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Inflation, Human Capital and Tobin's q

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- JOSEPH PEARLMAN

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Inflation, Human Capital and Tobin's q

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Abstract

A less well-known empirical finding for the US and UK is a pronounced low frequency negative relationship between inflation and Tobin's q; a normalized market price of capital. This stylized fact is explained within a dynamic stochastic general equilibrium model using three key features: (i) a Lucas and Prescott (1971) physical capital adjustment cost with a rising marginal cost of investment, (ii) production of human capital with endogenous growth and (iii) an inflation tax cash-in-advance economy. The baseline endogenous growth model matches the US inflation and q long term correlation, while comparable exogenous growth are unable to do this, and it outperforms the exogenous growth models in explaining business cycle volatilities of q and of stock returns.

Keywords: Low frequency, Tobin's q; inflation tax, endogenous growth

JEL: E31, E44, G12

Infláció, humán tőke és a Tobin-féle q

Parantap Basu - Max Gillman - Joseph Pearlman

Összefoglaló

Az Egyesült Államokra és az Egyesült Királyságra vonatkozik az a kevésbé ismert empirikus megállapítás, hogy az infláció és a Tobin-féle q, azaz a tőke normált piaci értéke között alacsony frekvencián megfigyelhető egy erős negatív összefüggés. Ez a stilizált tény egy dinamikus sztochasztikus általános egyensúlyi modellel írható le, amely három kulcstényezőre támaszkodik. Ezek a Lucas - Prescott (1971) féle fizikai tőke növekvő marginális beruházási költségek melletti alkalmazkodási költsége, a humán tőke létrehozása endogén növekedéssel, valamint az inflációs adóval és készpénzelőleggel működő gazdaság. Az exogén növekedési modellel ellentétben az endogén növekedési modell alapváltozata elvégzi az USA inflációjának és a q-nak a hosszú távú korrelációját, ráadásul felülmúlja az előbbit a q és a tőkehozam üzleti ciklusban megfigyelt változékonyságának magyarázatában is.

Tárgyszavak: alacsony frekvencia, Tobin-féle q, inflációs adó, endogén növekedés

JEL: E31, E44, G12

Inflation, Human Capital and Tobin's q^*

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Abstract

A less well-known empirical finding for the US and UK is a pronounced low frequency negative relationship between inflation and Tobin's q, a normalized market price of capital. This stylized fact is explained within a dynamic stochastic general equilibrium model using three key features: (i) a Lucas and Prescott (1971) physical capital adjustment cost with a rising marginal cost of investment, (ii) production of human capital with endogenous growth and (iii) an inflation tax cash-in-advance economy. The baseline endogenous growth model matches the US inflation and q long term correlation, while comparable exogenous growth are unable to do this, and it outperforms the exogenous growth models in explaining business cycle volatilities of q and of stock returns.

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*We are grateful for comments from seminars at the Columbia Business School, St Louis Federal Reserve Bank, University of Iowa, and the CDMA Conference at St. Andrews for useful comments, with special thanks to John Donaldson, B. Ravikumar and Marc Giannoni for valuable comments. The first author acknowledges research leave support from Durham Business School.

1 Introduction

The negative association between firm value and inflation in general equilibrium has been the focus of work at least since Danthine and Donaldson (1986), who use a money-in-the-utility function and an endowment economy. This focus is motivated for example by Figure 1, in which US postwar data illustrate a negative correlation between the inflation rate and Tobin's q, a normalized market price of capital.[1] This association remains to be explained within a production-based dynamic stochastic general equilibrium ($DSGE$) model economy. This paper explains the empirical link as resulting from inflation causing less growth, lower human and physical capital accumulation rates, a lower marginal cost of physical capital investment, and subsequently a lower q.

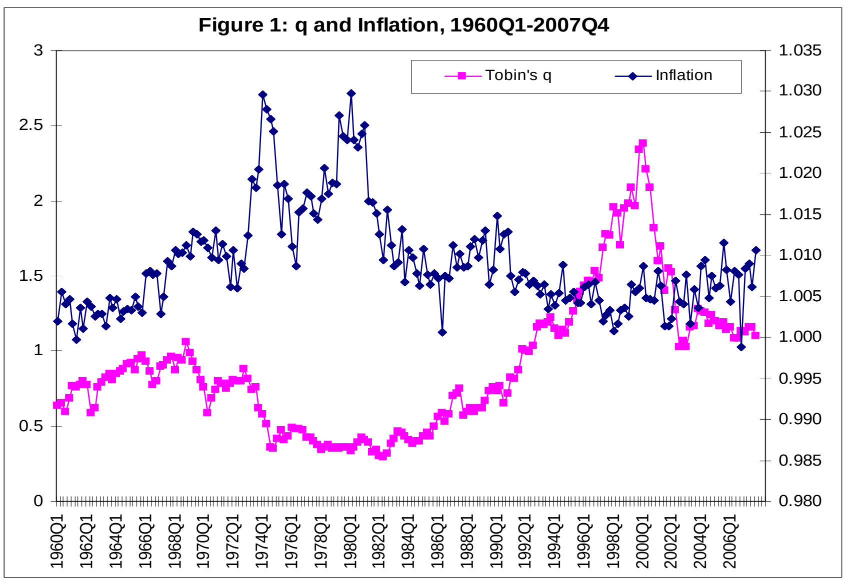


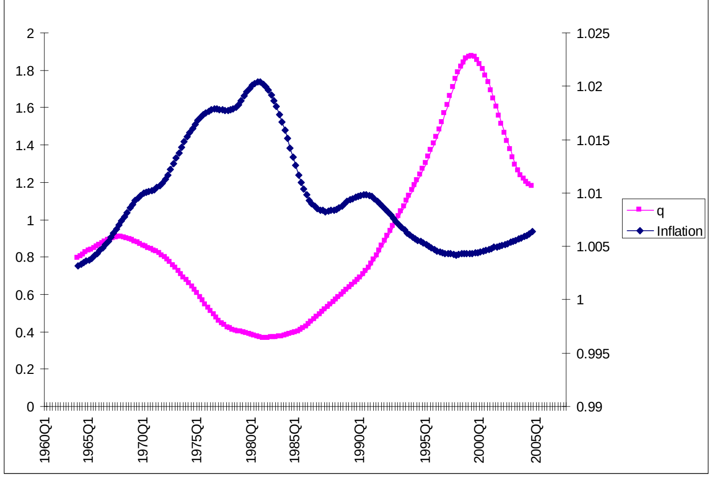
Figure 1 has a sample period of 1960:Q1 to 2007:Q4; its negative correlation is particularly pronounced starting in the mid $1960's$. Tobin's q bottoms out around the early $1980's$ when inflation peaks. The subsequent rise of q coincides with an era of disinflation and high economic growth. Then q

[1] Figure 1 data for q is from Smithers & Co (http://www.smithers.co.uk/); the negative correlation also holds using the Tobin's q estimates of Hall (2001).

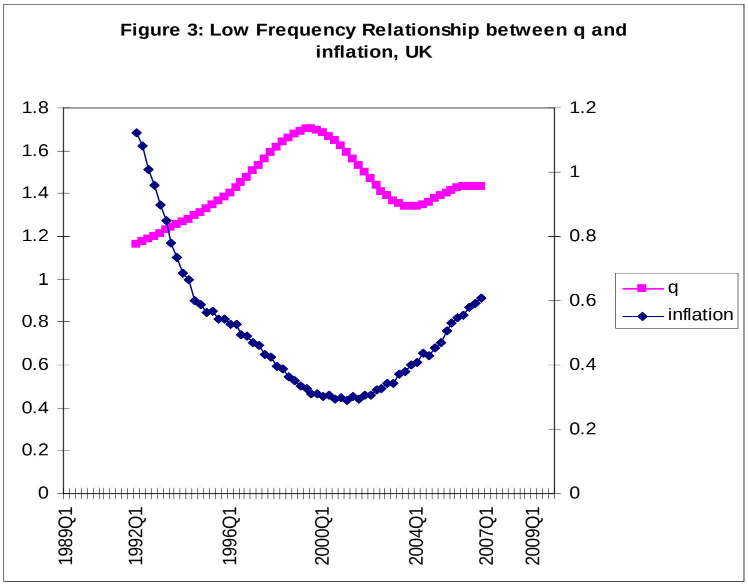
reaches an all time high before the stock market crash when inflation falls to about 1%.

The negative relationship between inflation and q is particularly a low frequency phenomenon. Using a Baxter and King (1999) band-pass filter, with the low frequency component having a periodicity of longer than 32 quarters, Figure 2 plots the low frequency components of the US inflation and q series, for the same sample period as in Figure 1, and with a window of 12 quarters that loses the first and last three years. The negative correlation is -0.76, statistically significant at a 5% level using Newey and West (1987) heteroskedasticity-adjusted standard errors. At a business cycle frequency, with a periodicity of 6 to 32 quarters, the correlation coefficient between inflation and q is -0.07 and insignificant at a 5% level. UK quarterly data exhibits a similar low frequency negative correlation of -0.77, also significant at a 5% level. Figure 3 plots this low frequency q and inflation relation over the sample period 1989:Q1 to 2009:Q4.[2] Given the closeness of the US and UK correlation coefficient, the calibration is based on US data with the idea that the results may also be suggestive for the UK.

