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Lost in Translation: Unified Consumption Theory, Dynamic AS-AD, and Business Cycles

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Abstract

The paper shows how aggregate AS-AD can be derived within the standard neoclassical dynamic setting known as the Ramsey-Cass- Koopmans (RCK) model. AS-AD is the stationary equilibrium of the deterministic dynamic general equilibrium framework. The derivation builds a permanent income type consumption demand that corresponds to a consumption theory that depends on current income, illustrating analytically how to unify the alternative theories within RCK. With permanent income de.ned by the flow on time and goods endowments, the logic of changing both endowments simultaneously coincides with changing the external labor margin along with goods sector productivity in order to explain business cycles. This gives rise to a stylistic AS-AD explanation of the business cycle that is consistent with the primary features of the RBC literature and consumption theory.

Keywords: Ramsey-Cass-Koopmans, supply, demand, state variable, external labor margin

JEL classification: A22, A23, E13

A fogyasztáselmélet, a dinamikus aggregált kereslet és kínálat, valamint az üzleti ciklusok egyesítése

Max Gillman

Összefoglaló

A tanulmány bemutatja, hogyan vezethető le az aggregált kínálat és kereslet (AS–AD) standard neoklasszikus dinamikus feltételek között, amelyeket a Ramsey–Cass–Koopmans-modell (RCK-modell) írja le. Az aggregált kínálat és kereslet stacionárius egyensúlyi helyzetben van determinisztikus dinamikus általános egyensúlyi keretben. Az elemzés állandó jövedelem melletti fogyasztási kereslettel számol, amely megfelel a folyó jövedelemtől függő fogyasztás elméletének; és analitikusan bemutatja, hogyan egyesíthetők a különböző elméletek az RCK-modellben. Az állandó jövedelemet az idő- és jószágkészletek határozzák meg, és e kétféle készlet párhuzamos változásának logikája megegyezik a munkakínálat extenzív határának és az árutermelő szektor termelékenységének változásával. Ezek így megmagyarázzák az üzleti ciklusokat. Ilyen módon a stilizált aggregált kínálat és kereslet megmagyarázza az üzleti ciklusokat, és mindez konzisztens az üzleti ciklusok (real business cycle) és a fogyasztási elmélet szakirodalomának legfontosabb jellemzőivel.

Tárgyszavak: Ramsey–Cass–Koopmans-modell, kereslet és kínálat, statikus változók, a munkakínálat externális határa

JEL kódok: A22, A23, E13

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December 4, 2012

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1 Introduction

Lost in the transition from textbooks to research articles, the missing case of aggregate supply and aggregate demand ($AS - AD$) is clued for example by Colander (1995). He argues that common textbook aggregate supply and aggregate demand analysis is "incorrectly specified", lacks internal consistency and mixes analyses by combining a Keynesian demand with a classical supply curve. Also lost is a representation of real business cycle theory using $AS - AD$ within the standard dynamic general equilibrium model that is a cornerstone of modern macroeconomics. Meanwhile consumption theory at the heart of aggregate demand remains at odds because of differing neoclassical permanent income and Keynesian current income approaches.

Samuelson (1951) forms Keynesian demand using the Keynesian "cross". This is based upon consumption as a function of current expenditure with a constant and a slope less than one (Samuelson, p. 266). It remains a basis of the standard derivation of Keynesian consumption demand as a function of current income. And it underlies pathbreaking modern consumption research such as rule of thumb and credit constrained consumers. In contrast, Friedman (1957) offers the alternative of deriving a consumption demand that is a fraction of permanent income as based on Fisher's (1930) intertemporal analysis, also still a mainstay of current research including Campbell, and Mankiw (1987, 1990, 1991).

This paper sidesteps debate on textbook $AS-AD$ derivations. Instead it offers a single approach for deriving $AS - AD$ within the mainstream model of Ramsey (1928)-Cass (1965)-Koopmans (1965) (RCK). This approach is shown to be built upon a consumption function that is consistent equally with both a neoclassical and a Keynesian theoretic approach. The paper thereby exemplifies a "unification" of permanent income consumption theory with "current income" based consumption theory en route to building the aggregate output analysis. In particular the consumption theories are shown to be different perspectives of the same equilibrium within the model.

The $AS - AD$ analysis is conducted along the stationary equilibrium, or "balanced growth path" (BGP). It is not a trivial problem because of the so-called "state" variable of recursive economics, in this case the capital stock. The model is fully dynamic but the BGP capital stock is analytically solved and used for analysis. It is apparently also a non-trivial issue as to how aggregate supply and demand are important for modern research. Here it is suggested that use of $AS - AD$ as based on standard consumption theory gives a better view of the key role of the external labor margin in explaining business cycles, while perhaps invigorating real business cycle (RBC) analysis through a simple presentation of it.

Comparative statics are used to illustrate the basic facts of RBC analysis, aiming for a simply well-told story of basic business cycles. To do this, the analysis needs to pass the litmus "taste" test that one can use a stationary equilibrium model to examine business cycles in the first place. This qualification is met perhaps best by the "medium cycle" of Comin and Gertler (2006). They include three frequencies in their Medium Term cycle: the short run high frequency, the "business cycle" frequency and the longer run low frequency. Given how the economy adjusts to any parameter shock to a considerable extent during the medium term frequency, a fully deterministic comparative static change in parameters produces changes in equilibria that can characterize in a stylized way what happens in more complex stochastic economies over the medium term. This is the basis for the comparative static $AS - AD$ approach here: as a way to establish key features of the general equilibrium dynamic model in a way every economist can understand. This gives another, fundamental, tool in our box of building models that fit.

