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Socially optimal cap on pension contributions

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Socially optimal cap on pension contributions

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Abstract

In our model, the government operates a mandatory proportional pension system to substitute for the low life-cycle savings of the lower-paid myopes, while maintaining the incentives of the higher-paid, far-sighted in contributing to the system. The introduction of an appropriate cap on pension contribution (or its base) raises the optimal contribution rate, helping more the myopes and making more room for the saving of high-paid far-sighted workers. The cap has quite a weak impact on the social welfare in a relatively wide interval but the maximal welfare is higher than the capless welfare by 0.3–4.5%.

Keywords: pensions, contribution rate, contribution cap, maximum for taxable earnings

JEL classification: H53, H24

A nyugdíjjárulék-plafon társadalmi optimuma

Simonovits András

Összefoglaló

Modellünkben a kormányzat kötelező arányos nyugdíjrendszert működtet, amely helyettesíti a rövidlátó és alacsonyabb keresetűek elégtelen időskori megtakarítását, míg fenntartja az előre látó magasabb keresetűek érdekeltségét, hogy részt vegyen a rendszerben. Megfelelő járulékplafon (vagy járulékalap-plafon) bevezetése növeli az optimális járulékkulcsot, jobban segítve a rövidlátókat és nagyobb helyet hagyva az előre látóknak. A plafon egy tág tartományban alig hat a társadalmi jólétre, de a maximális jólét 0,3–4,5%-kal nagyobb, mint a plafonmentes rendszerben.

Tárgyszavak: nyugdíjak, járulékkulcs, járulékplafon, az adózó jövedelmek maximuma

JEL kód: H53, H24

Socially optimal cap on pension contributions

by

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October 22, 2013

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Abstract*

In our model, the government operates a mandatory proportional pension system to substitute for the low life-cycle savings of the lower-paid myopes, while maintaining the incentives of the higher-paid, far-sighted in contributing to the system. The introduction of an appropriate cap on pension contribution (or its base) raises the optimal contribution rate, helping more the myopes and making more room for the saving of high-paid far-sighted workers. The cap has quite a weak impact on the social welfare in a relatively wide interval but the maximal welfare is higher than the capless welfare by 0.3–4.5%.

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1. Introduction

Since the publication of World Bank (1994), the debate on privatization and prefunding of the unfunded public pension systems has focused attention on the socially optimal choice of the *contribution rates* to the arising two pillars and of the *degree of redistribution* achieved in the public pillar. In terms of gross wage, the public contribution rate varies across time and space dramatically: while the US social security contribution rate is as low as 12.4%, the Hungarian rate is as high as 34%. Such differences are due to differences in the breakdown to the employer's and the employee's rates, the dependency rates (the ratios of pensioners' number to the workers'), the replacement rates (the ratios of average benefits to averages wages) and the share of the private pillar in the total pension system. Similar differences exist among the degrees of redistribution in the public pillars (cf. Disney, 2004): in the Netherlands and Great Britain, the public pensions are flat or flat-rate, respectively; in Germany and most other European continental countries, the benefits are proportional to lifetime contributions or wages. In between, there are progressive systems with various degrees of redistribution: strong in the USA and weak in Hungary. Note the interaction between the degree of redistribution and the contribution rate: the more progressive the system, the lower is the contribution rate.

Much less attention has been paid to the socially optimal choice of the *contribution (base) cap*, officially called the maximum for taxable earnings.[1] Such a cap (or ceiling) implies an upper limit on the mandatory pension contributions as well as on the future benefits. While the contribution rate and the degree of progressitivity affect every individual, a well-designed cap only influences the higher-paid; nevertheless, the cap also deserves attention. We give only two historical examples to illustrate poor design of the cap. (i) In the 1950s, in Great Britain the cap was fixed at the minimum wage (probably about half the average wage), degrading the usually earning-related contributions into flat ones. It was only realized much later that such a solution reduces excessively the flat-rate benefit and then was replaced by a much higher cap, making the rise of the benefit and the redistribution possible within the public system. (ii) In Hungary, the ratio of the cap to the average gross wage sank from 3.3 to 1.6 between 1992 and 1996 just to grow from 1.6 to 3.1 between 1997 and 2005. From 2013, the cap is removed.[2]

Like other experts, Barr and Diamond (2008, p. 63) mention two roles for the cap. (i) Under certain conditions, the cap acts as a hidden personal income tax. For example, in some countries, the cap only applies to the employee's contribution and the proportional benefit, therefore from an economic point of view the employer's contribution above the cap is a pure personal income tax.[3] Similarly, for progressive benefits, the cap limits the otherwise unbounded redistribution from the higher-paid to the lower-

[1] For a summary statistics in terms of per capita GDP, see Table 1 in Valdés-Prieto and Schwarzhaupt (2011). The cap-to-gross wage-ratio is around 1.2 in Sweden and 1.8 in Germany. The usual statistics give the contribution rate as well as the cap's relative value with respect to the gross wage rather than to the total wage. Note that these rates depend on the economically meaningless break-up of contribution to employee's and employer's contributions.

