MAGYAR TUDOMÁNYOS AKADÉMIA
KÖZGAZDASÁGTUDOMÁNYI INTÉZET

INSTITUTE OF ECONOMICS
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK **DISCUSSION PAPERS**

MT-DP – 2008/21

Riskiness, Risk Aversion, and Risk Sharing: Cooperation in a Dynamic Insurance Game

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BUDAPEST, 2008

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Riskiness, Risk Aversion, and Risk Sharing:
Cooperation in a Dynamic Insurance Game

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2008 September

ISBN 978 963 9796 39 3
ISSN 1785 377X

Kockázatosság, kockázatkerülés és kockázatmegosztás: Együttműködés egy dinamikus biztosítási játékban

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Összefoglaló

Ez a tanulmány azt vizsgálja, hogy az együttműködés egy biztosítási játékban hogyan függ a kockázatkerülés mértékétől és a jövedelem kockázatosságától. Egy olyan dinamikus játékot vizsgálok, amelyet korlátotozott elköteleződés jellemez. Az együttműködés szintjét úgy definiálom, hogy az egyenlő a diszkontfaktorral, amely felett a tökéletes kockázatmegosztás önfenntartó. Amikor nincs aggregát kockázat, az együttműködés mértéke nagyobb, ha (i) a hasznosságfüggvény konkávabb, és ha (ii) a jövedelem kockázatosabb, ha a jövedelemeloszlás kockázatosságának kritériuma az átlagtartó spread, vagy a másodrendű sztochasztikus dominancia (SSD). Viszont ha a biztosírás nem teljes, (ii) nem mindig igaz, az egyéni és aggregát kockázat kölcsönhatása miatt. CARA (CRRA) preferenciák esetén az együttműködés pozitívan függ az abszolút (relatív) kockázatkerülési koefficienstől és a jövedelemeloszlás szórásától (relatív szórásától), és független az átlagjövedelemtől. Ez a tanulmány az együttműködés szintjét összefüggésbe hozza a biztosítási transzferekkel és a fogyasztás simaságával abban az esetben is, amikor a tökéletes kockázatmegosztás nem önfenntartó.

Tárgyszavak: informális biztosítás, korlátozott elköteleződés, kockázatkerülés, kockázatosság, komparatív statika, dinamikus sztochasztikus játékok

JEL: C73, D80

as measured by the coefficient of variation, while cooperation does not depend on expected income.

Proof. In the appendix. □

Remark 2. *One may say that the correct measure of riskiness in the case of CRRA preferences is the coefficient of variation, since, along with risk aversion, this is what determines cooperation in the informal insurance game.*

Thus, $1/\delta^*$ is consistent with standard measures of risk aversion, and with measuring riskiness by the standard deviation (coefficient of variation), if preferences are of the CARA (CRRA) form. However, $1/\delta^*$ can be computed for any type of utility function, while it can still disentangle risk and expected value, and one can compare the riskiness of random variables with different means.

4 No aggregate uncertainty

Let us now turn to general, increasing and concave utility functions. This section looks at the case where, sharing risk perfectly, agents' consumption is completely smooth across states and over time. That is, agents only face idiosyncratic risk, aggregate income in the community is the same in all states of the world. For this, the two agents' incomes must be perfectly negatively correlated. Examining informal insurance in this case is related to a standard insurance setting, where a risk-averse agent can buy complete insurance from a principal, in other words, there is no background risk.

Since aggregate income is constant across states of the world and shared equally between the two agents, consumption of both agents is equal to per-capita income, $\overline{y}$. Then, (15) can be rewritten as

$$\begin{aligned}\frac{1}{\delta^*} &= 1 + \frac{u\left(\overline{y}\right) - \sum_s Pr\left(s\right) u\left(y_i\left(s\right)\right)}{u\left(y^h\right) - u\left(\overline{y}\right)} \\ &= 1 + \frac{u\left(\overline{y}\right) - u\left(CE^u\right)}{u\left(y^h\right) - u\left(\overline{y}\right)}, \qquad (19)\end{aligned}$$

where CE^u denotes the certainty equivalent of the distribution Y when preferences are described by the function $u()$.

Note that the complete insurance case means putting strong restrictions on the possible income distributions. In particular, if some income $y_i(s)$ is earned with probability $Pr(s)$, then there must be another income realization $y_i(-s) = 2\overline{y} - y_i(s)$, where $2\overline{y}$ is the constant aggregate income, and it must occur with the same probability, that is, $Pr(-s) = Pr(s)$. In other words, the distribution must be symmetric.

This section conducts a number of comparative static exercises on how the level of cooperation, given by equation (19), depends on the characteristics of the utility function, and the income distribution, Y. In particular, I examine how $1/\delta^*$ depends on the concavity of the utility function, that is, on risk aversion. I also study how $1/\delta^*$ changes, if the riskiness of the income distribution changes in terms of a mean-preserving spread, and when ranking the riskiness of distributions is based on second-order stochastic dominance (SSD).

