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Optimal linear redistributive tax and pension systems with flexible labor supply

ANDRÁS SIMONOVITS

INSTITUTE OF ECONOMICS, RESEARCH CENTRE FOR ECONOMIC AND REGIONAL STUDIES,
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Author:

András Simonovits
research advisor
Institute of Economics
Research Centre for Economic and Regional Studies
Hungarian Academy of Sciences
also Institute of Mathematics
Budapest University of Technology and Economics
also Department of Economics, CEU
Email: simonovits.andras@krtk.mta.hu

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Optimal linear redistributive tax and pension systems with flexible labor supply

András Simonovits

Abstract

The tax system redistributes labor incomes among workers, the pension system redistributes incomes from workers to pensioners. We consider a linear transfer system, where workers pay pension contributions and personal income taxes and pensioners receive proportional benefits, while workers and pensioners enjoy basic income. Every worker maximizes his discounted lifetime utility function, depending on young- and old-age consumption plus leisure. The government chooses a transfer system which maximizes the undiscounted social welfare function. Our major result is as follows: The optimal transfer system balances the efficiency of proportional pensions and the guarantee of the basic income.

Keywords: personal income tax, pension system, optimal redistribution

JEL classification: H24, I31, J22, J26

Optimális lineáris adó- és nyugdíjrendszer rugalmas munkakínálat esetén

Simonovits András

Összefoglaló

Az adórendszer újraelosztja a dolgozók között a munkajövedelmeket, a nyugdíjrendszer pedig a nyugdíjasok között a nyugdíjjövedelmeket. Az egyszerűség kedvéért lineáris transzferrendszert vizsgálunk, amelyben a befizetett nyugdíjjárulékért és személyi jövedelemadóért cserébe az idősek keresetarányos nyugdíjat, a dolgozók és a nyugdíjasok pedig egyaránt alapjövedelmet kapnak. A munkakínálat maximalizálja a dolgozó leszámítolt életpálya-hasznosságát, amely a fiatal- és időskori fogyasztás mellett a szabadidőtől is függ. A kormányzat olyan transzferkulcsot és alapjövedelmet választ, amely maximalizálja a leszámítolásmentes társadalmi jóléti függvényt. Fő eredményünk: e transzferrendszer egyensúlyt teremt a keresetarányos nyugdíjrendszer hatékonysága, valamint az alapjövedelem szerinti újraelosztás között.

Tárgyszavak: személyi jövedelemadó, nyugdíjrendszer, optimális újraelosztás

JEL kódok: H24, I31, J22, J26

Optimal linear redistributive tax and pension systems with flexible labor supply*

András Simonovits
Institute of Economics, CERS Hungarian Academy of Sciences
also Mathematical Institute of Budapest University of Technology,
and Department of Economics of CEU
Budapest, Budaörsi út 45, Hungary
e-mail: simonovits.andras@krtk.mta.hu, telephone: 36-1-325-5582, fax: 36-1-319-3631

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Abstract

The tax system redistributes labor incomes among workers, the pension system redistributes incomes from workers to pensioners. We consider a linear transfer system, where workers pay pension contributions and personal income taxes and pensioners receive proportional benefits, while workers and pensioners enjoy basic income. Every worker maximizes his discounted lifetime utility function, depending on young- and old-age consumption plus leisure. The government chooses a transfer system which maximizes the undiscounted social welfare function. Our major result is as follows: The optimal transfer system balances the efficiency of proportional pensions and the guarantee of the basic income.

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1. Introduction

Personal income taxation and public pensions are two important pillars of the modern welfare states. For example, in the simplest personal income tax system, every worker pays a tax proportionally to his earning and receives the same basic income. A proportional (contributive) pension is a mandatory life-cycle saving tool, while the flat part can be considered as an extension of the personal income tax from the workers to the pensioners. (The only difference is that at least part of the pension contributions are capped.) These two pillars interact in a number of ways. The study of these interactions is a demanding but important task.

Nowadays, there are a lot of complex models studying this interaction (cf. Auerbach–Kotlikoff, 1987 and Fehr–Habermann, 2008). In this paper, in contrast, we analyze a quite simple model, concentrating on the most important qualitative features of the interaction. Our model is easy to program and yields numerical answers to a number of major questions. At the same time, we neglect many salient issues: for example, private savings, the fine tuning of entry pensions, the indexation of ongoing pensions, the tax credit for earnings and other pensions and the redistribution between generations.