[2] Note also that the cap on the contributions to the tax-favored *voluntary* pension system is much lower and it interacts with the mandatory one; Simonovits (2011).

[3] In Hungary, the former contribution rate is 10% of the gross wage, while the latter is 24%. Until

paid.[4] (ii) The government has no mandate to force high old-age consumption on high-earners and removing the cap would further increase the perverse redistribution from the poor to the rich caused by the strong correlation between lifetime earning and life expectancy.

To present a third role of the cap in the mandatory proportional public system, we apply two widely accepted idealizations: (a) the higher the wage, the higher is the discount factor and (b) any dollar saved privately rather than in a public pension system raises the old-age consumption. Then the third role with a dual task can be formulated as follows: *the capped pension contribution ensures sufficient mandatory savings for the low-earning short-sighted but leaves sufficient room for the more efficient voluntary savings for the high-earning far-sighted.* For practical reasons, it would be politically impossible to legislate different contribution rates for different types, therefore as a second-best solution, the government may introduce a cap on the contributions.[5] For any fixed contribution rate, the cap reduces the *effective* contribution rate (i.e. the ratio of contribution to earning) of workers earning above the cap, making room for more private savings.

We are aware that our idealizations are only approximately valid. Ad a) In addition to the pairs described above, there are low-paid who are far-sighted and there are high-paid who are short-sighted. Nevertheless, the correlation between discount factors and wages appears to be strongly positive, therefore the atypical combinations can be neglected in a first approximation. (Becker and Mulligan (1997) provide a rich theory on the endogenous determination of time preferences.) Ad b) In fact, the voluntary (private) saving may also be less rather than more efficient than the mandatory (public) one, see Barr and Diamond (2008, Chapter 6). We only use idealization (b) to allow for certain incentive problems—e.g. flexible labor supply—occurring in the mandatory pension system.

Correspondingly we consider a very simple model, where workers only differ by their wages and discount factors but have the same age when they start working, retire or die. Using a *paternalistic* utilitarian social welfare function, where individual discounting is eliminated, a socially optimal pension system successfully combines the dual task.[6]

For the sake of simplicity, we confine our analysis to *proportional* (contributive or defined contribution) pension systems (and write contribution rather than pay-roll tax), where the benefit is proportional to the net covered wage. Such systems have much stronger incentives to supply labor and report earnings than the progressive ones, therefore these problems can be neglected.

As a starting point, we start the analysis with the capless system (cf. Simonovits, 2013). If everybody is totally myopic, then the socially optimal contribution rate is

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2013, the uncapped contributions alone provided 6% of the total pension contributions and would have provided 10% of the *de facto* personal income tax.

4 In the US, the progressivity of the Social Security disappears above the cap, the latter being about 2.5 times the average wage.

5 One might object that in a lot of countries, the self-employed have a significantly lower contribution rate than the employees have. Note, however, that this differentiation is connected more to the difficulty of auditing the former rather than the weaker rationality of the latter.

6 Note also that in an imaginary world, where the discount factor is a decreasing rather than an increasing function of the wage, the cap should be replaced by a floor.

obviously equal to the *maximal contribution rate*—equalizing the young- and old-age consumptions. Having accepted more or less far-sighted workers into the model, this equality was shown to be approximately valid by numerical illustrations for a relatively wide part of the parameter space. We shortly discuss its degenerated, single-type version: the socially optimal contribution rate is either zero (for a sufficiently far-sighted population) or maximal (for an insufficiently far-sighted population).

Introducing the cap, the social welfare function has two rather than one independent variables: the contribution rate and the cap. To obtain sharp results, we introduce a continuous-type model. We conjecture and show by numerical calculations that *having a cap, the socially optimal replacement rate is higher than without a cap.* To be more concrete, we shall work with the Pareto-2 distribution (where the minimum wage is just half the average wage and the wage variance is infinite) because it is analytically very convenient and approximates the distribution of high wages quite well (cf. Diamond and Saez, 2011). Furthermore, a sufficiently myopic population (represented by the discount factor-wage elasticity $\xi = 0.2$ in (15)) and a moderate annual real interest rate (3.7%), the socially optimal contribution rate is close to the maximal one (0.333) and the socially optimal cap is equal to the average wage, covering 71% of the workers (Tables 2 and 3). By diminishing the discount factor–wage curve or using alternative utility functions, the socially optimal cap can rise even higher (cf. Simonovits, 2012).

