MAGYAR TUDOMÁNYOS AKADÉMIA
Közgazdaság- és Regionális Tudományi Kutatóközpont

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Research Centre for Economic and Regional Studies
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MŰHELYTANULMÁNYOK DISCUSSION PAPERS

MT-DP – 2013/10

Competition and cooperation in a PFF game theoretic model of electrical energy trade

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HUNGARIAN ACADEMY OF SCIENCES BUDAPEST, 2013

Discussion papers
MT-DP – 2013/10

Institute of Economics, Research Centre for Economic and Regional Studies,
Hungarian Academy of Sciences

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March 2013

ISBN 978-615-5243-62-2
ISSN 1785 377X

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Abstract

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Any non-monopolistic proper subset of the generators may cooperate and harmonize their offered prices to increase their resulting profit. Since we allow the redistribution of profits among cooperating generators, a transferable utility game theoretic framework is used. Furthermore, as cooperation affects the outsiders as well, the resulting game is defined in partition function form. The model is able to demonstrate some interesting benefits of cooperation as well as the effect of market regulations and asymmetric information on the resulting profits and total social cost.

Keywords: partition function form games, electrical energy transmission networks

JEL classification: C71, L14, L94

Versengés és kooperáció az elektromosenergia-kereskedelem PFF játékelméleti modelljében

Csercsik Dávid

Összefoglaló

Egy kooperatív játékelméleti keretet vezetünk be, hogy tanulmányozzuk az elektromos energiaszolgáltatók kompetitív, illetve kooperatív viselkedését racionális árpreferenciával rendelkező fogyasztók esetén. A szolgáltatók viselkedését egy DC load flow model által leírt idealizált környeteben tanulmányozzuk, ahol a hálózat veszteségmentes és egy független hálózatirányító felügyelete alatt áll, amely biztosítja a hálózat stabilitását és a fogyasztási igények teljesülését, emellett figyelembe veszi a fogyasztók preferenciasorrendjét a termelőkre vonatkozóan. Egy iteratív folyamatot feltételezünk, melynek során a szolgáltatók egyidőben publikálják áraikat, és a fogyasztók preferenciasorrendje ezeknek megfelelően alakul ki. A modell figyelembe veszi a hálózat esetleges túlterhelését, amennyiben nem minden termelő–fogyasztó párosítás megengedett a hálózat stabilitásának biztosítása érdekében. A modell egyszerűségének megtartása érdekében nem veszünk figyelembe átviteli díjat, a termelők profitja a bevételük és a termelt mennyiség függvényében lineárisan csökkenőnek feltételezett termelési költség különbségeként számolható.

A termelők minden nem-monopolisztikus részhalmaza együttműködhet és összehangolhatja a kínált árakat a profit maximalizálásának érdekében. Mivel feltételezzük a profit újraoszthatóságát az együttműködő termelők között, a keletkező játék átruházható hasznosságú. Továbbá, mivel az együttműködés a kívülállókra is hatással van, a játékot PFF formában adjuk meg.

A modell alkalmas az együttműködés érdekes hatásainak bemutatására, csakúgy mint a piaci szabályozások hatásainak demonstrációjára.

Tárgyszavak: Partíciós függvényformájú játékok, elektromosenergia-átviteli hálózatok

JEL kódok: C71, L14, L94

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Dávid Csercsik*†

February 23, 2013

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A cooperative game theoretic framework is introduced to study the behavior of cooperating and competing electrical energy providers considering price-preference rational consumers. We analyze the interactions of generators in an idealized environment described by a DC load flow model where the network is lossless and is operated by an independent regulator who ensures network stability and fulfillment of consumption needs while taking into account the preferences of consumers over generators. We assume an iterative process in which the generators publish their price offers simultaneously in each step, based on which the consumers preferences are determined. The model deals with network congestion and safety as not every generator-consumer matching is allowed to ensure the fault tolerant operation of the transmission system. To make the model as simple as possible we do not deal with transmission fees, the profit of the generators is determined as the difference between their income, and their production cost which is assumed to be linearly decreasing with the produced quantity.

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1 Introduction

Because of its extreme importance, power system economics (Kirschen and Strbac, 2004) has been an intensively researched interdisciplinary area. The recent trends as liberalization of electricity markets and growing portion of renewable sources (which require high investments, but their production depends on weather factors), together with occasionally rapidly extending consumption in long term and consumption peaks in short term, put increasing load on system operators and authorities reliable for network operation and expansion.

If one wishes to analyze the electrical energy market as interactions of market participants, he has to take into account that the possible interactions are constrained by laws of physics as well as by market regulations. We study the interaction between the physical and economic aspects of the power transmission system operation focussing on the incentives for group formation.

When studying electric power transmission networks, most of the research in economics has focussed on the topics of competition, market power and regulation (Cardell, Hitt, and Hogan, 1997; Gilbert, Neuhoff, and Newbery, 2004; Neuhoff, Barquin, Boots, Ehrenmann, Hobbs, Rijkers, and Vázquez, 2005; Chen, Hobbs, Leyffer, and Munson, 2006) and very few authors study the market and the transmission issues in their whole complexity (Kirschen and Strbac, 2004).