Figure 2: Low Frequency Relationship between q and Inflation, US



[2] UK quarterly CPI data is from the Office of National Statistics; Tobin's q data is from the Bank of England, in which the methodology for computing q is described in Price and Schleicher (2005).



The *DSGE* model has three features which give rise to such a negative relation between q and inflation, namely (i) a Lucas and Prescott (1971) physical capital adjustment cost with a rising marginal cost of investment, (ii) human capital investment that endogenizes the balanced growth path equilibrium (*BGP*) growth rate, and (iii) a cash-in-advance inflation tax economy. A higher inflation rate, as a result of the model's shocks, induces agents to take more leisure since the proceeds from work are subject to the inflation tax (Gomme, 1993, and Gillman and Kejak, 2005). This reduces human capital utilization, the *BGP* growth rate, the rate of accumulation of both human and physical capital, the marginal cost of physical capital investment, and so also q.

A related paper is by McGrattan and Prescott (2005), who argue that the rise until 2000 of the stock price to GDP ratio is due to lower taxes on corporate distributions to shareholders. Given a stable output/capital ratio, the stock price/GDP also reflects the behavior of Tobin's q. McGrattan and Prescott focus on the role of intangible capital and explicit taxes in determining stock price behavior. The alternative focus here is on one form of such intangible capital, human capital, and implicit the inflation tax in determining q.

The model is a straightforward combination of four of Bob Lucas's papers: Lucas and Prescott (1971), Lucas (1978), Lucas (1980), and Lucas (1988), although with normal depreciation of capital as in Basu (1987) and Hercowitz and Sampson (1991). The result is that q is expressed as a simple function of the growth rate (Proposition 4), with a rising marginal cost of investment, and it is affected by endogenous changes in the growth rate including those induced by the inflation tax.[3] Two types of structural shocks are specified: real productivity shocks in each the goods and human capital investment sectors, and a money supply growth rate shock. Both productivity shocks tend to induce a negative correlation between inflation and q as well as between inflation and growth over time, but the human capital sector shock has an effect that is an order of magnitude stronger than that of the goods sector. The monetary shock induces a Tobin (1965) type effect of an increase in physical capital accumulation that initially weakens the negative q-inflation correlation, but then marginally strengthens this correlation over an extended period.

A comparison is made of the baseline endogenous growth model to an alternate endogenous growth model differing by only one parameter, and to two versions of an exogenous growth model. The baseline model best matches the low frequency correlation between q and inflation (Table 5), and the business cycle volatility of both q and a measure of the stock return (Table 6). The alternative endogenous growth model best fits the volatilities of the growth rate and the investment rate. A qualification is that all of the models overstate the inflation rate volatility, as the model was kept as simple as possible without price adjustment factors to focus on fundamentals affecting q over time.

Section 2 sets out the model, Section 3 the analytic *BGP* equilibrium q, and Section 4 the calibration and impulse response analysis. Section 5 presents the low frequency correlation and business cycle volatility results, while Section 6 concludes.

[3] Human capital based endogenous growth continues to find empirical support, ranging from US-UK times series work such as Kocherlakota and Yi (1997), to a *DSGE* setting with shocks to human capital productivity as in Maffezzoli's (2000) explanation of international business cycle facts.

2 The Model

2.1 The Representative Household

The representative household allocates time between leisure (x_t), work in the goods sector (l_{Gt}) at a nominal wage W_t, and work in the human capital investment (l_{Ht}) . Households own the human capital (h_t) and augment it through human capital investment. Firms own the physical capital (k_t) and accumulate it through physical investment (i_t).

At time t, households first trade in goods with the cash held in advance, M_t, and then they visit the asset markets to trade in stocks at the ex-dividend prices V_t and in nominal bonds at the price P_t^b . Nominal bonds B_t held at date t pay 1 unit of currency with certainty in the following periods. Money is used to buy goods, and is augmented by the central bank through a stochastic nominal lump-sum transfer N_t, which with market clearing in equilibrium equals $\mu_t M_{t-1}$; μ_t is the stochastic growth rate of money supply.[4]

At date t, the revenues of the household are nominal dividends per ownership share in the goods producer, D_t, factored by the shares z_t, plus wages $W_t l_{Gt} h_t$ and the lump sum transfer N_t. Expenses are investment in bonds, $P_t^b B_{t+1} - B_t$, in cash, $M_t - M_{t-1}$, and in stocks, $V_t(z_{t+1} - z_t)$, plus consumption purchases $P_t c_t$.

The household maximizes the following life time utility function:

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \psi \Gamma(x_t) \right\} \tag{1}$$

where $U(.)$ and $\Gamma(.)$ are monotonically increasing and strictly concave functions, with the parameter $\psi \geq 0$, subject to the flow budget constraint facing the household,

$$D_t z_t + W_t l_{Gt} h_t + N_t - \left(P_t^b B_{t+1} - B_t \right) - V_t(z_{t+1} - z_t) - (M_t - M_{t-1}) - P_t c_t = 0, \tag{2}$$

[4] Gillman et al. (2007) demonstrate how a related endogenous growth economy implies an equilibrium Taylor (1993) condition so that interest rate ("speed-limit") rules and exogenous money supply growth rate targets are synonymous.

time allocation,

$$1 = x_t + l_{Gt} + l_{Ht}, \tag{3}$$

and human capital accumulation and exchange constraints.

Human capital investment is linear in effective labor time $l_{Ht}h_t$ as in Lucas (1988), with a depreciation rate of δ_h and with A_{Ht} the exogenous sectoral productivity shock, giving the accumulation constraint of

$$h_{t+1} = (1 - \delta_h)h_t + A_{Ht}l_{Ht}h_t. \tag{4}$$

The exchange constraint requires money to purchase consumption such that

$$P_t c_t \leq M_{t-1} + N_t. \tag{5}$$

All equilibrium conditions are found in Appendix A, with the standard stochastic discount factor m_{t+1} facing the household given by

$$m_{t+1} \equiv \frac{\beta E_{t+1}\left[U'(c_{t+2})\frac{c_{t+2}}{c_{t+1}}\frac{1}{1+\mu_{t+2}}\right]}{E_t\left[U'(c_{t+1})\frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}\right]}. \tag{6}$$

Using the equations (A.13) and (A.14) in Appendix A, the stock price and bond price equations can be written typically as

$$1 = E_t m_{t+1}\left\{\frac{v_{t+1} + d_{t+1}}{v_t}\right\}, \tag{7}$$

and

$$p_t^b = E_t m_{t+1} \tag{8}$$

where v_t is the real share price, $v_t \equiv V_t/P_t$ and p_t^b is the real price of bond that satisfies $p_t^b \equiv \frac{P_t^b}{P_t}$.

2.2 The Firm's Problem

The firm produces output y_t with a Cobb-Douglas production function $A_{Gt}F(k_t, l_{Gt}h_t)$ in physical capital k_t and effective labor $l_{Gt}h_t$, with A_{Gt} the stochastic total factor productivity (TFP) at date t, and $\alpha \in (0,1)$, such that

$$y_t = A_{Gt}F(k_t, l_{Gt}h_t) = A_{Gt}k_t^{\alpha}(l_{Gt}h_t)^{1-\alpha}. \tag{9}$$

The firm costs are wages and the nominal physical capital investment $P_t i_t$. With λ_t the shadow price of the consumer's nominal income in equation (2), A_{Gt} the stochastic total factor productivity (TFP) at date t, and $\alpha \in (0,1)$, the firm solves

$$\underset{l_{Gt},}{Max} \quad E_0 \sum_{t=0}^{\infty} \lambda_t \left[P_t A_{Gt} k_t^{\alpha} (l_{Gt}h_t)^{1-\alpha} - W_t l_{Gt} h_t - P_t i_t \right] \tag{10}$$

subject to the physical capital accumulation constraint, for $\delta_k \in (0,1)$ and $\theta \in (0,1)$ of

$$k_{t+1} = k_t \left[1 - \delta_k + \frac{i_t}{k_t} \right]^{\theta}, \tag{11}$$

as in Basu (1987) and Hercowitz and Sampson (1991). The parameter θ represents the extent of adjustment cost; with $\theta = 1$ there is no adjustment cost.

The marginal cost of investment, MC_t, can be expressed by solving for i_t in equation (11), and differentiating with respect to next period capital, as

$$MC_t \equiv \frac{\partial i_t}{\partial k_{t+1}} = \frac{1}{\theta} \left(\frac{k_{t+1}}{k_t} \right)^{\frac{1-\theta}{\theta}} = \frac{1}{\theta} \left(1 - \delta_k + \frac{i_t}{k_t} \right)^{1-\theta}, \tag{12}$$

which is rising in $k_{t+1,}$ or in i_t. Figure 4 graphs the MC_t function for a varying $\frac{i_t}{k_t}$, given $\theta = 0.8$, and $\delta_k = 0.03$, as in the baseline calibration below. The marginal cost rises as the investment rate rises.