Note, however, that as is usual, the utilitarian social welfare hardly changes with the cap in the relevant interval. There is a wide interval $[0.75, 2]$, where—slightly raising the cap—the marginal gains of the lower-paid and short-sighted workers are approximately canceled by the marginal losses of the higher-paid and far-sighted workers (Table 3). For other parameter vectors, the *maximal efficiency gain* due to introducing the cap is about 0.3–4.5% with respect to the capless case. In other words: one has to increase proportionally all wages by 0.3–4.5% in a capless system to have the same social welfare as in the optimal capped system with the original wages.[7]

It is time to discuss the related literature. Evaluating various mandatory pension systems, Feldstein (1985) and (1987) emphasized the outstanding role of different discount factors in the accumulation of voluntary life-cycle savings and used a utilitarian paternalistic social welfare function for correcting individual myopia (originally suggested by Samuelson (1975)). As a short-cut, he neglected wage differences. In Feldstein (1985), the optimality of the maximal contribution rate was limited to extremely myope population. Otherwise, assuming very high real interest rates (around 11% per year) and workers are not excessively myopic, he established that having no pension is superior to the flat pension.

In addition to the determination of the socially optimal contribution rate, Feldstein (1987) compared the means-tested benefits to the flat benefits. He found that the optimal means-tested system is typically welfare superior to the flat one, because it limits the benefit provision to the myopes and leaves room for more efficient private savings. (Note the similarity between the limits in Feldstein's means-tested system and our capped system.)

Generalizing Feldstein (1985), Cremer, De Donder, Maldonaldo and Pestieau (2008)

[7] To have an idea about these numbers, we cite two similar "German" numbers from the calibrated mode of Fehr (2000, p. 436, Table 6): raising the retirement age from 60 to 62 years or reducing the replacement rate from 70 to 64% would increase the long-run welfare by 1 and 1.6%, respectively.

investigated the socially optimal contribution rate, the progressive linear system and flexible labor supply with continuous wage distribution. They confined attention to total myopia and total farsightedness, while assumed that this distinction is independent of earnings. They also assumed that the public and the private pension systems are equally efficient. In their socially optimal linear pension system, the level of benefits and the link between wage and benefits are typically increasing with the share of myopes. (Cremer and Pestieau (2011) surveyed the related literature!)

Recently, Valdés-Prieto and Schwarzhaupt (2011) analyzed the issue of the optimal coercion including the choice of cap. They considered a larger set of errors of judgments on remaining life span and future needs; furthermore, modeled various pension systems. They put the value of the optimal cap near the 80th percentile of the earning distribution.

Further research is needed to generalize the results for wider setting. We have already alluded to the lack of redistribution, of flexible labor supply and of the progressivity of the social welfare function in the present study. Deterministic and stochastic changes in the relative earnings position during one's lifetime may also be important. The dependence of type-specific life expectancy and private saving's efficiency on earning is also important in practice, and may call for progressive benefits.

Note that models of this type neglect real-life *dynamic* complications like growth, inflation and population aging. Therefore, in our static model, we cannot consider the dynamic problem of carving out a private pillar from a public one (Fehr, 2000; Diamond and Orszag, 2004 vs. Feldstein, 2005). Neither can we evaluate proposals like Diamond and Orszag (2004) who would phase-in a 3% tax on incomes above the cap to reduce the long-term imbalance of the US Social Security. The problem of time-inconsistency in private savings (e.g. Laibson, 1997) is also out of scope, though the paternalistic government makes up this omission. We do not follow Docquier (2002), who replaced the representative generation's social welfare function by a proper one, comprising the life-time utilities of all present and future generations.

The structure of the remainder is as follows. Section 2 presents the model, Section 3 displays numerical illustrations. Finally, Section 4 concludes.

2. Model

First we shall outline a framework, and then specify the two versions of the model.

Framework

We consider a very simple pension model, where workers only differ in wages and discount factors but do not vary with age. A type can be described by his total wage w and his discount factor δ, the joint distribution function of (w, δ) denoted by F and the corresponding expectations by $\mathbf{E}$. To replace the two- by a one-dimensional distribution, we assume that the discount factor is an increasing function of the wage: $\delta = \delta(w)$. The average wage is normalized as $\mathbf{E}w = 1$. Workers pay contributions τw up to $\tau \bar{w}$, where $0 < \tau < 1$ is the *contribution rate* to the mandatory pension system and $\bar{w}$ is the *wage ceiling or cap*. Neglecting personal income taxes, his *covered total wage and net* *wage* are respectively

$$\hat{w} = \min(w, \bar{w}) \quad \text{and} \quad v = w - \tau\hat{w}. \tag{1}$$

Calculating pension benefits, a related variable is used, the *net covered earning*:

$$\hat{v} = (1-\tau)\hat{w}. \tag{2}$$

Note that for wages lower than or equal to the cap: $w \le \bar{w}$, the cap's existence is indifferent; for wages higher than the cap: $w > \bar{w}$, the cap ensures that the *effective* contribution rate is lower than originally: $\tilde{\tau} = \tau\bar{w}/w < \tau$. The function of the cap is as follows: lower-paid myopic workers can be locked into a pension system with a high contribution rate, but higher-paid far-sighted workers pay a lower effective rate. At the same time, there is no reason to set the cap below the minimum wage, since by replacing the contribution rate τ by its multiple of $\bar{w}/w_m < 1$, namely $\tau' = \tau\bar{w}/w_m$, $(\tau, \bar{w})$ is equivalent to (τ', w_m).

