MAGYAR TUDOMÁNYOS AKADÉMIA
Közgazdaság- és Regionális Tudományi Kutatóközpont

MTA KRTK KTI

Centre for Economic and Regional Studies
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK DISCUSSION PAPERS

MT-DP – 2014/1

Optimal Child Allowances with Heterogeneous Fertilities

ANDRÁS SIMONOVITS

INSTITUTE OF ECONOMICS, CENTRE FOR ECONOMIC AND REGIONAL STUDIES,
HUNGARIAN ACADEMY OF SCIENCES BUDAPEST, 2014

Discussion papers
MT-DP – 2014/1

Institute of Economics, Centre for Economic and Regional Studies,
Hungarian Academy of Sciences

Optimal Child Allowances with Heterogeneous Fertilities

Author:

András Simonovits
research advisor
Institute of Economics
Centre for Economic and Regional Studies
Hungarian Academy of Sciences
also Mathematical Institute, Budapest University of Technology,
and Department of Economics, CEU
email:simonovits.andras@krtk.mta.hu

January 2014

ISBN 978-615-5447-09-9
ISSN 1785 377X

Optimal Child Allowances with Heterogeneous Fertilities

András Simonovits

Abstract

A child-allowance system is to raise fertility beyond the individual optimum. The more heterogeneous the population with respect to rearing costs, however, the stronger are the redistribution and polarization.

Keywords: Child allowance, Endogenous fertility, Heterogeneous fertility

JEL classification: J13

Acknowledgement:

I have received generous financial support from Hungarian Science Research Foundation's project K 81482. I express my indebtedness for Zsombor Cseres-Gergely, Róbert Iván Gál and Volker Meier for their useful comments. This usual disclaimer applies.

Optimális gyermektámogatás endogén termékenység esetén

Simonovits András

Összefoglaló

Egy gyermektámogatási rendszer egyik feladata az, hogy a termékenységet az egyéni optimum fölé emelje. Minél heterogénabb azonban a népesség a gyermeknevelési költségekben, annál erősebb az újraelosztás és a polarizáció.

Tárgyszavak: gyermektámogatás, endogén termékenység, heterogén termékenység

JEL kód: J13

Optimal Child Allowances with Heterogeneous Fertilities

by András Simonovits

January 2, 2014

Institute of Economics, CERS, Hungarian Academy of Sciences
also Mathematical Institute of Budapest University of Technology,
and Department of Economics of CEU
Budapest, Budaörsi út 45, Hungary
e-mail: simonovits.andras@krtk.mta.hu, telephone: 36-1-325-5582, fax: 36-1-319-3631

Abstract*

A child-allowance system is to raise fertility beyond the individual optimum. The more heterogeneous the population with respect to rearing costs, however, the stronger the redistribution and polarization.

Keywords: Child allowance, Endogenous fertility, Heterogeneous fertility

JEL Numbers: J13

* I have received generous financial support from Hungarian Science Research Foundation's project K 81482. I express my indebtedness for Zsombor Cseres-Gergely, Róbert Iván Gál and Volker Meier for their useful comments. This usual disclaimer applies.

1. Introduction

Since Becker (1960), a lot of theoretical papers studied the issue of *endogenous fertility*. The basic idea is simple: the rise in the number of children diminishes the parent's consumption through rearing costs and its utility but increases the utility of having children. At the optimum, the two "marginal" effects cancel each other. Reformulating the theory of fertility, Becker and Barro (1988) assumed that it is the utility of the offsprings rather than their mere number that enters the parent's utility function calculated the impact of interest rate, social security etc. on fertility. Other papers considered the positive impact of child allowances on endogenous fertility. (Allowances include everything: from cash transfers through tax credits to free schooling.) Some studies also analyzed the interaction of public pension (and private saving) and child allowances in connection with fertility (e.g. Groezen, Leers and Meijdam (2003), for short, GLM). In the simplest setting, the appropriate increase in fertility through child allowances raises the pension benefits, while the introduction of fertility-related pensions can also raise fertility. Note that somewhat surprisingly, the role of the *heterogeneity* of fertility has mostly been neglected.

