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Local Interaction in Tax Evasion

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Abstract

We study a model of tax evasion, where a flat-rate tax only finances the provision of public goods. Deciding on reported income, each individual takes into account that the less he reports, the higher is his private consumption but the lower is his moral satisfaction. The latter depends on his own current report and average previous reports of his neighbors. Under quite general assumptions, the steady state reported income is symmetric and the process converges to the steady state.

Keywords: tax evasion, steady state, asymptotic stability, symmetrization, networks, monotone maps

JEL: C62, H26

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We express our debts to Zs. Méder and J. Vincze.

Lokális kölcsönhatás adócsalás esetén

Garay M. Barnabás - Simonovits András – Tóth János

Összefoglaló

Egy adócsalási modellt elemzünk, ahol az arányos adó kizárólag közjavakat finanszíroz. A jövedelembevalláskor minden egyén figyelembe veszi, hogy minél kevesebb jövedelmet vall be, annál több pénze marad a fogyasztására, de annál kisebb lesz a morális elégedettsége. Ez utóbbi egyaránt függ idei bevallásától és környezete tavalyi átlagos bevallásától. Meglehetősen általános feltevések mellett a jövedelembevallások állandósult állapota szimmetrikus és a folyamat ehhez az állapothoz tart.

Tárgyszavak: adócsalás, állandósult állapot, aszimptotikus stabilitás, szimmetrizálás, hálózatok, monoton leképezések

JEL kódok: C62, H26

Local interaction in tax evasion

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Abstract

We study a model of tax evasion, where a flat-rate tax only finances the provision of public goods. Deciding on reported income, each individual takes into account that the less he reports, the higher is his private consumption but the lower is his moral satisfaction. The latter depends on his own current report and average previous reports of his neighbors. Under quite general assumptions, the steady state reported income is symmetric and the process converges to the steady state.

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1. Introduction

In the study of a tax system, tax avoidance and tax evasion should be considered. The first mathematical analysis by Allingham and Sandmo (1972) modeled tax evasion as a gamble: for a given audit probability and a penalty proportional to the undeclared income, what share of their income do risk averse individuals report? Of course, the lower the tax rate, the higher the share of reported income. Subsequent studies have discovered that the actual probability of audits and the penalty rates are insufficient to explain why citizens of healthier societies pay income taxes in the propensity they do. Therefore, another explanatory variable should be introduced, *tax morale*, as in Frey and Weck-Hannemann (1984). For example, we may assume that every individual has two parameters: his income and his tax morale. The tax system can also be characterized by two parameters: the marginal tax rate and the cash-back. Apart from stabilization, the tax system has two functions: income redistribution and financing of public goods. Simonovits (2010) studied the impact of exogenous tax morality on income redistribution and public services, while Frey and Torgler (2007) neglected income differences and redistribution and confined attention to financing public goods. A growing literature extends the analysis to agent-based models, see e.g. Lima and Zaklan (2008). Recently Méder, Simonovits and Vincze (2011) compared the classical and the agent-based approaches to tax evasion. (See further references therein.) Here we return to the second one of the three models, namely where individuals maximize utilities and observe only their neighbors' behavior. We prove a conjecture of that paper: under mild conditions, the steady state is unique, symmetric (everybody reports the same income) and globally asymptotically stable.

The structure of the paper is as follows: Section 2 outlines a simple model of tax evasion. Section 3 proves the existence and global asymptotic stability of the nontrivial symmetric steady state. Section 4 shows that by dropping concavity, the steady state can be asymmetric and stable periodic orbits can also emerge. Section 5 draws the conclusions.

2. A simple model of tax evasion

There are I individuals in the country, indexed as $i = 1, \ldots, I$. Individual i observes the behavior of his neighbors, whose nonempty set is denoted by $N_i \subset \{1, 2, \ldots, I\}$ and the number of its elements is n_i. Time is discrete and is indexed by $t = 0, 1, \ldots$.