By assumption, every pension benefit is proportional (to the net covered wage):

$$b = \beta\hat{v}, \qquad \beta > 0. \tag{3}$$

Everybody works for a unit period, and everybody spends a shorter (or equal) period in retirement with a common length μ, $0 < \mu \le 1$. Hence the pension balance is simply

$$\mu\beta\mathbf{E}\hat{v} = \tau\mathbf{E}\hat{w}, \quad \text{i.e.} \quad \mu\beta = \tau/(1-\tau). \tag{4}$$

Since the two periods' lengths are different, we use *intensities* in (5) below, i.e. quantities per a unit time period even if it is not always mentioned.

In addition to paying mandatory pension contributions, workers can also privately save for old-age: $s \ge 0$. Denoting the compound interest factor by $\rho \ge 1$, the intensity of the decumulated saving is approximately $\mu^{-1}\rho s$.

We can now describe the young and old-age consumption (intensities):

$$c = v - s \quad \text{and} \quad d = \beta\hat{v} + \mu^{-1}\rho s. \tag{5}$$

To determine the individually optimal savings, we introduce a discounted lifetime utility function. To obtain analytical formulas, we adopt and modify Feldstein's Cobb–Douglas utility function by scaling the utility of old-age consumption:

$$U(w, \delta, c, d) = \log c + \mu\delta \log d, \tag{6}$$

where δ is the discount factor, $0 \le \delta \le 1$. We must distinguish two cases: either 1) nonnegative saving intention or 2) negative saving intention. Inserting (5) into (6), the optimum condition of type 1 is $1/c = \delta\rho/d$. Hence equation

$$\delta\rho v - \delta\rho s = \beta\hat{v} + \mu^{-1}\rho s$$

yields the optimal saving intention:

$$s^i = \frac{\delta\rho v - \beta\hat{v}}{(\delta + \mu^{-1})\rho} \tag{7}$$

which materializes if it is positive or zero.

Hence the type-1 optimal consumption pair are

$$c_1^{\mathrm{o}}(\tau, \bar{w}, w, \delta) = \frac{\mu^{-1}\rho v + \beta\hat{v}}{(\delta + \mu^{-1})\rho} \quad \text{and} \quad d_1^{\mathrm{o}}(\tau, \bar{w}, w, \delta) = \delta\rho c_1^{\mathrm{o}}(\tau, \bar{w}, w, \delta). \tag{8}$$

If $s^i < 0$, then $s_2^{\mathrm{o}} = 0$, hence $c_2^{\mathrm{o}} = \hat{v}$ and $d_2^{\mathrm{o}} = \beta\hat{v}$.

Next we study the single- and the continuous-type models.

The single-type model

In the single-type model, every individual has the same wage w and the same discount factor δ. Here the cap has no function but the model is a useful ground for preparation.

Using formulas (4) and (7) above, we can deduce the optimal saving function

$$s^{\mathrm{o}} = \frac{[\delta\rho(1-\tau) - \mu^{-1}\tau]w}{(\delta+\mu^{-1})\rho} \ge 0 \quad \text{if} \quad 0 < \tau \le \tau_\delta = \frac{\delta\rho}{\delta\rho + \mu^{-1}},$$

where τ_δ is the δ-maximal contribution rate which allows for nonnegative saving intention. To avoid trivial cases with no pension at all, we assume $\delta\rho < 1$.

To determine the paternalistically optimal τ (where $\delta = 1$), we need to separate the two cases.

For the 1-type case, the paternalistic utility function is

$$u_1(\tau, w, \delta) = \log c_1^{\mathrm{o}}(\tau, w, \delta) + \mu \log[\delta\rho c_1^{\mathrm{o}}(\tau, w, \delta)]. \tag{9}$$

It is easy to show that $u_1(\cdot, w, \delta)$ is a decreasing function of the contribution rate. Indeed, inserting (8) into (9) and dropping the constant terms, the relevant function is equal to

$$g_1(\tau) = \log(\rho - (\rho - 1)\tau)$$

which is decreasing in the interval $[0, \tau_\delta]$.

For the 2-type case, the paternalistic utility function is

$$u_2(\tau, w) = \log((1-\tau)w) + \mu \log[\mu^{-1}\tau w].$$

It is easy to see that $u_2(\cdot, w)$ is an increasing function of the contribution rate in the interval $\tau_\delta < \tau \le \bar{\tau}$, where

$$\bar{\tau} = \frac{1}{1+\mu^{-1}} \tag{10}$$

is the *maximal contribution rate*, assuring unit net replacement rate. Indeed, dropping the constant terms,

$$g_2(\tau) = \log(1-\tau) + \mu \log \tau.$$

Taking its derivative,

$$g_2'(\tau) = -\frac{1}{1-\tau} + \frac{\mu}{\tau}$$

is positive in the interval $[\tau_\delta, \bar{\tau})$ and is negative in the interval $(\bar{\tau}, 1)$.