First, let us compare cooperation levels when risk aversion changes. A standard characterization states that agent j, with utility function $v()$, is more risk averse than agent i, with utility function $u()$, if and only if $v()$ is an increasing and concave transformation of $u()$. This is equivalent to saying that agent j's (Arrow-Pratt) coefficient of absolute risk aversion is uniformly greater than that of agent i. Denote by $\phi()$ the increasing and concave function that transforms $u()$ into $v()$, that is, $v() = \phi(u())$. Taking Y as given, denote by δ_v^* (δ_u^*) the discount factor above which perfect risk sharing is self-enforcing, if agents have utility function $v()$ ($u()$).

Proposition 2. *With no aggregate uncertainty, $1/\delta_v^* \geq 1/\delta_u^*$. That is, if agents are more risk averse in the sense of having a more concave utility function, then cooperation in the informal insurance game increases.*

Proof. Using the formula determining $1/\delta^*$ with no aggregate uncertainty, equation (19), $1/\delta_v^* \geq 1/\delta_u^*$ is equivalent to

$$\frac{v(\overline{y}) - v(CE^v)}{v(y^h) - v(\overline{y})} \geq \frac{u(\overline{y}) - u(CE^u)}{u(y^h) - u(\overline{y})}.$$

Replacing $\phi(u())$ for $v()$ yields

$$\frac{\phi(u(\overline{y})) - \phi(u(CE^v))}{\phi(u(y^h)) - \phi(u(\overline{y}))} \geq \frac{u(\overline{y}) - u(CE^u)}{u(y^h) - u(\overline{y})}. \tag{20}$$

Since $\phi()$ is increasing and concave, and $u\left(y^h\right) > u(\overline{y}) > u(CE^u) > u(CE^v)$, we know that

$$\frac{\phi(u(\overline{y})) - \phi(u(CE^v))}{u(\overline{y}) - u(CE^u)} \geq \frac{\phi(u(\overline{y})) - \phi(u(CE^u))}{u(\overline{y}) - u(CE^u)} \geq \frac{\phi\left(u\left(y^h\right)\right) - \phi(u(\overline{y}))}{u(y^h) - u(\overline{y})}.$$

Rearranging yields (20). □

Proposition 2 means that we have the desirable comparative static result between risk aversion and the level of cooperation, when complete insurance is achieved, using concavity of the utility function as the measure of risk aversion, and $1/\delta^*$ as the measure of cooperation. Proposition 2 is analogous to the well-known result that a more risk-averse agent is willing to pay more for formal, complete insurance, with the same measure of risk aversion.

In the case of formal insurance, we know that a decrease in wealth, or, equivalently, an increase in a lump-sum tax, makes risk-averse agents willing to pay more to avoid a given risk, if preferences exhibit nonincreasing absolute risk aversion (DARA). This comparative static result goes through to the informal insurance case as well, as the following corollary states.

Corollary 1. *If preferences are characterized by nonincreasing absolute risk aversion (DARA), then a decrease in wealth, or, an increase in a lump-sum tax, results in more cooperation.*

Proof. Follows from Proposition 2 and the well-known result that, under DARA, a decrease in wealth is equivalent to an increasing and concave transformation of the utility function. □

Let us now turn to riskiness. First, a mean-preserving spread on the income distribution is taken as the criterion for ranking the riskiness of random incomes. I examine how $1/\delta^*$ changes when riskiness according to this standard concept changes under either of the following two assumptions.

Assumption (a). Income may take maximum three values.

Assumption (b). The support of the income distribution is constant.

Under assumption (a) and no aggregate uncertainty, there are only three possible income states: hl (agent 1 earning high income y^h, and agent 2 getting y^l) and lh (the reverse), and both occur with probability Pr^{asym}[6], and in the third income state, both agents must earn $\overline{y}$.

To consider a mean-preserving spread in this case, let us define a new income distribution, $\widetilde{Y}$, as $\widetilde{y}^h = y^h + \epsilon$ and $\widetilde{y}^l = y^l - \epsilon$, with $\epsilon > 0$. Note that mean income does not change, that is, $\frac{\widetilde{y}^h + \widetilde{y}^l}{2} = \frac{y^h + y^l}{2} = \overline{y}$. In the third income state, nothing changes. Denote by $\widetilde{\delta}^*$ the corresponding discount factor above which perfect risk sharing is self-enforcing.

Under assumption (b), the extreme income realizations, y^h and y^l are kept constant, and the spread occurs on the "inside" of the distribution. Denote by $1/\widetilde{\delta}^*$, for this case as well, the level of informal insurance corresponding to the more risky income distribution, $\widetilde{Y}$.