Studying a model with general rather than logarithmic utility function, Cremer, Gahvari, and Pestieau (2008) also took into account that the government has no information on cost differences. Applying the methods of mechanism design, they also analyzed the problems of moral hazard (when parents have no full control over their fertility) and of adverse selection (when low-cost type pretend to be high-cost type). They found that under adverse selection, "linking benefits to fertility penalizes high-cost individuals" and this is undesirable.

In order to simplify the analysis while stressing the role of heterogeneity, in the present paper we leave aside the old-age problem, moral hazard and adverse selection. Considering heterogeneity with lower and higher rearing cost types, however, it becomes clear that the positive impact on average fertility is accompanied with a negative one: *polarization of fertility.* (In a twin paper, Simonovits (2013) studied the problem including old-age consumption, saving and pension.)

In our model, there are two types with different rearing costs, and the tax system transfers resources from the higher cost (and lower fertility) type to the lower cost (and higher fertility) type through child allowances. The government sets the tax rate to maximize a paternalistic social welfare function, which sets higher value to children than the parents do. Taking the average rearing cost per child as given, the more heterogeneous the population, probably the lower the socially optimal tax rate and the higher the net transfer. Therefore the system diminishes the high-cost type's endogenous fertility with respect to the case when they lived alone: polarization. We call the reader's attention to our model's limitations: sterility is neglected, the labor supply is fixed (see Fenge and Meier, 2009 for flexible labor supply) and differences in the utility functions, especially in the parameter of relative utility of child are glossed over (for heterogeneous preferences, see Simonovits, 2013). We emphasize again that—like in most but not all models—in our simplistic model, the type-specific per child cost is given. Wage heterogeneity and the role of social norms are also neglected. Taking into account these complications presumably would not change the qualitative message of the paper: beware of heterogeneity and polarization!

This type of fertility polarization has already been discussed in the sociological literature by Hakim (2003, p. 367), supporting rather than opposing polarization: "The government ... should focus instead on policies to support home-centered women, who have the highest fertility rates and can most easily be persuaded to increase their family size". (By the way, she stressed the increasing role of women in deciding on fertility since the spread of contraceptives and derides male demographers underestimating this change.)

2. Model

We start from the seminal paper by GLM (as reformulated in Simonovits (2013)), but leaving out old-age consumption and public pension (and saving), rather we concentrate on the interaction of family allowances and endogenous heterogeneous fertility. There is a single-sex population, where adults give birth to children and pay taxes to finance family allowances.

It is assumed that every adult has the same pre-tax income, normalized to 1. We distinguish two types by their costs of rearing a child, p_i, $i = 1, 2$, $p_1 > p_2 > 0$. (As an extreme case, we shall also consider $p_1 = p_2$.) Everybody can choose her fertility. Denote n_i the type-dependent number of children, or equivalently, fertility, and then the total cost of rearing n_i children is assumed to be equal to $p_i n_i$. Let us introduce the per capita child allowance: $\varphi > 0$, the tax rate which finances the child-allowance: θ, $0 \le \theta < 1$; all are given reals. Note that in reality, fertility is always an integer or its half (in our unisex world), but insisting on it would superfluously complicate the model. To put it another way, both types include a number of subtypes, with different integer or half integer subfertilities.

By definition, type i's own consumption is equal to

$$c_i = 1 - \theta - (p_i - \varphi)n_i, \qquad i = 1, 2. \tag{1}$$

Assume that type i chooses her fertility to maximize an additive logarithmic utility function

$$U(c_i, n_i) = \log c_i + \gamma \log n_i, \qquad i = 1, 2, \tag{2}$$

where γ is the relative individual utility of having children with respect to that of adult consumption. Inserting (1) into (2) yields the reduced utility function:

$$u(n_i) = \log(1 - \theta - (p_i - \varphi)n_i) + \gamma \log n_i, \qquad i = 1, 2. \tag{3}$$

Following GLM (p. 240) we assume that the workers neglect the impact of their decisions on the tax balance described in (5) below, i.e. they take θ and φ as given. Equating the marginal utility of having "one small unit" more children to zero: $u'(n_i) = 0$ yields the necessary and sufficient condition of individual optimum and the optimal fertilities as well:

$$\frac{p_i - \varphi}{1 - \theta - (p_i - \varphi)n_i} = \frac{\gamma}{n_i}, \quad \text{or} \quad n_i(p_1, p_2) = \frac{\gamma(1-\theta)}{(1+\gamma)(p_i - \varphi)}, \qquad i = 1, 2. \tag{4}$$

Note that $n_1, n_2 > 0$ and $0 < p_2 < p_1$ imply $0 < n_1 < n_2$: *higher rearing costs imply lower optimal fertility.* (1) provides then the optimal consumption $c_i(p_1, p_2)$.