The articles (Hobbs and Kelly, 1992; Bai, Shahidehpour, and Ramesh, 1997) already use game theory for transmission analysis. Hobbs and Kelly (1992) use static cooperative models to calculate the possible outcomes of short-run transmission games and noncooperative Stackelberg games to model long-run games in which the amount of transmission capacity is a decision variable. The paper (Bai, Shahidehpour, and Ramesh, 1997) describes an open access transmission method for maximizing profits in a power system, where Transmission losses are considered. The proposed method is based on the Nash bargaining game for power flow analysis in which each transaction and its optimal price are determined to optimize the interests of individual parties. The paper (Orths, Schmidtt, Styczynski, and Verstege, 2001) describes a game theoretic approach of a multi-criteria optimization problem related to transmission planning and operation. A strategic gaming approach is described in (Kleindorfer, Wu, and Fernando, 2001).

Gately (1974) was probably the first to apply cooperative game theory to planning investments of electrical power systems. In this paper the concepts of the core and the Shapley value are used to determine the mutually acceptable set of final payments The paper (Evans, Zolezzi, and Rudnick, 2003) describes a cost assignment model for electrical transmission system expansion is using Kernel theory. The methods of cooperative game theory have been applied for the analysis of the transmission expansion problem both in the case of centralized and decentralized environment (Contreras, 1997; Contreras and Wu, 1999, 2000; Contreras, Gross, Arroyo, and Muñoz, 2009).

Considering market regulations, Singh, Hao, and Papalexopoulos (1998) compare a nodal pricing framework with cost allocation procedures in the case of competitive electricity markets, and analyze some game-theoretic aspects of the proposed model. The paper of Ding and Fuller (2005) considers nodal, zonal and uniform marginal prices and emphasizes that the nodal marginal price correctly accounts for transmission constraints and losses in some cases.

In this article we consider an almost totally liberalized electricity market model, in which power providing companies compete for the consumers and determine their offered prices basically free of any central influence. Market regulations however may appear. First, we assume that the formation of monopolistic coalitions, who could raise their prices as high as they wish, is prohibited, and second, rules may apply for the offered prices of one provider to various consumers.

2 Materials and Methods

2.1 Game theoretic preliminaries

Let $N = \{1, 2, \ldots, n\}$ be the set of players, and its non-empty subsets are the coalitions, denoted by C. A *partition* P is a set of disjoint coalitions; $P = \{C^1, C^2, \ldots, C^m\}$, where their union is N, i.e., players in set K cooperate if and only if $K \in P$. The set of partitions is $\mathcal{P}$ and the set of partitions of $C \subseteq N$ is $\mathcal{P}(\mathcal{C})$. A partition function; $V : \mathcal{P} \to (2^N \to \mathbb{R})$ assigns a characteristic function (v) to each partition. A characteristic function $v : 2^N \to \mathbb{R}$ assigns to each coalition $C \subseteq N$ its *worth* or *payoff* $v(C)$, with the convention that $v(\emptyset) = 0$. A *cooperative game with transferable utility in partition function form*, or briefly *PFF-game* (Thrall and Lucas, 1963), is a pair (N, V), where the worth of a coalition may be different in each partition.

Externalities describe the effect of the formation of a coalition on third parties. Externalities may be positive if the payoff of a third party coalition increases when a particular coalition forms or negative if the payoff decreases.

2.2 The Model

We consider a model of the electricity market where the energy transfer implied by the trading transactions takes place on an energy transmission network. For the description of the underlying network we will use the DC load flow model and the corresponding terminology defined in (Csercsik and Kóczy, 2011) based on Oren, Spiller, Varaiya, and Wu (1995) and Contreras (1997). The most important feature of a DC load flow model is that if we make some key assumptions regarding the power grid (neglect the real part of line impedance, and assume the same peak voltage at every node) then the mathematical form of the equations describing the real power flow will be equivalent to the equations describing the flow of the DC current in a resistance network. The admittance values (Y) and the injected/consumed energy amounts of the nodes uniquely determine the energy flows on the edges (branches or lines) of the network, which can be obtained by the solution of a system of linear equations. In addition to its admittance value, each edge is characterized by a transfer capacity ($\overline{q}$), which corresponds to the maximal amount of energy which can be transferred on it. Furthermore, as a most simple approach we neglect transmission losses. For the sake of simplicity we will assume that every node of the energy transmission network is assigned to a certain generator or consumer. The most straightforward interpretation of the model is that we study the high voltage energy transportation networks, in which case consumers correspond to local energy providers who own mid-voltage networks.

In this paper we study how the behavior of energy providers is influenced by market regulations and asymmetric access to information. For the description of competitive and cooperative behavior of energy providers we define a game theoretic framework where energy providers (or generators in the following, which means that in this case every provider holds one generator) with different production characteristics offer energy for sale to multiple consumers.

In the following we summarize the assumptions regarding our model

- We assume n_g generators, which are considered as the players of the game, n_c consumers and n_l lines.
- Any consumer is allowed to buy energy from only one generator.
- As the focus of this study is to analyze the competition and cooperation of energy providers under various market regulations (prohibition or allowance of zonal pricing), we neglect the demand elasticity of consumers (it is supposed that consumers buy their total required amount of energy, even if not on the most preferred price), and assume perfectly predictable loads.
- Depending on the actual market regulation, each generator may offer nodal (consumer-dependent) or universal prices (same for all consumer), which determine the priority of consumers over the generators.
- Each generator has a limited production capacity, which may limit the number of consumers he can provide energy for.
- Generators act as the players of the game, while rational price-priority consumers are assumed.
- Generator j offers energy for sale to consumer k at po_k^j. If no zonal pricing is allowed $po_k^j = po^j \quad \forall\ k$. We summarize the offers in the price offer matrix PO, where $[PO]_{k,j} = po_k^j$.