Proposition 3. $1/\widetilde{\delta}^* \geq 1/\delta^*$*, that is, when there is no aggregate uncertainty and assumption (a) or (b) holds, if income is riskier in the sense of a mean-preserving spread, then cooperation is higher in the informal insurance game.*

Proof. Under assumption (a), (15) can be written as

$$\frac{1}{\delta^*} = 1 - Pr^{asym} + Pr^{asym} \frac{u(\overline{y}) - u(y^l)}{u(y^h) - u(\overline{y})}. \tag{21}$$

Thus, in this case, $1/\widetilde{\delta}^* \geq 1/\delta^*$ is equivalent to

$$\frac{u(\overline{y}) - u(\widetilde{y}^l)}{u(\widetilde{y}^h) - u(\overline{y})} \geq \frac{u(\overline{y}) - u(y^l)}{u(y^h) - u(\overline{y})}.$$

Replacing for $\widetilde{y}^h$ and $\widetilde{y}^l$ gives

$$\frac{u(\overline{y}) - u(y^l - \epsilon)}{u(y^h + \epsilon) - u(\overline{y})} \geq \frac{u(\overline{y}) - u(y^l)}{u(y^h) - u(\overline{y})}. \tag{22}$$

Now, since $u()$ is increasing and concave, we know that

$$\frac{u(\overline{y}) - u(y^l - \epsilon)}{\overline{y} - y^l + \epsilon} \geq \frac{u(\overline{y}) - u(y^l)}{\overline{y} - y^l},$$

[6] If, for example, agent i earned y^h with probability $\pi > Pr^{asym}$, agent $-i$ (the other agent) would get y^h with a smaller probability $1 - \pi < Pr^{asym}$, the two agents' expected incomes would differ, thus they would not be ex-ante identical.

and

$$\frac{u\left(y^{h}+\epsilon\right)-u\left(\overline{y}\right)}{y^{h}+\epsilon-\overline{y}}\leq\frac{u\left(y^{h}\right)-u\left(\overline{y}\right)}{y^{h}-\overline{y}}.$$

Then, using the fact that $y^h - \overline{y} = \overline{y} - y^l$, dividing gives (22).

Under assumption (b), $1/\widetilde{\delta}^{*} \geq 1/\delta^{*}$ is equivalent to

$$\frac{u\left(\overline{y}\right)-u\left(\widetilde{CE}^{u}\right)}{u\left(y^{h}\right)-u\left(\overline{y}\right)}\geq\frac{u\left(\overline{y}\right)-u\left(CE^{u}\right)}{u\left(y^{h}\right)-u\left(\overline{y}\right)}, \tag{23}$$

where $\widetilde{CE}^{u}$ is the certainty equivalent of the riskier distribution $\widetilde{Y}$. It is well known that $\widetilde{CE}^{u} < CE^{u}$, thus (23) holds. □

Thus, in the complete insurance case, $1/\delta^*$ is consistent with a mean-preserving spread as the measure of riskiness, assumptions (a) or (b) being sufficient conditions. Now, let us consider second-order stochastic dominance (SSD) as the measure of riskiness. With a constant mean, the above result naturally extends to SSD, since an SSD deterioration is equivalent to a sequence of mean-preserving spreads. The result still holds if the dominated process has a lower mean, as the following corollary states.

Corollary 2. *In the complete insurance case, under assumption (b), if income is riskier in the sense of an SSD deterioration, then there is more informal insurance.*

Proof. Follows from the proof of Proposition 3, noting that, if $\widetilde{Y}$ is dominated by Y in the sense of SSD, then $\widetilde{CE}^{u} < CE^{u}$ for any $u()$ increasing and concave. □

Thus assumption (b) is a sufficient condition for the desirable comparative static result, using SSD to compare the riskiness of income distributions. Future work should determine necessary conditions.

5 With aggregate uncertainty

This section examines the case where agents must bear some consumption risk, even though they share risk perfectly. The community faces aggregate risk as well, while agents can only provide insurance to each other against idiosyncratic risks. In particular, I assume that income is realized independently for the two agents. Remember that to have no aggregate uncertainty, as in section 4, income realizations have to be perfectly negatively correlated across agents. As in the standard insurance setting when the agent cannot buy complete insurance, one may also say that there is background risk. I am interested in what goes through from the results of section 4.

Let us consider risk aversion first. Remember that $u()$ and $v()$ are two utility functions, and we have assumed that an agent with utility function $v()$ is more risk averse than an agent with utility function $u()$. Remember also that δ_v^* (δ_u^*) denotes the discount factor above which perfect risk sharing is self-enforcing, if agents have utility function $v()$ ($u()$). The following assumption is sufficient to guarantee that the desirable comparative static result holds.

Assumption (c). $u\left(y_1\left(s\right)\right)+u\left(y_2\left(s\right)\right)\leq 2u\left(\overline{y}\right)$, where $\overline{y}=\frac{y^h+y^l}{2}$, for all s where $y_1\left(s\right)\neq y_2\left(s\right)$.

This assumption means that there is no asymmetric state where the expected utility in autarky would be higher than the utility from consuming $\overline{y}$.