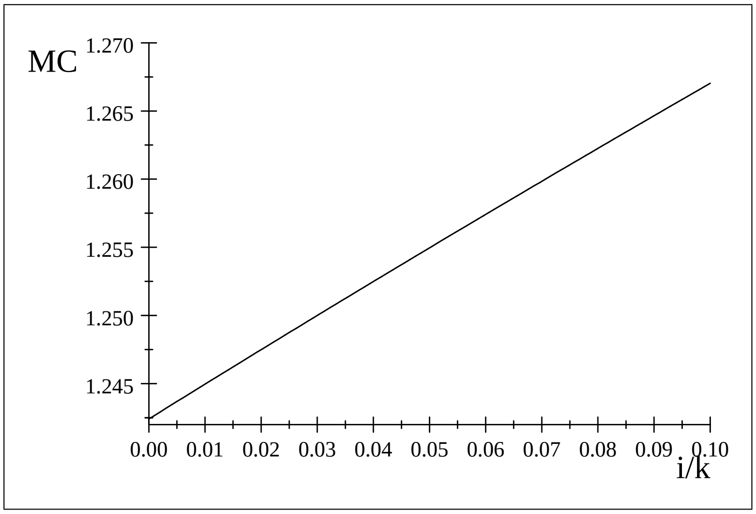


Figure 4: Marginal Cost of Investment Function, MC_t

This MC_t is almost linear in a range that is specified for reasonable growth rates, since on the BGP it is true that $\frac{i_t}{k_t} = (1+g)^{\frac{1}{\theta}} - 1 + \delta_k$, which equals $g+\delta_k$ as $\theta \to 1$. This standard near-linearity holds for most values of θ, such as for $\theta \in (0.15, 1)$, within the growth rate range, while for very low values of θ some concavity is evident.

In comparison, Belo et al. (2010) use a related adjustment cost function, whereby investment plus their adjustment cost, with the sum denoted by C_t, is given with their parameters of ν and a by

$$C_t \equiv i_t + \frac{a}{\nu}\left(\frac{i_t}{k_t}\right)^{\nu} k_t,$$

with a marginal cost of $\frac{\partial C_t}{\partial k_{t+1}} = 1 + a\left(\frac{i_t}{k_t}\right)^{\nu-1}$. Like Figure 4, this gives a rising marginal cost, but one that can be quite convex, mainly through the curvature parameter, ν. They find empirical support for significant convexity in their GMM estimation; however these interesting results are based on a partial equilibrium model that is not directly comparable to our $DSGE$, endogenous growth setting.

The endogenous growth setting in particular distinguishes our model for example from the $DSGE$ model of Christiano et al. (2008). Their comparable equation to our equation (11) is

$$\frac{k_{t+1}}{k_t} = 1 - \delta + \frac{i_t}{k_t} - z\left(\frac{i_t}{k_t} - \eta\right)^2,$$

where z is a parameter and η is the steady state investment to capital ratio. Their adjustment cost is therefore zero in the steady state, as in Lucas (1967), while in our specification in equation (11), the adjustment cost is positive along the balanced growth path equilibrium.

2.3 Forcing Processes

The exogenous variables A_{Gt} , A_{Ht}, μ_t follow the processes :

$$A_{Gt} - \bar{A}_G = \rho_G(A_{Gt-1} - \bar{A}_G) + \epsilon_t^G \tag{13}$$

$$A_{Ht} - \bar{A}_H = \rho_H(A_{Ht-1} - \bar{A}_H) + \epsilon_t^H \tag{14}$$

$$\mu_t - \bar{\mu} = \rho_\mu(\mu_{t-1} - \bar{\mu}) + \epsilon_t^\mu \tag{15}$$

where $\epsilon_t^G, \epsilon_t^H$ ϵ_t^μ are white noises with standard deviations σ_G, σ_H and σ_μ respectively. We assume zero contemporaneous covariances between these three shocks. Letters with a bar represent steady state values.

2.4 Characterization of Equilibrium

(E.1): Given the processes $\{P_t\}$, $\{W_t\}$, $\{D_t\}, \{A_{Ht}\}, \{V_t\}$, $\{P_t^b\}$, and $\{N_t\}$, the household maximizes utility in equation (1) subject to equations (2) to (5).

(E.2): Given the processes $\{P_t\}$, $\{W_t\}$, $\{A_{Gt}\}$, the goods producer maximizes (10) subject to (11).

(E.3) : Spot assets, goods, and money markets clear: $z_t = 1$, $B_t = 0$, and $N_t = u_t M_{t-1}$.

3 Tobin's q

The shadow price of physical capital investment normalized by the shadow price of consumption gives a standard expression for Tobin's q. Using the first order condition with respect to physical capital investment and equation (A.17) of Appendix A, one gets the expression for Tobin's q.

Proposition 1

$$q_t \equiv \frac{\omega_t}{P_t \lambda_t} = \frac{1}{\theta}\left[1 - \delta_k + \frac{i_t}{k_t}\right]^{1-\theta}. \tag{16}$$

Proof. This follows directly from the first order condition with respect to physical capital investment, equation (A.17) of Appendix A, where the shadow price of consumption $P_t \lambda_t$ is the shadow price of nominal income in equation (2) of the household problem as multiplied by the nominal price level P_t. ■

Corollary 2 *Tobin's q equals the marginal cost of investment, which is rising in* k_{t+1}.

Proof. By equations (16) and (11), $q_t = \frac{1}{\theta}\left(\frac{k_{t+1}}{k_t}\right)^{\frac{1-\theta}{\theta}}$, which by equation (12) is the marginal cost of investment; and $\frac{\partial q_t}{\partial k_{t+1}} > 0$. ■

As in a standard quadratic q model of investment, the marginal cost of investment here also equals the average q based on the stock market valuation equation (Obstfeld and Rogoff, 1996). In other words,

Proposition 3 *The marginal and average q are the same; in that*

$$q_t = \frac{v_t}{k_{t+1}}.$$

Proof. See Appendix B.1. ■

As investment increases its marginal and average cost rise. And this cost is closely connected to the economy's growth rate. Hereafter, log utility is specified, with $U(c_t) = \ln c_t$ and $\Gamma(x_t) = \ln x_t$. Along the BGP, the q depends positively on the growth rate, and in turn on the return to capital.

Proposition 4 *Along the balanced growth path, Tobin's q is a simple rising function of the growth rate and a falling function of the adjustment cost parameter θ whereby*

$$q = \frac{1}{\theta}(1+g)^{\frac{1-\theta}{\theta}}, \tag{17}$$

and this can be expressed through g in terms of either the return on physical or on human capital.

Proof. From Corollary 1, and given that $\frac{k_{t+1}}{k_t} = 1 + g$ along the BGP, then $q = \frac{1}{\theta}(1+g)^{\frac{1-\theta}{\theta}}$ and $\frac{\partial q}{\partial g} > 0$, and $\frac{\partial q}{\partial \theta} < 0$. Further, as shown in Appendix $B.2$, the balanced growth rate in this economy is given in terms of the physical capital net return $\bar{A}_G F_1 - \delta_k$ by ■

$$1 + g = \left[\frac{\beta\theta \left(1 + \bar{A}_G F_1 - \delta_k \right)}{1 - \beta\left(1 - \theta\right)} \right]^{\theta}, \tag{18}$$

and by in terms of the human capital net return of $\bar{A}_H \left(1 - x\right) - \delta_h$ by

$$1 + g = \beta[1 + \bar{A}_H \left(1 - x\right) - \delta_h]. \tag{19}$$

And so

$$q = \frac{1}{\theta} \left[\frac{\beta\theta \left(1 + \bar{A}_G F_1 - \delta_k \right)}{1 - \beta\left(1 - \theta\right)} \right]^{1-\theta} = \frac{1}{\theta} \left(\beta \left[1 + \bar{A}_H (1 - x) - \delta_h \right] \right)^{\frac{1-\theta}{\theta}}. \tag{20}$$

A higher BGP return on capital, with the return on human and physical capital equal along the BGP, causes a higher growth rate and a higher q. A persistent shock that lowers the growth rate on the BGP is likely to cause a low frequency decrease in q. For example, an increase in the productivity factors $\bar{A}_G$ and $\bar{A}_H$ cause the BGP q to rise. A persistent positive money supply rate increase, of $\bar{\mu}$, causes higher inflation over time, substitution from goods to leisure, a lower human capital utilization rate of $1 - x$, and a lower return on both human and physical capital. This cause Tobin's q to fall over time, which should be reflected in low frequency data. With exogenous growth, or without an adjustment cost of physical capital (if $\theta = 1$ and $q = 1$), there is no interaction between growth, the capital return and q that produces the low frequency inflation and q correlation found in the data.

4 Calibration

In calibrating a standard DSGE growth model, typically only business cycle properties are matched, using exogenous growth models. Endogenous growth also allows examination of long run, low frequency, properties of the simulated model relative to the data. This additional step involves setting the structural parameters to calibrate the growth component of the model, along with low frequency and business cycle aspects.

4.1 Data

Following Baxter and King (1999), the low frequency component of a series has a periodicity of longer than 32 quarters, the business cycle component a periodicity of 6 to 32 quarters, and the high frequency component a periodicity of 2 to 6 quarters, given a minimum duration of a cycle as being 2 quarters. Therefore the low frequency component is identified using a band pass filter to filter out the periodicity of 2 to 32 quarters.