Therefore there are two extreme candidates for the optimum: $\tau = 0$ and $\tau = \bar{\tau}$. To choose between them, we introduce the *critical discount factor* δ^* which equalizes the two paternalistic utilities, satisfying the implicit equation

$$u_1(0, w, \delta^*) = u_2(\bar{\tau}, w). \tag{11}$$

Note that if the private saving is efficient, i.e. $\rho > 1$, then $(0 <)\delta^* < 1$.

We have proved the following theorem.

Theorem 1. *In the single-type model with efficient private savings, no cap is needed and the optimal contribution rate is maximal [(10)] for subcritical discount factors and zero for supercritical discount factors; furthermore it is equal to the maximum and zero for the critical discount factor:*

$$\tau^* = \begin{cases} \bar{\tau} & \text{if } 0 \le \delta < \delta^*; \\ 0,\ \bar{\tau} & \text{if } \delta = \delta^*; \\ 0 & \text{if } \delta^* < \delta \le 1. \end{cases}$$

Remark. In the limit, when the private saving is as efficient as the mandatory pension system, i.e. $\rho = 1$, then $\delta^* = 1$. By $\delta\rho < 1$, now $\delta < 1$, in the optimum, there is no place for private savings. From now on, $\rho > 1$ is always assumed.

We can now formulate a trivial

Corollary. *For multi-type models, a) if all discount factors are lower than the critical one, then the Pareto-optimal contribution rate and the corresponding cap are equal to the maxima; b) if all discount factors are higher than the critical one, then the Pareto-optimal contribution rate is zero, while the corresponding cap is indifferent.*

Note that both cases cover quasi-single-type populations.

Continuous distributions of types

Next we turn to the more realistic and genuinely multi-type model, namely with *continuous* wage and discount factor distributions. (Following Feldstein (1985) and (1987), in an earlier version, Simonovits (2012) discussed two- and three-type models as well, while working with discounted Leontief utility functions.)

Let the discount factor $\delta(w)$ be a monotone increasing function of the wage w in the interval $[w_m, w_M]$ with $0 < w_m < w_M \le \infty$ and be also continuous. Let the wage distribution have a positive density function f and a corresponding distribution function

$$F(w) = \int_{w_m}^{w} f(\omega)\, d\omega$$

with $F(w_m) = 0$ and $F(w_M) = 1$. Furthermore, $\delta_m = \delta(w_m)$ and $\delta_M = \delta(w_M)$, $0 \le \delta_m < \delta_M \le 1$. To avoid triviality (when either there is no pension system or no private saving) we assume that the lowest earner does not save enough and the highest earner would save enough in the absence of pension system. In formula

$$\rho\delta(w_m) < 1 < \rho\delta(w_M).$$

Before defining the social welfare function, we reformulate the indirect paternalistic utility function, dropping the notational distinction between the two regimes and suppressing the third argument $\delta(w)$:

$$u[\tau, \bar{w}, w] = \log c[\tau, \bar{w}, w] + \mu \log d[\tau, \bar{w}, w].$$

If it were possible to apply a wage-dependent contribution rate $\tau(w)$, then by Theorem 1, the Pareto-optimum would be trivial: to request maximal participation for subcritical earners ($\delta(w) < \delta^*$) and relieve participation for supercritical earners ($\delta(w) > \delta^*$), where criticality is defined already by (11).

We now turn realistically to a uniform contribution rate τ. The government chooses this parameter and the contribution base cap $\bar{w}$ to maximize the expected value of the *paternalistic*, undiscounted indirect utility functions, i.e.

$$V(\tau, \bar{w}) = \mathbf{E}u[\tau, \bar{w}, w] \to \max .$$

The socially optimal cap will frequently be an interior point in (w_m, w_M).

We shall need the notion of critical wage $w^* = w^*(\tau, \bar{w})$, which makes the saving intention 0. Here the two regions' utility functions are equal:

$$u_1[\tau, \bar{w}, w^*] = u_2[\tau, \bar{w}, w^*]$$

If there is no such a wage, i.e. for a high enough contribution rate, the saving intention is negative for any wage, then by definition, the critical wage is identified with the maximal wage.

We can now reformulate the social welfare function as follows:

$$V(\tau, \bar{w}) = \int_{w_m}^{w^*} u_2[\tau, \bar{w}, w] f(w)\, dw + \int_{w^*}^{w_M} u_1[\tau, \bar{w}, w] f(w)\, dw.$$

Further analysis would yield another theorem on the necessary condition for optimality but it would have only of limited use. As Simonovits (2013) demonstrates, even in the capless case, the first-order condition can determine not only the locally maximal but also locally minimal contribution rate. We stay content with a

Conjecture. *Assume that the suboptimal contribution rate in the capless system is less than the maximum: $\tau_\infty < \bar{\tau}$. Then the socially optimal contribution rate τ^* with cap w^* lies between them: $\tau_\infty < \tau^* < \bar{\tau}$.*

As a heuristic justification, note that the introduction of a cap reduces the effective contribution rates of those earners whose wages are higher than the cap and leaves the others unaffected. If a high cap is introduced and the capless optimal contribution rate is slightly raised, then the gains of the low-paid and of the high-paid more than compensate for the losses suffered by those who are around but above the critical wage w^*, whose savings and utilities are slightly diminished.