Our starting point is Feldstein (1987), who built a very simple two-period OLG model, where the young work and the old are retired, maximizing discounted Cobb–Douglas utility functions. A significant part of the workers are shortsighted and voluntarily would not save enough for their old age. There is a government, which operates a mandatory public pension system, to maximize a paternalistic (undiscounted) social welfare function. Feldstein concentrated on the choice between a universal flat and means-tested system, therefore he neglected wage differences and flexible labor supply. Maximizing a utilitarian social welfare function, he showed that generally the means-tested system is socially superior to the flat system.

Cremer–De Donder–Maldonaldo–Pestieau (2008) introduced wage heterogeneity and flexible labor supply into Feldstein's model, and complemented the flat benefits with proportional ones (cf. Disney, 2004). Discussing the two cases of credit constrained and free credit life-cycle systems, and applying more general individual utility and social welfare functions, they determined the optimal redistribution within the pension system.

In the present paper we try to combine and enrich the two models in a new way. We neglect private savings (i.e. we assume that they are hardly more efficient than the mandatory pensions) but treat the labor disutility without the quasilinearity assumption of Cremer et al. We adopt the heterogeneity of discount factors but assume they are dependent on rather than independent of the wage rates. Finally, we introduce a linear personal income tax system which is unified with the pension system and extend the pensioners' flat benefits to the workers as well.

Our major results are as follows: Qualitatively, the socially optimal system balances the efficiency advantages of proportional pensions and the redistributive advantages of a basic income, given to workers and pensioners alike. We risk the following conjecture: pure flat benefits are socially not optimal. Turning to the quantitative dimensions, we observe that the socially optimal tax-and-pension rate (for short: transfer rate) is an increasing function of three key parameters: pre-tax wage inequality, the ratio of the retirement period's length to the working period's and the discount factor (note, however, that higher discount factor means more farsighted workers). Note, however, that below a critical value of the wage rate inequality, there is no optimal pure tax at all. (There is a reassuring exception: when individuals work until death, the critical value is 1, i.e. however small wage rate differences exist, in the optimum, there is some income redistribution!)

We only give a single numerical example in the Introduction: to have the same social welfare, one must raise the wage rates uniformly by 3 and 5 percents in the suboptimal proportional and the flat systems as in the optimal mixed transfer system. (Note that by assumption, the value of the labor disutility parameter is independent of age.)

Note that the foregoing models (Feldstein, Cremer et al. and the present one) neglected the much subtler earning and pension credit systems (Sefton–van de Ven–Weale, 2008 and Simonovits, 2012), skipped over the insurance provided by social security (Varian, 1980) and overlooked underreporting earnings (Simonovits, 2009). Though these issues are important, their analysis would make the model analytically impractical.

The structure of the remaining part is as follows. Section 2 presents and analyzes the model. Section 3 numerically illustrates the model. Section 4 draws the conclusions.

2. Model

We distinguish the micro and macro sides of the model. Quantities first appearing in the model are positive (or zero) real numbers. It is assumed that the population as well as the economy is stationary and there is no inflation.

Micromodel

Let w be the total wage rate cost (or full compensation) of a certain type of workers, T is his time limit, l is his labor supply: $0 < l < T$ and wl is his earning. It is assumed that every individual works for a time period with unitary length and spends another period of length μ in retirement, $0 < \mu \le 1$. Let t be the tax-and-pension rate and βwl be the proportional (or contributive) part, β being the accrual rate. Every worker and pensioner receive the same basic income $\gamma \ge 0$ per period. There is no private saving in this model. Introducing $\bar{t} = 1 - t$, the worker and pensioner consumption (intensities) are respectively

$$c = \gamma + \bar{t}wl \quad \text{and} \quad d = \gamma + \beta wl. \tag{1}$$

We turn to the determination of the optimal labor supply. Let ξ be the ratio of the utilities of leisure and of young-age consumption, and let δ be the discount factor, $0 < \delta \le 1$. Then the lifetime utility of the individual with wage rate w and discount factor δ is

$$U(w, \delta, c, l, d) = \log c + \xi \log(T - l) + \mu\delta \log d. \tag{2}$$

Substituting the formula (1) of the consumption pair into the lifetime utility function (2), we arrive at a reduced utility function with two individual parameters w and δ and one individual variable l:

$$u(w, \delta, l) = \log(\gamma + \bar{t}wl) + \xi \log(T - l) + \delta\mu \log(\gamma + \beta wl), \qquad \bar{t} = 1 - t. \tag{3}$$

Note that economically it is more suitable to assume that in the labor disutility, the leisure should be multiplied by the wage rate. Since $\log[w(T-l)] = \log w + \log(T-l)$, the optimal decision is not affected but this difference should be taken into account in welfare comparisons below.