In addition to the neoclassical growth and business cycle focus on the goods sector productivity parameter (TFP), the key "second" parameter change for explaining basic RBC facts in this paper is one that captures the "external margin" of labor supply. For example, King and Rebelo (1999) focus on this as necessary for explaining real business cycles in the neoclassical framework. Many others have presented this margin as a key for frontier business cycle research. Here, in its simplest presentation, a Beckerian (1965) "allocation of time" approach allows consideration of how a change in the time endowment for work and leisure affects labor supply. In conjunction with a goods sector productivity change this leads to a simple business cycle story. The time allocation side thereby plays a key role in the $AS - AD$ explanation.[1]

Section 2 sets out the RCK model, Section 3 the consumption theory, and Section 4 the stylized explanation of business cycles as based on the baseline calibration. Section 5 presents the business cycle in terms of output market production possibility curves and input market isoquants and isocosts. Section 6 discusses the consumption theory in conjunction with the business cycle theory and Section 7 concludes.

2 Representative Agent Economy

Let the representative agent act as both firm and consumer. The firm rents capital k_t from the consumer at the real competitive rate r_t and pays wages for labor time l_t at the competitive rate w_t. The firm's production technology for goods output y_t is Cobb-Douglas with the productivity parameter $A \in R_+$

[1] The key role of the external margin follows the literature for example of Hansen (1984), Rogerson (1988), Benhabib et al (1991) and Greenwood et al. (1991).

output	consumption	leisure	labor	capital	wage	interest
y_t	c_t	x_t	l_t	k_t	w_t	r_t
utility	time	leisure pref.	time pref.	depreciation	labor share	discount rate
V	T	α	ρ	δ_k	γ	β

Table 1: Variable and Parameter Notation

and the labor share parameter $\gamma \in [0,1]$, whereby $y_t = A(l_t)^{\gamma}(k_t)^{1-\gamma}$. The firm profit Π_t maximization yields that the wage rate and capital rental rate equal their respective marginal products:

$$\begin{aligned} \underset{l_t,k_t}{Max}\, \Pi_t &= A(l_t)^{\gamma}(k_t)^{1-\gamma} - w_t l_t - r_t k_t; \\ w_t &= \gamma A(l_t)^{\gamma-1}(k_t)^{1-\gamma}, \quad (1) \\ r_t &= (1-\gamma) A(l_t)^{\gamma}(k_t)^{-\gamma}. \quad (2) \end{aligned}$$

The consumer's period t utility u is of log form in goods c_t and in leisure x_t, such that with the leisure preference parameter $\alpha \geq 0 : u(c_t, x_t) = \ln c_t + \alpha \ln x_t$. The consumer spends time working for the firm, l_t, and time in leisure, such that the total time endowment is equal to T :

$$T = l_t + x_t. \quad (3)$$

The consumer's goods budget constraint sets expenditure on consumption c_t equal to income from wages $w_t l_t$ and capital rental $r_t k_t$ minus investment in capital i_t. With $\delta_K \in [0,1]$ the depreciation rate on capital stock, i_t is given by $i_t = k_{t+1} - k_t(1-\delta_K)$. The goods constraint is then

$$c_t = w_t l_t + r_t k_t - k_{t+1} + k_t(1-\delta_K). \quad (4)$$

With $\beta \in (0,1) \equiv \frac{1}{1+\rho}$ being the discount factor, and $V(k_t)$ the recursive utility, the consumer maximization problem is

$$V(k_t) = \underset{l_t,k_{t+1}}{Max} \ln\left[w_t l_t + r_t k_t - k_{t+1} + k_t(1-\delta_K)\right] + \alpha \ln(T - l_t) + \beta V(k_{t+1}).$$

The equilibrium conditions reduce to the goods and time constraints plus the intratemporal and intertemporal margins, which are

$$\frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}} = w_t, \; \frac{c_{t+1}}{c_t} = \frac{1 + r_t - \delta_k}{1+\rho}. \quad (5)$$

Table 1 provides a summary of key variable and parameter definitions.

2.1 Aggregate Demand: AD

Use the allocation of time constraint to solve for labor supply $l_t = T - x_t$. Substitute in for $x_t = \frac{\alpha c_t}{w_t}$ from the intratemporal margin so that $l_t = T - \frac{\alpha c_t}{w_t}$. Then substitute in for l_t in the budget constraint: $c_t = w_t \left(T - \frac{\alpha c_t}{w_t}\right) - k_{t+1} + k_t \left(1 + r_t - \delta_K\right)$. Solving for consumption demand, $c_t = \frac{w_t T - k_{t+1} + k_t (1 + r_t - \delta_K)}{1+\alpha}$. Bringing together terms such that $c_t = \frac{w_t T + k_t \left(-\frac{k_{t+1}}{k_t} + 1 + r_t - \delta_k\right)}{1+\alpha}$, the BGP solution sees all non-stationary variables growing at the same rate, say g; the real interest rate r is constant along the BGP. Then $k_{t+1}/k_t = 1 + g$ and consumption demand is

$$c_t = \frac{w_t T + k_t \left(r - \delta_k - g\right)}{1 + \alpha}. \tag{6}$$

The intertemporal margin along the BGP implies that $1 + g = \frac{c_{t+1}}{c_t} = \frac{1+r-\delta_k}{1+\rho}$, so that $r - \delta_k - g = \rho\left(1 + g\right)$, and $c_t = \frac{w_t T + k_t \rho (1+g)}{1+\alpha}$.