For the target variables below in Table 1, the data are annual averages of quarterly post-1960 US data, from the National Income and Product Accounts, except q which is from Hall (2001), and leisure which is from the Bureau of labor Statistics (BLS). For the average values of target variables the data period is 1960 to 1999, since we are constrained by the need to target a plausible historical q that is greater than one; for 1960 to 2007 data, q falls below unity. However for the volatility data, found below in Table 5, the data is quarterly from 1960 to 2007. For the q volatility, the data is from Smithers and Co. (2007), which is computed using the methodology of Wright (2004). Note that the business cycle and low frequency properties of both the Hall and Smithers and Co. q series are similar.

One exception to the 1960 to 1999 period for the historical averages of target values in Table 1 is leisure since the BLS data starts in 1964 instead of 1960. Here, the average leisure is estimated at 0.55 by following Gomme and Rupert (2007), who have a calibrated value of 0.505. In particular, using the annual average weekly hours of work, with the total daily time of 16 hours, and a 5 day working week, normalized leisure is [16-(average weekly hours of work/5)]/16.

4.2 Target Variables and Parameter Values

Table 1 presents the target variables with values given from the data and the steady state calibrated model. The value of q is 1.26, while the data value of g and π are 3.4% and 4.01%. The average share in GDP of consumption plus government spending, which is abstracted from in the model and considered as consumption, is 84%. The calibrated model is close to the target values.

Table 1: Values of the Growth Model Target Variables: Actual and Model

Target Variables, 1960 – 1999	Data	Model
GDP Growth(g)	3.4%	3.26%
Rate of Inflation(π)	4.01%	4.03%
c/y	0.84	0.79
i/y	0.16	0.21
q	1.26	1.26
Leisure (x)	0.55	0.52

Table 2 gives the baseline model parameter values. Standard values are chosen for β, α, and ψ. The mean money supply growth rate, μ is chosen to be consistent with the 4.01% annual average inflation rate of the data. The human capital technology parameters $\bar{A}_H$ and δ_h are fixed to target the 3.4% annual average GDP growth rate and a human capital utilization rate $1-x$ equal to 0.45 based on equations (4) and (19). The physical capital depreciation rate is fixed at 0.03 in line with Benk et al (2009). Calibration of the adjustment cost parameter θ is novel given the partial depreciation of the model. Substituting into the q equation (17) the average growth rate g

and the average q, from the data in Table 1, the result is that $\theta = 0.80$, which then made the baseline value of θ. In contrast, for example, Hercowitz and Sampson (1991) assume 100% depreciation of physical capital and estimate $\theta = 0.44$.

Table 2: Baseline Structural Parameter Values

β	α	δ_k	δ_h	ψ	θ	$\bar{A}_G$	$\bar{A}_H$	μ
0.96	0.36	0.03	0.024	1.84	0.8	1.2	0.21	0.0745

4.3 Shock Process Parameters

Table 3 reports the baseline values of the shock processes. The three forcing processes described in (13) through (15) involve six parameters, namely three autocorrelation parameters, ρ_G, ρ_H, ρ_μ, and three standard deviation parameters, σ_G, σ_H, σ_μ. The money supply parameters ρ_μ and σ_μ are 0.72 and 0.004, as estimated from an $AR(1)$ regression of quarterly seasonally adjusted currency supply growth from the Federal Reserve Bank of St. Louis database for 1960 to 2007.

For the other two shocks, the closest paper may be Maffezzoli (2000) who employs similar stochastic goods and human capital technologies, although Maffezzoli has an international focus, plus human capital spillover and the use of both physical and human capital in the Cobb-Douglas production of human capital. As in Maffezzoli, ρ_G and ρ_H are both set to 0.96, and $\sigma_G = 0.001$. The human capital shock standard deviation, σ_H, is set at 0.003 in the baseline, with an alternate endogenous growth model calibration using $\sigma_H = 0.001$ as in Maffezzoli.

Table 3: Baseline Second Moment Parameter Values

ρ_G	ρ_H	ρ_μ	σ_G	σ_H	σ_μ
0.96	0.96	0.72	0.001	0.003	0.004

5 Results

Figures 5 to 7 describe the model's impulse responses, while Table 4 presents low frequency simulations of q under alternative shock assumptions. And then Table 5 presents a fuller comparison across alternative models of both low frequency values and business cycle volatilities.

5.1 Impulse Response Analysis

The impulse responses to orthogonalized shocks to A_G, A_H and μ are based on the log-linearization of the full equation system (A.19) through (A.24) that is given in Appendix A. In Figures 5 to 7, the notation is "iy"$\equiv i/y$, "kh"$\equiv k/h$, "lg"$\equiv l_G$, "lh"$\equiv l_h$, "infl"$\equiv \pi$.

Figure 5 shows that a positive productivity shock in the goods sector makes agents substitute away from human capital investment time and leisure towards labor. This effort shocks upwards the physical capital investment rate (iy), with a consequent gradual increase in the physical capital to human capital ratio (kh). The output growth rate (g) falls as the physical capital investment rate rises. The greater productivity also raises the real wage and lowers the relative price of output, causing the inflation rate (infl) to be initially shocked downwards. The q initially rises, as the investment rate and the labor in the goods sector are shocked upwards, as can be seen in equation (21), which is derived simply by using the average product of capital $\frac{y_t}{k_t}$ and equation (16):

$$q_t = \frac{1}{\theta}\left[1 - \delta_k + \frac{i_t}{y_t} A_{Gt} \left(\frac{k_t}{h_t}\right)^{\alpha-1} l_{Gt}^{1-\alpha}\right]^{1-\theta} \tag{21}$$

However as $\frac{k_t}{h_t}$ gradually rises, this pushes q down. As $\frac{k_t}{h_t}$ begins to fall, the investment rate falls below its baseline and so does q. Meanwhile the inflation rate rises over time, moving in negative correlation to the q effects.

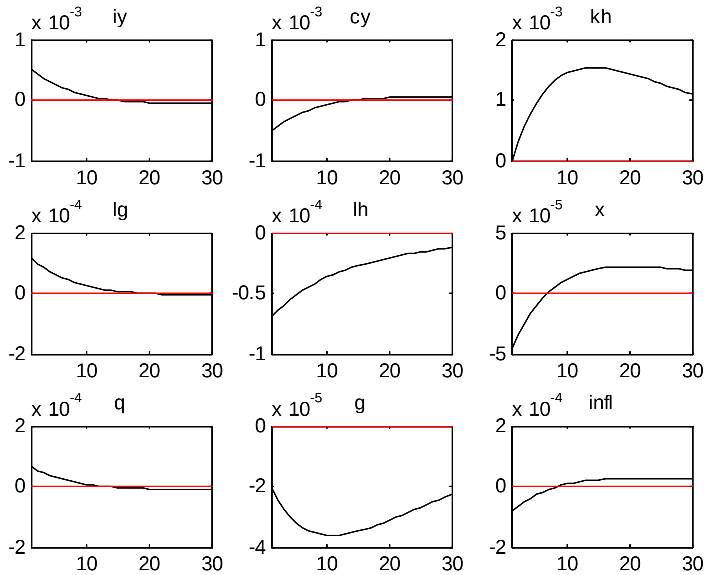


Figure 5: Impulse responses with respect to an orthogonalized TFP shock

Figure 6 shows that a positive shock to A_H causes agents to switch from leisure and labor in goods production towards human capital investment time, causing the growth rate to rise. The physical investment rate declines as the consumer shifts towards human capital investment and a lower $\frac{k_t}{h_t}$. A lower l_G and investment rate shock q downwards, again as in equation (21). Inflation falls over time as the increased human capital time leaves less time for goods production, causing a higher wage rate and lower relative price of output. Over time the q and inflation rate effects are relatively strong in their negative correlation, as compared to the A_G shock above.

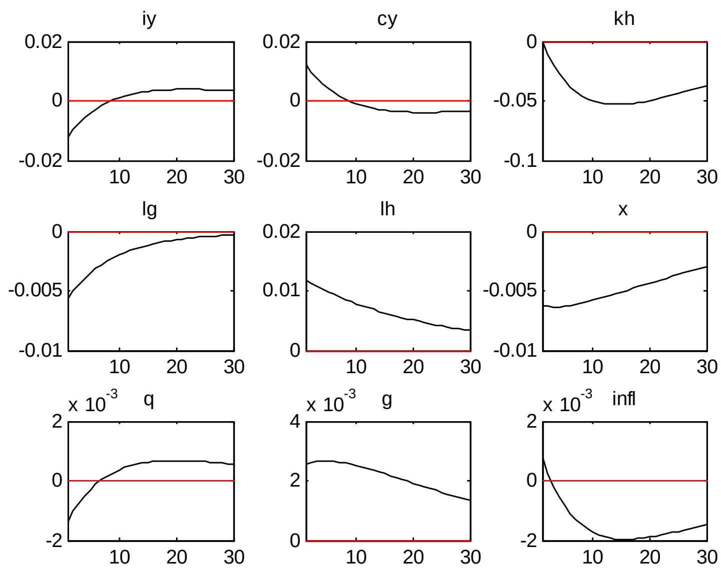


Figure 6: Impulse responses with respect to an orthogonalized human capital shock

In Figure 7, a positive monetary shock raises the inflation rate, thereby inducing substitution from goods to leisure and human capital investment, which are not subject to the inflation tax. The initial rise in the investment rate (iy) corresponds to the gradual rise in the physical capital to human capital ratio (kh), and a rise in q. As the capital ratio begins falling, the investment rate (iy) and q fall somewhat, even as inflation is still shocked upwards. This produces some additional negative correlation over time in the q and inflation rate effects.