We assume that every individual has the same income, for simplicity, unity. Then there is no reason for income redistribution, the tax only finances the provision of public goods.

Let $x_{i,t}$ be individual i's income report in period t, $0 \le x_{i,t} \le 1$. (By the logic of the theory, the government also knows that everybody's income is unity, nevertheless, it tolerates underreporting.)

In period t the average nationwide reported income is equal to

$$\bar{x}_t = \frac{1}{I} \sum_{i=1}^{I} x_{i,t}.$$

At the same time, individual i observes his local average

$$\bar{x}_{i,t} = \frac{1}{n_i} \sum_{j \in N_i} x_{i,t}.$$

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Let $\theta \in (0,1)$ be the tax rate. Let $c_{i,t}$ denote the individual i's consumption: $c_{i,t} = 1 - \theta x_{i,t}$, its *traditional* utility is then $u(c_{i,t})$. Let the *moral* utility be $z(x_{i,t}, \bar{x}_{i,t-1})$, the utility derived from his own report $x_{i,t}$ and influenced by this neighbors' previous average report $\bar{x}_{i,t-1}$. Finally let the per capita public expenditure be $\theta\bar{x}_t$, whose individual utility is $q(\theta\bar{x}_t)$.

The individual i's utility at time $t+1$ is the sum of three terms:

$$U^*_{i,t+1} = u(c_{i,t+1}) + z(x_{i,t+1}, \bar{x}_{i,t}) + q(\theta\bar{x}_{t+1}).$$

Of course, maximizing $U^*_{i,t}$, the individual neglects the third term, because this depends on the simultaneous decisions of many other individuals. In other words, in period $t+1$ individual i reports such an income which maximizes his *narrow utility*:

$$U_{i,t+1}(x_{i,t+1}, \bar{x}_{i,t}) = u(1 - \theta x_{i,t+1}) + z(x_{i,t+1}, \bar{x}_{i,t}) \to \max .$$

It is reasonable to assume that the maximum is attained at a unique $x_{i,t+1} =: F(\bar{x}_{i,t}) \in [0,1]$. This complies with property

$$U''_{11}(x,\bar{x}) = \theta^2 u''(1-\theta x) + z''_{11}(x,\bar{x}) < 0 ,$$

a direct consequence of the usual strict concavity assumption on the utility functions u and $z(\cdot,\bar{x})$. Here, of course, $U(x,\bar{x}) = u(1-\theta x) + z(x,\bar{x})$ and U''_{11}, z''_{11} (and U'_1, U'_2, U''_{12} etc.) stand for the respective partial derivatives.

Now we are in a position to define the transition rule by letting

$$\mathbf{x}_{t+1} = \mathbf{F}(\mathbf{x}_t), \qquad t = 0, 1, \ldots$$

where starting from the initial state $\mathbf{x}_0 = (x_{1,0}, \ldots, x_{I,0})$, $\mathbf{x}_t = (x_{1,t}, \ldots, x_{I,t})$ is the vector of reported incomes at time t, and

$$(\mathbf{F}(\mathbf{x}_t))_i := F(\bar{x}_{i,t}) \quad i = 1, \ldots, I.$$

We are interested in the asymptotic behavior when iterating the transition rule $\mathbf{F}$ as a self–map of the I–dimensional unit cube $[0,1]^I$. Note that steady states of tax evasion are just fixed points of $\mathbf{F}$.

Our main question is as follows: are the reported incomes eventually the same, regardless of initial state and individuals? In other words:

1. do we have an asymptotically stable steady state with the same reported incomes?
2. are all nontrivial initial states attracted to it?

The next section is devoted to determine a natural class of transition rules for which both answers are affirmative.

3. Examples and an abstract mathematical result

We keep the notation introduced in the previous section for time, individuals, state and transition rule. Dependence of function F on parameters will sometimes be suppressed.