In the baseline case, set the exogenous growth rate to zero, so that $g = 0$; then $r = \rho + \delta_k$, w is constant, and $c = \frac{wT + k\rho}{1+\alpha}$. Consumption can be considered a fraction of "permanent income". Here this is denoted by y_p, and if we use the standard concept of the permanent wage and rental flow from time and capital, we can view the consumption demand in this way: this definition and notation y_p is used only here to illustrate the implicit, well-known, permanent income hypothesis within the model.

$$y_p \equiv wT + \rho k; \tag{7}$$

$$c = \frac{1}{1+\alpha}\left(wT + \rho k\right) \equiv \frac{y_p}{1+\alpha}. \tag{8}$$

Two qualifications arise relative to Friedman (1957). First it is the Beckerian "full income" T that yields the flow on the time side of the endowment, at the rate of w. Second, it is the degree to which the consumer values leisure, given by α, that determines the fraction of "permanent income" consumed; Friedman made this fraction some exogenous constant while here it is a function of "structural" parameters (just α), as in the Lucas (1976) "critique" vein. This then gives a fair sense in which the permanent income hypothesis of consumption plays a central role within the deterministic RCK model.

Adding stationary investment demand means adding the maintenance of capital or $\delta_k k$ to consumption demand. This gives the BGP aggregate demand. For clarity of concept although at the risk of additional notation, this demand can be denoted by y^d, with aggregate supply denoted by y^s, and where in equilibrium $y^d = y^s = y$; such superscripts will also be added for the labor market. In the case with g assumed to be zero, all time subscripts can be dropped as all variables are stationary:

$$AD : y^d = \frac{1}{1+\alpha} (wT + \rho k) + \delta_k k. \tag{9}$$

The relative price of goods to leisure can be solved for so that a typical demand graph can ensue in price-quantity space, where the price is the relative price of goods to leisure:

$$\frac{1}{w} = \frac{T}{y^d (1+\alpha) - k [\rho + (1+\alpha) \delta_k]}. \tag{10}$$

Given k and the parameter values, a downward sloping demand (hyperbola) results. The solution to k can be found by using the $AS - AD$ analysis, in particular by setting AD equal to AS, although only after the AS analysis has been presented.

2.2 Aggregate Supply: AS

Aggregate supply of goods is derived from the firm's equilibrium conditions. From equation (1), labor demand is $l_t = \left(\frac{\gamma A}{w_t}\right)^{\frac{1}{1-\gamma}} k_t$. Substituting this l_t into the firm's production function $y_t = A (l_t)^{\gamma} (k_t)^{1-\gamma}$ gives the aggregate supply AS as a function of the relative price $\frac{1}{w_t}$ and the capital stock k_t. Again with $g = 0$, dropping time subscripts, and denoting aggregate supply as y^s :

$$AS : y^s = A^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} k. \tag{11}$$

Solving for the relative price along the BGP with zero growth,

$$\frac{1}{w} = \frac{1}{\gamma A^{\frac{1}{\gamma}}} \left(\frac{y^s}{k}\right)^{\frac{1-\gamma}{\gamma}}. \tag{12}$$

Given k and the parameter values, the supply slopes upward with convexity if $\gamma < 0.5$, with linearity if $\gamma = 0.5$ and with concavity if $\gamma > 0.5$. This means that a standard looking supply, in the sense of an increasing marginal cost as output expands, must have a labor intensity that is less than one-half. This would be consistent with the Mankiw, Romer and Weil (1992) estimation of the capital share near 0.60. Having an increasing marginal cost as output rises might even by viewed as an angle that adds support to the nature of their conclusions. Regardless, it is this approach taken in the calibration below in which $\gamma = \frac{1}{3}$ so as to yield a convex supply curve.[2]

Note that here as in microeconomics the price of goods is indeed the marginal cost of output. Let total cost be denoted by TC_t, where $TC_t = w_t l_t + r_t k_t$. From the production function of output, $l_t = \left(\frac{y_t}{A}\right)^{\frac{1}{\gamma}} (k_t)^{\frac{\gamma-1}{\gamma}}$. On the BGP, $r = \rho + \delta_k$, and k and w are known equilibrium values. Then the BGP total cost in terms of aggregate output is $TC = \frac{w}{A^{\frac{1}{\gamma}}} (y^s)^{\frac{1}{\gamma}} (k)^{\frac{\gamma-1}{\gamma}} + (\rho + \delta_k) k$. Taking the partial derivative with respect to y defines the standard marginal cost of goods (MC) is

$$MC = \frac{\partial (TC)}{\partial y^s} = \frac{w}{\gamma A^{\frac{1}{\gamma}} (k)^{\frac{1-\gamma}{\gamma}}} (y^s)^{\frac{1-\gamma}{\gamma}}. \tag{13}$$

With price equal to marginal cost, normalize the goods price to unity so that $MC = 1$. Make the price a relative one by dividing by the shadow price of leisure, the real wage: $\frac{1}{w} = \frac{(y^s)^{\frac{1-\gamma}{\gamma}}}{\gamma A^{\frac{1}{\gamma}} (k)^{\frac{1-\gamma}{\gamma}}}$ as in the AS equation (12) above. The only difference from standard microeconomics is that k is the endogenous stationary solution rather than an assumed fixed capital stock as in Varian (1978, p. 22).

2.3 Labor Market

With the BGP consumption demand $c = \frac{wT + \rho k}{1+\alpha}$, the intratemporal margin $x = \frac{\alpha c}{w}$, and the allocation of time constraint, labor supply is $l = T - \frac{\alpha}{1+\alpha}\left[T + \left(\frac{\rho}{w}\right) k\right]$ and labor demand is given from the firm's marginal product condition. The solution for the relative price of labor (leisure) to goods is the real wage divided by 1. Using the s and d superscripts for labor supply and demand, as solved for w,

$$w = \frac{\alpha \rho k}{T - (1+\alpha) l^s}, \tag{14}$$

$$w = \gamma A \left(\frac{k}{l^d}\right)^{1-\gamma}. \tag{15}$$

[2]The use of the Solow exogenous growth model and the implied shares of labor versus capital is contentious. Bernanke and Gürkaynak (2001) argue that the Mankiw, Romer and Weil (1992) results are consistent but in an endogenous growth approach. Note that one of the original versions of the so-called output gap arose from an AS curve drawn horizontally up until "full capacity" at which point it became vertical: this is a limiting case of the RCK convex marginal cost of output as $\gamma \to 0$ (and so $\gamma < 0.5$). An economy had an output gap if the presumed downward-sloping AD crossed this AS at a point less than full capacity.