It is not enough to determine the social optimum, we must evaluate the efficiency gain of having an optimal pension system $(\tau^*, \bar{w}^*)$ with respect to having a suboptimal one (without cap). Since the numerical value of V has no direct meaning, we define the *relative efficiency* ε of $(\tau^*, \bar{w}^*)$ with respect to (τ_∞, ∞) as follows: multiplying the earnings by a positive scalar ε such that the social welfare of the no cap system is the same as the social welfare of the optimal cap with the original wages. Denoting the dependence of the welfare on the (average) wage ε, we have the following definition for efficiency:

$$V[\varepsilon, \tau_\infty, \infty] = V[1, \tau^*, \bar{w}^*].$$

Due to the specific utility function, $(1 + \mu) \log \varepsilon$ can be separated in the LHS, i.e.

$$V[1, \tau_\infty, \infty] + (1 + \mu) \log \varepsilon = V[1, \tau^*, \bar{w}^*], \quad \text{i.e.} \quad \varepsilon = \exp \frac{V[1, \tau^*, \bar{w}^*] - V[1, \tau_\infty, \infty]}{1 + \mu}.$$

Pareto-distribution

We assume that wages follow a *Pareto-distribution* with a density function

$$f(w) = \sigma w_m^\sigma w^{-1-\sigma} \quad \text{for} \quad w \geq w_m,$$

where $\sigma > 1$ is the exponent of the distribution and w_m is the minimum wage. It is easy to give an explicit formula for the distribution function:

$$F(w) = \int_{w_m}^{w} f(\omega)\, d\omega = 1 - w_m^\sigma w^{-\sigma} \quad \text{for} \quad w \geq w_m. \tag{12}$$

Hence $F(w_m) = 0$ and $F(\infty) = 1$, and its expectation can explicitly be calculated:

$$\mathbf{E}w = \int_{w_m}^{\infty} w f(w)\, dw = \frac{\sigma w_m}{\sigma - 1}.$$

Since we normalized the expected wage as unity, the minimum wage is given as

$$w_m = \frac{\sigma - 1}{\sigma}.$$

In practice, $\sigma \approx 2$, then $w_m \approx 1/2$. We also display the second moment of the Pareto-distribution:

$$\mathbf{E}w^2 = \frac{\sigma w_m^2}{\sigma - 2} = \frac{(\sigma - 1)^2}{\sigma(\sigma - 2)} \quad \text{for} \quad \sigma > 2 \quad \text{and} \quad \mathbf{E}w^2 = \infty \quad \text{otherwise.}$$

For our unbounded distribution, let w_N be the maximal value at which the wage distribution is cut in the numerical illustrations and we represent all the wages above w_N by a cleverly chosen w_K. By definition,

$$1 = \int_{w_m}^{w_N} w f(w)\, dw + [1 - F(w_N)] w_K. \tag{13}$$

The expected censored wage (given in the integral in (13)) is equal to

$$\mathbf{E} \min(w, w_N) = 1 - \frac{w_m^\sigma w_N^{-\sigma+1}}{\sigma - 1}, \tag{14}$$

hence (12), (13) and (14) yield

$$w_K = \frac{\sigma}{\sigma - 1} w_N.$$

For example, for $\sigma = 2$, the representative highest wage is double of the "maximum": $w_K = 2w_N$.

3. Numerical illustrations

Even though our framework is very elementary, our problem is quite involved, therefore we turn to numerical illustrations.

Critical discount factor

Unlike others (e.g. Feldstein (1985), (1987) and Cremer et al. (2008)) we have distinguished the lengths of the working and of the retirement periods. (On the other hand, we have confined attention to a stationary population and economy!) Assuming 40-year working and 20-year retirement periods, the length-ratio is 1/2 rather than 1. By this way, we receive more realistic numbers. For example, even in our stationary economy and population, the socially optimal contribution rate $\bar{\tau} = 1/(1 + \mu^{-1})$ in (10) drops from 1/2 to 1/3 as we replace $\mu = 1$ by 0.5. If we took into account that the socially optimal discount factor is less than one (e.g. labor disutility, reduced family size, etc., as postulated by Cremer et al. (2008)), then we could reduce the contribution rate further, even to 1/4.

We shall calculate as if the whole saving and dissaving occurred at the middle points of the working and retirement periods, namely at adult ages 20 and 50 years, respectively.

We start with the tabulation of the critical discount factor as a function of the interest factor. Recall that for workers with subcritical discount factors, the maximal contribution rate is optimal; while for workers with supercritical discount factors, the zero contribution rate is optimal (cf. Theorem 1). It is qualitatively obvious that the lower the interest factor, the higher the critical discount factor. Table 1 gives the quantitative answer. For example, for the modest annual interest factor of 1.023, the critical annual discount factor is quite high: 0.947, while for the super high annual interest factor of 1.089, the critical annual discount factor is quite low: 0.883.