Example 1. Simonovits (2010) and Méder, Simonovits and Vincze (2011) consider the utility function

$$U(x,\bar{x}) := \log(1-\theta x) + m\bar{x}(\log x - x)$$

defined for $x \in (0,1]$, $\bar{x} \in [0,1]$, where the new parameter $m > 0$ represents the *exogeneous tax morale*. Equation

$$U'_1(x,\bar{x}) = 0 \quad \Leftrightarrow \quad \frac{-\theta}{1-\theta x} + m\bar{x}\left(\frac{1}{x} - 1\right) = 0$$

can be extended to all $x, \bar{x} \in [0,1]$ in the form

$$E(x,\bar{x}) := m\bar{x}\theta x^2 - (\theta + m\bar{x} + m\bar{x}\theta)x + m\bar{x} = 0 . \qquad (1)$$

Function E is quadratic in x and satisfies $E(0,\bar{x}) = m\bar{x} \geq 0$, $E(1,\bar{x}) = -\theta < 0$. Thus equation (1) has a unique solution $x =: F(\bar{x}) = F(\bar{x}, m, \theta) \in [0,1)$ and $0 = F(0)$ is a fixed point of mapping $F : [0,1] \to [0,1)$.

Actually, for $\bar{x} = 0$, (1) simplifies to $x = 0$. For $\bar{x} > 0$ the discriminant of (1) is positive, and the classical formula for the smaller solution of a quadratic polynomial applies. Smoothness of function F at $\bar{x} = 0$ is a consequence of the implicit function theorem because $E(0,0) = 0$, and $E'_1(0,0) = -\theta \neq 0$.

In order to find additional fixed points of F (if $\bar{x} > 0$) one has to solve equation

$$E(x,x) = x(m\theta x^2 - (m + m\theta)x + m - \theta) = 0 .$$

Here again, function $\mathcal{E}(x) := \frac{1}{x}E(x,x)$ is quadratic in x and satisfies $\mathcal{E}(0) = m - \theta$, $\mathcal{E}(1) = -\theta < 0$. Thus the existence of a second fixed point $x^{\mathrm{o}} \in (0,1)$ is equivalent to $m > \theta$.

Remark 1. Evaluated at $\bar{x} = 0$, implicit differentiation gives that

$$F = F'_2 = 0 \, , \; F'_1 = \tfrac{m}{\theta} \, ,$$
$$F''_{11} = -2m^2 \tfrac{1+\theta}{\theta^2} < 0 \, , (F''_{12})^2 - F''_{11}F''_{22} = \tfrac{1}{\theta^2} \neq 0.$$

It is crucial that $F'_1 = 1$ whenever $m = \theta$. For $\theta \in (0,1)$ arbitrarily given, Theorem 9.3 in Glendinning (1994) yields that the one-parameter family of discrete-time dynamical systems $\{\bar{x} \to F(\bar{x}, m, \theta)\}_{m>0}$ undergoes a transcritical bifurcation at $m = \theta$.

Returning to Example 1, we obtain by implicit differentiation that $F'_1(\bar{x}) > 0$ and $F''_{11}(\bar{x}) < 0$ for all $\bar{x} \in [0,1]$.

Now we are in a position to formulate our

Standing assumption

1. *Analytic part:* function $F : [0,1] \to [0,1]$ is continuous, increasing and has exactly two fixed points, namely 0 and $x^{\mathrm{o}} \in (0,1)$.
2. *Connectivity part:* there is an integer $T \geq 1$ with the property that any two (not necessarily distinct) individuals are connected by a chain of $T - 1$ consecutive neighbors.

By the *analytic part of the standing assumption*, $F(x) > x$ whenever $x \in (0, x^{\mathrm{o}})$ and $F(x) < x$ whenever $x \in (x^{\mathrm{o}}, 1]$. In addition, $F^t(x) \to x^{\mathrm{o}}$ as $t \to \infty$ for each $x \in (0,1]$. Here F^t stands for the tth iterate of F. (Note that the analytic part of the standing assumption is automatically fulfilled for a continuous, increasing and concave function $F : [0,1] \to [0,1]$ having two fixed points in $[0,1)$.)

