan egy ilyen "irracionális" viselkedés? Célunk ebben a tanulmányban az, hogy felírjunk egy olyan modellt, amiben a megtakarítások "esetlegesek". Ebben felhasználjuk a két típus gondolatát, és megindokoljuk azt, hogy miért maradhat fenn ezek egymás mellett. Kiindulva egy hagyományos termelési struktúrájú és hagyományos tényezőpiacokkal rendelkező heterogén ágenses makromodellből, egy ágens-alapú modellt írunk fel. Ebben létezik olyan szelekciós mechanizmus, amely azokat a magatartásokat részesíti előnyben, amelyek nagyobb hosszú távú átlagos fogyasztást realizálnak. A modell nem ergodikus viselkedést mutat, ahol a stacionaritás nem elfajult tulajdonság. A modell stacionáriussá válik, amikor a szerelciós nyomás nagyon nagy, és a tücskök eliminálódnak. Bár igaz, hogy általában a tücskök átlagosan jobban el vannak adósodva, mint a hangyák és átlagos fogyasztásuk kisebb, találtunk olyan eseteket, ahol az egész gazdaság fogyasztása tücskökkel átlagosan nagyobb, mint tücskök nélkül.

Kulcsszavak: fogyasztási-megtakarítási rejtély, korlátozott racionalitás, ágens-alapú makromodell

JEL kód: E03, E14, E27

1 Introduction

Saving is one of the central themes in economics. In growth theory it is noticed that it is through savings that nations become rich, and understanding fluctuations in savings is an important component of business cycle analysis, too. More recently Akerlof–Shiller (2009) suggested that the arbitrariness of savings can be one cause of the unpredicatbility of macroeconomic outcomes.

Saving behaviour exhibits heterogeneity across ages and nations, and within nations, too. The existence of differential saving rates is an important issue in the economics of poverty (Banerjee–Duflo, 2011), and it is thought to have bearing on the ever changing wealth distribution within developed countries (Piketty, 2014). Large changes in saving rates have been observed in the last decades that led to big changes in the international economy (see the US savings glum), and transformed our views on the way societies must deal with the needs of their older citizens.

Theoretical explanations of modifications in savings behaviour are not easy to arrive at. Recently Dobrescu–Kotlikoff–Motta (2012) proposed models to account for the large negative changes in saving rates in developed countries, and concluded that the cause must be changes in social preferences. This explanation is, however, leaves unanswered the question why preferences have been unstable.

For a long time empirical modelling of savings-consumption has had to face the problem of individual heterogeneity. There is a long tradition that separates people into two groups: those whose behaviour can be described by some version of the utility maximizing rational expectations model (originated by Fisher (1930), and those who seem to behave in a purely consumption oriented manner, largely disregarding the future consequences (see Hall–Mankiw (1988), Laibson (1997)). In other words some people seem to behave soberly, while others in an extremely short-sighted fashion. The latter behaviour has been attributed first to liquidity-constraints, then to hyperbolical discounting coupled with naivete, and more recently some authors suggested that certain behavioural features (e.g. limited cognitive abilities, or a mild form of "schizophrenia" (dual-self)) can also be invoked for an explanation (see e.g. Fudenberg–Levine (2006)). The behavioural economics literature takes us farther and farther from the traditional model, without giving up the idea that people (or their selves) try to maximize some preference functional—consistently in a non-traditional sense—over their lifetime. We believe that some facts are difficult to fit into any maximizing model, and they will ever be. For instance the extensive literature on 401(k) accounts (see Madrian–Shea, 2001) would hardly fit into any preference-based theory.

Can we build a savings model that yields endogenously non-stationary aggregate saving behaviour, while exhibiting the persistent coexistence of foresighted savers (ants), and of people focusing solely on current consumption (crickets)? Of course, we do not believe that mankind can so easily be dichotomized. Still as a first approximation this simple dichotomy is convenient. If a preliminary investigation along these lines brought fruits, it would give us hope that more refined distinctions of the same genre are worth looking for.

Our model has a simple production side based on neoclassical assumptions concerning production, labour and capital service markets. The main challenge is how to operationalize the idea of ant and cricket saving behaviour. Our solution is that both ants and crickets plan their consumption using their own behavioural rule as follows: ants try to figure out their total lifetime wealth, and intend to achieve perfect consumption smoothing, while crickets focus on immediate consumption after determining their reference consumption level in a given period. Consumption plans may not be individually feasible, but there is a credit market that intermediates between plans and actual consumption. Ants use statistical learning rules to forecast returns on capital and labour income, while shorttermist crickets do not bother with the future. Ants are not rational in the traditional sense as they have no "rational expectations". However, in contrast to crickets they care for the future and strive to forecast relevant variables. There exists also a social adaptation-selection mechanism that favours the type of behaviour that obtains more in terms of accumulated consumption. Thus our model has an evolutionary flavour with "objective" utility (fitness). The evolutionary process results in modifications in the ant-cricket composition of the economy, the relative proportion of ants and crickets can continuously change, and there is a possibility that one of them is driven out completely.

After setting up the model we carry out simulations to study two questions. First, we address the arbitrariness of saving, that is whether saving rates are "predictable in the long run" in this setting. Unpredictability takes a weak and a strong form: non-stationarity and non-ergodicity, respectively. Secondly, we ask whether the model is robust with respect to learning parameters.

The next section gives a few notes on the various types of literature that can be regarded as direct antecedents of our work. Section 3 presents the model, and Secton 4 reports the simulation results. The concluding section summarizes our findings, and indicates directions for further research.