Table 1. *Critical discount factor as a function of interest factor*

Compound interest factor ρ	Annual interest factor $\rho(1)$	Compound critical discount factor δ^*	Annual critical discount factor $\delta^*(1)$
2	1.023	0.197	0.947
3	1.037	0.118	0.931
5	1.055	0.066	0.913
7	1.067	0.046	0.902
9	1.076	0.035	0.894
11	1.083	0.029	0.889
13	1.089	0.024	0.883

Pareto-2 distribution and the role of the cap

We shall assume that wages are distributed along a Pareto-distribution. To give a flavor of the behavior of the Pareto-2 distribution, we display selected values of the distribution function and the covered expected earnings. The median wage is about 0.71. Note how fast the probability of being fully covered converges to 1 as the relative value of the cap goes to 4, and how slowly the share of the covered earnings does so. For example, 1.6% of all the earners still have 12.5% of the total earnings (last row).

Table 2. *Pareto-probabilities and covered earnings for varying caps*

Earning cap $\bar{w}$	Probability $F(\bar{w})$	Share of covered earnings $F(\bar{w})\mathbf{E}(w\vert w < \bar{w})$
0.707	0.500	0.250
1.0	0.750	0.500
1.5	0.889	0.667
2.0	0.938	0.750
2.5	0.960	0.800
3.0	0.972	0.833
4.0	0.984	0.875

Remark. $\sigma = 2$.

Recall that in our model, the discount factor is an increasing function of the wage: $\delta = \delta(w)$. To map an infinite interval into a finite one, we assume the simple relation

$$\delta(w) = (\delta_m - \delta_M)e^{\xi(w_m - w)} + \delta_M, \tag{15}$$

where $\xi > 0$ stands for the discount factor–wage elasticity. Note that for any finite w_N, $\delta(w_N) < \delta_M$, but for high w_N/w_m, the error is small. We shall work with $\delta_m = 0$, $\delta_M = 1$. For this special choice, $\xi = -\delta'(w)/\delta(w)$ is indeed the discount factor–wage elasticity and typically we shall work with $\xi = 0.2$.[8]

Table 3 displays the impact of the choice of the cap between 0.5 and 2 for $\rho = 3$ and $\xi = 0.2$ with the optimal contribution rate $\tau^* = 0.33$. Relative efficiency is calculated with relation of no cap (i.e. $\bar{w} = \infty$). Note that in this example, the relative efficiency is lower than 1 for the minimum wage $\bar{w} = 0.5$: $\varepsilon = 0.993$ and rises above 1 until it reaches 1.01 for the average wage and then slowly declines. Even at the double of the average wage, the relative efficiency is still 1.005. The critical wage (at which the saving intention turns from negative to positive) is always greater than the cap if and only if the cap is less than 2.5.

[8] We shall divide the interval $[w_m, w_M]$ into $n = 200$ subintervals such a way that the division points w_i form a geometrical sequence: and at integration, the representative points are the geometrical means of the subsequent points: $w_{i+1} = qw_i$ and $z_i = \sqrt{w_i w_{i+1}}$. The mass of the remaining infinite part is $1 - F(w_M) = 0.0001$ (with $w_M = 50$) and the earning $w_K = 100$ represents the average highest wage.

Table 3. *The impact of cap on efficiency and savings*

Cap $\bar{w}$	Relative efficiency ε	Expected saving $\mathbf{E}s^*$	Critical wage w^*
0.50	0.993	0.056	1.094
0.57	1.000	0.054	1.199
0.75	1.008	0.051	1.315
1.00	1.010	0.046	1.510
1.25	1.009	0.043	1.734
1.50	1.007	0.040	1.901
2.00	1.005	0.035	2.183
2.50	1.003	0.032	2.506
3.00	1.002	0.027	2.506
...	...	...	...
∞	1	0	–

Remark. $\xi = 0.2$, $\rho = 3$, $\rho(1) = 1.037$ and $\tau^* = 0.33$.

In the remainder we shall investigate the sensitivity of the social optimum to the parameter values, namely to the wage elasticity of the discounting factor and to the interest factor.

Optimal contribution rate without cap

First we display the impact of the contribution rate in a capless system. Fixing first the 30-year interest factor as $\rho = 3$, Table 4 shows that as the discount factor–wage elasticity ξ rises from 0.1 to 0.7, the socially optimal contribution rate sinks from 0.331 to 0.257, and the corresponding net replacement rate drops from 0.99 to 0.692.

Table 4. *The socially optimal contribution and replacement rates, changing elasticity*

Wage elasticity of discount factor ξ	Contribution rate τ_∞	Net replacement rate β	Expected saving $\mathbf{E}s^*$
0.1	0.331	0.990	0.008
0.2	0.325	0.963	0.015
0.3	0.317	0.928	0.023
0.4	0.306	0.882	0.031
0.5	0.293	0.829	0.039
0.6	0.278	0.770	0.048
0.7	0.257	0.692	0.059

Remark. $\rho = 3$, $\rho(1) = 1.037$.