Considering the prices offered by the generators, the consumers (who are assumed to be price-preference rational) set up preferences ($Pref$) regarding from which generator they want to purchase their required amount of energy. Since the prices offered by the consumers may be the same, the preference ranking is not necessary strictly monotonic.

Based on these preferences and network characteristics, the independent network regulator determines a matching of generators and consumers.

Definition 1 *We call a matching between generators and consumers* feasible *is every generator's capacity allows him the fulfillment of all those consumers's requirements who are assigned to him.*

Definition 2 *We call a matching* stable *if the resulting network configuration implies a stable state of the network. Under network stability we mean that no line is overloaded, and no instantaneous fail of any line may lead to overload of any other line in the network.*

A matching can be easily summarized by a matrix in which columns are corresponding to pairs of consumers and generators (in which a generator may appear multiple times).

First the independent network regulator identifies if the most preferred matching (the matching which implies the lowest resulting total cost for the consumers) is feasible and stable. In case it is, it will be the resulting matching. If the most preferred matching is not stable or not feasible, the independent network regulator analyzes all matchings where the preferences are violated in one case (one consumer is assigned to his second most preferred generator), and from the stable and feasible matchings he chooses the one which brings the least additional cost to the consumer who's subject of preference violation. If multiple more matchings exist in which the additional cost is nearly equal, based on the stability margin of the configuration (for the definition of the stability margin, see appendix A), the more stable one will be chosen. If no such matching exists, the matchings with two preference violation will be analyzed, etc. If there is at least one feasible and stable configuration, the algorithm will stop.[1]

[1]A possible alternative for this matching method is when the independent network regulator calculates

This matching will determine the resulting prices. p_k^j denotes the price which is paid by consumer k to generator j for one unit of energy. p_k^j=po_k^j for all {k,j} pairs in the matching.

In other words, the independent network regulator ensures the fulfillment of the matching most preferred by the consumers under the constraints implied by the network stability and feasibility requirements. The parameters of this configuration depend on the actual prices offered by the generators.

- The generators are fully aware of the independent network regulator's matching algorithm.

- We assume that as more of the generation capacity is utilized, the more efficient the energy production is. Furthermore, we assume that generation cost per unit is linear decreasing function of generated quantity: $c^j = a^j - m^j Q^j$ where a^j and $m^j > 0$ are the constants describing the production characteristics of generator j (which depend on the applied technology), while Q^j is the total energy quantity produced by the generator. The total generation cost of a generator can be formulated as: $C^j = c^j Q^j$

- The income of a certain generator is the sum of the incomes regarding various consumers:

$$I^j = \sum_{k \in S^j} p_k^j Q_k$$

  where S^j is the set of consumers who buy energy from generator j. Q_k denotes the quantity bought by consumer k.

- Profit of player j denoted by $\$^j(p_k)$ can be calculated as the difference of income and generation cost for the player: $\$^j(p_k) = I^j(p_k) - C^j(Q^j)$. Basically, in the case of total competition it is assumed that all players try to maximize their own expected profit.

- One straightforward interpretation of coalitions is to assume that the generators of a certain coalition are belonging to the same company (we would like to emphasize

all possible matchings, and chooses that stable and feasible one, which implies the lowest total cost for the consumers. The required computational effort in this case is similar if the consumption demands are consant (like in our case), but it may be significally different if the demands change during the iterations.

that still in this case we assume that consumers buy energy from generators, not from the coalitions). The worth or payoff of a given coalition embedded in a partition is determined as follows. Coalitions determine their price offers jointly to maximize their expected *overall* profit. A basic assumption of transferable utility coalitional game theory applies here: we assume that the members of a coalition may freely redistribute their profits among themselves. We have to note that if we consider a different interpretation of coalition formation, and assume that different generators belong to different but cooperating companies, this is not necessary true in a realistic economical environment.

Based on the price offers the independent network regulator determines the matching. The sum of the profits in the for the members of a coalition determines its value in the current partition P:

$$V(C, P) = \sum_{j \in C} \$_j$$

Since we allow the redistribution of profits among cooperating generators, and cooperation may affect agents non included in the coalition (externalities), the resulting game is defined in partition function form. Furthermore, we assume that anti-cartel regulations exclude the formation of monopolistic coalitions.

Definition 3 *We call a coalition* monopolistic*, if the presence of at least one member of the coalition is unavoidable in any matching which is stable and feasible.*

The exclusion of monopolistic coalitions will result in the fact that despite any allowed cooperation, there will be at least two alternatives (considering coalitions) for any consumer, which implies a true competition.

Example In a 3 generator (nodes 1,2,3) 2 consumer (nodes 4,5) network, in which only the matchings $\begin{pmatrix} 4 & 5 \\ 1 & 1 \end{pmatrix}$, $\begin{pmatrix} 4 & 5 \\ 2 & 2 \end{pmatrix}$, $\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$, $\begin{pmatrix} 4 & 5 \\ 1 & 3 \end{pmatrix}$ and $\begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}$ are stable and feasible, none of the singleton coalitions or the coalitions $\{1, 3\}$ or $\{2, 3\}$ are monopolistic while the coalition $\{1, 2\}$ is.

3 Results and discussion

To demonstrate the game considered in section 2.2 and some possibly arising phenomena we use the example network depicted in Fig. 1.