Proposition 4. *With aggregate uncertainty, under assumption (c),* $1/\delta_v^*\geq 1/\delta_u^*$. *That is, if agents are more risk averse in the sense of having a more concave utility function, then cooperation is higher in the informal insurance game.*

Proof. Using the formula determining $1/\delta^*$, equation (15), for $1/\delta_v^*\geq 1/\delta_u^*$ to hold it is sufficient that

$$\frac{v\left(\frac{y_1(s)+y_1(-s)}{2}\right)-\frac{1}{2}\left[v\left(y_1\left(s\right)\right)+v\left(y_1\left(-s\right)\right)\right]}{v\left(y^h\right)-v\left(\overline{y}\right)}\geq\frac{u\left(\frac{y_1(s)+y_1(-s)}{2}\right)-\frac{1}{2}\left[u\left(y_1\left(s\right)\right)+u\left(y_1\left(-s\right)\right)\right]}{u\left(y^h\right)-u\left(\overline{y}\right)},\forall s. \tag{24}$$

Denote by $Ey\left(s\right)$ mean income at state s, and by $CE^u\left(s\right)$ the certainty equivalent at state s, when preferences are described by the utility function $u()$, that is, $u\left(CE^u\left(s\right)\right)=\frac{1}{2}u\left(y_1\left(s\right)\right)+\frac{1}{2}u\left(y_1\left(-s\right)\right)$. Then, (24) can be written as

$$\frac{v\left(Ey\left(s\right)\right)-v\left(CE^v\left(s\right)\right)}{v\left(y^h\right)-v\left(\overline{y}\right)}\geq\frac{u\left(Ey\left(s\right)\right)-u\left(CE^u\left(s\right)\right)}{u\left(y^h\right)-u\left(\overline{y}\right)}.$$

To complete the proof, one may use the same argument as in the proof of Proposition 2. □

Here I put a restriction on the income process that is a sufficient condition for more risk aversion to increase voluntary insurance. One could also follow another approach, like Ross (1981) in the case of formal insurance, to find a stronger measure of risk aversion.

Let us now turn to riskiness, in particular, how $1/\delta^*$ changes if there is a mean-preserving spread on the income distribution. I provide counterexamples to the expected comparative static result. It turns out to be sufficient to examine the simplest possible income distribution.

Suppose that income may only take two values, high or low, denoted y^h and y^l, respectively, as before. Let π denote the probability of earning y^h. Then $Pr^{asym} = \pi(1-\pi)$. Now, let us define a new, more risky income distribution, $\widehat{Y}$, in the sense of a mean-preserving spread. Let the new high income realization be $\widehat{y}^h = y^h + \epsilon$, with $\epsilon > 0$. To keep mean income constant, $\widehat{y}^l$ must equal $y^l - \frac{\pi}{1-\pi}\epsilon$, with $\epsilon < \frac{1-\pi}{\pi}y^l$. Note that in this case consumption in the asymmetric states is $\frac{\widehat{y}^h+\widehat{y}^l}{2} = \frac{y^h+y^l}{2} + \frac{1-2\pi}{1-\pi}\frac{\epsilon}{2}$. Denote the corresponding level of cooperation by $1/\widehat{\delta}^*$.

Proposition 5. *It is not true in general that $1/\widehat{\delta}^* \geq 1/\delta^*$. That is, with aggregate uncertainty, a mean-preserving spread on incomes may result in less cooperation, even when income may take only two values.*

Proof. Let us construct a counterexample. Take $y^h = 1.5$, $y^l = 0.55$, $\pi = 0.6$ (so mean income is $0.6 \cdot 1.5 + 0.4 \cdot 0.55 = 1.12$), thus $Pr^{asym} = \pi(1-\pi) = 0.6 \cdot 0.4 = 0.24$, and $\epsilon = 0.2$. It follows that $\frac{y^h+y^l}{2} = 1.025$, and $\widehat{y}^h = 1.7$, $\widehat{y}^l = 0.25$, and $\frac{\widehat{y}^h+\widehat{y}^l}{2} = 0.975$. The mean is now $0.6 \cdot 1.7 + 0.4 \cdot 0.25 = 1.12$. Thus the distribution $\widehat{Y}$ is indeed a mean-preserving spread of Y. Consider the utility function

$$u(c) = \begin{cases} c^{0.8} & \text{if } c < 1 \\ c^{0.1} & \text{if } c > 1 \end{cases},$$

and smooth it appropriately in a small neighborhood of 1. This utility function could represent the preferences of a loss-averse agent. Replacing the above values in (16), we have

$$\frac{1}{\delta^*} = 1 - 0.24 + 0.24\frac{1.025^{0.1} - 0.55^{0.8}}{1.5^{0.1} - 1.025^{0.1}} = 3.12,$$

and

$$\frac{1}{\widehat{\delta}^*} = 1 - 0.24 + 0.24 \frac{0.975^{0.8} - 0.25^{0.8}}{1.7^{0.1} - 0.975^{0.8}} = 2.85,$$

which contradicts $1/\widehat{\delta}^* \geq 1/\delta^*$.