In Table 5, we discuss the sensitivity of the outcomes to the interest factor. As the interest factor ρ rises from 2 to 13 (at an annual level, from 1.023 to 1.089), the socially optimal contribution rate drops from 0.331 to 0 and the corresponding net replacement rate sinks from 0.99 to 0.

Table 5. *The socially optimal rates, changing interest factor*

Interest factor ρ	Annual interest $\rho(1)$	Contribution rate τ_∞	Net replacement rate β	Expected saving $\mathbf{E}s^*$
2	1.023	0.331	0.990	0.008
3	1.037	0.325	0.963	0.015
5	1.055	0.303	0.869	0.027
7	1.067	0.268	0.732	0.037
9	1.076	0.224	0.577	0.046
11	1.083	0.168	0.404	0.056
13	1.089	0	0	0.081

Remark. $\xi = 0.2$.

Optimal contribution rate and cap

We have already conjectured that the introduction of the cap raises the optimal contribution rate with respect to the capless case. Comparing Table 6 to Table 4, our conjecture is confirmed. When ξ rises from 0.1 to 0.7, the socially optimal contribution rate sinks much slower than in the capless case, but the socially optimal cap drops from 1.6 to 0.5. (For $\xi = 0.04$, the socially optimal cap rises to 3.6!) Here we also display the relative efficiency of the capped system to the corresponding capless one: it rises from 1.003 to 1.036.

Table 6. *The socially optimal cap, changing* ξ

Wage elasticity of discount factor ξ	Cap $\bar{w}^*$	Relative efficiency ε	Expected saving $\mathbf{E}s^*$
0.1	1.6	1.003	0.027
0.2	1.0	1.010	0.046
0.3	0.8	1.017	0.062
0.4	0.7	1.023	0.074
0.5	0.6	1.029	0.087
0.6	0.6	1.032	0.095
0.7	0.5	1.036	0.107

Remark. $\rho = 3$, $\tau = 0.33$.

Finally, we check the impact of the interest factor ρ for a fixed elasticity, again $\xi = 0.2$. As the 30-year-period interest factor rises from 2 to 13 in Table 7, the socially optimal contribution rate drops slowly but then suddenly from 0.33 to 0.23. The decrease in the socially optimal cap starts immediately: from 1.5 to 1 and then to the minimum wage, namely 0.5. The relative efficiency of the capped system with respect to the corresponding capless one rises from 1.03 to 1.048 and then drops back to 1.033.

Table 7. *The socially optimal contribution rate and the cap, changing interest factor*

Compound interest ρ	Annual factor $\rho(1)$	Contribution rate τ^*	Cap $\bar{w}$	Relative efficiency ε	Expected saving $\mathbf{E}s^*$
2	1.023	0.333	1.5	1.003	0.036
3	1.037	0.332	1.0	1.010	0.046
5	1.055	0.329	0.7	1.026	0.056
7	1.067	0.318	0.6	1.039	0.062
9	1.076	0.319	0.5	1.048	0.066
11	1.083	0.279	0.5	1.048	0.068
13	1.089	0.228	0.5	1.033	0.071

Remark. $\xi = 0.2$.

We have not commented the changes in saving as a result of imposing a cap (last columns of Tables 4–7). Typically the socially optimal expected saving is higher than the suboptimal one: $\mathbf{E}s^* > \mathbf{E}s_\infty$ but for the unrealistically high compounded interest factor $\rho = 13$, the capless suboptimal contribution rate is so much lower than the capped one: $0 < 0.228$, that the order of saving is reversed: $0.071 < 0.081$.

In summary, some results (especially on the optimal contribution rate) are quite robust, while other results (notably on the socially optimal cap) are very sensitive to the key parameters of the model, namely to the discount factor–wage elasticity (ξ) and the interest factor (ρ).

4. Conclusions

We have constructed a very simple model of the proportional pension system to analyze the impact of the socially optimal contribution rate and especially of the contribution (base) cap on the social welfare (and private savings). We have concentrated on the contradiction between the needs of low-earning myopic and of high-earning far-sighted types: the former need a high contribution rate to make up for their low saving intentions; the latter need a low contribution rate to make room for their high and efficient saving. Under certain plausible conditions, the socially optimal contribution rate in a capless system is close to the maximal one, approaching the old-age consumption of the lower-paid, shorter-sighted to their young-age one. A politically convenient compromise is the introduction of an appropriate cap on the contribution: a well-chosen cap does not diminish the contribution as well as the utility of the myopes but relieves the far-sighted from a part of the contribution burden.

This model is just the beginning. It neglects very important issues: the heterogeneity of the life spans and of the interest factors. In fact, the expected life span and the interest factor rise with lifetime wages. This may suggest the introduction of progressive pension systems, for example, the proportional part is complemented by a uniform basic benefit. Then the analysis of the progressive personal income tax also comes to the fore. The flexibility of the labor supply and the underreporting of the true labor income are other important issues, which have been studied with other, related elementary models (Cremer et al., (2008) and Simonovits (2009)).

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