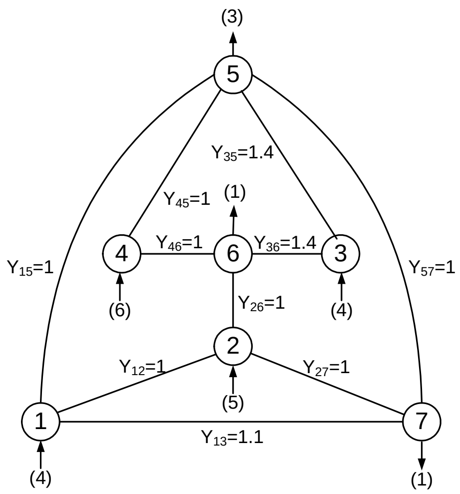

Figure 1: Topology and parameters of network 1. The Y_{st} values denote the admittance values of the line connecting node s and t. We suppose that $\overline{q}_{st}$ (the transmission capacity constraint, the maximum amount of energy which can be transmitted via the corresponding transmission line) is proportional to the admittance as $\overline{q}_{st} = 2.4Y_{st}$ The numbers next to the nodes indicate the available generation amounts and required consumption quantities denoted by inward and outward arrows respectively.

3.1 Cooperation may be necessary among generators to divert consumers from previous providers

In the following, first without the explicit analysis of profits and prices, we'll demonstrate how cooperating players may overcome network stability related limitations during the extension of their client-set while diverting a consumer from a third generator. The further aim of this example is to demonstrate how the stability properties of the network may be a barrier for some matchings between generators and consumers.

As it is indicated in Fig. 1 by the inward/outward arrows at the nodes, nodes 1, 2, 3 and 4 correspond to generators and 5, 6 and 7 correspond to consumers. First let us identify the monopolistic coalitions in the case of network 1, since they are not allowed according to our assumptions. Since e.g. the following matchings

$$\begin{pmatrix} 5 & 6 & 7 \\ 2 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 5 & 6 & 7 \\ 1 & 3 & 3 \end{pmatrix} \quad \begin{pmatrix} 5 & 6 & 7 \\ 3 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 5 & 6 & 7 \\ 3 & 4 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 6 & 7 \\ 2 & 4 & 4 \end{pmatrix} \quad \begin{pmatrix} 5 & 6 & 7 \\ 1 & 4 & 4 \end{pmatrix}$$

are stable and feasible, we may conclude that all 2-player coalitions are non-monopolistic. If we analyze further matchings, the calculations show that none of the generators is able to supply the consumers alone in a feasible and stable way. This implies that every coalition with at least 3 members is monopolistic.

Let us suppose the initial stable and feasible matching depicted in Fig. 2

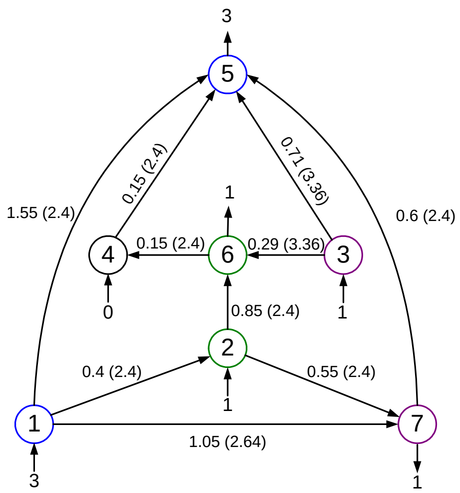

Figure 2: Flows in the case of network 1, and the stable and feasible generator-consumer matching 5-1, 6-2, 7-3. Matchings are labeled with different colors. The numbers on the edges in parentheses denote the maximal flow value on the edge.

3.1.1 Coalition structure {1} {2},{3},{4}

Let us consider first the all-singleton partition $\{1\}, \{2\}, \{3\}, \{4\}$, the initial stable and feasible matching depicted in Fig. 2 and the possible cooperation of generators 1 and 2. Let us assume that generator 1 offers a price for consumer 7 which is lower than the price offered by generator 3, his current supplier. In general, this can be a rational decision for generator 1, as the increase of his load leads to more effective utilization of his capacities and this may imply even higher profits for him, even in the case of lower prices offered.

However, independent of the exact offered prices and potential profit change, although the capacity of generator 1 is sufficient to supply both consumers 5 and 7, and no lines would be basically overloaded, this configuration is not allowed by the network regulator, because of stability issues depicted in Fig. 3.

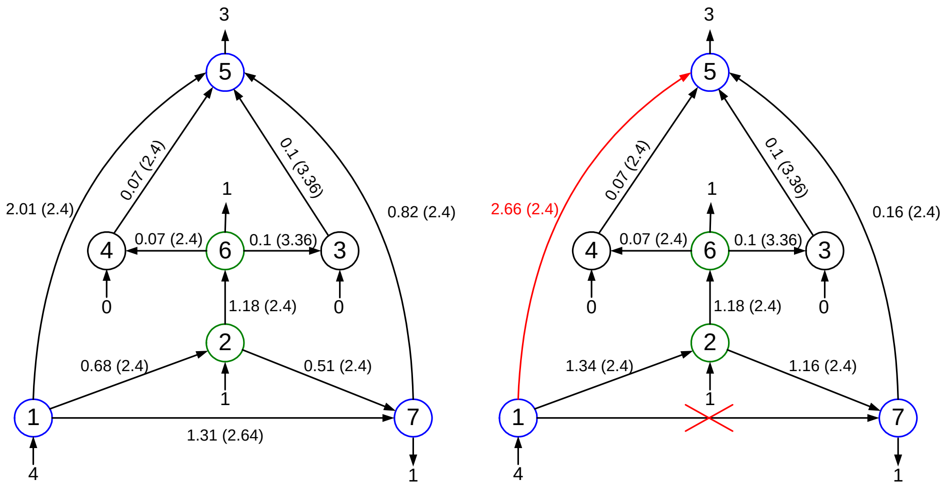

Figure 3: If generator 1 would supply consumer 5 and 7, the network would lose its stability: The fail of line 1-7 would lead to the overload of line 1-5, so this configuration is not allowed by the independent network regulator. However not included in the figure, further stability issues arise regarding this configuration: The fail of line 2-6 or 5-7 would also result in the overload of line 1-5.