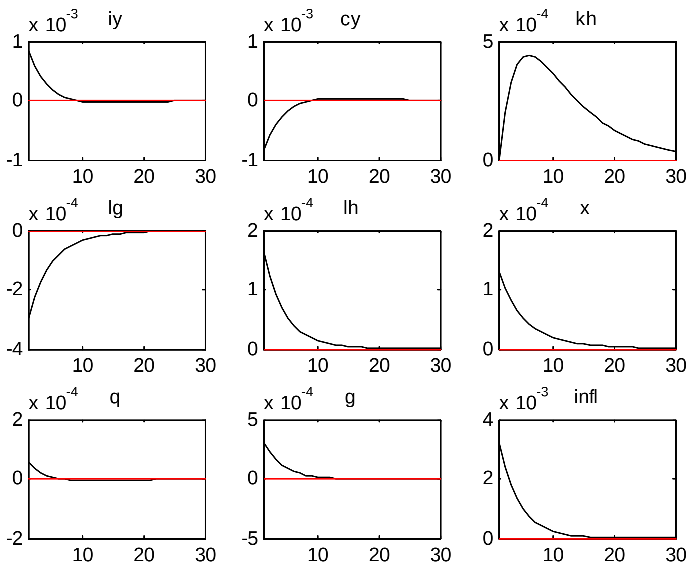


Figure 7: Impulse responses with respect to an orthogonalized money shock

5.2 Low Frequency Correlation

Table 4 presents the correlations between q and inflation for the baseline model and variations in the standard deviation of the two productivity shocks. The baseline model in the first simulation row does well in reproducing the data. The Table further shows in the last three simulation rows that the productivity shock to human capital investment is critical in affecting the level of the negative correlation. In contrast, changes in the productivity shock to the goods sector, in the first three simulation rows, have negligible effects. This reflects in part that the inflation and q effects of the impulse responses to the productivity shocks, in Figures 5 and 6, are an order of magnitude higher for the human capital shock than for the goods sector shock. to evaluate the model's performance against the data.

The results imply that a shock to human capital technology is key in determining the oscillations of q and inflation, in particular their low frequency correlation. Given the central objective of understanding this low frequency relation, Table 4 indicates success of the model in predicting these cross correlation properties of q-inflation data.

Table 4: Low Frequency Correlation between q and Inflation

US (UK) Data		-0.76 (-0.78)
Model: Calibration		Simulation
σ_H	σ_G	
0.003	0.001	-0.75
0.003	0.002	-0.76
0.003	0.003	-0.78
0.002	0.001	-0.69
0.001	0.001	-0.45
Note: Other parameters as in Table 3		

5.3 Alternative Model Comparison

Table 5 presents low frequency correlation and business cycle volatility results for the baseline plus three alternative models, which comprise the rows of the table. The first data column of numbers is the inflation - q low frequency correlation. The next four number columns are the standard deviation of four of the six target variables in Table 1, at a business cycle frequency. And the last column is the standard deviation of the stock return, derived in the next subsection.

The alternative models are one alternative endogenous growth model that differs only by the value of one parameter, plus two exogenous growth versions of the model. For the alternative endogenous growth model, the standard deviation of the human capital shock innovation is set to 0.001 as in Maffozolli (2000), instead of 0.003 as in the baseline. The two exogenous growth versions of this model are (i) a fixed labour supply model where l_H and x are fixed at their steady state levels as in the baseline growth model, and (ii) a variable labour supply model where only l_H is fixed at its steady level. Also in these exogenous growth variants, the standard deviation of the A_H shock is set to zero, with the human capital productivity parameter fixed at its steady state level $\bar{A}_H$.

Since both l_H and A_H are fixed at their steady state levels, the endogenous growth channel is shut down, with human capital growing exogenously at a balanced growth rate of 3.26% as in the baseline endogenous growth model. Model (i) is observationally equivalent to a standard exogenous growth model with inelastic labour; model (ii) reduces to an exogenous growth model with variable labour supply. Both have comparable BGP properties to the baseline model.

Both exogenous growth models have two forcing processes, $\{A_{Gt}\}$ and $\{\mu_t\}$ which evolve according to the $AR(1)$ representations in equations (13) and (15). Since the exogenous growth models lacks one shock process compared to the endogenous growth model, to make a fair comparison we set a higher level for the standard deviation of A_G than the baseline. Here $\sigma_G = 0.008$ in line with Prescott (1986) and Hansen (1985).

Table 5 shows that the endogenous growth baseline model clearly outperforms the exogenous growth models with respect to the low frequency correlation of inflation and q, in the first column. The baseline model also is closest to the data's standard deviation of q, in the second data column, but still falls short by an order of magnitude. The variable labor exogenous growth model comes closest to the data for the standard deviation of g and of i/y, in the third and fourth data columns, while the baseline model overstates these. The alternative endogenous growth model does best in simulating the standard deviation of the inflation rate in the fifth data column.

Table 5: Low Freq. Corr. and Bus. Cycle Volatility in Alternative Models

	Low Freq Correlation:	Business Cycle Frequency				
	$(\pi : q)$	sd(q)	sd(g)	sd(i/y)	sd(π)	sd(R_m)
US Data	-0.76	0.11	0.005	0.009	0.002	0.014
Baseline Endog Growth	-0.75	0.004	0.013	0.03	0.012	0.011
Alternate Endog Growth	-0.45	0.001	0.004	0.009	0.006	0.003
Exog: Fixed Labour	-0.04	0.0007	0.0009	0.006	0.005	0.0013
Exog: Variable Labour	-0.15	0.0008	0.001	0.008	0.005	0.0013
Note: For Exog models, $\sigma_G = 0.008$. Other parameters are fixed as in Table 3						

Note that there is an interesting trade-off indicated between the baseline model and the alternative endogenous growth model, with a lower standard deviation of the human capital shock innovation, at $\sigma_H = 0.001$, instead of $\sigma_H = 0.003$ as in the baseline. The alternative lowers the inflation- q correlation, and the q and R_m volatilities towards a worse fit with the data, but raises the g volatility, and lowers the i/y and π volatility towards a better fit. This implies that any extension of the model that enables a lower σ_H while maintaining the q and R_m facts, can improve across this whole spectrum. Such extensions are proposed in the concluding section.

5.4 Stock Returns

The baseline model does better than the two versions of the exogenous growth models in replicating the q volatility at a business cycle frequency, getting 36% of its magnitude. However, this happens at the cost of overstating the magnitude of the volatility of growth, the investment ratio and inflation, while exogenous growth models underestimate the volatility of q and g, and also overestimate the volatility of inflation. A related facet for comparison robustness is the business cycle volatility of stock returns.

The last data column of Table 5 shows data and simulation results for the standard deviation of the stock price return, which can be derived within the baseline model. The real stock return, denoted by R_{mt}, is typically defined by:

$$R_{mt+1} = \frac{v_{t+1} + d_{t+1}}{v_t}. \tag{22}$$

By deflating the numerator and denominator of equation (22) by the capital stock and using equation (16), the following relationship between q and stock return results.

$$R_{mt+1} = \frac{(1-\theta)\theta^{\frac{\theta}{1-\theta}} q_{t+1}^{\frac{1}{1-\theta}} + 1 + MPK_{t+1} - \delta_k}{q_t} \tag{23}$$

Appendix $B.3$ presents this derivation.

Equation (23) shows that the adjustment cost parameter θ drives a wedge between stock return and gross marginal product of capital. In the absence of adjustment cost, with $q_t = 1$, the stock return equals to the gross marginal product of capital, $1 + MPK_{t+1} - \delta_k$. With $q_t > 1$, volatility in q would impact on the volatility of the stock return.

The last column of Table 5 reports the standard deviation of stock returns for data and the model. Data in the last column of Table 5 for stock return is from Robert Shiller's online databank, with monthly series converted to quarterly. The baseline model does well in matching the data, while the other models do not. For variants of the endogenous growth model, this points out that when the key parameter of the standard deviation of the human capital shock innovation, σ_H, is set so that the model matches the inflation-q correlation data, the result is that the model also nearly matches the stock return volatility data. And note that the failure of exogenous growth models in matching such stock volatility data is pointed out by Gomme et al (2008).

5.5 Exogenous Growth

Table 5 shows that the exogenous growth models (i) and (ii) give rise to a slight-to-modest negative correlation between q and inflation. In Appendix C, Figures 8 and 9 present the TFP and money supply impulse responses in model (i), with fixed labor, and Figures 10 and 11 give the impulse responses in model (ii) with variable labor. The results show that there are mostly similar effects amongst the growth rate, q, and the inflation rate, but the difference that stands out in comparison to the endogenous growth model is the low order of magnitude of these effects. The main missing ingredient is that seen in Figure 6, in which a human capital shock causes a relatively big growth rate response, and inflation and q negatively correlated response over time.