Let $A\{a_{ij}\}^I_{i,j=1}$ denote the adjacency matrix of the network (defined by letting $a_{ij} = 1$ if $j \in N_i$ and 0 if $j \notin N_i$). The *connectivity part of the standing assumption* is equivalent to requiring that A^T is a positive matrix.

Theorem 1. Both $\mathbf{0} = (0, \ldots, 0)$ and $\mathbf{x}^o = (x^o, \ldots, x^o)$ are fixed points (trivial and nontrivial, respectively) of the iteration dynamics induced by $\mathbf{F}$. The nontrivial fixed point $\mathbf{x}^o$ is asymptotically stable and, given an initial state $\mathbf{x}_0 \in [0,1]^I \setminus \{\mathbf{0}\}$ arbitrarily, $\mathbf{x}_t \to \mathbf{x}^o$ as $t \to \infty$.

Proof. For $j = 1, \ldots, I$ let $\mathbf{e}^j \in [0,1]^I$ denote the jth element of the standard basis, i.e., let $e_i^j = 1$ if $i = j$ and 0 if $i \neq j$. The crucial observation is that

$$\mathrm{sgn}((F^t(\mathbf{e}^j))_i) = \mathrm{sgn}((A^t)_{ij}) \quad \text{for } i, j = 1, \ldots, I;\ t = 0, 1, \ldots .$$

It follows that there exists $\gamma > 0$ such that, given an initial state $\mathbf{x}_0 \in [0,1]^I \setminus \{\mathbf{0}\}$ arbitrarily,

$$\min_{1 \le i \le I} x_{i,T} \ge \gamma \cdot \max_{1 \le i \le I} x_{i,0} > 0 .$$

Constant γ depends on F and the finer connectivity properties of the network.

Now a simple monotonicity argument results in the inductive chain of inequalities

$$F^t(\min_{1 \le i \le I} x_{i,T}) \le \min_{1 \le i \le I} x_{i,T+t} \le \max_{1 \le i \le I} x_{i,T+t} \le F^{T+t}(\max_{1 \le i \le I} x_{i,0})$$

valid for each $t \in \mathbb{N}$. By letting $t \to \infty$, we are done. □

Our next example shows that the connectivity part of the standing assumption cannot be weakened to connectivity.

Example 2. Keeping the analytic part of the standing assumption, we consider the case of $I \ge 3$ individuals located on a circle and assume that everybody knows only his next-door neighbors' reported income from the previous year. (To calculate the income report based only on these data means that the person is *other-directed* using the terminology of Riesman (1950).) With convention $I + 1 = 1$ and $0 = I$, this means that $N_i = \{i-1, i+1\}$, $n_i = 2$. Then $\bar{x}_{i,t} = (x_{i-1,t} + x_{i+1,t})/2$. It is immediate that the connectivity part of the standing assumption is satisfied if and only if I is odd (and then integer T can be chosen for $I - 1$ and $\min_{1 \le i \le I} x_{i,I-1} > 0$ for each $\mathbf{x}_0 \in [0,1]^I \setminus \{\mathbf{0}\}$).

Remark 3. The analytic part of the standing assumption is implied by assuming that function $F : [0,1] \to [0,1)$ is twice continuously differentiable, $F(0) = 0$, $F'(0) > 1$, and $F'(x) > 0$, $F''(x) < 0$ for all $x \in [0,1]$. These stronger assumptions lead to a simple convergence estimate. In fact, by continuity, there exists an $x_* \in (0, x^o)$ with the property that $F'(x^o) < q = F'(x_*) < 1$. It follows that the restriction of $\mathbf{F}$ to the I-dimensional cube $[x_*, 1]^I$ is a contraction with constant q in the topology of the ℓ_∞ norm. In fact, the collection of inequalities $x_* \le x_j \le 1$, $j = 1, \ldots, I$ implies that

$$x_* < F(x_*) \le F\left(\frac{1}{n_i}\sum_{j \in N_i} x_j\right) < 1 , \quad i = 1, \ldots, I .$$