As illustrated in Fig. 4 neither is Generator 2 able to supply consumer 7 in addition to his already established client, consumer 6.

3.1.2 Coalition structure {1,2},{3},{4}

However, if Generators 1 and 2 are able to somehow arrange their prices in order to exchange their former consumers (5 and 6) between themselves, generator 1 is able to supply consumer 7 in addition to his former client 6 in a stable and feasible manner, as depicted in Fig 5.

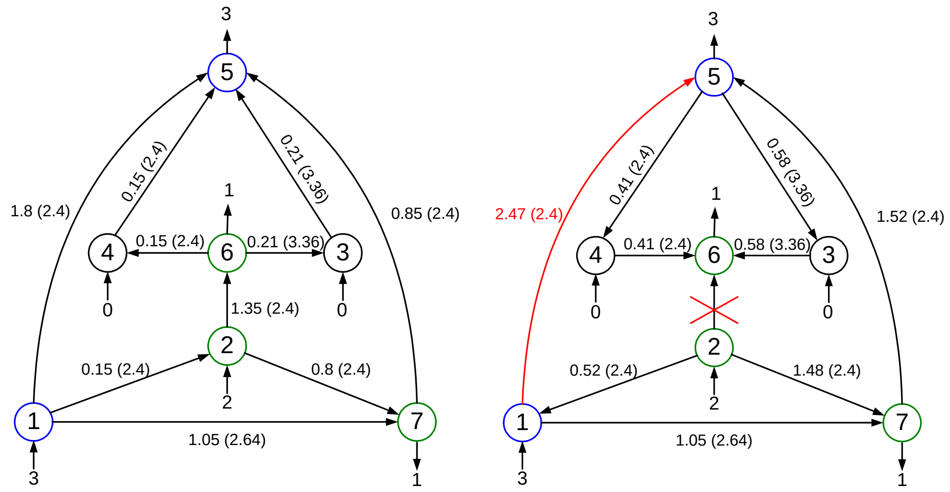

Figure 4: If generator 2 would supply consumer 6 and 7, the network would also lose its stability: The fail of line 2-6 would lead to the overload of line 1-5, so this configuration is neither allowed by the independent network regulator.

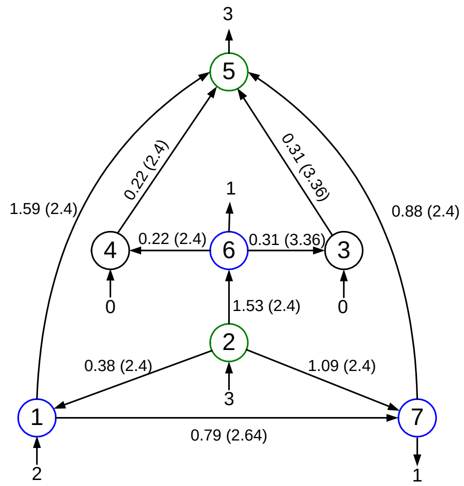

Figure 5: Via cooperation, generators 1 and 2 are able to exchange their former consumers among themselves and divert consumer 7 from generator 3.

It is important to note, that even in the case of cooperating generators, the consumers are still assigned to generators not to coalitions of generators. The cooperating generators may however design their price offers jointly and thus influence the preference of consumers in a way to reach a beneficial resulting configuration.

3.2 Consecutive offers and the PFF game of generators over the network

In the following we will analyze the PFF game defined in section 2.2 over the given network (network 1) and player set. As mentioned, the value of a certain coalition embedded in a partition will be defined straightforwardly via the sum of its member's total profits. The total resulting profit of the players is derived via the evaluation of consecutive simulation steps. In each step, generators are assumed to give price offers, based on which consumers determine their preference over the generators. Based on these preferences the independent network regulator determines a matching between consumers and generators and the corresponding transactions will be completed according to this matching.

The most important step in the process is the determination of the generators' price offers, since these values will determine the consumers preferences. According to our model the generators publish their price offers simultaneously. Furthermore, at first we assume that non-cooperating players have no information about each other's production characteristics (a and m) but they are fully aware of the consumers preference setting principles, all offered prices in the previous step, and the independent network regulator's matching algorithm. We call this a scenario of *symmetric information*.

This implies that every coalition tries to maximize its expected profit while assuming that the other players of the game will offer the same prices as in the previous step. Taking into account that the objective function in non-continuous (the consumer's preferences change abruptly) and multiple local extrema may exist (profits may be increased either by raising prices, or via the possible reduction of prices which may lead to multiple consumers, higher production rate and so higher production efficiency), this is a non-trivial optimization problem for which (especially if we limit the computation time to keep simulations tractable) different optimization approaches may result in different solutions.

Our approach will be to incorporate the possible flaws of certain optimization algorithms in the model and handle the optimization process as a model variable. Thus our model will be capable of comparing different optimization methods for the pricing problem as well.