6 Conclusion

The paper contributes to an explanation of the empirical stylized negative correlation between Tobin's q and inflation through a $DSGE$ endogenous growth model that identifies plausible fundamentals. The importance of this study is that while there is an emerging literature that shows how monetary policy, for example, affects the stock market through sticky wages and inflation targeting (Christiano et al, 2008), less is known of the long run effect of inflation taxes on the stock market through endogenous growth.

The paper develops closed form expressions for Tobin's q with physical capital adjustment cost to understand the relationship between inflation, q and human capital utilization along the balanced growth path. The impulse response analysis helps reveal the transmission mechanism of the productivity and monetary shocks through the "human capital channel". The simulation results then provide an explanation in particular for the observed low frequency negative correlation between q and inflation. Comparison of the baseline to alternative models including exogenous growth variants highlights the success of the baseline in this respect, while showing an ability to capture a good portion of q business cycle volatility as well as most of the stock return volatility. And this indicates that the human capital sector and its productivity shock is key to the overall results.

Extensions could involve introducing convexity into the model in at least two key ways. The q function itself can be made convex through factors such as those introduced as in Belo et al (2010). And the effect of inflation on growth can be made significantly more convex by introducing an exchange credit alternative to money for making transactions, as in Gillman and Kejak (2005). These factors can strengthen the translation of inflation effects onto q. In low inflation economies the negative growth effect of inflation would be marginally stronger, causing a bigger fall off of q; and this q decline could be even more pronounced with q convexity as the investment rate declines by more when the long term growth rate is decreased by the inflation tax.

Such convexities may combine to allow for an even smaller variance of the human capital shock standard deviation to be specified, in order to replicate the low frequency data correlation between inflation and q. Results presented indicate that this would improve the model's overall business cycle performance. And the convexities may allow for distinguishing between developed low inflation economies and developing high inflation economies in terms of the strength of the inflation - q correlation.

A second direction in extensions would be introducing explicit financial intermediation, both for exchange credit and for intertemporal credit via savings and investment intermediation (eg., Gillman, 2010). This would allow for bank crisis effects through a stochastic bank productivity factor that could help lower simulated inflation rate volatility and lesson the need to introduce sticky prices. The financial intermediation effect on q during bank crises may cause a less negative inflation - q correlation, as inflation and growth both fall during bank crises, and q also falls because of the bank crisis effect on equity markets. But during normal times, low inflation and high growth can combine with a rising bank productivity to cause q to be even higher and the low frequency inflation - q correlation to be more negative. As a third type of extension, the ability to explain q through the current model's shocks could be illustrated further by backing out the implied shocks of the model over time using data series as in Nolan and Thoenissen (2010).

References

[1] Basu, P., (1987), "An Adjustment Cost Model of Asset Pricing," International Economic Review, Vol. 28, No. 3, 1987, pp. 609–621.

[2] Baxter, M., and R. King, (1999), "Measuring Business Cycle Fluctuations: Approximate Band Pass Filter for Economic Time Series," Review of Economics and Statistics, 81, 4, 575-593.

[3] Belo F, C. Xue and L. Zhang, (2010), "The Cross Section of Stock Valuation Ratios," Mimeo, U of Minnesota.

[4] Benk, S, M. Gillman, M. Kejak, (2009), "US Volatility Cycles of Output and In‡Inflation, 1919-2004: A Money and Banking Approach to a Puzzle," CEPR Discussion Paper, No. 7150.

[5] Christiano, L., C. Ilut, R. Motto and M. Rostagno, (2008). "Monetary policy and stock market boom-bust cycles," Working Paper Series 955, European Central Bank.

[6] Danthine J. and J. Donaldson, (1986), "Inflation and Asset Pricing in an Exchange Economy", Econometrica, 54, 585-605.

[7] Gillman, M., (2010), Advanced Modern Macroeconomics, Financial Times Press, Pearson Education, forthcoming December.

[8] Gillman, M., and M. Kejak, (2005), " Inflation and Balanced-Path Growth with Alternative Payment Mechanisms", Economic Journal, Vol 115 (January): 247-270.

[9] Gillman, M, P. Minford, and V.P.M. Le, (2007), "An Endogenous Taylor Condition in an Endogenous Growth Monetary Policy Model," Cardiff Economics Working Papers E2007/29.

[10] Gomme, P., (1993). "Money and growth revisited : Measuring the costs of inflation in an endogenous growth model," Journal of Monetary Economics, Elsevier, vol. 32(1), pages 51-77, August.

[11] Gomme, P, and P. Rupert (2007), "Theory, Measurement and Calibration of Macroeconomic Models," Journal of Monetary Economics, 32: 51-77.

[12] Gomme P., B. Ravikumar and P. Rupert, (2008), "The Return to Capital and Business Cycle," Concordia University Department of Economics Working Paper Series, 08-002, Montreal.

[13] Hall, R.E., (2001), "Stock Market and Capital Accumulation."American Economic Review, December 2001, 91(5): 1185-1202.

[14] Hansen, G.D., (1985), "Indivisible Labour and the Business Cycle," Journal of Monetary Economics, 16, 309-327.

[15] Hercowitz, Zvi and Sampson, Michael, (1991). "Output Growth, the Real Wage, and Employment Fluctuations," American Economic Review, American Economic Association, vol. 81(5): 1215-37, December.

[16] King, R. G. and S.T. Rebelo, (1993), "Low Frequency Filtering and Real Business Cycles," Journal of Economic Dynamics and Control, 17, 207-231.

[17] Kocherlakota, Narayana R and Yi, Kei-Mu, (1997), "Is There Endogenous Long-Run Growth? Evidence from the United States and the United Kingdom," Journal of Money, Credit and Banking, Blackwell Publishing, vol. 29(2), pages 235-62, May.

[18] Lucas, Robert E., Jr., (1967), Adjustment Costs and the Theory of Supply, The Journal of Political Economy, 75 (4, Part 1, August): 321-334.

[19] Lucas, Robert E, Jr., (1978), "Asset Prices in an Exchange Economy," Econometrica, Econometric Society, vol. 46(6), pages 1429-45, November.

[20] Lucas, Robert E, Jr., (1980), "Equilibrium in a Pure Currency Economy," Economic Inquiry, Oxford University Press, vol. 18(2), pages 203-20, April.

[21] Lucas Robert E., Jr., (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics, 22:1: 3-42.

[22] Lucas, R.E., Jr., and E.C. Prescott, (1971), "Investment under Uncertainty," Econometrica, 39, 5: 659-681.

[23] Maffezzoli, M., (2000), "Human Capital and International Real Business Cycles," Review of Economic Dynamics, 3, 137-165.

[24] McGrattan, Ellen, and Edward C. Prescott, (2005), "Taxes, Regulations, and the Value of U.S. and U.K. Corporations", Review of Economic Studies 72, 767–796.

[25] Newey, Whitney K., and Kenneth D. West, (1987), "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," Econometrica, Vol. 55, Issue 3 (May), pp. 703–08.

[26] Nolan, C. and C. Thoenissen, (2010), "Financial shocks and the US business cycle", Journal of Monetary Economics, 56: 596–604.

[27] Obstfeld, M. and K. Rogoff, (1996), Foundation of International Macroeconomics, The MIT Press.

[28] Prescott, E.C., (1986), "Theory Ahead of Business Cycle Measurement," Carnegie Rochester Conference Series on Public Policy 27 (Autumn):403-420.

[29] Price, S. and C., Schleicher, (2005), "Returns to Equity, Investment and Q: Evidence from the UK,"Manchester School, 73(1), pp. 32-57.

[30] Tobin, J., (1965), Money and Economic Growth, Econometrica, Vol. 33, No. 4 (Oct.), pp. 671-84.

[31] Wright, S., (2004), Measures of Stock Market Value and Returns for the US Nonfinancial Corporate Sector, 1900-2002, Review of Income and Wealth, 50(4), pp. 561-584.