3 BGP Consumption Theory

With goods market clearing along the BGP such that $y = y^d = y^s$, let the total quantity of goods demanded equal the total quantity of goods supplied:

$$y^d - y^s = \frac{wT + k\left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha} - A^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} k = 0 \qquad (16)$$

Eliminate the wage rate w and solve for k by using that $r = \rho + \delta_k$ and that the marginal product of capital is $r = (1-\gamma)\,A\left(\frac{l}{k}\right)^{\gamma}$. This gives the equilibrium input ratio $\frac{l}{k} = \left[\frac{\rho+\delta_k}{(1-\gamma)A}\right]^{\frac{1}{\gamma}}$, which can be substituted back into the firm's marginal product of labor $w = \gamma A\left(\frac{l}{k}\right)^{\gamma-1} = \gamma A\left[\frac{\rho+\delta_k}{(1-\gamma)A}\right]^{\frac{\gamma-1}{\gamma}}$. Substituting in this solution for w into equation (16), and solving for k, gives the explicit closed form solution for the capital stock, independent of time and the initial capital stock at time 0.

$$k = \frac{T\gamma A^{\frac{1}{\gamma}}\left[\frac{(1-\gamma)}{\rho+\delta_k}\right]^{\frac{1}{\gamma}}}{\gamma + \alpha - \alpha\delta_k\left(\frac{(1-\gamma)}{\rho+\delta_k}\right)}. \qquad (17)$$

Proposition 1 *Consumption c is a fraction BT of the real wage w, where $B \equiv \frac{\gamma\delta_k+\rho}{\gamma\delta_k+\rho\left(\gamma+\alpha+\frac{\gamma\alpha\delta_k}{\rho}\right)}$ and $c = w$ in the case with $T = \gamma+\alpha+\frac{\gamma\alpha\delta_k}{\rho} = 1$. At the same time, consumption is a fraction of permanent income, in particular $c = \frac{1}{1+\alpha}\left(wT + \rho k\right)$, with the flow income on (human capital) time and (physical capital) goods endowments (T and k) defining permanent income.*

Proof: This follows from using the BGP solution for the wage rate and capital stock and substituting these into the consumption demand function.

Analytically this implies that in general the consumption level c is a multiple B of the real value of the time endowment, or $c = BwT$. If $\gamma + \alpha + \frac{\gamma\alpha\delta_k}{\rho} = 1$, $B = 1$ and $c = wT$, as in the baseline calibration given in the next Section 3. For the case with zero leisure preference, $\alpha = 0$, $c = wT$ if $\gamma = 1$ so that there is no physical capital in production: $k = T\left[\frac{A(1-\gamma)}{\rho+\delta_k}\right]^{\frac{1}{\gamma}} = 0$.

Corollary 2 *Consumption in the RCK model can also be written as $c = a + bw$ with $b < 1$.*

Proof: With $c = \frac{wT+\rho k}{1+\alpha}$, then $c = a + bw$, where $a = \frac{\rho k}{1+\alpha}$ and $b = \frac{T}{1+\alpha}$. Given $T = 1$ as is commonly assumed, then $b = \frac{1}{1+\alpha} < 1$ given any positive leisure preference.

The consumption theory that underlies the RCK model is consistent with both permanent income views and the current income view. As a fraction of the flow of "human" and physical capital, consumption is given by the permanent income hypothesis. Taking the steady state capital as given in the constant term, consumption is the Keynesian consumption function with $b < 1$. The third view is that consumption, in the end with the analytic solution, is a simple fraction of the wage income, with a fraction that can be equal to one. This gives a consumption theory based on the permanent income hypothesis but which looks equally just like a theory based on current income, albeit this is the "full wage income" based on the time endowment as in Becker (1965).

The unified view makes sense by realizing that the time endowment is often set to 1, and so is not visible, while the capital stock is not a typical part of consumption estimation and so it is swept into the constant term. Within this unified view, the T is also similar to the "index of human capital" as in an extension to endogenous growth with human capital . This quality index analogy helps view the model in permanent income terms as a function of the flow of human and physical capital, even as seen in the exogenous growth world presented here. And this view leads to a "resolution" of how to explain the basic business cycle facts in the simple RCK model, made resonant by viewing this with $AS - AD$ analysis.[3].

4 Calibrated AS-AD with Business Cycle

With a baseline calibration the AS and AD can be graphed and comparative statics conducted. In particular, increasing the goods endowment for a given production function involves simply increasing the goods productivity parameter A, the key parameter change in the RBC revolution ushered in by Kydland and Prescott (1982). Here this causes output, the real wage, and the capital stock to rise but has no effect on the employment of labor as the income and substitution effects exactly offset each other. An increase in the time endowment T causes output, the capital stock and the employment to all rise, while the real wage stays constant. Combining such an increase in both goods and time endowments causes a business cycle type increase in output, the capital stock, the real wage and employment; this captures basic elements by which we describe an expansion in the business cycle. Decreasing both endowments mimics what we think of as a contraction, or downturn, in the business cycle. The following calibration and application

[3]See Gillman (2011) for the endogenous growth with human capital extension; changing the human capital investment sectoral productivity parameter leads to an endogenous change in the time T that is divided between work and leisure.

differs from Gillman (2011) by making the goods and labor endowments the same, at unity. This makes it easier to illustrate how the consumption theory is both a permanent income and current income theory, while underlying the $AS - AD$ analysis of RBC theory without conflict on this issue.