The result does not hinge on the fact that $\pi > \frac{1}{2}$, and that therefore consumption in the asymmetric states, hl and lh, decreases. Take $y^h = 1.5$, $y^l = 0.495$, $\pi = 0.1$, and $\epsilon = 0.6$, and consider the same utility function as above. This specification provides another counterexample, since $1/\delta^* = 1.84$ and $1/\widehat{\delta}^* = 1.35$. $\square$

The intuition behind this result is the following. In the case of incomplete insurance, when income becomes riskier in the sense of a mean-preserving spread, not only the spread between the high and low income realizations changes, but also consumption in the asymmetric states. As a result, the transfers $\frac{y^h+y^l}{2} - y^l = y^h - \frac{y^h+y^l}{2}$ are not just increased to $\frac{y^h+y^l}{2} - y^l + \frac{1}{1-\pi}\frac{\epsilon}{2} = y^h - \frac{y^h+y^l}{2} + \frac{1}{1-\pi}\frac{\epsilon}{2}$, but they also occur at consumption levels that are shifted by $\frac{1-2\pi}{1-\pi}\frac{\epsilon}{2}$ at the mean. Because of this shift, the utility gain of insurance represented by $u\left(\frac{y^h+y^l}{2}\right) - u\left(y^l\right)$, and the loss of insurance represented by $u\left(\frac{y^h+y^l}{2}\right) - u\left(y^h\right)$ are evaluated at a different consumption level for the income distribution Y than for $\widehat{Y}$. The curvature of the utility function may differ sufficiently at the two consumption levels, so that the ratio between the utility gain and loss of informal insurance changes in an ambiguous way, when a mean-preserving spread occurs on the income distribution. In particular, the level of cooperation may decrease.

This result points out that, when agents share risk informally, determining how much consumption variability they have to deal with is a rather complex issue, since the link between income risk, in some standard sense, and consumption risk is not straightforward. This is the consequence of the interplay of idiosyncratic and aggregate risk. See also Attanasio and Ríos-Rull (2000), who show that aggregate insurance may reduce welfare, when agents share (idiosyncratic) risk informally.

How to reconcile this negative result? Aggregate risk should be kept constant, while idiosyncratic risk increases. To do this, some negative correlation between the income realizations of the two agents has to be reintroduced. This can indeed work, as the following example demonstrates.

Example. Let us reconsider the first example of the proof above. The original income distribution, Y, was $y^h = 1.5$, $y^l = 0.55$, with the probability of the high income realization $\pi = 0.6$, that is, $Pr^{asym} = 0.24$. The second, more risky income distribution, $\widehat{Y}$, was $\widehat{y}^h = 1.7$, $\widehat{y}^l = 0.25$, with $\pi = 0.6$ still. Expected individual income is 1.12 for both agents, while expected aggregate income is 2.24 for both income distributions. We wanted to increase idiosyncratic risk, however, aggregate risk has also increased. In particular, the standard deviation of the distribution of aggregate income has increased from 0.5472 to 0.8352.[7] Now, let us introduce some negative correlation between the income realizations of the two agents for $\widehat{Y}$, to match the standard deviation of Y. This can be achieved my setting $Pr^{asym} = 0.364$, and decreasing the probability of the hh and ll states by 0.124 each. Let us denote the level of informal insurance by $1/\check{\delta}^*$ in this case. Then $1/\delta^* = 3.12$ as before, but $1/\check{\delta}^* = 3.81$, thus, keeping aggregate risk constant, cooperation in the informal insurance game increases as a result of a mean-preserving spread on the income distribution.

6 Discussion on measuring informal insurance

Risk theorists have devoted a lot of attention to formal insurance contracts, that occur between a risk-averse agent and an insurance company. The first issue is to measure the level of insurance, that is, "how much?" insurance occurs. In the case of formal insurance, the answer is simple: we can measure insurance in money units. The second issue is to relate insurance to risk preferences and the riskiness of a random variable, or gamble. With appropriate measures of risk aversion and riskiness, we would like to have comparative static results like "if the agent is more risk averse, she is willing to pay more to avoid a given gamble", and "a risk-averse agent is willing to pay more to avoid a riskier gamble". See Pratt (1964), Arrow (1965), Hadar and Russell (1969), Rothschild and Stiglitz (1970), Ross (1981), Jewitt (1987, 1989), and others, and Gollier (2001) for a summary.

[7] Note that speaking about the standard deviation or the coefficient of variation is equivalent here, since the mean doesn't change.

This paper, considering an informal insurance game, addresses similar issues. In particular, it examines how risk preferences and riskiness of agents' income together determine cooperation in the case of voluntary insurance, and aims to establish the type of comparative static results that exist for the case of formal insurance. In this section I look at how δ^*, the discount factor above which perfect risk sharing is self enforcing, is related to the complicated object, the set of state dependent intervals, that is the solution of the informal insurance game for any discount factor. I also examine how it relates to the insurance transfers and the smoothness of consumption across income states.

To do this, let us reconsider the numerical example presented in Ligon, Thomas, and Worrall (2002). Suppose that there are two agents with isoelastic utility and with a coefficient of relative risk aversion equal to 1, that is, $u() = ln()$. Income is independently and identically distributed (i.i.d.) across agents and time, and it may take two values, high ($y^h = 20$, say) or low (y^l=10)[8]. The probability of the low income realization is 0.1. Remember that when $\delta = \delta^*$, or whenever perfect risk sharing occurs and Pareto weights are equal, aggregate income should always be shared equally. This means that in the asymmetric states a transfer of 5 should be made, thus both agents consume 15.

Let us also consider an alternative scenario where the income distribution is as before, but agents are more risk averse. Denote the new utility function by $v()$. Let the coefficient of relative risk aversion be constant and equal to 1.5, thus $v(c) = c^{1-\sigma}/(1-\sigma) = c^{-0.5}/-0.5$.