A Appendix: Equilibrium Conditions

Define the lagrange multipliers associated with the consumer's flow budget constraint (2) as λ_t, the human capital technology (4) as η_t and the cash-in-advance constraint (5) as γ_t. The consumer's first order conditions are:

$$c_t : \beta^t U'(c_t) - P_t(\lambda_t + \gamma_t) = 0, \tag{A.1}$$

$$M_t : -\lambda_t + E_t\{\lambda_{t+1} + \gamma_{t+1}\} = 0, \tag{A.2}$$

$$z_{t+1} : -\lambda_t V_t + E_t \lambda_{t+1}\{V_{t+1} + D_{t+1}\} = 0, \tag{A.3}$$

$$B_{t+1:} - P_t^b \lambda_t + \lambda_{t+1} = 0, \tag{A.4}$$

$$h_{t+1} : -\eta_t + E_t \lambda_{t+1} l_{Gt+1} W_{t+1} + E_t \eta_{t+1}(1 - \delta_h + A_{Ht+1} l_{ht+1}) = 0, \tag{A.5}$$

$$l_{Gt:} - \psi \Gamma'(1 - l_{Gt} - l_{Ht})\beta^t + \lambda_t W_t h_t = 0, \tag{A.6}$$

$$l_{Ht} : -\psi\Gamma'(1 - l_{Gt} - l_{Ht})\beta^t + A_{Ht}\eta_t h_t = 0. \tag{A.7}$$

Using (A.1) and (A.2)

$$\lambda_t = \beta^{t+1} E_t \frac{U'(c_{t+1})}{P_{t+1}}, \tag{A.8}$$

which upon substitution in (A.3) and (A.4) yields

$$V_t E_t \left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\{V_{t+1} + D_{t+1}\}\right]; \tag{A.9}$$

$$P_t^b E_t \left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\right]. \tag{A.10}$$

A binding cash in advance constraint means that (5) reduces to

$$\frac{M_t}{P_t} = c_t, \tag{A.11}$$

which implies that

$$\frac{P_t}{P_{t+1}} = \frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}. \tag{A.12}$$

Upon substitution into (A.9) and (A.10) it results that

$$v_t E_t \left[U'(c_{t+1})\frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}\right] = \beta E_t \left[E_{t+1}\left(U'(c_{t+2})\frac{c_{t+2}}{c_{t+1}}\frac{1}{1+\mu_{t+2}}\right)(v_{t+1} + d_{t+1})\right], \tag{A.13}$$

and

$$p_t^b E_t \left[U'(c_{t+1})\frac{c_{t+1}}{c_t}\frac{1}{1+\mu_{t+1}}\right] = \beta E_t \left[E_{t+1}\left(U'(c_{t+2})\frac{c_{t+2}}{c_{t+1}}\frac{1}{1+\mu_{t+2}}\right)\right], \tag{A.14}$$

where v_t = real share price (V_t/P_t), $p_t^b = \frac{P_t^b}{P_t}$, and w_t denotes the real wage (W_t/P_t).

Using (A.11) and (6), one obtains the following compact expression for m_{t+1} :

$$\frac{\lambda_{t+1}P_{t+1}}{\lambda_t P_t} = m_{t+1}. \tag{A.15}$$

For the goods producer, define ω_t as the Lagrangian multiplier associated with the adjustment cost technology (11). The firms' first order conditions are

$$l_{Gt}^f : \frac{W_t}{P_t} = A_{Gt}F_2(k_t, l_{Gt}^f h_t), \tag{A.16}$$

$$i_t : \lambda_t P_t = \theta\omega_t \left(1 - \delta_k + \frac{i_t}{k_t}\right)^{\theta-1}, \tag{A.17}$$

$$\begin{aligned} k_{t+1} \quad : \quad & 0 = -\omega_t + E_t\left(P_{t+1}\lambda_{t+1}A_{Gt+1}F_{1t+1}\right) \\ & +E_t\omega_{t+1}\left[(1-\theta)\left\{1-\delta_k+\frac{i_{t+1}}{k_{t+1}}\right\}^{\theta} + \theta(1-\delta_k)\left\{1-\delta_k+\frac{i_{t+1}}{k_{t+1}}\right\}^{\theta-1}\right] \end{aligned} \tag{A.18}$$

The model can then be summarized by the following equations:

Tobin's q equation,

$$q_t = E_t m_{t+1}\left[\alpha A_{Gt+1} l_{Gt+1}^{1-\alpha}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{-(1-\alpha)} + 1 - \delta_k + (1-\theta)\theta^{\theta/(1-\theta)} q_{t+1}^{1/(1-\theta)}\right]; \tag{A.19}$$

the l_G equation,

$$\begin{aligned} & \frac{A_{Gt}}{A_{Ht}} l_{Gt}^{-\alpha}\left[\frac{k_t}{h_t}\right]^{\alpha} = E_t\left[m_{t+1}A_{Gt+1}l_{Gt+1}^{1-\alpha}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha}\right] + \\ & E_t\left[m_{t+1}l_{Gt+1}^{-\alpha}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha}(1-\delta_h + A_{Ht+1}l_{ht+1})\frac{A_{Gt+1}}{A_{Ht+1}}\right]; \end{aligned}$$

the x equation,

$$\frac{\psi}{x_t} - (1-\alpha)\beta E_t \left[\frac{1}{1+\mu_{t+1}} A_{Gt} l_{Gt}^{-\alpha} \left(\frac{k_t}{h_t}\right)^{\alpha-1} \left(\frac{c_t}{k_t}\right)^{-1} \right] = 0; \tag{A.20}$$

the k/h equation,

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\{(1-\delta_k)(k_t/h_t) + A_{Gt} l_{Gt}^{1-\alpha}(k_t/h_t)^{\alpha} - (c_t/k_t)(k_t/h_t)\}^{\theta}(\frac{k_t}{h_t})^{1-\theta}}{1-\delta_h + A_{Ht}(1-l_{Gt}-x_t)}; \tag{A.21}$$

the output growth equation,

$$\frac{y_{t+1}}{y_t} = \left[\frac{A_{Gt+1}}{A_{Gt}}\right] \left[\frac{k_{t+1}/h_{t+1}}{k_t/h_t}\right]^{\alpha} \{A_{Ht} l_{Ht} + 1 - \delta_h\} \left[\frac{l_{Gt+1}}{l_{Gt}}\right]^{1-\alpha}; \tag{A.22}$$

the inflation equation,

$$\frac{P_{t+1}}{P_t} = \frac{1+\mu_{t+1}}{\{(c_{t+1}/k_{t+1})/(c_t/k_t)\}\{(k_{t+1}/h_{t+1})/(k_t/h_t\}\{A_{Ht} l_{Ht} + 1 - \delta_h\}}; \tag{A.23}$$

and the discount factor equation,

$$m_{t+1} = \beta \frac{\{1+(1+\rho)\bar{\mu} - \rho\mu_{t+1}\}}{\{1+(1+\rho)\bar{\mu} - \rho\mu_t\}} \frac{(c_t/k_t)}{(c_{t+1}/k_{t+1})} \frac{(k_t/h_t)}{(k_{t+1}/h_{t+1})} \frac{1}{(1-\delta_h + A_{Ht} l_{Ht})}. \tag{A.24}$$

Equation (A.19) follows from (A.18), (16) and (A.15). Equation (**??**) follows from (A.5), (A.6), (A.7), (A.8), A.15) and (A.16). Equation (A.20) follows from (A.6), (A.8) and (A.16). Equation (A.21) follows by combining (4) (9) and (11). The growth equation (A.22) follows from (4) and (9). To obtain the inflation equation (A.23) rewrite the cash-in-advance constraint

(5) using (4) as

$$\frac{P_{t+1}}{P_t} = \frac{(1+\mu_{t+1})(A_{Ht}l_{Ht} + 1 - \delta_h)^{-1}}{\frac{c_{t+1}}{k_{t+1}}\frac{k_{t+1}}{h_{t+1}}\left(\frac{c_t}{k_t}\frac{k_t}{h_t}\right)^{-1}}.$$

For equation (A.24), use the log utility specification and equation (4) to rewrite this as

$$m_{t+1} = \frac{\beta\frac{c_t}{k_t}\frac{k_t}{h_t}}{\frac{c_{t+1}}{k_{t+1}}\frac{k_{t+1}}{h_{t+1}}} \frac{E_{t+1}\left(\frac{1}{1+\mu_{t+2}}\right)}{E_t\left(\frac{1}{1+\mu_{t+2}}\right)} \frac{1}{(1-\delta_h + A_{Ht}l_{Ht})}.$$

Next take a first order approximation around the steady state and use the forcing process for money supply growth in equation (15) to get the expression in equation (A.24).

The balanced growth equilibrium solution then follows. Based on (4), (18), the resource and time constraints of equations (**??**) and (A.20), the steady state can be represented as

$$1+g = 1-\delta_h + \bar{A}_H l_h = (1-\delta_k + \frac{i}{k})^{\theta}, \qquad (A.25)$$

$$1+g = \beta(1-\delta_h + \bar{A}_H(1-x)), \qquad (A.26)$$

$$\frac{c}{k} + \frac{i}{k} = \frac{y}{k} = \bar{A}_G\left(\frac{l_G h}{k}\right)^{1-\alpha}, \qquad (A.27)$$

$$\frac{c}{k}\frac{\psi}{x} = \frac{(1-\alpha)\beta}{(1+\bar{\mu})}\frac{y}{k}\frac{1}{l_g}, \qquad (A.28)$$

$$\beta\theta(\alpha\frac{y}{k} + 1 - \delta_k) = [1-\beta(1-\theta)](1+g)^{\frac{1}{\theta}}, \qquad (A.29)$$

$$1 = x + l_G + l_H. \qquad (A.30)$$

Equating the $1+g$ terms in the first equality of (A.25) and (A.26), and using equation (A.30), yields a linear relationship between l_G in terms of x

as follows:

$$(1-\delta_h)(1-\beta) = \bar{A}_H[l_G - (1-\beta)(1-x)]. \tag{A.31}$$

From equation (A.28) and the first part of equation (A.27), obtain $\frac{y}{k}$ in terms of $\frac{i}{k}$ and $\frac{x}{l_g}$. Substituting this into (A.29), and then writing $\frac{i}{k}$ in terms of g from (A.25), yields a further expression for g in terms of $\frac{x}{l_g}$. Finally replace l_G by its representation in terms of x, and g in terms of x from equation (A.26), to get an equation solely in x:

$$\begin{aligned}
&\frac{\theta(1-\beta)}{\bar{A}_H}(1-\delta_k)(1-\alpha)(1-\delta_h+\bar{A}_H(1-x))\psi - \frac{\theta(1-\delta_k)(1-\alpha)\beta}{1+\bar{\mu}}x \\
&+\frac{(1-\beta(1-\theta))(1-\alpha)}{1+\bar{\mu}}\beta^{\frac{1}{\theta}}(1-\delta_H+\bar{A}_H(1-x))^{\frac{1}{\theta}}x \\
&-\frac{(1-\beta+\beta\theta(1-\alpha))(1-\beta)}{\bar{A}_H}\psi\beta^{\frac{1}{\theta}-1}(1-\delta_h+\bar{A}_H(1-x))^{1+\frac{1}{\theta}} = 0.
\end{aligned} \tag{A.32}$$

Once x is solved from (A.32), l_G can be solved from (A.31). The remaining endogenous variables are just functions of l_G and x and can be computed.