4.1 Calibration

Targeting a one-third investment to output ratio, leisure at 0.5 as in Gomme and Ruppert (2007), and with a standard range of annual parameter choices for developed countries such as the US, let $\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\rho = 0.03$, $T = 1$, $\delta_k = 0.03$ and $A = 0.1134$. Then $k = 1.00$, with $c = w = 0.06 = \frac{2}{3}y$; investment i is one-third of output $y = 0.09$, and employment is $l = 0.5$.

The AD and AS are given respectively as

$$\frac{1}{w} = \frac{1}{y^d (1 + 0.5) - (1) [0.03 + (1.5) 0.03]}, \tag{18}$$

$$\frac{1}{w} = \frac{(y^s)^2}{\frac{1}{3} (0.1134)^3 (1)^2}. \tag{19}$$

Similarly the labor market equations are given by

$$w = \frac{0.5 (0.03) (1)}{1 - (1.5) l^s}, \tag{20}$$

$$w = \frac{1}{3} (0.1134) \left(\frac{1}{l^d} \right)^{\frac{2}{3}}. \tag{21}$$

4.2 Comparative Statics of a Business Cycle

First consider the mainstream exercise in RBC (and Solow-growth) theory to see the well-known results. Let A rise 5% from (0.1134) to $(0.1134)(1.05) = 0.1191$ with no other parameter changes. The capital stock rises from one to $k = 1.1587$. This implies the permanent income sense along the BGP in which an increase in A leads to an increase in the goods endowment. The higher capital stock creates a subsequently higher permanent yield of interest to use for consumption, even as the share of consumption in output does not change.

Consumption and output, along with the real wage, rise by exactly the same 15.87% as does the capital stock. The supply and demand equations in the goods and labor markets become adjusted accordingly by the new higher A and k. Figure 1 graphs the baseline $AS - AD$ equations (18) and (19) plus these equations with A increased by 5% and with k subsequently rising. Both AD and AS shift outwards from the red baseline to the new

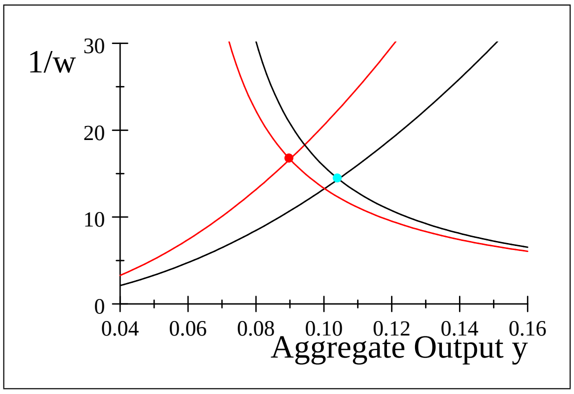

Figure 1: RCK $AS - AD$ with 5% Increase in A_G (in black) Compared to Baseline (in red).

black curves as a result of the goods sector productivity increase. The rise in k shifts out both curves directly while the increase in A acts to diminish the degree to which the AS shifts out. The AS shifts out by more than the AD curve and the relative price of goods $\frac{1}{w}$ falls as in Harberger's (1998) graph on economic growth, where he states that we should expect a falling relative price of output with such Solow type productivity increases. This relative price fall, if set through a continual A change, is what might be called a Harberber-Solow- growth fact". It appears to be consistent with certain "medium term" business cycle frequency-based evidence on the aggregate price.[4]

Figure 2 similarly graphs the baseline labor market of equations (20) and (21) when A rises by 5% and the wage rate rises. The employment does not change, reflecting the key challenge of the standard RBC model in which employment insufficiently changes relative to the data. Here the result is robust for homothetic utility and production.

In contrast, changing the time endowment instead of goods productivity causes employment to rise. From the baseline, a 5% increase in time endowment from 1 to 1.05 increases the capital stock by 5% to $k = 1.05$, and the labor employment by 5% to 0.525. The wage rate remains unchanged as it depends on the unchanged capital to labor ratio; or seeing this from the solution for the wage ($w = \gamma A \left[\frac{\rho+\delta_k}{(1-\gamma)A}\right]^{\frac{\gamma-1}{\gamma}}$), the wage does not depend on T. And since consumption is $c = BwT$ and B is not a function of T,

[4]den Haan and Sumner (2004) present evidence that comovement of aggregate prices is significantly negative at lower frequencies during the postwar period for G7 countries, using VAR forecast errors and frequency domain filters; Comin and Gertler (2006) include such lower frequencies in their "medium term" cycle.

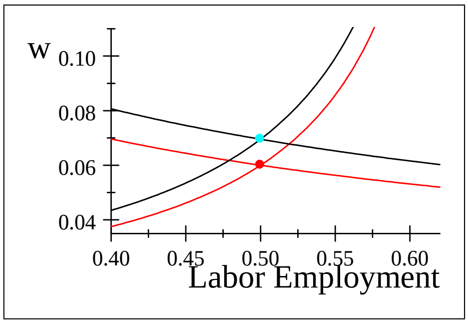

Figure 2: RCK Labor Market: 5% Increase in A_G (in black) Compared to Baseline (in red).