1.1 Notes on the literature

Stylized facts to account for

Akerlof–Shiller (2009, Ch. 9) point out the arbitrariness of saving, and its variability. Across nations savings rates as high as 1/3 coexist with negative ones. The personal saving rate in the US was reduced from 10% in the early 1980s to negative rates in the XXIst century. They notice also the vast variety of savings rates across individuals that bring in huge differences in retirement wealth. They emphasize the inability of the traditional theory to account for this fact. Their preferred explanation is "Animal Spirits", of course. They muster observations indicating that savings decisions are made on "irrelevant cues", and their anecdotal description of the role of stories (one may say "culture") seems plausible. However, in this paper we do not pursue this line of research. We want to develop a model where saving seems arbitrary, but we have no idea how to incorporate a role for irrelevant information, or for stories as a way of cultural transmission of behavioural patterns.

Banerjee–Duflo (2011) consider the puzzling fact that though many poor people over the world might increase their well-being substantially with a little bit of saving, there are very few who accomplish this. Banerjee and Duflo enlist several explanations but their favourite one rests on the lack of self-control, with the addition of a positive biological feedback in case of those who are poor. Of their description we underline one major point: people say that money at home cannot be kept, because there will be some purpose to spend it in no time. We interpret this that many people have consumption plans without much regard to the future, thus their saving is something accidental. We acknowledge that the self-control approach is an interesting hypothesis to understand saving behaviour, but it entails a personality which is more complicated than the ones we want to model as a first approximation. Lack of self-control means that someone cannot achieve some of her own goals, because of her own weakness, and not because external conditions make it impossible.

Behavioural theories of saving

The idea that the presence of "liquidity constrained" households, who behave myopically, can explain aggregate saving behaviour (see Hall–Mishkin (1988) can be construed as the first behavioural theory of savings that makes a strict distinction between two types of households, those who behave with foresight and in a rational manner, and those who simply consume as much as they can. The modern variant that makes use of psychological facts about "time discounting" was rendered in its now usual format by Laibson (1997). Here the distinction is between rational long-run maximizers and naive agents who discount their future utilities hyperbolically, i.e. those who have time-inconsistent preferences. More recent theories are based on other — more complete — psychological theories according to which we have a dual-self, and sometimes we are unable to behave in our best long-run interests (Fudenberg–Levine, 2006). All of these theories rely on a persistent coexistence of rational and irrational behaviour with respect to saving, and leave unanswered the problem why people cannot learn to act rationally, or why (cultural?) evolution cannot eradicate an inferior behavioural pattern.

Agent-based modelling

Agent-based models have been established in economics for at least 20 years now. The consumption-saving decision has been given relatively little attention in this literature. One reason for this, perhaps, is that the *par excellence* topic of agent-based models is markets where there is obviously a lot of interaction. In contrast traditionally saving was considered as a subfield of individual decision theory, surely to be incorporated into a general equlibrium model, but where particularly exciting questions of market equlibrium do not emerge. In agent-based macromodels households save, generally, by following some fixed rule, almost like automata (see Deissenberg et al. (2008) or Delli Gatti et al. (2011)). These rules may be sophisticated, and are devised so that they approximate the behaviour of traditional utility-maximizing agents. What is missing for us in these models is the evolution or adaptation of behavioural patterns.

2 The ants-crickets model

2.1 The production-distribution side

The production–primary income distribution side of the model follows Aiyagari (1994). There are N *ex ante* identical infinitely living households. Labour supply of each household is characterized by the same two-state Markov-chain, with the following transition matrix:

	L_1	L_2
L_1	p	$1-p$
L_2	$1-q$	q

Here $L_1 < L_2$ and $p < q$. Thus, if the number of households is large, aggregate labour supply uncertainty is small, but individual uncertainty can still be substantial. Labour markets always clear, and work is rewarded according to its marginal product. The economy's aggregate production function is Cobb–Douglas, with aggregate labour and aggregate capital as its arguments, and α is labour's share. Summarizing the assumptions:

$$L_t = \sum_k L_{t,k}, \qquad K_t = \sum_k K_{t,k} \qquad \text{and} \qquad Y_t = AK_t^{1-\alpha} L_t^{\alpha},$$

where L_t is aggregate labour, $L_{t,k}$ is labour supply of household k, K_t is aggregate physical capital, $K_{t,k}$ is capital owned by household k, and Y_t is aggregate output. The wage rate (w_t), and the compensation of capital services (r_t^K) can be expressed implicitly as

$$w_t L_t = \alpha Y_t \qquad \text{and} \qquad r_t^K K_t = (1-\alpha) Y_t.$$

Capital depreciates at rate δ per period.

At the beginning of each period idiosyncratic labour shocks are realized, then production and the distribution of primary income take place. Total funds of individual households can be written as

$$A_{t,k} = (1 - \delta + r_t^K) K_{t,k} + (1 + r_t) B_{t,k} + w_t L_{t,k} - (1 + r_t) D_{t,k},$$

where $A_{t,k}$ is funds of household k (inclusive of current labour and capital income minus depreciation), r_t is the rate of interest on private debt, $B_{t,k} \geq 0$ is credits to other households, and $D_{t,k} \geq 0$ is debt due to other households. (About lending and borrowing see below.)

2.2 The consumption side

Every household belongs to either of two types in each period:

$$T_{t,k} = a \qquad \text{or} \qquad T_{t,k} = c,$$

where a stands for Ant, and c for Cricket. Each type has its characteristic consumption planning rule, whose parameters, however, depend on the actual state of household k.