The aim of this exercise is to compare the solution of the risk sharing with limited commitment model in these two cases. In particular, we first look at the optimal state-dependent intervals on the ratio of marginal utilities, that fully characterize the solution, as a function of the discount factor. I consider discount factors between 0.84 and 0.99. Then I examine what δ^*, tells us about the solution, and how it is related to the insurance transfers and the consumption allocation. The computations have been done using the software R (see www.r-project.org).

The black lines in Figure 1 reproduce figure 1 in Ligon, Thomas, and Worrall (2002), that

[8]The graph is the same for any y^h and y^l, if $y^l = 0.5y^h$ holds. The transfers and consumptions will be different, of course.

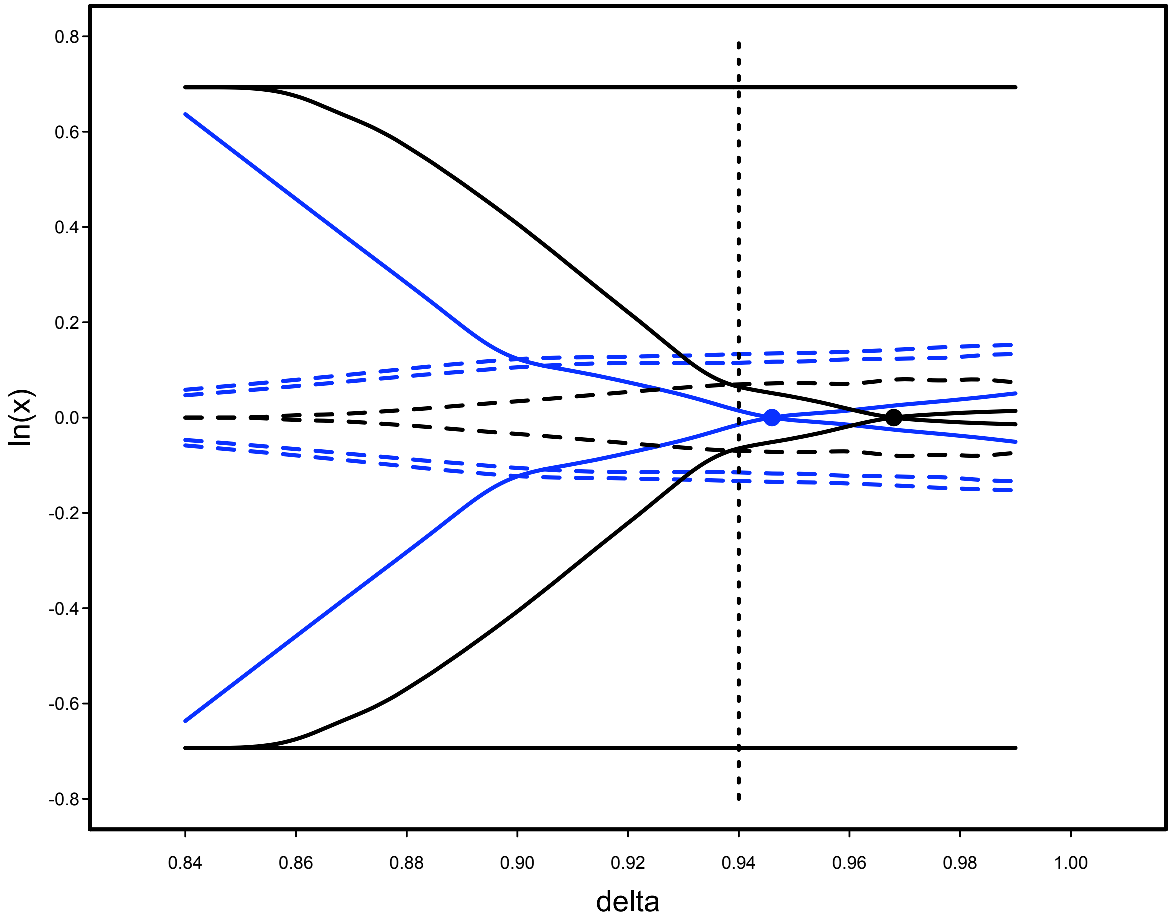

Figure 1: The optimal intervals of $ln\,(x)$ as a function of δ. The black lines show the optimal intervals on the (logarithm of the) ratio marginal utilities for the utility function $ln(c)$ (as in Ligon, Thomas, and Worrall (2002)), and the blue lines for $c^{1-\sigma}/(1-\sigma)$ with $\sigma = 1.5$. The dots represent δ^*. See more details in the main text.

represents the logarithm of x, the ratio of marginal utilities, as a function of the discount factor, δ. The dashed lines represent the optimal intervals for the symmetric states, hh and ll (the two coincide with logarithmic utility), while the solid lines are the intervals for the asymmetric states. The blue lines in Figure 1 show the corresponding intervals when $\sigma = 1.5$, that is, when agents are more risk averse.