B Appendix: Proofs

B.1 Proposition 3

Divide (A.13) through by k_{t+1} to get

$$\frac{v_t}{k_{t+1}} = E_t m_{t+1}\left[\left(\frac{v_{t+1}}{k_{t+2}}\right)(k_{t+2}/k_{t+1}) + (d_{t+1}/k_{t+1})\right].$$

Noting that $[A_{Gt}F(k_t, l_{Gt}h_t) - (W_t/P_t)l_{Gt}h_t - i_t] = d_t$,

$$\frac{v_t}{k_{t+1}} = E_t m_{t+1}\left[\left(\frac{v_{t+1}}{k_{t+2}}\right)\frac{k_{t+2}}{k_{t+1}} + \lambda_{t+1}A_{Gt+1}F_{2t+1} - \frac{i^k_{t+1}}{k_{t+1}}\right].$$

Now use the adjustment cost equation (11) to rewrite the above as:

$$\frac{v_t}{k_{t+1}} = E_t m_{t+1} \left[(A_{Gt+1} F_{2t+1} + 1 - \delta_k) + \left(\frac{v_{t+1}}{k_{t+2}} \right) \left(\frac{k_{t+2}}{k_{t+1}} \right) - \left(\frac{k_{t+2}}{k_{t+1}} \right)^{1/\theta} \right]. \tag{B.33}$$

Using the definition of q_t (21) rewrite this again as

$$\frac{v_t}{k_{t+1}} = E_t m_{t+1} \left[(A_{Gt+1} F_{2t+1} + 1 - \delta_k) + \left(\frac{v_{t+1}}{k_{t+2}} \right) (\theta q_{t+1})^{\theta/(1-\theta)} - (\theta q_{t+1})^{1/(1-\theta)} \right]. \tag{B.34}$$

Next verify that (B.33) collapses to (A.19) if $q_t = \frac{v_t}{k_{t+1}}$.

B.2 Proposition 4

Note that from equations (6) and (A.8), along the BGP,

$$m_{t+1} = \frac{P_{t+1}\lambda_{t+1}}{P_t \lambda_t} = \frac{\beta}{1+g}. \tag{B.35}$$

Using (16), (A.19) , and (B.35), and imposing the balanced growth condition, $\frac{i_t}{k_t} = \frac{i_{t+1}}{k_{t+1}}$ one obtains that

$$\left(1 - \delta_k + \frac{i_t}{k_t}\right)^{1-\theta} = \frac{\beta\theta}{1+g} \bar{A}_G F_1 + \frac{\beta}{1+g} \left[(1-\theta) \left(1 - \delta_k + \frac{i_t}{k_t}\right) + \theta(1-\delta_k) \right]. \tag{B.36}$$

Use the adjustment cost function (11) to write

$$\frac{i_t}{k_t} = (1+g)^{1/\theta} - 1 + \delta, \tag{B.37}$$

which after substituting into equation (B.36) yields the proposition result of equation (18). Also it is straightforward to verify from equation (**??**) the standard result as in such Lucas (1988) human capital models with leisure, that $1 + g = \beta[1 - \delta_h + A_H(1-x)]$.

B.3 Stock Return Equation

Rewrite (22) as

$$R_{mt+1} = \frac{q_{t+1} + (d_{t+1}/k_{t+1})(k_{t+1}/k_{t+2})}{q_t} \left(\frac{k_{t+2}}{k_{t+1}} \right).$$

Noting that $d_{t+1} = (\alpha y_{t+1}/k_{t+1}) + 1 - \delta_k - (k_{t+2}/k_{t+1})^{1/\theta}$, and using (16) the above can be rewritten as:

$$R_{mt+1} = \frac{q_{t+1} + \{(\alpha y_{t+1}/k_{t+1}) + 1 - \delta_k - (\theta q_{t+1})^{\frac{1}{\frac{-\theta}{1-\theta}}}\}(\theta q_{t+1})^{\frac{-\theta}{1-\theta}}}{q_t},$$

which after simplifying reduces to (23).

C Appendix: Exogenous Growth Model Impulse Responses

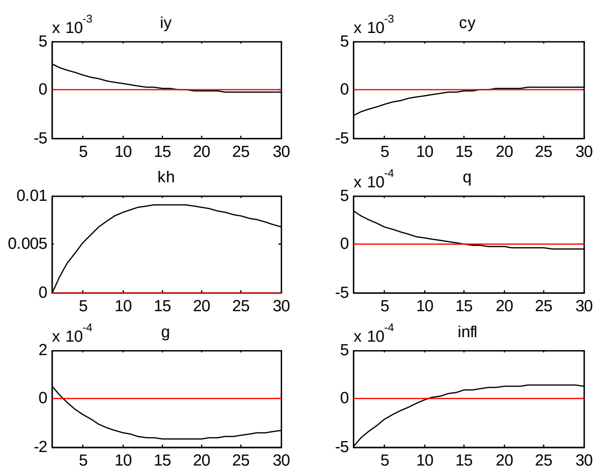


Figure 8: Effects of an orthogonalized TFP shock: Fixed Labour Supply Model

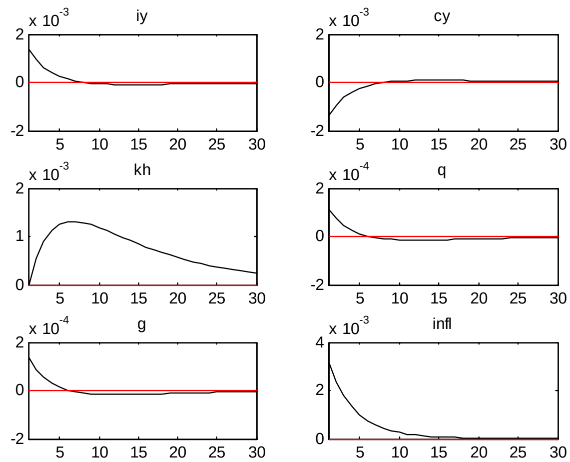


Figure 9: Effects of an orthogonalized money shock: Fixed Labour Supply Model

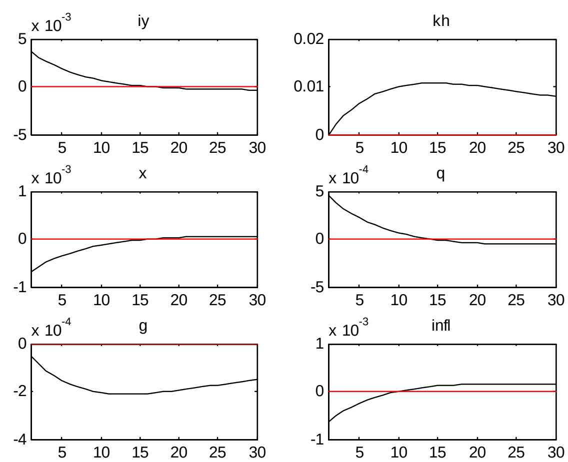


Figure 10: Effects of an orthogonalized TFP shock: Variable Labour Supply Model

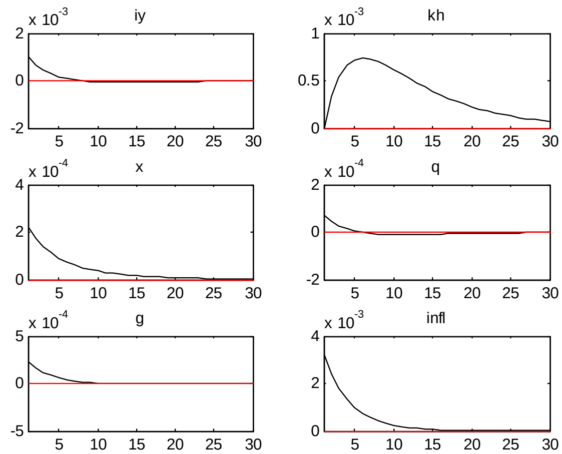


Figure 11: Effects of an orthogonalized money shock: Variable Labour Supply Model

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