Consumption plans of ants Ants try to forecast their lifetime wealth, and plan to achieve constant (non-negative) consumption:

$$C_{t,k}^P = \max\left(0, \frac{\widetilde{E}_{t,k}(r)}{1 + \widetilde{E}_{t,k}(r)} A_{t,k} + \frac{1}{1 + \widetilde{E}_{t,k}(r)} \widetilde{E}_{t,k}(LW_{t,k})\right),$$

where $C_{t,k}^P$ is the planned consumption of household k, if $T_{t,k} = a$, and $\widetilde{E}_{t,k}(LW_k)$ and $\widetilde{E}_{t,k}(r)$ are "expectations" of average labour income and of the interest rate, respectively, by household k in period t. These expectations are weighted averages of the observed variables, with exponentially declining weights. They can be recursively defined as:

$$\begin{aligned}\widetilde{E}_{t,k}(LW_k) &= \sigma \widetilde{E}_{t-1,k}(LW_k) + (1-\sigma) w_t L_{t,k} \\ \widetilde{E}_{t,k}(Er_k) &= \sigma \widetilde{E}_{t-1,k}(Er_k) + (1-\sigma) r_t.\end{aligned}$$

and initially

$$\begin{aligned}\widetilde{E}_{1k}(LW_k) &= w_1 L_{1k} \\ \widetilde{E}_{1k}(r_k) &= r_1.\end{aligned}$$

Consumption plans of crickets Crickets focus exclusively on consumption. They want to consume according to their perceived social status, which is defined as their quartile of individual consumptions in the previous period. Concretely, if $T_{t,k} = c$, and $Q_{t-1}^{i-1} < C_{t-1,k} \leq Q_{t-1}^{i}$ then

$$C_{t,k}^{P} = \max(0, Q_{t-1}^{i}),$$

where Q_{t-1}^{j} denotes the jth quartile of individual consumption levels in period $t-1$, and $i = 1, 2, 3, 4$. ($Q_{t-1}^{0} = 0$ and Q_{t-1}^{4} is the maximum of individual consumption levels.)

Consumption If $C_{t,k}^{P} \leq A_{t,k}$ then

$$C_{t,k} = C_{t,k}^{P},$$

and the household's supply of funds becomes

$$W_{t+1,k} = A_{t,k} - C_{t,k} \geq 0.$$

Otherwise:

$$C_{t,k} = \min(C_{t,k}^{P}, \max(0, A_{t,k} + \frac{1}{1+r_{t+1}} \overline{D}_t)),$$

where $\overline{D}_t$ is a universal debt constraint (see below), and r_{t+1} is the interest rate on loans maturing in period $t+1$.

The debt constraint is calculated as the capital value of wage income in the "worst case scenario", where the current rate of interest is perturbed to be positive in any case.

The incipient borrowing demand of household k is:

$$D_{t+1,k} = \max\left(0, \min\left(\frac{1}{1+r_{t+1}} \overline{D}, C_{t,k}^{P} - A_{t,k}\right)\right).$$

2.3 The credit market

The interest rate is determined as

$$r_{t+1} = \omega_t + \left(\max\left(0, r_t^{K} - \delta\right)\right),$$

where ω_t in $(0, \omega\frac{D^2_{t+1}}{B^2_{t+1}})$. This is a naive (centralized) credit market, where some premium is added to the net current return on holding physical capital.

If the feasibility condition

$$\sum_k W_{t+1,k} \geq \sum_k D_{t+1,k}$$

is satisfied, the supply of funds is consistent with borrowing demand. In that case

$$B_{t+1,k} = W_{t+1,k} \frac{\sum\limits_k D_{t+1,k}}{\sum\limits_k W_{t+1,k}},$$

$$K_{t+1,k} = W_{t+1,k} - B_{t+1,k}.$$

In other words: the portfolio weights with respect to loans and physical capital are the same for each household, whenever a household's supply of funds is positive.

When the feasibility condition is not satisfied, the credit market collapses, and there is no lending or borrowing, and all debts are cancelled. (This did not happen in the simulations we report in the following section.)

2.4 Adaptation-selection

Agents accumulate their consumption "experience" (a measure of fitness if you like) as follows (Brock–Hommes, 1997):

$$U_{t,k} = \lambda U_{t-1,k} + (1-\lambda)C_{t,k}, \qquad 0 < \lambda < 1.$$

In each period there is a "small" chance (ρ) for any agent that a change in its type may occur. If this chance is realized, the agent examines his neighbours (there is a stochastic neighbourhood relationship that gives an average neighbourhood size of $N/5$), and identifies the ant and cricket with the highest value U_{t-1} in each subgroup. Let it be $U^*_{t-1,a(k)}$ and $U^*_{t-1,c(k)}$, respectively. Then the type of agent k becomes τ ($\tau = c$ or a) with probability

$$\Pr(T_{t,k} = \tau) = \frac{\exp\left(\frac{U^*_{t-1,k(\tau)}}{\Upsilon}\right)}{\exp\left(\frac{U^*_{t-1,k(a)}}{\Upsilon}\right) + \exp\left(\frac{U^*_{t-1,k(c))}}{\Upsilon}\right)},$$

where a very large $\Upsilon > 0$ means that success is almost irrelevant, and Υ close to zero implies that there is a high probability that the more successful type "wins". (We will call Υ sometimes "temperature" for brevity, referring to its origin in thermodynamics.) Whatever case is realized, household k "inherits" the success of his winning neighbour, and this inheritance is immediately eroded, whenever $\lambda < 1$.

3 Simulations

Some of the parameters were held constant in all the simulations we report. Table 1 shows them, and their numerical values.

Table 1: Fixed parameters

Parameter	N	L_2	L_2	A	p	q	ω	σ	α	δ	ω
Value	100	0.1	1	1	0.4	0.95	0.02	0.8	0.67	0.005	10^{-6}

3.1 The baseline parameterization

The baseline setting contains the following parameter values in addition to those described in Table 1: $\Upsilon = 1$, $\lambda = 0.9$ and $\rho = 0.01$.

For this parameter setting we ran 10 simulations, in each case for 5000 periods. For every run we dropped the first 1000 periods to eliminate any effects coming from the particular assumptions that we used to start the simulation, and calculated all statistics on the basis of the last 4000 simulated ”observations” only. Table 2 contains the average values of the averages of 5 variables of interest, as well as the mimima and maxima of their respective averages. (For instance the min value of S/Y (the savings ratio) 0.04 means that of the 10 runs, the lowest average savings ratio was 0.04.)