On the other hand, the ℓ_∞ matrix norm (i.e. the maximum absolute row sum) of the Jacobian $\mathbf{J}(\mathbf{x})$ for each $\mathbf{x} \in [x_*, 1]^N$ is not greater than q because

$$(\mathbf{J}(\mathbf{x}))_{i,j} = \begin{cases} \frac{1}{n_i} \cdot F'\left(\frac{1}{n_i}\sum_{j \in N_i} x_j\right) & \text{if } j \in N_i \\ 0 & \text{otherwise} \end{cases} , \quad i = 1, \ldots, I.$$

It follows that $\mathbf{x}^o$ is an exponentially stable fixed point of $\mathbf{F}$.

Note also that the monotonicity-concavity assumption is a consequence of a finite collection of inequalities in terms of the first, second, and third order (mixed partial) derivatives of the utility functions u and z. In particular, the most convenient general assumption of guaranteeing existence and uniqueness for $x = F(\bar{x})$ (i.e., for the solution of equation $U_1'(x, \bar{x}) = 0$ with $U(x, \bar{x}) = u(1 - \theta x) + z(x, \bar{x})$) is that

$$U_1'(0, \bar{x}) \ge 0 \ , \ U_1'(1, \bar{x}) \le 0 \ \text{ and } \ U_{11}''(x, \bar{x}) < 0$$

whenever $x, \bar{x} \in [0,1]$. Moreover, in view of identity

$$U_{11}''(F(\bar{x}), x) \cdot F_1'(\bar{x}) + U_{12}''(F(\bar{x}), x) = 0 ,$$

property $F_1'(\bar{x}) > 0$ is a consequence of the additional inequality $U_{12}''(x, \bar{x}) < 0$ required for all $x, \bar{x} \in [0,1]$. A similar result holds true for concavity.

Returning to Example 1 again (and assuming the connectivity part of the standing assumption), we note that $m \le \theta$ implies global asymptotic stability for the trivial (and then unique) fixed point $\mathbf{0}$ of the iteration dynamics induced by $\mathbf{F}$. In particular, for $\theta \in (0,1)$ arbitrarily given, the I-dimensional discrete-time dynamical system $\mathbf{F} : [0,1]^I \to [0,1]^I$ (while keeping all essential features of mapping $F : [0,1] \to [0,1]$) undergoes transcritical bifurcation at the $m = \theta$-value of the bifurcation parameter m. As a function of m, x^o is strictly increasing on $(0,1)$ and $x^o \to 1$ as $m \to \infty$.

4. Remarks on the underlying theory of monotone maps

Throughout this section, we consider the transition rule $\mathbf{F} : [0,1]^I \to [0,1]^I$ under the condition that function $F : [0,1] \to [0,1]$ is continuous, $F(0) = 0$ and $F'(x) > 0$ for each $x \in [0,1]$.

This is an essential weakening of the analytic part of the standing assumption (which corresponds to dropping concavity of the utility function in our tax evasion model) which makes the existence of asymptotically stable asymmetric steady states and also the existence of asymptotically stable nontrivial periodic orbits possible, see Examples 2A, 2B below.