Due to the strict concavity of u, the optimal labor supply is given by the root of the marginal utility–labor supply function:

$$0 = u'_l(w,\delta,l) = \frac{\bar{t}w}{\gamma+\bar{t}wl} - \frac{\xi}{T-l} + \frac{\delta\mu\beta w}{\gamma+\beta wl}. \tag{4}$$

This yields a quadratic equation

$$a_2 l^2 + a_1 l + a_0 = 0,$$

where

$$a_2 = -(1+\xi+\delta\mu)\bar{t}\beta w^2,$$
$$a_1 = \bar{t}w(T\beta w - \gamma) - \xi w(\gamma\beta + \bar{t}\gamma) + \delta\mu\beta w(\bar{t}wT - \gamma),$$

and

$$a_0 = \bar{t}wT\gamma - \xi\gamma^2 + \delta\mu\beta w\gamma T$$

Obviously, the larger root gives the optimum.

Because of importance and simplicity, it is worthwhile considering two special cases: (i) the *proportional* pension (P) without personal income tax ($\gamma = 0$) and (ii) the flat income and pension benefit, defined by $\beta = 0$. In both cases, the quadratic equation simplifies to a linear one, moreover, in case (i), the wage and pension parameters disappear. The corresponding labor supplies are respectively

$$l_0^P = \frac{T}{1+\xi} < l^P = \frac{(1+\delta\mu)T}{1+\xi+\delta\mu} < T \quad \text{and} \quad 0 < l_w^F = \frac{T - \xi\gamma/(\bar{t}w)}{1+\xi} < l_0^P, \tag{5}$$

where l_0^P is the optimal labor supply of Feldstein's myopes with $\delta = 0$. This number is a lower bound on the optimal labor supply in the proportional system and the upper bound of that in the flat system. To have a positive labor supply in the flat system, one must assume that

$$\bar{t}wT > \xi\gamma. \tag{6}$$

In general, the optimal labor supply (hence the net earning) is a complex function of the parameter values, therefore we shall speak of inequality of wage-rates rather than wages.

Here we use the method of comparative static, and determine the dependence of labor supply on various parameters. Let p be an arbitrary parameter of the model, then

$$u'_l[p,l] = 0. \tag{4p}$$

Applying the implicit function theorem, while excluding the degenerate case $u''_{ll}[p,l] = 0$, the function $l(p)$ exists and is smooth, its derivative is given by

$$l'(p) = -\frac{u''_{lp}[p,l]}{u''_{ll}[p,l]}.$$

Because of the sufficient condition of optimality, $u''_{ll}[p,l] < 0$, therefore

$$\mathrm{sgn} l'(p) = \mathrm{sgn} u''_{lp}[p,l].$$

Assuming $\gamma > 0$ and taking the partial derivative of ($4p$) with respect to p, gives the signs. The optimal labor supply is a decreasing function of the transfer rate t, and the basic income γ, while is an increasing function of the wage rate w, of the length ratio μ, of the discount factor δ and of the accrual rate β.

Macromodel

Until now we have considered a single type, with a wage rate w and a discount factor δ. Now we turn to the variety of types. Except for a single numerical example, it is also assumed that types only differ in wage rates and discount factors, consequently, in labor supplies, but they retire at the same age, and die at the same (but later) age, independently of w. (In reality, the higher the wage rate, the lower is the labor disutility, the higher is the discount factor and the longer is life span, but we skip over these heterogeneities in general. Also we know of individuals with low wage rate and high discount factor and vice versa but we neglect them.) Denoting by $\mathbf{E}$ the expected value operator of the wage distribution, the balance equation of the transfer system is

$$t\mathbf{E}(wl) = (1+\mu)\gamma + \mu\beta\mathbf{E}(wl), \quad \text{i.e.} \quad \gamma = (1+\mu)^{-1}(t-\mu\beta)\mathbf{E}(wl), \qquad t \geq \mu\beta. \quad (7)$$

Choosing an appropriate unit of measurement, the average wage rate can be taken unity: $\mathbf{E}(w) = 1$.