For simplicity, we assume that the initial population share of the two types are 50–50. Then the average fertility n and the tax balance are related as follows:

$$n = \frac{n_1 + n_2}{2} \quad \text{and} \quad \theta = \varphi n. \tag{5}$$

The two scalar equations in (4) are coupled through the balance of child allowances (5).

What is the socially optimal value of the tax rate? We shall introduce a paternalistic social welfare function $V(\theta)$ á la Feldstein (1985), but correct the relative utility of a child rather than the discount factor. The individual utility coefficient γ of a child is replaced by another coefficient γ^*, then the socially optimal tax rate can also be determined:

$$V(\theta) = \frac{1}{2}\sum_{i=1}^{2}[\log c_i(p_1, p_2) + \gamma^* \log n_i(p_1, p_2)] \to \max .$$

Though the paternalistic coefficient may be lower than the individual one (like in present-day China), we shall confine our attention to the other, more relevant case, when $\gamma^* \ge \gamma$.

As a prelude to the general discussion, we shall consider the case of homogeneous rearing costs.

Example 1. In the *homogeneous case*, $p_1 = p_2 = p$, the optimal fertilities are the same, the net transfers are zeros, the fertility and the adult consumption equations become explicit:

$$n[p] = \frac{\gamma + \theta}{(1+\gamma)p} \quad \text{and} \quad c[p] = \frac{1-\theta}{1+\gamma}.$$

Note, however, that—at least, in a first-best setting—the tax is totally superfluous here: the adults simply receive back what they have paid in.

The social welfare function is as follows:

$$V(\theta) = \log\frac{1-\theta}{1+\gamma} + \gamma^* \log\frac{\gamma+\theta}{(1+\gamma)p} = \log(1-\theta) - \log(1+\gamma) + \gamma^* \log(\gamma+\theta) - \gamma^* \log((1+\gamma)p).$$

Condition $V'(\theta) = 0$ yields the socially optimal tax rate, which is independent of p:

$$\theta^* = \frac{\gamma^* - \gamma}{1+\gamma^*}. \tag{6}$$

Returning to the heterogeneous case, it can be shown that in a large domain in the (p_1, p_2)-plane, there are unique solutions to the pair of nonlinear equations (4)–(5). More precisely,

Lemma 1. *a) For $\theta > 0$, the average fertility n is the larger positive root of the quadratic equation*

$$p_1 p_2 n^2 - (\theta + \delta)(p_1 + p_2)n + \theta(\theta + 2\delta) = 0, \quad \text{where} \quad \delta = \frac{(1-\theta)\gamma}{2(1+\gamma)}. \tag{7}$$

b) Then the type-specific optimal fertilities $n_1(p_1, p_2)$ and $n_2(p_1, p_2)$ can be determined from (4)–(5).

Remark. In the excluded case of $\theta = 0$, $n_1(p_1, p_2)$ and $n_2(p_1, p_2)$ are those of the corresponding homogeneous ones:

$$n[p_i] = \frac{\gamma}{(1+\gamma)p_i}$$

and the average fertility is determined by their average.

Proof. Insert (5b) into (4):

$$n_i = \frac{\gamma(1-\theta)n}{(1+\gamma)(p_i n - \theta)}, \qquad i = 1, 2. \tag{8}$$

Add up (8-1) and (8-2) and divide the sum by $2n$:

$$1 = \frac{\gamma(1-\theta)}{2(1+\gamma)} \left[\frac{1}{p_1 n - \theta} + \frac{1}{p_2 n - \theta} \right], \quad \text{where} \quad 0 < \theta < p_2 n. \tag{9}$$

After rearrangement of (9), (7) obtains. The lower root of (7) lies in the interval $\theta/p_1 < n < \theta/p_2$, to be excluded. ∎

In the forthcoming proofs one can rely on the explicit formula for n derived from (7) but it is not needed, 'geometric' ideas in (9) yield simpler proofs. First we shall prove the following theorem.