By letting $\mathbf{x} \le \mathbf{y}$ if and only if $x_i \le y_i$ for each i, a closed partial order on $[0,1]^I$ is introduced. We write $\mathbf{x} \prec \mathbf{y}$ if $x_i < y_i$ for each i. Clearly $\mathbf{F}(\mathbf{x}) \le \mathbf{F}(\mathbf{y})$ whenever $\mathbf{x} \le \mathbf{y}$. In the terminology of Hirsch and Smith (2005), $\mathbf{F}$ is a discrete-time monotone dynamical system or monotone map. Monotonicity is strong if $\mathbf{F}(\mathbf{x}) \prec \mathbf{F}(\mathbf{y})$ whenever $\mathbf{x} \le \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. If only $\mathbf{F}^t(\mathbf{x}) \prec \mathbf{F}^t(\mathbf{y})$ for some t, then $\mathbf{F}$ is eventually strongly monotone. Nonzero elements of the Jacobian are positive and arranged in the same pattern determined solely by the topology of the network. For each $\mathbf{x} \in [0,1]$, $(\mathbf{J}(\mathbf{x}))_{i,j}$ is nonzero if and only if $j \in N_i$, $i, j = 1, \ldots, I$.

The connectivity part of the standing assumption implies that, from a certain exponent onward, powers of the Jacobian are positive matrices. Hence $\mathbf{F}$ is eventually strongly monotone and Theorem 5.26 in Hirsch and Smith (2005) applies. The conclusion is that, for an open and dense set of the starting points $\mathbf{x} \in [0,1]^I$, $\mathbf{F}(\mathbf{x})$ is converging to some periodic orbit.

Remark 4. In case the connectivity part of the standing assumption is violated, $\mathbf{F}$ cannot be eventually strongly monotone: with Q denoting the union of facets of the unit cube $[0,1]^I$ anchored at vertex $\mathbf{0}$, there exists a $j^* \in \{1,\dots,I\}$ with the property that $\mathbf{F}^t(\mathbf{e}^{j^*}) \in Q$ for each $t \in \mathbb{R}$. This is a consequence of the crucial observation we made in proving Theorem 1. In fact, if $A^{t_j}\mathbf{e}^j$ is a positive vector for each j, then A^T is a positive matrix with $T = \prod_{1\le j\le I} t_j$. In particular, consider case $I = 4$ of Example 2. Then, for each $x \in (0,1]$, the $\mathbf{F}$-trajectory starting from $\mathbf{x}_0 = (0,x,0,x)$ satisfies $\mathbf{F}^t(\mathbf{x}_0) = (F^t(x),0,F^t(x),0)$ for $t \in \mathbb{N}$ odd and $\mathbf{F}^t(\mathbf{x}_0) = (0,F^t(x),0,F^t(x))$ for $t \in \mathbb{N}$ even. Similar examples can be given for $I = 6,8,10,\dots$.

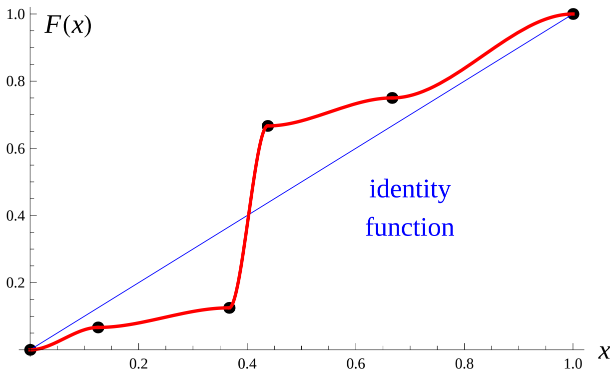

Figure 1: Example 2A: function F generating a monotone (and eventually strongly monotone) $\mathbf{F}$ with an asymptotically stable two-periodic point

Example 2A. (Asymptotically stable nontrivial two-periodic orbit) Let $I = 3$ in Example 2 and consider only the special case $x_1 = x_2$. Clearly, $\mathbf{x} = (x_1,x_1,x_3)$ is a two-periodic point of mapping $\mathbf{F}$ if and only if

$$F\left(\frac{F(\frac{x_1+x_3}{2}) + F(x_1)}{2}\right) = x_1 \quad \text{and} \quad F\left(F\left(\frac{x_1+x_3}{2}\right)\right) = x_3. \quad (2)$$