First of all, let us look at the case where $\delta = 0.94$. For this discount factor, all the intervals overlap, except for the ones for states hl and lh (see the intervals along the vertical, dotted line in Figure 1). Then, the ratio of marginal utilities, after a sufficient number of periods, will only take two values, $\overline{x}^{hl}$ and $\underline{x}^{lh} = 1/\overline{x}^{hl}$. For the utility function $u()$, these numbers are 0.940 any 1.064. When agents' preferences are described by the more concave function $v()$, they equal 0.990 and 1.010. It follows from the first order conditions that the insurance transfers in the asymmetric states, hl and lh, are 4.53 and 4.92, for the utility functions $u()$ and $v()$, respectively. This also means that, if agents are more risk averse, consumption is smoother across states, so agents achieve more insurance. Note also that we are very close to the first-best transfer, 5, in both cases.

Now, notice that for any discount factor, the blue intervals, that belong to the case when agents are more risk averse, are wider. This means that a wider range of x's are possible with voluntary participation, in other words, agents cooperate more. Remember that, in the case where perfect risk sharing is self-enforcing, all the intervals overlap. On the other hand, when no informal insurance is possible, that is, when agents stay in autarky, each interval is just one point. Thus, if the intervals are wider, we may say that there is more insurance.

Finally, in Figure 1, the dots represent the discount factor above which perfect risk sharing is self-enforcing, δ^*. The black dot represents $\delta^* = 0.964$ for the utility function $u()$, while the blue dot is $\delta^* = 0.943$ that belongs to the more concave utility function $v()$. Notice that, as the dot moves to the left, the optimal intervals also move to the left, thus they become wider. Thus, one may capture the changes in the intervals, and thereby the changes in the transfers and the consumption allocation, by the scalar δ^*. Future research should determine how well δ^* may characterize the solution in more complicated settings.

7 Conclusion

This paper has shown a way to characterize cooperation in a widely-used informal insurance game, and made a first attempt to relate it to riskiness and risk aversion. In particular, I defined the level of cooperation, denoted $1/\delta^*$, as the reciprocal of the discount factor above which perfect risk sharing is self-enforcing. Comparative static results include that, if the utility function is more concave, that is, agents are more risk averse, $1/\delta^*$ is higher. However, in the case with aggregate uncertainty, a mean-preserving spread on the income process may decrease cooperation. This is because of the interplay of idiosyncratic and aggregate risk.

This paper has also shown that, in a simple setting, the comparative static results relating to the concavity of the utility function and the riskiness of the distribution of income go through to insurance transfers and the smoothness of consumption.

Let me conclude with a remark on measuring risk. Consider two simple distributions, Y and Z, that both have two possible realizations, high or low, determined by the toss of a fair coin. Y yields 1 or 2 euros, while Z gives 3 or 100. SSD or the recent measure of riskiness proposed by Aumann and Serrano (2008) tell us that Y is more risky, since it yields a lower payoff in all states of the world. However, Z seems to involve more variation. The standard deviation or the coefficient of variation would tell us that Z is indeed more risky, but these are right measures only for the CARA and CRRA cases, respectively. Supposing either preferences, $1/\delta^*$ gives a ranking that is consistent with the right measure of riskiness, the standard deviation or the coefficient of variation. It may also say something about the risk agents actually want and can insurance against in the case of more general preferences.

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8 Appendix

Proof of Claim 2:

Replace the utility function (18) and $y^h = y$ and $y^l = (1-q)\,y$ in the equation determining $1/\delta^*$ in the case of two possible income realizations, equation (21). For $\sigma \neq 1$, this gives

$$\frac{1}{\delta^*} = \frac{1}{2} + \frac{1}{2}\frac{\frac{\left(\left(1-\frac{q}{2}\right)y\right)^{1/\sigma}}{1-\sigma} - \frac{((1-q)y)^{1/\sigma}}{1-\sigma}}{\frac{y^{1-\sigma}}{1-\sigma} - \frac{\left(\left(1-\frac{q}{2}\right)y\right)^{1/\sigma}}{1-\sigma}},$$

which can be rewritten as

$$\frac{1}{\delta^*} = \frac{1}{2}\frac{(1-q)^{1-\sigma} - 1}{\left(1-\frac{q}{2}\right)^{1-\sigma} - 1}. \tag{25}$$

For $\sigma = 1$, we have $1/\delta^* = ln\,(1-q)\,/\left(2ln\left(1-\frac{q}{2}\right)\right)$. Thus $1/\delta^*$ only depends on σ and q, and is independent of y, that is, of mean income.

Now, I want to show that $\frac{\partial 1/\delta^*(\sigma,q)}{\partial\sigma} > 0$ and $\frac{\partial 1/\delta^*(\sigma,q)}{\partial q} > 0$. Let us suppose that $\sigma \neq 1$. The results generalize to $\sigma = 1$ taking limits. Let us differentiate equation (25) with respect to σ first. This gives