Theorem 1. *Fix the tax rate $\theta > 0$. The average fertility of the heterogeneous population is higher/lower than that of the homogeneous population with higher/lower rearing cost, while the type-specific fertility is lower/higher:*

$$n_1(p_1, p_2) < n[p_1] < n(p_1, p_2) < n[p_2] < n_2(p_1, p_2), \quad \text{where} \quad 0 < p_2 < p_1.$$

Remark. The best way to understand the situation is to look at the *net transfer* received by type i:

$$t_i = \varphi n_i - \theta, \qquad i = 1, 2.$$

Obviously, $t_1 = 0.5\varphi(n_1 - n_2) = -t_2 < 0 < t_2$.

Proof. From (9). ∎

To highlight the role of heterogeneous costs, fix the average rearing cost: $p_1 + p_2 = 2p$, and change the difference between the two rearing costs: i.e. $2\pi = p_1 - p_2$, i.e. $p_1 = p + \pi$ and $p_2 = p - \pi$, where π is called the *degree of (cost) heterogeneity.* We prove now two results.

Theorem 2. *Fix the tax rate $\theta > 0$ and the average rearing cost p. The average fertility $n(p + \pi, p - \pi)$ is an increasing function of the degree of heterogeneity π.*

Proof. From (7). ∎

Theorem 3. *The higher the tax rate, the higher the average fertility but also the divergence between the high and the low fertilities.*

Proof. From (7). ∎

What happens if the government adjusts the socially optimal tax rate to the varying heterogeneity?

Conjecture. *The higher the degree of heterogeneity π, the lower the socially optimal tax rate $\theta(p+\pi, p-\pi)$ and the optimal fertility of the less fertile type, $n_1(p+\pi, p-\pi)$.*

Remark. This polarization limits the attractiveness of pronatalistic policies, where the paternalistic parameter is greater than the individual one: $\gamma^* > \gamma$.

3. Numerical illustrations

Since the formulas do not inform us about the importance of the quantitative effects, we shall present some numerical experience. Start from the homogeneous case and set the relative child utility parameter: $\gamma = 0.4$, rearing cost: $p = 0.4$ and no taxation: $\theta = 0$. Then the optimal fertility is 0.714, well below the stationary case. Experimenting with the paternalistic coefficient $\gamma^* = 0.8$, yields the socially optimal tax rate: $\theta^* = 0.222$ [(6)], implying an expanding population with $n = 1.111$.

Moving to the heterogeneous rearing costs: first we fix them as $p_1 = 0.48$ and $p_2 = 0.32$ with average rearing cost $p = 0.4$. To study the impact of the paternalistic coefficient on the optimum, change γ^* from 0.4 to 1 step-by-step. To use Theorems 2 and 3, for the time being, we apply the socially optimal tax rate derived for the average homogeneous population [(6)] to the heterogeneous population as well. For example, in the last but one line of Table 1, $\gamma^* = 0.8$ yields the homogeneous socially optimal tax rate $\theta^* = 0.222$. Then in separated homogeneous populations, the fertilities would be $n[0.48] = 0.926$ vs. $n[0.32] = 1.389$, while in the heterogeneous population, the optimal fertilities are $n_1(0.48, 0.32) = 0.754$ and $n_2(0.48, 0.32) = 1.647$, polarization.

Table 1. *Dependence of fertilities on paternalism: homogeneity vs. heterogeneity*

Paternalistic coefficient γ^*	Homogeneous optimal tax rate θ^*	Homogeneous fertility Low $n[p_1]$	Homogeneous fertility High $n[p_2]$	Heterogeneous fertility Low $n_1(p_1, p_2)$	Heterogeneous fertility High $n_2(p_1, p_2)$
0.4	0.000	0.595	0.893	0.595	0.893
0.6	0.125	0.781	1.172	0.705	1.286
0.8	0.222	0.926	1.389	0.754	1.647
1.0	0.300	1.042	1.563	0.766	1.976

Remark. $p = 0.4$.