Note that we have a *general equilibrium* model, where the type-specific labor supply in (4) depends on the parameter-triple (t, β, γ), and in turn, the balance condition (7) depends on the labor supplies. In the case of the proportional pension system without taxation, the balance condition (7) is trivial: $t^P = \mu\beta$. The condition is not too complicated with the flat system ($\beta = 0$), either. Substituting the type-specific labor supply l_w [(5b)] into the balance condition (7), imply $(1+\mu)\gamma = t\mathbf{E}(wl)$. Using $\mathbf{E}(w) = 1$, the following fixed-point equation is obtained:

$$(1+\mu)\gamma = t\frac{T}{1+\xi} - t\frac{\gamma}{(1+\xi)\bar{t}}. \qquad (8)$$

With rearrangement, the basic income is equal to

$$\gamma^F(t) = \frac{T}{(1+\mu)(1+\xi)t^{-1} + \xi\bar{t}^{-1}}. \qquad (9)$$

Using notation $\nu = (1+\xi)(1+\mu)$, the maximal value of the basic income is achieved at

$$0 < t_M = \frac{\sqrt{\nu}}{\sqrt{\nu}+\sqrt{\mu}} < 1. \qquad (10)$$

Substituting $\gamma^F(t)$ [(9)] into condition (6) ($l_w > 0$) yields the minimal feasible wage:

$$w > \frac{\xi}{\nu t^{-1}\bar{t} + \xi}, \quad \text{or} \quad w > \frac{1}{\sqrt{\nu/\xi} + 1} \qquad (t = 1/2).$$

To determine the socially optimal system, a *paternalistic* social welfare function (cf. Feldstein, 1987) is applied, where δ is replaced by 1 in the individual utility functions:

$$V(t,\beta) = \mathbf{E}U(w,1,c,l,d) = \mathbf{E}[\log c + \xi \log(T-l) + \mu \log d], \tag{11}$$

where γ is balanced [(7)].

Considering a proportional pension system ($\gamma = 0$) and using the balance condition $\beta = \mu^{-1}t$, the optimal transfer rate is equal to

$$t_P^* = \frac{\mu}{1+\mu}. \tag{12}$$

Note that here the optimal accrual rate is $\beta_P^* = 1/(1+\mu)$. Sometimes the conversion of gross into net accrual rate makes the results clearer: here $\beta_P^n = 1$.

In fact, dropping the superfluous constants, the indirect utility function is equal to

$$U[t] = \log(1-t) + \mu \log(t) \to \max.,$$

To find the optimal transfer rate, one solves

$$U'[t] = -\frac{1}{1-t} + \frac{\mu}{t} = 0.$$

Then the optimal transfer rate is indeed τ_P^* and the corresponding consumption pair are

$$c_P^* = d_P^* = \frac{w}{1+\mu}.$$

Turning to the flat system, inserting $\gamma^F(t)$ [(9)] into $V(t,0)$ [(11)] yields a complicated function whose local maximum cannot be simply determined. We repeat the conjecture mentioned in the Introduction: pure flat benefits are socially not optimal.

3. Numerical illustration

Apart from the two pure systems, our formulas are too complicated to be used in analytical investigations. Therefore we must rely on numerical illustrations.

For the sake of simplicity, we shall only distinguish two types: the lower paid (L) and the higher paid (H), who are myopes and farsighted, respectively. Their frequencies are $f_L = 2/3$ and $f_H = 1/3$, and in the base runs, we shall consider a wage-rate profile with a strong wage inequality: $w_L = 0.5$ and $w_H = 2$. We choose the ratio of the length of the work period (40 years) to that of the retirement period (20 years), for short, the *length ratio* as $\mu = 1/2$. The corresponding discount factors are $\delta_L = 0.4$ and $\delta_L = 0.7$. Having 30 years between the centers of the labor and pensioner stages, the annual discount factors are approximately 0.970 and 0.987, respectively. To have simple numbers, after some experimentation, $T = 2$ and $\xi = 1.5$ are chosen. Here the maximal basic income is achieved at $t_M = 0.613$ [(10)], $\gamma_M = 0.2$, and $l_L > 0$ of (6) is equivalent to $w_L > 0.387$.