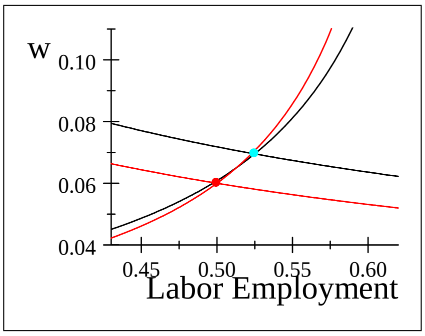

Figure 3: RCK Economic Expansion (in black) Compared to Baseline (in red).

consumption also rises by 5% as does output y.

In Figure 3, combining a 5% goods productivity and time endowment increase sees the same 5% rise in employment along with an increase in the wage as in a business cycle expansion. Consumption and output also rise with the $AS - AD$ graph being qualitatively similar to that of Figure 1. Figure 3 shows how labor supply twists slightly such that is appears to look like a movement up the labor supply curve. In fact the labor supply has shifted out from the time endowment increase while shifting back from the goods productivity increase, with the end result of what looks like a slight pivoting. Included in the graph, in this case of both A and T rising by 5%, the capital stock rises from one to $k = 1.2167$, close to a sum of the increases of 15.87% and 5% from each effect alone.

A similarly generated contraction occurs in the labor market when the goods and time endowment fall by 5%. The demand for labor shifts down while the supply for labor twists somewhat, causing employment to fall.

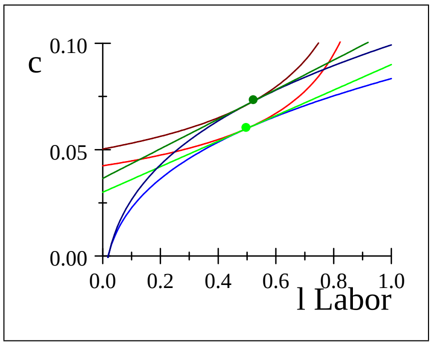

Figure 4: RCK PPC and Indifference Curves under Economic Expansion (upper curves) Compared to Baseline (lower curves).

5 RCK Output, Input Markets

Given the general equilibrium formulation of the $AS-AD$ analysis, the RCK model allows for exactly drawn "output space" production possibility curve output diagrams and "input space" isoquant-isocost diagrams. This allows for another view of the business cycle explanation through the two simultaneous comparative static changes in goods productivity and time endowment.

5.1 Indifference and Production Possibility Curves

Proceeding without the s and d superscripts in this part, and starting with the baseline calibration, consumption can be written in terms of the production function and the utility level, and graphed in (c, l) space: $c = y - i = A\,(l)^{\gamma}\,(k)^{1-\gamma} - \delta_k k$. Substituting in the baseline parameters, $k = 1$, and $A = 0.1134$, this gives a "production possibility curve" (PPC) of $c = 0.1134\,(l)^{\frac{1}{3}} - 0.03$. Similarly, the utility level can be found in the baseline as $\ln\left(0.1134\,(0.5)^{\frac{1}{3}} - 0.03\right) + 0.5 \ln 0.5 = -3.16$. Solving for c, the indifference curve is $c = \frac{e^{-3.16}}{(1-l)^{0.5}}$. The BGP budget line in equilibrium is $c = wl + \rho k = (\ 0.06)\,l + (\ 0.03)\,(\ 1)$.

For an economic expansion, add in both of the 5% increases in A and T, to 0.01191 and 1.05 respectively; c rises to 0.073, k to 1.2167, and l to 0.0525. Figure 4 graphs both the baseline and the expansion in terms of each of the three sets of lines: the PPC, indifference curve, and budget line. The lower set of curves are the baseline and the upper set is the expansion. The upper set shifts up such that employment rises.

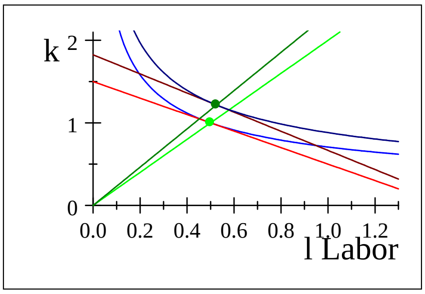

Figure 5: RCK Isoquants and Isocosts in Economic Expansion (upper curves) Compared to Baseline (lower curves).

5.2 Input Space: Isoquants and Isocosts

The isoquant curve is based on output, $y = A(l)^{\gamma}(k)^{1-\gamma}$, which for the baseline calibration is written as $0.09 = 0.1134(k)^{\frac{2}{3}}(l)^{-\frac{1}{3}}$. Solving for k gives the isoquant curve $k = (0.707)\, l^{-0.5}$. The isocost line is $y = wl + rk$, which in baseline is $0.09 = 0.06l + (0.06)\, k$; solving for k gives the baseline isocost line of $k = 1.5 - l$. The factor input ratio is $\frac{k}{l} = \frac{1}{0.50} = 2$. Figure 5 draws the lower set of baseline input space curves plus the upper set for the economic expansion, with A and T rising by 5% from the baseline values. It shows how employment rises with the capital-labor ratio rising, the isocost and isoquant shifting up, and a higher wage causing a steeper sloped isocost.

6 Consumption Theory and Business Cycles

Consider the consumption theory outlined in Proposition 1 and Corollary 2, which takes us back to the original competing theories of consumption based on Keynes (1930,1936)-Samuleson (1951) and Friedman (1957). Further let us consider them in light of business cycle analysis as Keynes did in his Treatise. There Gillman (2002) posits how Keynes (1930) presented a mathematical theory of the price level based on Marshallian fundaments which became the basis of the Keynesian cross and IS-LM analysis in subsequent work. And in particular Keynes (1930) uses this theory with consumption to tell a business cycle story in terms of expansion when investment exceeds savings and contraction in the converse. In the RCK model above investment equals savings along the BGP. But what is known as Keynes's consumption theory can be well analyized in Samuelson graphical terms using the comparative static changes associated with modern business cycle theory.