Next we take into account that the socially optimal tax rate in a heterogeneous population is different from that of the average homogeneous one, according to Conjecture, lower. First we repeat the calculations of Table 1 with varying paternalistic coefficient. In fact, the higher the paternalistic coefficient, the greater the reduction of the heterogeneous population's optimal tax rate from the homogeneous population's. For example, in the last row in Table 2, $\theta(p_1, p_2)$ drops from 0.3 to 0.248. Similarly, the fertilities rise slightly slower with γ^* than in the homogeneous case.

Table 2. *Dependence of optimal tax rate and fertilities on paternalism*

Paternalistic coefficient γ^*	Optimal tax rate $\theta(p_1, p_2)$	Fertility: Low $n_1(p_1, p_2)$	Fertility: High $n_2(p_1, p_2)$
0.4	0.000	0.595	0.893
0.6	0.113	0.697	1.245
0.8	0.191	0.742	1.525
1.0	0.248	0.760	1.752

Remark. $p = 0.4$ and $\pi = 0.08$.

Finally we fix γ^* at 0.8 and let the degree of cost heterogeneity vary. Note that as π rises from 0 to 0.12 in Table 3, the socially optimal tax rate drops from 0.222 to 0.171. As a result, the lower optimal fertility sinks from 1.111 to 0.63 while the higher optimal fertility jumps from 1.111 to 1.743, lifting the average fertility from 1.111 to 1.187. The last column shows the rise of the transfer paid by the low fertility type to the other, from 0 to 0.08.

Table 3. *Dependence of optimal tax rate and fertilities on the degree of heterogeneity*

Degree of heterogeneity π	Optimal tax rate $\theta(p_1, p_2)$	Fertility: Low $n_1(p_1, p_2)$	Fertility: High $n_2(p_1, p_2)$	Fertility: Average $n(p_1, p_2)$	Transfer paid $t(p_1, p_2)$
0.00	0.222	1.111	1.111	1.111	0.000
0.04	0.213	0.903	1.329	1.116	0.041
0.08	0.191	0.742	1.525	1.134	0.066
0.12	0.171	0.630	1.743	1.187	0.080

Remark. $\gamma^* = 0.8$, $p = 0.4$, $p_1 = p + \pi$ and $p_2 = p - \pi$.

4. Conclusions

We have analyzed a very simple model of heterogeneous and endogenous fertility. We have obtained the following results. The presence of child allowances raises the fertility of the type with lower private rearing cost and diminishes the fertility of the type with higher rearing cost with respect to a homogeneous population of the given low and high rearing costs, while increasing the average fertility with respect to the average of two homogeneous populations. This limits the scope of acceptable family allowance schemes to raise fertility.

References

Becker, G. S. (1960): "An Economic Analysis of Fertility ", *Demographic and Economic Change in Developed Countries*, eds: R. Esterline, Princeton, Princeton University Press.

Becker, G. S. and Barro, R. (1988): "A Reformulation of the Economic Theory of Fertility", *Quarterly Journal of Economics 103*, 1–25.

Cremer, H., Gahvari, F. and Pestieau, P. (2008): "Pensions with Heterogeneous Agents and Endogenous Fertility", *Journal of Population Economics 21*, 961–981.

Feldstein, M. S. (1985): "The Optimal Level of Social Benefits", *Quarterly Journal of Economics 100*, 302–320.

Fenge, R. and Meier, R. (2009): "Are Family Allowances and Fertility Related Pensions Perfect Substitutes?", *International Taxation and Public Finance 16*, 137–163.

Hakim, C. (2003): "A New Approach to Explaining Fertility Patterns: Preference Theory", *Population and Development Review 29*, 349–374.

van Groezen, B.; Leers, Th. and Meijdam, L. (2003): "Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins", *Journal of Public Economics 87*, 233–251.

Simonovits, A. (2013): Savings, Child Support, Pensions and Endogenous (and Heterogeneous) Fertility, IE-CERS-HAS Working Paper 35.