To satisfy the balance condition in the general case, it is worthwhile starting from a pair (t, β) such that $t \geq \mu\beta$, leaving room for the basic income. To determine the balancing value γ, we shall use the method of successive approximation of (7b). Starting with $\mathbf{E}(wl_0) = 1$, the first approximation yields $\gamma_0 = (1+\mu)^{-1}(t - \mu\beta)$. In iteration m, the wage-dependent optimal labor supply $l(\gamma_{m-1}, w)$ is determined as a function of γ_{m-1} and determine the new approximation of the basic income $\gamma_m = (1+\mu)^{-1}(t - \mu\beta)\mathbf{E}(wl(\gamma_{m-1}, w))$, $m = 1, 2, \ldots$. In our experiments, the convergence $\gamma_m \to \gamma$ is very fast.

The aggregate characteristics of socially optimal proportional, flat and general systems are displayed in Table 1, while the type-specific data are displayed in Table 2, respectively. Only the relative efficiency needs some explanation: to compare the two values of the social welfare function has no economic meaning, therefore we use a roundabout method. Denote $V_X(e)$ the value of the social welfare function of system X when the original wage rates are uniformly multiplied by a positive scalar e. If $V_A(1) < V_B(1)$, then there is generally a scalar $e > 1$ such that $V_A(e) = V_B(1)$ and we call $1/e$ the relative efficiency of system A in terms of system B. In fact, we shall calculate the efficiency of P and F in terms of M.

Table 1. *Optimal characteristics in three systems*

Type	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Relative efficiency $1/e$
Proportional (P)	0.33	0.000	0.66	0.928	0.97
Flat (F)	0.49	0.189	0.00	0.578	0.95
Mixed (M)	0.52	0.158	0.40	0.741	1.00

Remark: $w_H/w_L = 4$, $\delta_L = 0.4$ and $\delta_H = 0.7$.

Table 2. *Individual optima in the three systems*

Type	Low labor supply l_L	Low worker consumption c_L	Low pensioner consumption d_L	High labor supply l_H	High worker consumption c_H	High pensioner consumption d_H
Proportional	0.889	0.296	0.296	0.947	1.264	1.262
Flat	0.356	0.280	0.189	0.689	0.892	0.189
Mixed	0.512	0.281	0.260	0.856	0.980	0.843

We have already analytically determined the socially suboptimal *proportional* transfer (contribution) rate: $\tau = 1/3$. Apart from rounding-off errors, this achieves the equality of young- and old-age consumption, for both types (Table 2, row 1). The "only" shortcoming of this system is the lack of redistribution, the original wage differences remain.

The socially suboptimal flat system (Table 2, row 2) suffers from another mistake: it eliminates any differences in the old-age consumption, giving only one sixth of the high-earner consumption achieved in the proportional system. At this point the lack of private saving becomes unacceptable! Small wonder that charging so high a transfer rate, the aggregate wage (and labor supply) is very low, especially of the lower-earners. It is noteworthy that the low pension benefit is much lower than in its proportional counterpart: $0.19 < 0.3$. In fact, the main gain is more leisure rather than more consumption: $0.89 > 0.36$.

The socially optimal solution is a good combination of the two suboptima (Table 2, row 3). On the one hand, it preserves the high transfer rate of the flat system, but paying 2/3 of the proportional system's pension, thus it does not destroy the labor supply incentives, and pays acceptable incomes to everybody in both periods. Finally, we list the relative efficiency of the suboptimal systems: proportional = 0.97, flat = 0.93.

We shall explore the sensitivity of the model outcomes to key parameters of the model. The basic setup will be italicized in the tables.

We shall start with the labor disutility parameter value ξ. As was expected, the higher the labor disutility parameter value, the lower is the labor supply, and correspondingly the lower is the optimal transfer rate. It was, however, surprising that its impact was not dramatic (Table 3): the socially optimal transfer rate only dropped from 0.54 to 0.52 to 0.51 when the disutility parameter value rose from 1 to 1.5 to 2! It is true that the basic income dropped from 0.21 to 0.16 to 0.13, while the average wage sunk from 0.92 to 0.74 to 0.62.

Table 3. *The impact of labor disutility on the social optimum*

Coefficient of labor disutility ξ	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
1.0	0.54	0.211	0.39	0.918	0.363	0.340
1.5	*0.52*	*0.158*	*0.40*	*0.741*	*0.281*	*0.260*
2.0	0.51	0.127	0.41	0.623	0.229	0.212

Here we shall explore the impact of the more flexible parameters, namely the wage rate inequality (ω), the length ratio (μ) and the individual discount factor (δ) on the optimal transfer system.