Figure 6 shows the consumption function of the baseline calibration $c = w$ as graphed in (c, w) space. This gives the 45% line of the diagram. In addition, the equivalent form of the function $c = a + bw$ is also graphed, with $a = \frac{\rho k}{1+\alpha} = \frac{0.03(1)}{1+0.5}$ and $b = \frac{T}{1+a} = \frac{1}{1+0.5} < 0$. Thus with the baseline we have reproduced the well-viewed idea of the equilibrium consumption being determined at the intersection of the 45% line and the upward sloping consumption function with a positive constant and a slope less than one.

Now add in the business cycle comparative statics. First let A rise 5% as in the Section above from the baseline value to $A = 0.1191$, as in a change in the goods sector TFP that underlies modern analysis. This does not affect either the 45% line or the slope coefficient b of the $c = a + bw$ representation of consumption. However, the capital stock k rises to 1.1587, causing the vertical-axis intercept to increase, and causing in turn the consumption function in the $c = a + bw$ form to rise to $c = a' + bw$ and shift upwards in a parallel slope-preserving fashion. This is represented by the blue solid line, with the new equilibrium at $c = w = 0.69523$.

In Keynesian parlance the shift upwards might be accomplished by the government turning unused savings during a deep banking crisis and recession into investment by government spending on for example infrastructure. But here the shift up is simply due to the goods sector productivity rising as in a business cycle expansion with the result that k increases. A key part of this resulting expansion-based shift up in the consumption function is that the "propensity to consume" out of "current income" w, in the $c = a' + bw$ form, does not change. This "propensity is given by the parameter b which is constant since neither A not k affects it endogenously. This slope parameter being unchanged means that this definition of the "propensity to consume" does not change either in the expansion or contraction, which could be viewed as inconsistent with notions of rule of thumb or credit constrained consumers in this growing new dynamic literature examining consumption behavior.

Second consider the experiment outlined above of increasing both A and T by 5% so as to give the salient facts of a basic business cycle. This results in a again rising as the capital stock rises from the baseline value of one to 1.2167. And the difference from just changing A alone is that the slope parameter b also increases, because of the increase in T, representing the external labor margin (or human capital index if one prefers). The slope goes from $b = \frac{1}{1+0.5}$ to $b' = \frac{1.05}{1+0.5}$ which is still well below one. In Figure 6 the green solid line graphs this representation of the business cycle expansion. The slope has become steeper while the function has shifted up a bit more due to the higher capital stock. This is a key difference because the combination of the "goods and time endowment" changes, as one might characterize the comparative static combination, causes a larger percent of "current income" seemingly to be consumed during an expansion. Conversely a smaller percent would be consumed during a contraction, for example as in a credit constrained outcome.

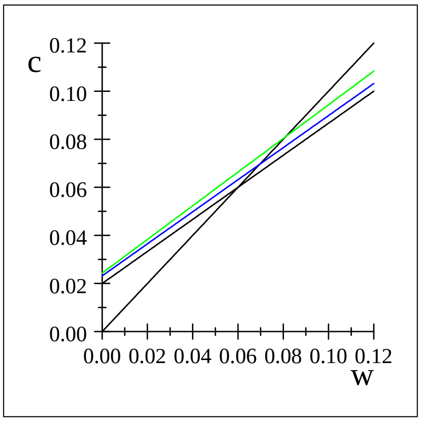

Figure 6: RCK Consumption Theory and Busines Cycles: Change in A and in T

Thus the consumption theory of RCK and the business cycle theory presented here within RCK creates a view of the consumption function consistent with Keynesian notions even while it is equally viewed in permanent income terms. There is no inconsistency in the views and further the unification of these views within this business cycle explanation is consistent with modern theory behind why the parameter b might rise and fall with the business cycle. The dynamic recursive nature of the RCK model in turn gives an endogenous explanation of why the constant a rises and falls with the business cycle, in particular because the state variable capital stock rises and falls with the business cycle.

7 Conclusion

The paper derives aggregate supply and demand in the Ramsey framework from the basis of the RCK consumption function. Equilibrium consumption demand is shown to be consistent with both a permanent income hypothesis as well as a Keynesian consumption theory. Motivated by the permanent income approach, a business cycle expansion is explained by combining two comparative static changes, in the goods sector productivity and in the time endowment. This reproduces the most basic salient facts of a business cycle expansion and contraction. The results are graphed both in goods $(AS - AD)$ and labor markets separately and jointly within output and input spaces. The distinction between microeconomics and macroeconomics is minimal given the representative agent convention. While Stiglitz (2012) argues against exactly such "Standard Models" on the basis that they cannot explain the recent bank crisis, the $AS - AD$ RCK model here forms a basis for extension in many directions. A considerable literature now involves the inclusion of banking. Such an extension to this paper's framework is possible, for example, with intermediation of savings and investment through a bank sector and with a large decrease in bank productivity causing a large capital stock and employment decrease as in a bank crisis.[5]

Extension of the model to Uzawa (1965)-Lucas (1988) endogenous growth can make endogenous the time allocated to work and leisure because of the additional allocation of time to human capital accumulation (and including research and development time, viewing this approach more broadly). Changing the human capital investment sector productivity parameter causes the time left for work and leisure (T) to change endogenously.[6]

Using a consumption theory that is consistent with both "opposing" neoclassical and Keynesian views helps ease $AS - AD$ back into acceptance within the foundation research model of macroeconomics. Aiming here to show power in simple RBC theory, more generally this approach may be important both for clarifying research investigations in various extensions of the model and for teaching. It would be challenging to show how the $AS - AD$ is reconfigured as a result of the many additional features that have been used in dynamic macroeconomic research. And it might prove rewarding to illustrate the effect of these extensions using aggregate supply and demand.