Simulations detailed in the following were done using MATLAB.

3.3 Analysis of market regulations: Zonal versus global price offers

The first question we aim to analyze with our model is how the possibility of zonal pricing affects the profits of the generators and the total social cost. If zonal pricing is allowed each generator may offer his energy to each consumer on a different price.

3.3.1 All-singleton coalitions, zonal pricing, reference case

The scenario detailed next will serve as a reference for our future analysis to show some details of the evolution of profits. In this setup all players form singleton coalitions $P = \{1\}, \{2\}, \{3\}, \{4\}$. Furthermore, in this reference case all generators are allowed to offer zone-dependent prices. In other words, they may offer their generated energy for sale for each individual consumer at a different price.

We assume the following production characteristic parameters: $a^1 = 0.65$, $a^2 = 0.36$, $a^3 = 0.68$, $a^4 = 0.7$, $m^1 = 0.1$, $m^2 = 0.07$, $m^3 = 0.08$, $m^4 = 0.04$. For the optimization each player uses simulated annealing (van Laarhoven and Aarts, 2008) (as implemented in the MATLAB function *simulannealbnd* with iteration time limit 30s). Furthermore, let us assume the initial offered zonal prices described by the following matrix

$$PO^{init} = \begin{pmatrix} 0.7 & 0.8 & 0.9 & 1 \\ 0.85 & 0.75 & 0.95 & 1.05 \\ 0.97 & 1.07 & 0.87 & 0.77 \end{pmatrix}$$

If we assume 10 iteration steps and calculate the price offers and the resulting matchings the evolution of generator profits will be as depicted in Fig. 6. The number of iteration steps was determined to keep the computations feasible. The optimization takes 30 s for each player or coalition and this has to be done in every partition and every step. As we will see these 10 steps are enough to show the phenomena we are interested in.

As we can see in Fig. 6 (a) in each step 2 or 3 generators supply the consumers with energy. At least one generator is lacking consumers in every step, which implies a profit of 0. According to our simulation results the game does not tend to converge to a Nash equilibrium even if the number of iterations is increased by several orders of magnitude.

We can see that generator 4 whose m parameter is the smallest amongst the 4 (he can increase his efficiency with the produced quantity at least) is usually able to reach the low

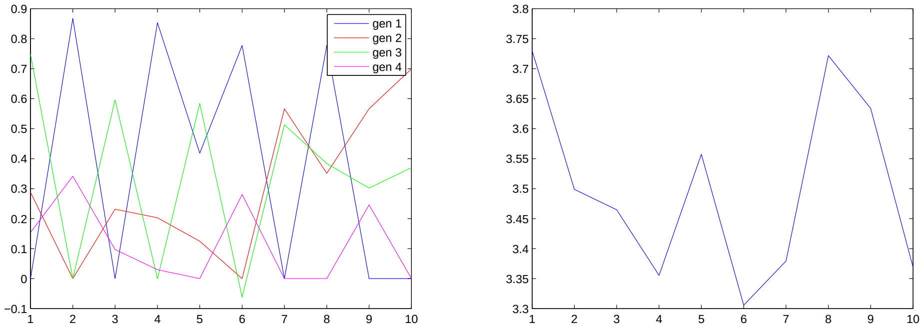

Figure 6: (a) Evolution of generator profits in the reference case. Players do not cooperate, have no information about each other's production characteristic and are allowed to offer zonal prices for consumers. (b) The evolution of total social cost during the simulation.

profits. Furthermore, it can be seen in this figure that profits may be also negative (as in the case of generator 3 in step 6). This can be explained by the fact that price offers of generators are based on the prices of competitors in the previous step. It is possible that a generator expects that he may supply multiple consumers and so utilize the more efficient part of his production characteristics, but the real scenario results only in a single customer for him. In this case, on the offered price, the income may not be even enough to cover the cost of energy production - this, as we see, is however not a typical scenario.

The evolution of the total social cost SC_T (the total amount of money the consumers pay to the generators in order to supply their needs) during the 10 simulation steps is depicted in Fig. 6 (b). The value of the total social cost integrated for the 10 time steps is 35.02 in this case.

3.3.2 Zonal pricing

If zonal pricing is allowed the resulting partition function, single profits and total social costs will be as summarized in Table 1.

Before the comparison of these results with the ones without zonal pricing available let us make a few observations. First, let us note that cooperation is almost always beneficial for the players. We use the game theoretic concept of *superadditivity* for the analysis whether a cooperation is beneficial for its members or not. Formally if $\mathcal{P}$ and $\mathcal{Q}$ are partitions and $(\forall P \in \mathcal{P})(\exists Q \in \mathcal{Q})(P \subseteq Q)$, we say that $\mathcal{P}$ is a refinement of $\mathcal{Q}$. In this