Start with the impact of wage inequality, i.e. reduce the ratio $\omega = w_H/w_L$ from 4 to 1. It is to be expected that the socially optimal transfer rate is dropping, but only the calculations give the measures: namely the socially optimal transfer rate drops from 0.5 to 0.33 and the average wage rises from 0.73 to 0.9 but it is quite surprising that redistribution disappears at all, namely slightly below $\omega = 2$ (exactly at 1.8)! At the same time, the accrual rate steeply rises and reaches the value of the optimal proportional system.

Table 4. *The impact of earning inequality on the social optimum*

Ratio of wage rates ω	Low wage rate w_L	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
4	0.5	0.52	0.158	0.40	0.741	0.281	0.260
3	0.6	0.46	0.111	0.51	0.811	0.337	0.324
2	0.75	0.35	0.015	0.65	0.905	0.440	0.440
1	1	0.33	0.000	0.66	0.908	0.596	0.587

Remark. $\mu = 0.5$, $\delta_L = 0.4$ and $\delta_H = 0.7$.

Turning to the impact of the length ratio on the social optimum, note that when $\mu = 0$, we encounter a pure tax system. Here we get a relatively low tax rate, namely 0.36. After fixing the discount factors and raising the length ratio from 0 to 0.5 to 1, the socially optimal transfer ratio rises from 0.36 to 0.52 to 0.62, while the basic income (intensity) drops from 0.21 to 0.16 to 0.13. At the same time, the average earning rises from 0.6 through 0.74 to 0.85. We note that how much distorting the usual but statically superfluous specification $\mu = 1$! (At the same time, in dynamic models all the time periods should have the same length, otherwise the demographic relations become untractable!)

Table 5. *The impact of length ratio on the social optimum*

Length ratio μ	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
0.00	0.36	0.215	0	0.598	0.342	–
0.25	0.45	0.181	0.46	0.676	0.308	0.287
0.50	*0.52*	*0.158*	*0.40*	*0.741*	*0.281*	*0.260*
0.75	0.58	0.141	0.36	0.798	0.258	0.241
1.00	0.62	0.128	0.32	0.852	0.242	0.224

Remark. $\omega = 4$.

Note that these results only depend on the labor disutility coefficient and the length ratio but are independent of the absolute length of the working and the retirement periods. In the developed world of our era, the lengthening of the life expectancy at birth is mainly due to the drop in old-age mortality, If one accepts the common assumption that labor disutility remains low for an interval whose length rises with the adult life expectancy, then Table 5 suggests the indexation of the full-benefit retirement age (Andersen, 2012).

We also explore the impact of the individual discount factor on the social optimum. We shall study the impact of a rise in the average discount factor $\delta = f_L\delta_L + f_H\delta_H$ from 0.25 to 1, changing the lower and the higher discount factors in a ration 2:1. We see that it implies a rise in the socially optimal transfer rate, from 0.49 to 0.5 to 0.59. While the basic income also rises from 0.14 through 0.16 to 0.19, the accrual rates remain quite stable, around 0.4! The average wage increases from 0.72 through 0.74 to 0.76, like other indicators.

Table 6. *The impact of the individual discount factors on the social optimum*

Average discount factor δ	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
0.25	0.49	0.144	0.38	0.720	0.268	0.237
0.50	0.52	0.158	0.40	0.741	0.281	0.260
0.75	0.55	0.174	0.41	0.758	0.294	0.283
1.00	0.59	0.194	0.42	0.765	0.303	0.305

Remark. $\omega = 4$, $\mu = 0.5$, $\delta = f_L\delta_L + f_H\delta_H$

Finally we present a calculation concerning the impact of life expectancy heterogeneity on the social optima. Retaining the average relative life span $\mu = 0.5$, we desaggregate the type-specific ones as $\mu_H = \varepsilon\mu_L$, where ε is the ratio of the long and short time spent in retirement. Then the previous balance equation generalizes into

$$t\mathbf{E}(wl) = (1 + \mathbf{E}(\mu))\gamma + \beta\mathbf{E}(\mu wl).$$

Turning to years, this may be read as $D_i = 60 + 20\mu_i$, $i = L, H$. Diminishing the lower life span from 80 to 76.7 years, most characteristics change very little. For example, the socially optimal transfer rate and the basic income hardly change, while the accrual factor β drops from 0.4 to 0.32.