Table 2: Baseline results

	Mean	Min	Max
S/Y	0.14	0.04	0.32
C	939	667	1211
proportion of crickets in total population	0.47	0.09	0.84
consumption of crickets in total consumption	0.41	0.06	0.83
debt of crickets in total debt	0.63	0.31	0.89

The average savings ratio is well within the range of observed ratios for actual economies, and this was not calibrated. Capital was on average about 30 % higher in the ”best” run than in the ”worst”, this difference is large, non-ergodicity turns up clearly in this figure. The last three rows of the table draw attention to an interesting feature: crickets were sometimes many and sometimes few, but their average consumption did not seem to be much different from the average consumption of ants. Though it is somewhat lower, corresponding to expectations. Their average level of debt is more pronouncedly higher, in fact again satisfying our preconception about the contrast between ants and crickets, but not outrageously. The following charts show the time series from some characteristically different runs, illustrating visually that much beyond the differences among averages, runs were qualitatively quite heterogenous.

Figure 1: Saving rate, capital and proportion of crickets. Simulation 1.

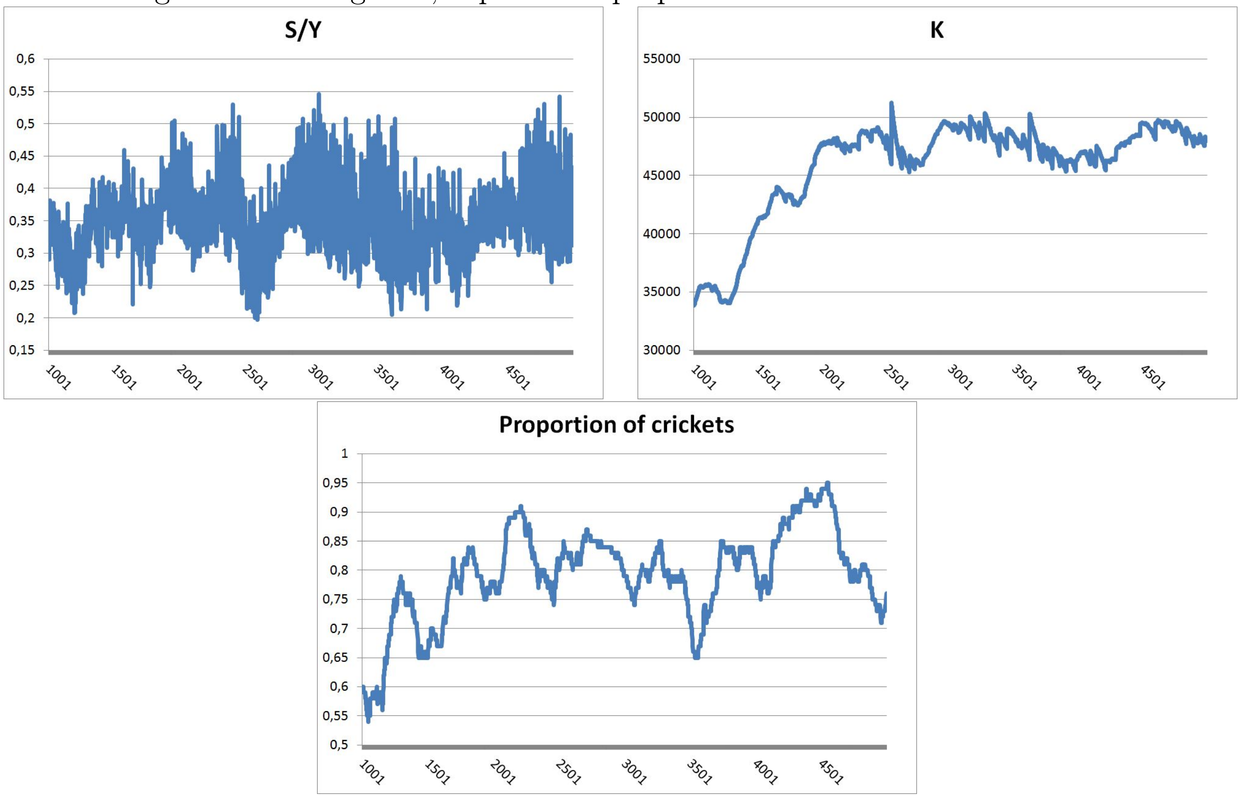
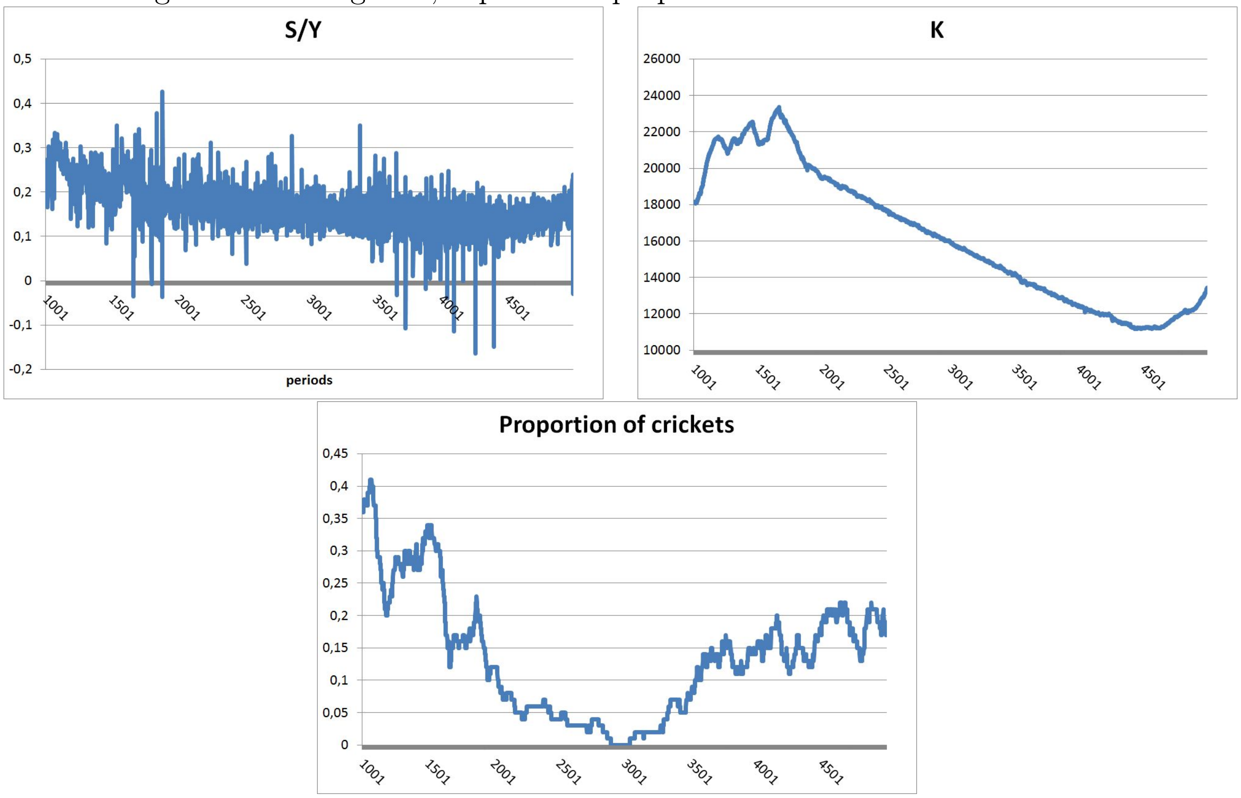