Property (2) can be satisfied by letting $x_1 = x_2 = \frac{1}{8}, x_3 = \frac{3}{4}$ and

$$F\left(\tfrac{1}{8}\right) = \tfrac{1}{15}, \quad F\left(\tfrac{11}{30}\right) = \tfrac{1}{8}, \quad F\left(\tfrac{7}{16}\right) = \tfrac{2}{3}, \quad F\left(\tfrac{2}{3}\right) = \tfrac{3}{4}.$$

Now it is easy to extend F to the interval $[0,1]$ in such a way that $F(0) = 0, F(1) = 1, F$ is smooth and $F'(x) > 0$ for each $x \in [0,1]$. By the construction, $\mathbf{F}$ is monotone and $\mathbf{x} = (\frac{1}{8},\frac{1}{8},\frac{2}{3})$ is a two-periodic point of $\mathbf{F}$. Asymptotic stability can be ensured by choosing $F'(\frac{1}{8}), F'(\frac{11}{30}), F'(\frac{7}{16}), F'(\frac{2}{3})$ sufficiently small, see Fig. 1. A great variety of similar examples (arbitrary periods, various moving averages, various networks) will be presented in Garay and Várdai (2011).

Note that $\mathbf{x} = (\frac{1}{8},\frac{1}{8},\frac{2}{3})$ is a steady state of $\mathbf{F}^2$, and a two-periodic point of $\mathbf{F}^3$. Note also that $\mathbf{F}^2$ and $\mathbf{F}^3$ are strongly monotone.

Remaining at case $I = 3$ of Example 2 it is worth mentioning that monotonicity of F alone implies that each steady state of $\mathbf{F}$ has the same coordinates. (In fact, $x_1 \le x_2$ is equivalent to $x_1 = F(\frac{x_3+x_2}{2}) \ge F(\frac{x_3+x_1}{2}) = x_2$. The very same argument leads to $x_2 = x_3$ as well.) The same holds true for $I = 4$, as well.

Example 2B. (Asymptotically stable asymmetric steady state) Let $I = 5$ in Example 2 and apply the method used in Example 2A. Starting from

$$F\left(\tfrac{4}{8}\right) = \tfrac{8}{16},\ F\left(\tfrac{1}{8}\right) = \tfrac{6}{16},\ F\left(\tfrac{2}{8}\right) = \tfrac{7}{16},\ F\left(\tfrac{6}{8}\right) = \tfrac{9}{16},\ F\left(\tfrac{7}{8}\right) = \tfrac{10}{16},$$

we arrive at the conclusion that $\mathbf{F}$ is monotone and $(\frac{4}{8},\frac{1}{8},\frac{2}{8},\frac{6}{8},\frac{7}{8})$ is a steady state for $\mathbf{F}$, see Fig. 2.

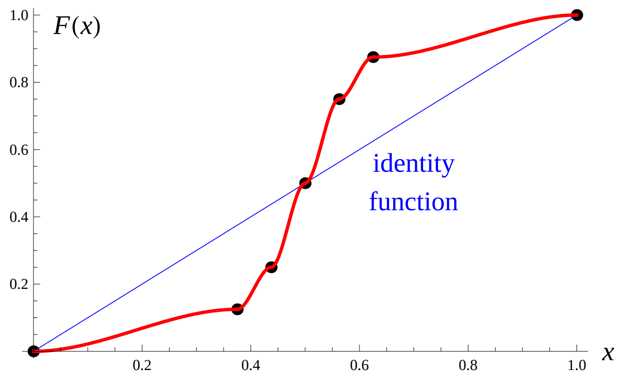

Figure 2: Example 2B: function F generating generating a monotone (and eventually strongly monotone) $\mathbf{F}$ with an asymptotically stable asymmetric steady state

5. Conclusions

A tax evasion model leading to discrete time network dynamics with local interactions is presented. Under quite natural assumptions (somewhat weaker than the usual concavity assumption on the utility functions), uniqueness, symmetry and global asymptotic stability of the nontrivial steady state is proved.

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