Table 1: Simulation results: Zonal Pricing

Partition	Values	$\$^1$	$\$^2$	$\$^3$	$\$^4$	SC_T
1,2,3,4	3.69, 3.03, 3.4, 1.15	3.69	3.03	3.4	1.15	35.02
{1,2},3,4	11.59, 5.42, 0.94	5.81	5.78	5.42	0.94	40.88
{1,3},2,4	8.55, 7.12, 0.94	4.63	7.12	3.93	0.94	38.62
{1,4},2,3	3.34, 5.60, 4.44	2.39	5.60	4.44	0.95	36.83
1,{2,3},4	6.35, 11.30, 3.22	6.35	8.31	2.99	3.22	43.61
1,{2,4},3	5.12 , 9.37, 0.45	5.12	8.92	4.36	0.45	41.25
1,2,{3,4}	4.21, 8.72, 5.33	4.21	8.72	5.00	0.33	40.40
{1,2},{3,4}	12.92, 9.67	10.52	2.40	7.52	2.15	45.56
{1,3},{2,4}	11.23, 8.63	7.47	6.42	3.76	2.21	41.71
{1,4},{2,3}	4.84, 10.96	3.33	7.78	3.18	1.52	38.06

case, under *superadditivity* we mean that $v(P_1, \mathcal{P}) + ... + v(P_k, \mathcal{P}) \leq v(Q, \mathcal{Q})$. For example if we consider the emerging cooperation between player 1 and 2, we have to analyze the superadditivity in two cases. First if players 3 and 4 are acting independently: in this case the total payoff of players 1 and 2 increases from 6.72 to 11.59, which is clearly beneficial for them. Second if players 3 and 4 are forming a coalition, the payoff of players 1 and 2 practically does not change (12.92 vs. 12.93).

We can observe that subadditivity (the lack of superadditivity) appears in the case when players 1 and 4 are cooperating independent of the behavior of other players. In every other cases cooperation is beneficial (or at least not disadvantageous) for the cooperating players, furthermore the gain is almost always greater if the remaining players do not cooperate. In other words however cooperation is usually necessarily beneficial for the generators, this is not always the case: The network topology and production characteristics potentially determine if players may increase their total payoff via cooperation or not[2].

Furthermore, we may point out how important the assumption of transferable utility is. Although in general cooperation implies higher individual profits as well, this is not necessary. In the case eg. of the cooperation of players 2 and 4, players 1 and 3 may raise their resulting profit with cooperation from 9.48 to 11.23 but regarding the single

[2]Of course cooperation implies that cooperating generators increase (or at least not decrease) their *expected* profit. However, simulations show that in some cases this can be done only in ways which systematically fail in the case when the new offers of other competitors are realized.

generators while generator 1 benefits from the cooperation, generator 3 has to face a lower individual profit. In this case if generator 1 can not entirely transfer a part of his profit for the payoff to generator 3 entirely free, the cooperation may be reconsidered.

Regarding externalities it can be easily seen (eg. the formation of $\{1,2\}$ from singletons) that coalition formation may imply both negative and positive externalities on the remaining players.

In addition, we may compare the average total social cost in the cases where no multi-generator companies appear on the market (35.02), if one multi-generator company appears (40.27), and if 2 multi-generator companies appear (41.77). We can see that as expected, the presence of larger firms dampens the competition, and results in higher prices for the consumers.

3.3.3 General pricing

If no zonal pricing is allowed, every generator offers his energy to every consumer on the same price. In this case, the initial conditions are

$$PO^{init} = \begin{pmatrix} 0.85 & 0.8 & 0.9 & 1 \\ 0.85 & 0.8 & 0.9 & 1 \\ 0.85 & 0.8 & 0.9 & 1 \end{pmatrix}$$

In this case Table 2 summarizes the results.

Table 2: Simulation results: General Pricing

Partition	Values	$\$^1$	$\$^2$	$\$^3$	$\$^4$	SC_T
1,2,3,4	2.15, 2.49, 1.47, 0	2.15	2.49	1.47	0	27.45
{1,2},3,4	4.97, 3.58, 3.09	1.39	3.58	3.09	0.42	31.37
{1,3},2,4	4.67, 6.39, 1.55	3.12	6.39	1.55	1.42	34.35
{1,4},2,3	2.68, 3.51, 1.63	1.57	3.51	1.63	1.11	28.79
1,{2,3},4	3.51, 5.35, 0.45	3.51	4.99	0.36	0.45	30.68
1,{2,4},3	2.30, 2.80, -0.45	2.30	2.50	-0.45	0.31	25.87
1,2,{3,4}	2.42, 7.46, 5.32	2.42	7.46	3.63	1.69	36.97
{1,2},{3,4}	22.33, 12.32	8.29	14.04	9.49	2.83	56.78
{1,3},{2,4}	14.54, 17.55	6.09	15.39	8.44	2.16	52.98
{1,4},{2,3}	13.34, 16.07	9.36	13.61	2.46	3.98	50.70

First, it is upfront that the aggregated profit may be also negative in extreme cases as in the case of partition $1, \{2, 4\}, 3$. This case also demonstrates that the cooperation of generators is not necessary deteriorative for the consumers. The total social cost in this case is lower compared to total competition. This can be explained with the fact, that the combination of price-suppressing competition and the more efficient utilization of generation capacities may lead to lower consumption prices.

Regarding superadditivity, in this case all of the coalitions meet the requirement, however the additional benefit of the merger $1, 4$ is still not very high if the other players are acting competitively.

The most important issue regarding the total social cost is that while the average prices in the case of none or one multi-generator company appears on the market (27.45 and 31.33 respectively) is significantly lower than in the case of zonal pricing, the competition of two multi-generator companies result in very high average social cost (53.49) in the case of general pricing. We may state that our model simulations suggest the following: *In the case of a highly competitive market, general pricing is beneficial for the consumers, while in the case of a market dominated by few multi-generator companies, zonal pricing may decrease the total social cost.*

3.4 Asymmetric information

As we have seen in the examples before, generator 4, thanks to his disadvantageous production characteristics, was almost always able to reach only the lowest profits. In the following we will examine how much does it help for him if he is aware some of the other players offers. We suppose full competition (all-singleton coalitions) in this case.