Figure 2: Saving rate, capital and proportion of crickets. Simulation 2.



3.2 Stationarity and ergodicity

Testing for stationarity and ergodicity

Stationarity is a familiar concept for economists. Its essential meaning is that each observation in a time series can be regarded as coming from the same distribution. Stationarity is important because it is necessary for the consistent estimation of the moments of the time series in question.

Ergodicity rarely arises as a problem in economics. In practical terms ergodicity implies that two distant observations on the same time series are almost independent. Economists having one realization of a time series usually do not distinguish between ergodicity and stationarity, as the distinction can be detected only if we have more than one realization at hand. Ergodicity means that all realizations are alike, while the lack of ergodicity implies that different realizations behave qualitatively differently. The simplest example of a non-ergodic stationary process is a Markov-chain with two different absorbing states. In one realization the process ends-up in one of these states, and and in another realization it may end-up in the other state. Clearly the observed trajectories are qualitatively different.

There exist traditional parametric tests to explore the stacionarity of time series. The augmented Dickey–Fuller test (Dickey–Fuller, 1979) employs an autoregressive representation of the time series to check whether they have a unit root, while the Phillips–Perron-test (Phillips–Perron, 1988) use kernel estimators for the nuisance parameters implied by the short-run dynamics of the process. In the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests

(Kwiatkowski et al., 1992) the series is expressed as the sum of deterministic trend, random walk, and stationary error, and it tests the hypothesis that the random walk has zero variance. All the tests described above are parametric in the sense that they need assumptions about the stochastic process generating the tested time series. Any conclusion derived from using parametric tests is valid only if the underlying assumptions are valid. However, modeling an economy by using an agent-based model implies that no explicit mathematical form can explain the behavior of the economy. The impossibility of having an analytical form for the data generating process requires nonparametric tests.

The nonparametric test we use to check the stacionarity, and later the ergodicity, of the time series is an application of the Wald–Wolfowitz test (Wald–Wolfowitz, 1940) by Grazzini (Grazzini, 2012).

Given a time series and a function that is meant to explain the time series (in our case the function of the unweighted mean of the saving rates), the observations should be randomly distributed above and below the function, regardless of the distribution of errors. The Wald–Wolfowitz test (the Runs Test) investigates whether the null hypothesis of randomness can be rejected or not. Given the estimated function, a 1 is assigned to the observations above or on the fitted line, and a 0 to the observations below the fitted line. The statistics used to test the null hypothesis is the number of runs, where a run is defined as "a succession of one or more identical symbols which are followed and preceded by a different symbol or no symbol at all" (Gibbons, 1985). The number of runs, too many or too few runs, may reflect the existence of non-randomness in the sequence. The Runs Test tests the null-hypothesis that a given set of observations is randomly distributed around a given fitted function.

In order to check whether the mean is stationary, we have to check whether the first moment is constant in time. We simulate our model for 101,000 periods for each Υ values (from which the first 1000 periods is neglected as a "warm up" period) and divide the long series into 100 windows. For each window we compute the mean, and check whether the moments of the samples are above or below the mean of the whole time series (except the "warm up period"). Then we perform the Runs Test, described in the previous paragraph. Were the sample moments randomly distributed around the overall moment, we would conclude that the hypothesis of stationarity for the first moment can't be rejected.

To test the ergodic property of a process we use the Runs Test again, but in the original form, proposed by Wald and Wolfowitz (1940). The first steps of the ergodicity test are similar to the stacionarity test: with the help of our agent-based model we simulate again a 101,000 periods long time series, divide the long series into 1000 windows and we compute the mean of each window. The first sample of the test (x_t) is formed by the mean of the 100 sub-samples. As a second step, 100 time series are generated with different random seeds. The number of periods are 2000 in each, and we compute the mean of each time series, which constitutes the second sample of the test (x_t), but we ignore again the first 1000 periods. We merge x_t and y_t and create a set Z which sorts them into ascending order of magnitude. Eventually, a sequence V is created defined as follows: $v_i = 0$ if $z_i \epsilon x_t$ and $v_i = 1$ if $z_i \epsilon y_t$. Given the set V the Runs Test is used as described above. Under the null hypothesis, samples x_t and y_t have the same mean and we can't reject that the process is ergodic.