References

Becker, Gary S., 1965, "A Theory of the Allocation of Time," The Economic Journal, 75(299, September): 493-517.

Benhabib, Jess, Richard Rogerson, and Randall Wright, 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," Journal of Political Economy, 99(6, December): 1166-87.

Bernanke, Ben S. and Refet S. Gürkaynak, 2001. "Is Growth Exogenous? Taking Mankiw, Romer and Weil Seriously," NBER Working Papers 8365.

Campbell, J.C., and N. G. Mankiw, 1987, "Are output fluctuations transitory?". Quarterly Journal of Economics 102, 857-880.

Campbell, J.C., and N. G. Mankiw, 1990. "Permanent income, current income, and consumption". Journal of Business and Economic Statistics, Vol. 8, No. 3, 265-279.

Campbell, J.C., and N. G. Mankiw, 1991. "The response of consumption to income. A cross-country investigation". European Economic Review 35, 723-767.

[5] See Gillman (2011).

[6] Dang et al. (2011) follow this approach in a stochastic setting to help explain central *RBC* puzzles as closely related to Maffezzoli's (2000) approach to international *RBC* puzzles.

Cass, David, 1965, "Optimum Growth in an Aggregative Model of Capital Accumulation", Review of Economic Studies, 37 (3, July): 233–240.

Colander, David, 1995. "The Stories We Tell: A Reconsideration of AS/AD Analysis," Journal of Economic Perspectives, 9(3, Summer): 169-88.

Comin, D and M. Gertler (2006), "Medium-term Business Cycles, American Economic Review, 96(3): 523-551.

Dang, Jing, Max Gillman, Michal Kejak, 2011. "Real Business Cycles with a Human Capital Investment Sector and Endogenous Growth: Persistence, Volatility and Labor Puzzles," Cardiff Economics Working Papers E2011/8.

den Haan, Wouter J. and Steven W. Sumner, 2004. "The comovement between real activity and prices in the G7," European Economic Review, 48(6, December): 1333-1347.

Fisher, Irving, 1930. The Theory of Interest The Macmillan Company, New York.

Friedman, Milton, 1957. "A Theory of the Consumption Function," NBER Books, National Bureau of Economic Research, Inc.

Gillman, Max, 2002, "On Keynes's Theory of the Aggregate Price Level in the Treatise: Any Help for Modern Aggregate Analysis?," European Journal of the History of Economic Thought, Vol.9, No.3 (September): 430-451.

Gillman, Max, 2011, Advanced Modern Macroeconomics, Pearson Financial Times, London.

Gomme, Paul and Rupert, Peter, 2007. "Theory, measurement and calibration of macroeconomic models," Journal of Monetary Economics, 54(2, March): 460-497.

Greenwood, Jeremy and Zvi Hercowitz„ 1991. "The Allocation of Capital and Time over the Business Cycle," Journal of Political Economy, 99(6, December): 1188-1214.

Hansen, Gary D., 1985. "Indivisible labor and the business cycle," Journal of Monetary Economics, vol. 16(3, November): 309-327.

Harberger, Arnold C. 1998, "A Vision of the Growth Process." American Economic Review, 88(1): 1-32.

Keynes, John Maynard, 1930, A Treatise on Money, Macmillan and Co., Limited, London.

Keynes, John Maynard, 1936, The General Theory of Employment, Interest and Money, Macmillan and Cambridge University Press.

King, Robert G. and Rebelo, Sergio T., 1999. "Resuscitating real business cycles," Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, vol. 1, chapt. 14, pp. 927-1007, Elsevier.

Koopmans, T. C., 1965, "On the Concept of Optimal Economic Growth", in The Econometric Approach to Development Planning, Pontif. Acad. Sc. Scripta Varia 28: 225-300.

Kydland, Finn E and Edward C. Prescott, 1982. "Time to Build and Aggregate Fluctuations," Econometrica, vol. 50(6, November): 1345-70.

Lucas, Robert E. Jr., 1976. "Econometric policy evaluation: A critique," Carnegie-Rochester Conference Series on Public Policy, 1(1, January): 19-46.

Lucas, Robert E. Jr., 1988. "On the mechanics of economic development," Journal of Monetary Economics, vol. 22(1, July): 3-42.

Maffezzoli, Marco, 2000. "Human Capital and International Real Business Cycles", Review of Economic Dynamics 3: 137-165.

Mankiw, N Gregory, Romer, David and Weil, David N, 1992. "A Contribution to the Empirics of Economic Growth," The Quarterly Journal of Economics, 107(2, May): 407-37.

Ramsey F.P., 1928, "A Mathematical Theory of Saving," Economic Journal, 38(152): 543-559.

Rogerson, Richard, 1988. "Indivisible labor, lotteries and equilibrium," Journal of Monetary Economics, 21(1, January): 3-16.

Samuelson, Paul A., 1951, Economics: An Introductory Analysis, Second Edition, McGraw-Hill Publishing Company Ltd, London.

Solow, Robert, 1956, "A Contribution to the Theory of Economic Growth," The Quarterly Journal of Economics, 70(1,February): 65-94.

Stiglitz, Joseph E., 2012, "Rethinking Macroeconomics: What Failed and How to Repair it", Journal of the European Economics Association, 9(4,August): 591-645.

Uzawa,.Hirofumi, 1965, "Optimal Technical Change in An Aggregative Model of Economic Growth", International Economic Review, 6(1, January): 18-31.

Varian, Hal R., 1978, Microeconomic Analysis, W. W. Norton & Company Inc., New York.