In addition to the symmetric information case, we will analyze 3 scenarios. Player 4 may be aware of one, two or all three price offers. If eg. player 4 has information of one other player's price offer, this player can be player 1,2 or 3. Table 3 summarizes the average profits of the players and the average resulting total social cost in various cases of additional information of player 4.

We can see that although the additional information of one or two price offers increases the profit of player 4, the real advantage is the case when he is aware of all other offers. In this case in long term he is able to reach the highest profit despite the most disadvantageous production characteristics.

Table 3: Simulation results: Zonal Pricing, asymmetric information, full competition - additional information of player 4

Information	$\$^1$	$\$^2$	$\$^3$	$\$^4$	SC_T
Symmetric	3.69	3.03	3.4	1.15	35.02
One additional offer	3.68	4.86	1.91	1.47	35.06
Two additional offers	3.69	4.53	3.11	1.58	35.79
Three additional offers	3.31	2.51	1.22	4.63	35.82

Table 4: Simulation results: General Pricing, asymmetric information, full competition - additional information of player 4

Information	$\$^1$	$\$^2$	$\$^3$	$\$^4$	SC_T
Symmetric	2.15	2.49	1.47	0	27.45
One additional offer	1.12	2.62	0.48	0.11	25.56
Two additional offers	1.77	2.80	0.58	0.36	27.27
Three additional offers	2.11	3.20	0.23	0.80	28.08

Table 4 shows that under general pricing player 4 can not take so much advantage of the additional information as in the case of zonal pricing. The disadvantageous production characteristics and the low degree of freedom implied by the uniform prices can not be balanced by this advantage.

4 Conclusions

In this article a model framework is proposed, which is able to analyze the effect of cooperation, asymmetric information and market regulations on the profit of generator companies and on the cost of price preference consumers in the case of various pricing strategies. In the demonstrated cases all players/coalitions were about to maximize their profit with numerical optimization based on the available information, however more simple strategies (eg. leader-follower) can also be assumed, which do not need this amount of computational effort. The simulated annealing optimization method applied during the simulations includes stochastic elements, which implies that the determined optimum may be not the same considering repetitive runs. According to our experiences this variability is relatively small and does not affect the observed trends.

Furthermore, we have to note that even considering cooperation, the model remains competitive. The exclusion of monopolistic coalitions ensures, that each consumer will have at least two alternatives for bargaining, regarding coalitions.

It is important to emphasize that the proposed model is far from being strictly realistic considering both the physical and the economical part. The transmission losses, which are currently not included in the model would imply an excess in the total injected energy, furthermore they would penalize generator-consumer pairs which are far from each other, and thus such matchings may result in completely different network flow configurations and profits.

In addition in the case of real scenarios, energy suppliers dominantly use various generating units with diverse production characteristics to meet their consumers needs. While in the case of generating units with great inertia the rescheduling itself may imply significant costs, units of other types (eg. gas turbines) may be set up easily but can not be shut down and restarted economically. Of course one can not avoid the problem of renewable sources, which dominantly operate at very low production costs, while they are subject to weather conditions thus bring uncertainty to the system. One straightforward future direction for the development of the current model is to include these more complex production characteristics and probability type variables in the system. We have to note that cooperative game theory already offers tools (Habis and Herings, 2011) to manage uncertainties.

Furthermore, our model also does not handle voltage stability issues, which could be an additional factor to take into account while determining stable matchings of the network (Van Cutsem and Vournas, 1998).

As our model uses an iterative approach, one straightforward generalization assumption may be that the consumption needs are changing over time. This may affect the resulting profits multiple ways (smaller consumption amounts may be supplied by multiple providers, while maybe the competition boil down to duopolies if the needed amount becomes greater).

Regarding economic aspects, also multiple future development directions can be considered. First of all, the model completely neglects the fee which is paid for the usage of the transmission network. The determination of transmission prices has been a subject of several studies (see eg. Christie, Wollenberg, and Wangensteen (2000); Kirschen, Allan, and Strbac (1997); Wijayatunga (2003)). The proposed model can be quite easily extended to include transmission prices, and the effect of various transmission pricing strategy can be analyzed in the case of different scenarios. At second, if we consider realistic energy providers, we have to assume that one company may hold multiple generation units, and the transaction takes place between not a consumer and a generator but between the consumer and the provider owning the generation unit. In this case the problem will be more complex, since the quantity which has to be fed into the network by the company to meet his consumers needs may be generated at different nodes - it is likely that the generation configuration optimal for the company will not match the generation configuration optimal for network stability.

We hope that the future extensions of the proposed model will be available to study the above problems also in the case of models of real power networks.

5 Acknowledgements

This work was supported by the Hungarian Academy of Sciences under its Momentum Programme (LP-004/2010) and by the Hungarian National Fund (OTKA NF-104706).

A Stability margin and matching

A resulting stability margin for a matching may be defined as follows. If the matching is unstable, the stability margin is 0. If the matching is stable, the value of the stability margin can be defined as

$$\sum_i \sum_{j \neq i} \overline{q}_j - q_j^i$$

where q_j^i is the flow on line j in the case of the fail of line i, and $\overline{q}_j$ is the maximal allowed flow on line j.

If multiple stable and feasible similarly preferred matchings exist (the prices offered by various providers may be the same), the independent network regulator uses the stability margin values to determine the resulting matching between consumers and generators.

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