Results of the tests

Firstly we performed the most commonly used traditional parametric test, the augmented Dickey–Fuller test of stationarity (Table 3) on our saving rate time series with different Υ values. The optimal lag structure of the model was determined according to the Akaike information criteria. The tests with greater Υ values ($\Upsilon = 1$ and $\Upsilon = 0.01$) rejects the null hypothesis of unit root, but the tests with lower Υ values ($\Upsilon = 0.001$, $\Upsilon = 0.005$ and $\Upsilon = 0.0001$) can't reject it. The high number of optimal lags in the differences of the savings rate (not reported) and the high MacKinnon approximate p-value of the tests with greater Υ values are unusual results, and suggest that the traditional parametric augmented Dickey–Fuller test fails to capture the real data generating process underlying the agent-based economy.

Table 3: ADF unit root test of the saving rate with different Υ values

Υ	1	0.01	0.001	0.005	0.0001
MacKinnon approximate p-value	0.0021	0.0000	1.0000	1.0000	1.0000

In order to not rely on any parametric assumption on the data generator process, we turn to the nonparametric Wald–Wolfowitz test of stacionarity, described in the previous subsection (Table 4). Parallel to the p-values of the test we report also the average ratio of the crickets in the model, as the stacionarity property of the economy is obviously related to it. In contrast to the results of the augmented Dickey–Fuller test the results of the nonparametric test indicate that as the selection process intensifies (the Υ value decreases), the economy becomes more stacionary and crickets die out. There is a small,transitional interval between $\Upsilon = 0.025$ and $\Upsilon = 0.009$, however, where interestingly the crickets dominates the ants and the later type dies out.

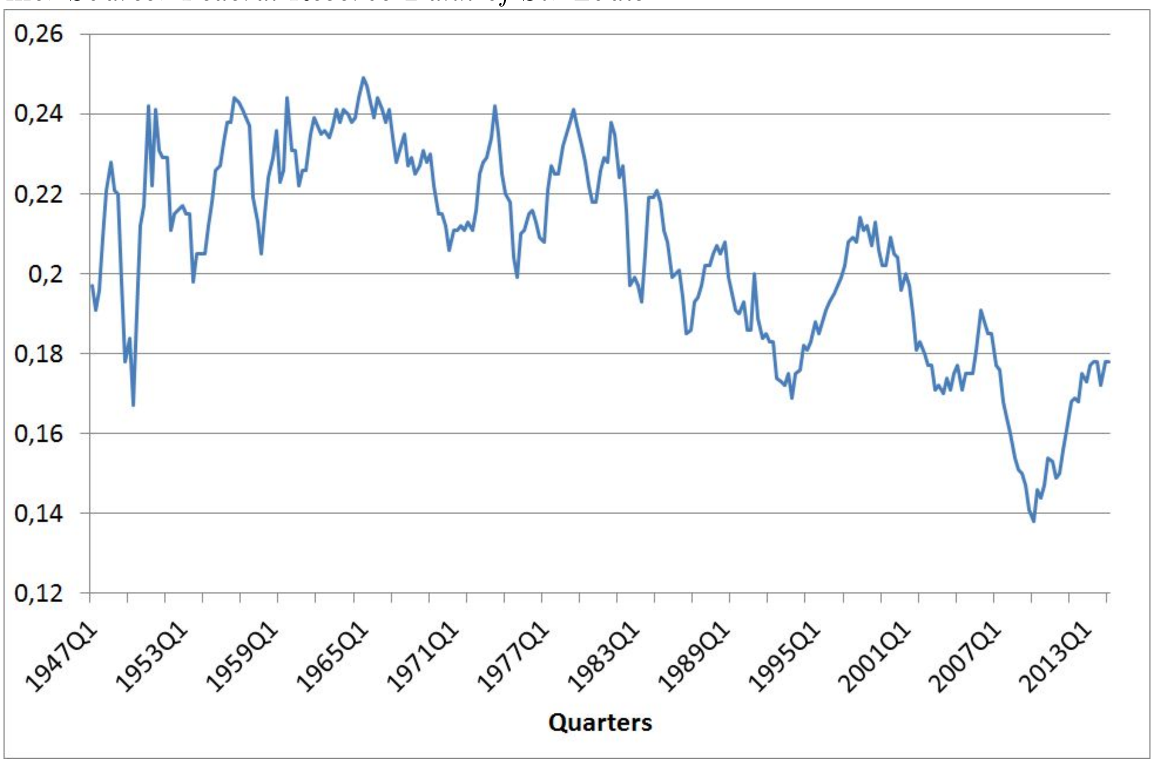
Table 4: Wald–Wolfowitz stacionarity test of the saving rate with different Υ values

Υ	p-value	ratio of crickets
1	0.000	0.5524
0.01	0.000	1
0.005	0.035	0
0.001	0.063	0
0.0001	0.067	0

At the end we perform the nonparametric Wald–Wolfowitz test to check whether the saving rate is ergodic. For parameter values $\Upsilon = 1$ and $\Upsilon = 0.001$ the p-values of the test are practically zeros, so the null hypothesis of ergodicity is flatly rejected.