$$\begin{aligned} sign\left(\frac{\partial 1/\delta^*(\sigma,q)}{\partial\sigma}\right) &= sign\left(\frac{1}{2}\left[(1-q)^{1-\sigma}\,ln\,(1-q)\,(-1)\left(\left(1-\frac{q}{2}\right)^{1-\sigma} - 1\right)\right.\right. \\ &\left.\left. - \left(1-\frac{q}{2}\right)^{1-\sigma} ln\left(1-\frac{q}{2}\right)(-1)\left((1-q)^{1-\sigma} - 1\right)\right] \Big/ \left(\left(1-\frac{q}{2}\right)^{1-\sigma} - 1\right)^2\right) \\ &= sign\left(ln\left(1-\frac{q}{2}\right)\left(1-(1-q)^{\sigma-1}\right) - ln\,(1-q)\left(1-\left(1-\frac{q}{2}\right)^{\sigma-1}\right)\right) \\ &= sign\left(\frac{1-(1-q)^{\sigma-1}}{ln\,(1-q)} - \frac{1-\left(1-\frac{q}{2}\right)^{\sigma-1}}{ln\left(1-\frac{q}{2}\right)}\right), \end{aligned}$$

where the third line follows after dividing by $(1-q)^{1-\sigma}\left(1-\frac{q}{2}\right)^{1-\sigma} > 0$, and the last line follows dividing by $ln\,(1-q)\,ln\left(1-\frac{q}{2}\right) > 0$. We know that $0 < \frac{q}{2} < q < 1$, thus $0 < 1-q < 1-\frac{q}{2} < 1$. What remains to be shown is that the function

$$f(z) \equiv \frac{1-z^{\sigma-1}}{ln(z)}$$

is decreasing in z, $z \in (0,1)$. To do this, let us differentiate $f(z)$ with respect to z. This gives

$$sign\left(\frac{\partial f(z)}{\partial z}\right) = sign\left(\frac{-(\sigma-1)\,z^{\sigma-2}ln(z) - \frac{1}{z}\left(1-z^{\sigma-1}\right)}{(ln(z))^2}\right)$$
$$= sign\left(\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1\right).$$

Note that $\lim_{z\to 1}\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1=0$. Now, to show that $\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1<0$, we only have to establish that $g(z)\equiv\left(1-(\sigma-1)\,ln(z)\right)z^{\sigma-1}-1$ is increasing in z for $z\in(0,1)$. Taking derivatives with respect to z gives

$$sign\left(\frac{\partial g(z)}{\partial z}\right) = sign\left((\sigma-1)\,z^{\sigma-2}-(\sigma-1)\left(\frac{1}{z}z^{\sigma-1}+ln(z)\,(\sigma-1)\,z^{\sigma-2}\right)\right)$$
$$= sign\left(-(\sigma-1)^2\,ln(z)\,z^{\sigma-2}\right).$$

The first term is positive, the second is negative, the third is positive, and all this is multiplied by (-1), thus $\frac{\partial g(z)}{\partial z}$ is positive. It follows that $\frac{\partial f(z)}{\partial z}$ is negative, and that $\frac{\partial 1/\delta^*(\sigma,q)}{\partial\sigma}$ is positive.

Now, let us differentiate equation (25) with respect to q. This gives

$$sign(\frac{\partial 1/\delta^*(\sigma,q)}{\partial q}) = sign\left(\frac{1}{2}(1-\sigma)\left[(1-q)^{-\sigma}(-1)\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)-\right.\right.$$
$$\left.\left.-\left((1-q)^{1-\sigma}-1\right)\left(1-\frac{q}{2}\right)^{-\sigma}\left(-\frac{1}{2}\right)\right]/\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)^2\right)$$
$$= sign\left((1-\sigma)\left[\frac{1}{2}\left((1-q)^{1-\sigma}-1\right)\left(1-\frac{q}{2}\right)^{-\sigma}-\right.\right.$$
$$\left.\left.-(1-q)^{-\sigma}\left(\left(1-\frac{q}{2}\right)^{1-\sigma}-1\right)\right]\right)$$
$$= sign\left((1-\sigma)\left[\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)\right]\right).$$

The last line follows after dividing by $(1-q)^{-\sigma}>0$ and $\left(1-\frac{q}{2}\right)^{-\sigma}>0$. We have to consider two cases.

- $\sigma<1$. Now $1-\sigma>0$, so we have to show that $\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)>0$.
- $\sigma>1$. In this case $1-\sigma<0$, so we have to show that $\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1}{2}\left((1-q)^{\sigma}+1\right)<0$.

We may rewrite this last expression as

$$\left(1-\frac{q}{2}\right)^{\sigma}-\frac{1^{\sigma}+(1-q)^{\sigma}}{2}. \tag{26}$$

Note that $1 - \frac{q}{2}$ is the mean of 1 and $1 - q$. Let us define $h(z) \equiv z^{\sigma}$. So what we are comparing is the mean (a convex combination) of the values $h(1)$ and $h(1-q)$ to the value at the mean, that is, $h\left(\frac{1+1-q}{2}\right) = h\left(1 - \frac{q}{2}\right)$.

- $\sigma < 1$. $h(z)$ is concave, thus $h\left(1 - \frac{q}{2}\right) > \frac{h(1)+h(1-q)}{2}$. It follows that (26) is positive, as I wanted to show.
- $\sigma > 1$. $h(z)$ is convex, thus $h\left(1 - \frac{q}{2}\right) < \frac{h(1)+h(1-q)}{2}$. It follows that (26) is negative, and this is what I wanted to show.