Table 7. *The impact of the heterogeneity of life expectancy on the social optimum*

Lower life expectancy D_t	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
80.000	0.52	0.158	0.40	0.741	0.281	0.260
76.667	0.53	0.165	0.32	0.744	0.273	0.239

Remark. $\varepsilon = 1.6$, $\mathbf{E}(\mu) = 0.5$ with possible heterogeneity.

Next, we explore the impact of neglecting the pension system or assuming that everybody works until his death. Table 8 demonstrates that decreasing the wage-rate ratio from 4 to 1, the socially optimal transfer rate diminishes from 0.36 to 0. But finer tabulation shows that the critical ratio is just 1, i.e. however small wage rate differences exists, in a system without pensions, there is some redistribution.

Table 8. *The impact of earning inequality on the social optimum: no pension*

Ratio of wage rates ω	Transfer rate t	Basic income γ	Average wage $\mathbf{E}(wl)$	Low wage consumption c_L
4	0.36	0.215	0.598	0.342
3	0.28	0.182	0.649	0.418
2	0.14	0.102	0.729	0.557
1	0.00	0.000	0.800	0.800

Remark. $\mu = 0$.

László Halpern suggested that following the logic of the GDP calculation, investigate the case when the government excludes the leisure from its social welfare function. Since the wage rate ratio had the greatest impact, we repeat the calculations in this case. A priori it could be expected that the role of incentives becomes stronger but the numerical results were surprising. It turned out that for every wage rate ratio considered, no redistribution is optimal.

Table 9. *Impact of wage rate inequality without leisure utility*

Wage rate ratio ω	Low wage rate w_L	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	Low pensioner consumption d_L
4	0.5	0.33	0	0.66	0.928	0.298	0.293
3	0.6	0.33	0	0.66	0.924	0.357	0.352
2	0.75	0.33	0	0.66	0.918	0.447	0.440
1	1	0.33	0	0.66	0.908	0.596	0.587

Until now we have only considered partial changes. Now we consider simultaneous changes, proceeding from the worst case tot the best case. Then the partial effects are added up and may counteract with each other. In the least favorable case, where the wage rate inequality is highest, the discount factor is the lowest and the length ratio is the lowest, the optimal transfer rate is very high: 0.59, the flat income is relatively high: 0.17 and the average earnings (i.e. the weighted labor supply) is low: 0.68. In the most favorable case, where the wage rate inequality is lowest, the discount factor is the highest and the length ratio is the shortest, the optimal transfer rate is very low: 0.30, the flat income is relatively low: 0.08 and the average earnings (i.e. the weighted labor supply) is high: 0.82.

Table 10. *Impact of simultaneous changes*

Wage rate ratio ω	Lower discount factor δ_L	Length ratio μ	Transfer rate t	Basic income γ	Accrual rate β	Average wage $\mathbf{E}(wl)$	Low worker consumption c_L	pensioner d_L
6	0.1	0.75	0.59	0.170	0.2	0.678	0.196	0.183
4	*0.4*	*0.50*	*0.52*	*0.158*	*0.4*	*0.741*	*0.281*	*0.260*
2	0.7	0.25	0.30	0.082	0.7	0.821	0.497	0.497

We could have continued our calculations but we stop here. We hope that further investigations will confirm these results.

4. Conclusions

In our minimal model, the age-specific redistribution was achieved as a combination of a proportional pension system *and* a universal basic income for everybody, young and old alike. While a socially suboptimal proportional system assures a relatively high labor supply and reduces the old-age low consumption; the socially suboptimal flat system depresses the high-earner old-age consumption but at assures low labor supply, i.e. a lot of leisure. The socially optimal transfer system harmonizes these two features. In our model, the socially optimal transfer rate is such that it equalizes the consumption of the young and the old in the proportional system and is an increasing function of the wage inequality, of the length ratio and the discount factor. Allowing for voluntary private savings probably would lead to a more redistributive system with higher labor supply.

At closing the paper we emphasize that our model is static and does not take int account dynamics. For example, the dynamic model of Anderson (2012) tries to answer the question: Should we Save or Work More? To answer this question correctly one must apply models with sufficiently fine demographic models!

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