Are our simulated series similar to observed saving rates? Though we do not have actual savings rate series quite as long as the series we based our tests on, the seasonally adjusted quarterly gross savings rate series in the United States from 1947.01.01 to 2014.07.01 (US. Bureau of Economic Analysis, 2014) contains 271 observations offering a possibility to test the stationarity of the saving rate on empirical data 3. The traditional augmented Dickey–Fuller test cannot reject the null hypothesis of a unit root, even at the significance level of 1%. Although the relatively small number of observations – in contrast to our artificial data in the agent-based model – decrease the power of the nonparametric test, we also performed the Wald-Wolfovitz test, dividing the first 270 observations into 18 windows. In this case the Runs Test gives the same result as the parametric test, as it rejects the null hypothesis of randomness with p=0.001. Comparing visually Figure 3 to Figure 2 or Figure 1 we can see that the simulated savings rate series seem to be much more volatile in the short term than the real ones. Clearly our simple behavioural assumptions must be augmented to enable the model to account for short term volatility as well.

Figure 3: US. Bureau of Economic Analysis, Gross saving as a percentage of gross national income. *Source: Federal Reserve Bank of St. Louis*



What is the content of non-ergodicity? Ergodic processes are such that different realizations of the process are essentially the same, thus one can discover the properties of a process from a single (long enough) realization as well as from an infinity (very large number) of different realizations. Economists usually do not care about ergodicity, saying that we do not, and will never, have different realizations, thus ergodicity is fundamentally untestable in empirical work. For this reason, finding non-ergodic behaviour in an artificial economy (each macroeconomic model is an artificial economy, not only ours) has a philosophical, rather than an empirical message. This message could tell us that long run historical explanations of general economic development are futile. This is a negative conclusion, one that is not very well received usually. Of course, at this moment we cannot give any compelling reason to accept it.

Our finding of non-stationarity is more nuanced, and has potential positive applications even empirically. First of all, it seems that the model becomes stationary when the selection pressure is very high, and crickets are eliminated. But the road towards stationarity appears to be long and winding, as there appears a narrow range of high, but not very high, selection pressure that eliminates ants, rather than crickets. Anyway, why would be stationarity a desirable feature of an economy? Though in general ants have somewhat higher per capita consumption than crickets have, and are less indebted, we have found cases where the total average consumption is higher with many crickets than without them. We do not yet understand the underlying mechanisms, only recognized that there is here an interesting question for further investigation.

3.3 Alternative settings

So far we have shown that saving in the model is arbitrary, in the sense of non-ergodicity, and even of non-stationarity for many parameter settings. However, one may object that probably infinitely many models can be set up with this property, and we should show also that it is not an "anything goes" model. Do our results exhibit a certain consistency at all? We were especially interested in the effects of the learning parameters, as their inclusion is the main novelty of our approach, and the traditional theory does not offer any clue concerning them. In the following we report the results of this sensitivity analysis, where we explored the effects of changing a single parameter with respect to the baseline at each time. For each parameter combination, we ran 10 simulations, and calculated the average of the means.

Table 5: Alternative parameter settings

	average of S/Y	average of C/average of C in baseline	average of the proportion of crickets
Baseline	0.14	1.00	0.47
$\Upsilon = 0.001$	0.16	1.14	0.00
$\Upsilon = 0.1$	0.25	1.22	0.56
$\Upsilon = 10$	0.06	0.72	0.43
$\Upsilon = 1000$	0.11	0.94	0.41
$\lambda = 0.6$	0.16	0.96	0.48
$\lambda = 0.8$	0.15	0.97	0.46
$\lambda = 0.95$	0.36	1.26	0.85
$\lambda = 0.99$	0.19	0.99	0.37
$\rho = 0.01$	0.26	1.21	0.40
$\rho = 0.05$	0.18	1.05	0.46
$\rho = 0.15$	0.21	1.15	0.68
$\rho = 0.2$	0.22	1.12	0.67

The "temperature", Υ is important for our model. This is a key parameter of the adaptation-selection process. A high Υ amounts to little selection pressure, when the survival of behavioural patterns becomes independent of their relative success. Giving Υ a high value achieves this: ants and crickets are present with equal probability in the population in the long run. It is good news for the model, the elimination of selection pressure apparently reduces consumption.

The memory parameter λ also affects the adaptation process. Short memory may be convenient in case of non-stationarity, when the ability to forget may be useful. On the other hand too quick forgetting may make adaptation too short sighted. On the whole λ seems to have weak effects either on consumption or on savings, but the results may indicate that there is an intermediate level of λ that brings about the highest consumption on average.

The behavioural adaptation process is also influenced by the parameter ρ. Its role is not clear-cut. A higher ρ means more frequent competition for survival, which can be interpreted as increased selection pressure, but it may mean that strategies do not have enough time to express their true long run potential. The effect of ρ is almost nil on consumtpion, despite that a higher ρ increases the share of crickets in the economy. Inspecting the whole table shows the relative number of crickets and total consumption do not correlate.

Learning parameters do not seem to have drastic effects on the model, which is good because it shows that the model is relatively robust to a feature which is not well understood. But, as we have shown in the previous paragraph, one learning parameter (the ”temperature”) may have a key role, and may drive the model into qualitatively different behavioral patterns. Our guess is that learning does not affect the model's behaviour within one regime, but may cause regime changes endogenously.

4 Conclusion

This paper documented the results of a first attempt to model ”ant” and ”cricket” type savings behaviour in an agent-based model, where agents follow behavioural rules that require the use of local and global information, and where a selection-adaptation process makes the distribution of behavioural patterns endogenous. Our goal to create a model that exhibits ”arbitrary” aggregate savings has been reached.

Traditional macroeconomics has the preconception that economies can be described as stationary and ergodic stochastic processes. (This includes economies with positive growth rates, where there exists a transformation that stationarize the economy in question.) Our paper has shown that some simple, but plausible, devations from the traditional assumptions generate non-stationary and non-ergodic behaviour generically.

We have to emphasize that non-stationarity does not amount to ”unpredictability”, a fearful word for many economists and the public, it means only that our forecasts tend to be more and more imprecise with the horizon increasing. This is an empirical hypothesis that can be tested, and it is one of our next task we set to ourselves.

How realistic is our model as a ”map” for the macroeconomy? Certainly not less than many highly stylized models explored in the literature. Staying largely outside the mainstream of modern behavioural economics Gigerenzer–Selten (2002) initiated a research program that introduced the fundamental concept of ecological rationality (see also Todd–Gigerenzer, 2007). In this approach economic agents do not maximize any preference function consistently, still do not behave arbitrarily. They have goals, and the survival of behavioural patterns (heuristics) depend on whether these heuristics help people achieve their goals in a given environment. Though there is a large body of psychological research that follows this research program, its application in economics is rather meagre. Our approach can be regarded as a first attempt to apply this research program to saving, and as we have indicated in Section 2, the behavioural assumptions are by and large consistent with previous findings and observations. Our labour and goods markets are copied from the traditional Bewley-type models, so in that respect we did not bring in anything new. The only market where we had to deviate from the standard approach is the credit market. Here we pictured an essentially cautious and conservative lending mechanism, debt levels to GDP (unreported) stay well below the figures currently observed in actual economies. Thus it is very unlikely that we created non-stationarity through modelling the credit market in an unduly irrational way.

As we have mentioned, it came as a surprise that crickets were not driven out of existence for most parameter settings. There were even cases that indicated that better overall long-run performance is accompanied with a higher share of crickets. The trajectories of our model are too complicated to deduce from them an easy answer to the question of "why crickets are useful" for the society as a whole. One may guess that crickets may be substantial savers unintentionally, and ants can overconsume if their long-term view on their own possibilities is too optimistic. The dichotomy between ants and crickets is clearly an oversimplification. In future work we would like to refine these concepts, allowing for more variability in behaviour. For instance, forecasts by ants can be optimistic or more conservative, and different type of crickets may possess different references for their consumption plans. Hopefully, by following the logic of ecological rationality, we will be able to restrict the range of behavioural patterns, and find out which patterns can coexist.

Taking our model to data would be premature. We think that an important message of our approach to empirical work is that new types of data must be discovered if we want to make sense of wild changes in savings rates. People are social animals who do very few things in isolation. Saving-consumption must be affected by social relationships. The effects on savings of the family, of peer groups or of the media have not been in the focus of empirical economic analyses as far as we know. We do not know of any research that addressed the question of heritability: do ants or crickets pass on their behaviour to their descendants or can people adapt by learning about the successes or failures of their parents?

A final word about interventions. Our model describes a private economy without government. We know very well that in most actual economies governments have long influenced savings through running pension systems. One of our most immediate goals is to extend the model with a government or public pension sector. Do different pension systems promote ant or cricket behaviour, and how do they influence capital accumulation? Can a system be devised with the purpose of stationarizing (if it is a goal) the economy, and of providing agents a higher level of consumption?

References

[1] Akerlof, G. and Shiller, R. (2009): Animal Spirits. Princeton University Press.

[2] Aiyagari, S. Rao. 1994. Uninsured Idiosyncratic Risk and Aggregate Saving. Quarterly Journal of Economics, Vol. 109(3), pp. 659–684.

[3] Banerjee, A. V. and Duflo, E. (2011): Poor economics: a radical rethinking of the way to fight global poverty. PublicAffairs.

[4] Bewley, T. F. (1980): The permanent income hypothesis and long-run economic stability. Journal of Economic Theory, 22(3), 377-394.

[5] Brock, W., A. and Hommes, C. (1997): A Rational Route to Randomness. Econometrica, vol. 65, issue 5, 1059-1096

[6] Deissenberg, C., Van Der Hoog, S. and Dawid, H. (2008): EURACE: A massively parallel agent-based model of the European economy. Applied Mathematics and Computation, 204(2), 541-552.

[7] Delli Gatti, D., Desiderio, S., Gaffeo, E., Cirillo, P. and Gallegati, M. (2011): Macroeconomics from the Bottom-up. New economic windows. Springer.

[8] Dickey, D. A. and Fuller, W. A. (1979): Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association 74 (366a): 427-431.

[9] Dobrescu, L. I., Kotlikoff, L. J., Motta, A. (2012): Why aren't developed countries saving? European Economic Review 56.6 1261-1275.

[10] Fudenberg, D. and Levine, D. K. (2006): A dual-self model of impulse control. The American Economic Review, 1449-1476.

[11] Gigerenzer, G. and Selten, R. [eds.] (2002): Bounded rationality: the adaptive toolbox. MIT Press.

[12] Grazzini, J. (2012): Analysis of the Emergent Properties: Stationarity and Ergodicity Journal of Artificial Societies and Social Simulation 15 (2) 7

[13] Hall, R. E. and Mishkin, F. S. (1982) The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households, Econometrica, 50(2), 461-481.

[14] Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., Shin, Y. (1992): Testing the null hypothesis of stationarity against the alternative of a unit root. Journal of Econometrics 54 (1-3): 159-178.

[15] Laibson, D. (1997): Golden Eggs and Hyperbolic Discounting. Quarterly Journal of Economics, 112(2), 443–477.

[16] Madrian, B. C. and Shea, D. F. (2001). The power of suggestion: Inertia in 401 (k) participation and savings behavior. The Quarterly Journal of Economics, 116(4), 1149-1187.

[17] Phillips, P. C. B. and Perron, P. (1988): Testing for a Unit Root in Time Series Regression. Biometrika 75 (2): 335-346.

[18] Piketty, Th. (2014): Capital in the Twenty-first Century. Harvard University Press.

[19] Todd, P., M. and Gigerenzer, G. (2007): Environments That Make Us Smart Ecological Rationality. Current Directions in Psychological Science 16.3 (2007): 167-171.

[20] US. Bureau of Economic Analysis, Gross saving as a percentage of gross national income [W206RC1Q156SBEA], retrieved from FRED, Federal Reserve Bank of St. Louis `https://research.stlouisfed.org/fred2/series/W206RC1Q156SBEA/`, December 8, 2014.