Energy transmission networks and cooperative game theory II: TFF games and externalities

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Energy transmission networks (review)

Matrix form of equations:

$$AQ = P$$

A - Node-branch incidence matrix, Q - power flow vector ($[q_{12}...]$),
P - power injection vector ($[q_1, q_2...]$)

$$B(Y)\Theta = P$$

B(Y) - susceptance matrix whose elements are $B_{kl} = -Y_{kl}$ for the off-diagonal terms and

$$B_{kk} = \sum_{l \neq k \in \Omega} B_{kl}$$

Θ - vector of nodal voltage angles. Ω -actual column

$$|Q| = |B^D A^T \Theta| < \bar{Q}$$

$|\bar{Q}|$ branch power flow limit vector, B^D - diagonal matrix with $B^D_{kk} = Y_{ij}$

The rescheduling problem in general

As we know

$$B\Theta = P$$

B is not invertable, but only the differences of the elements of the vector *Theta* are important, so we can write

$$\Theta = B^{+}P$$

where B^{+} is the Moore-Penrose pseudoinverse of B. Constraint:

$$|B^{D}A^{T}\Theta| = |B^{D}A^{T}B^{+}P| < \bar{Q}$$

suppose the initial power generation/consumption vector P^{init}
and initial flows Q^{init}

$$s_P = -sign(P^{init}) \quad s_Q^D = diag(sign(Q^{init}))$$

The rescheduling problem in general II

The general form of linear programming problem:

$$\min_{x} f'x \qquad subject \;\; to: \;\; A_{ineq}x \leqq b_{ineq}, \;\; A_{eq}x = b_{eq}$$

here (supposing that the direction of line flows do not change!):

$$f = s_P \;\; A_{ineq} = s_Q^D B^D A^T B^+ \quad b_{ineq} = \bar{Q} \quad A_{eq} = [1 \;\; 1...1] \quad b_{eq} = 0$$

corresponding to the maximum load of lines and the equality of total inlet and outlet power. Further linear constraints can be added to the problem, describing the minimal and maximal values of nodal power values, corresponding to maximum generator capacity, and minimum consumption at a node.

PFF Games

- N: set of players - subsets: coalitions
- $\Pi(S)$: The set of partitions of S. Typical element: $\mathcal{P}_S$
- CFF or TU game: (N, v) where $v : 2^N \rightarrow \mathbb{R}$
- Partition function (PF): $V : \Pi \rightarrow (2^N \rightarrow \mathbb{R})$ assigns a CF to each partition
- PFF game: (N, V)

A possible PFF game on energy transmission networks

Assumptions

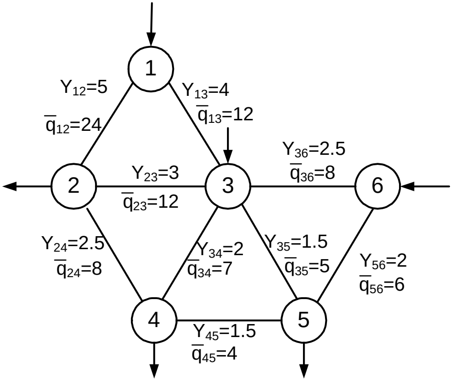
- An initial configuration of the network is given with generation and consumption values
- The overall power inlet/outlet of the coalition has to be in balance ($\sum_c P = 0$) this can be implemented in the LP formalism, as defining additional constraints: see eg the game of 4 players, with the coalitions $\{1,2\},\{3,4\}$

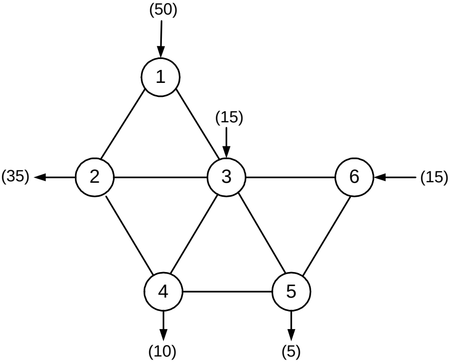
$$A_{eq} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad b_{eq} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

- We assume an independent network regulator, who determines the injectable/consumable quantities, according to the maximum possible overall consumption (for a given coalition structure, and the implied constraints, the amount of the total transmitted energy is optimized)

Example

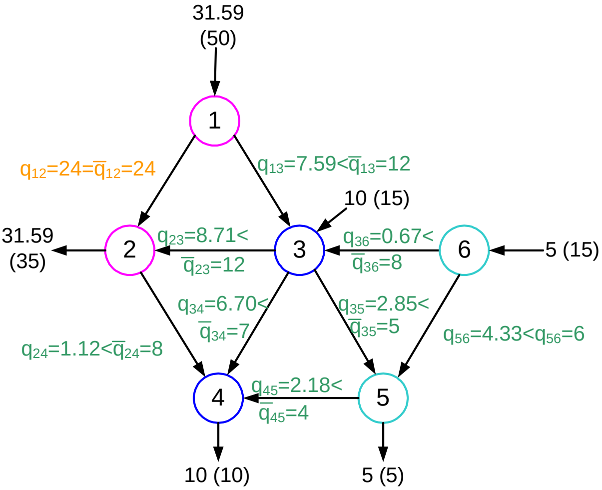
Structure, network parameters (admittances and load limits), generator capacities and demands:





Example II

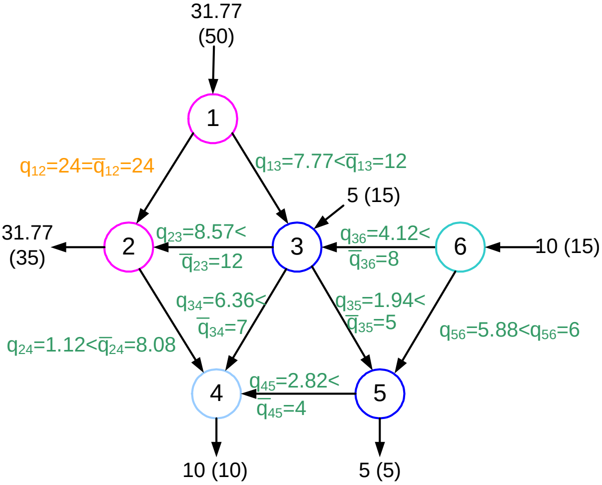
Partition:{ {1,2}, {3,4}, {5,6}}.



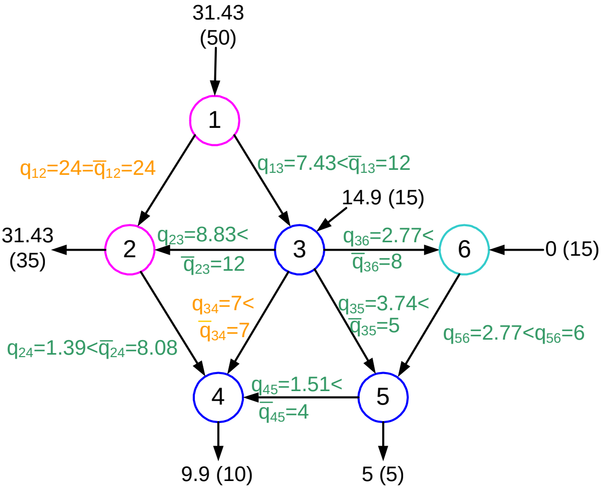
v({1,2})=2*31.59, v({3,4})=2*10, ect...

Example III

Partition:{ {1,2}, {3,5}, {4,6}}.



Partition:{ {1,2}, {3,4,5}, {6}}.



Stability issues

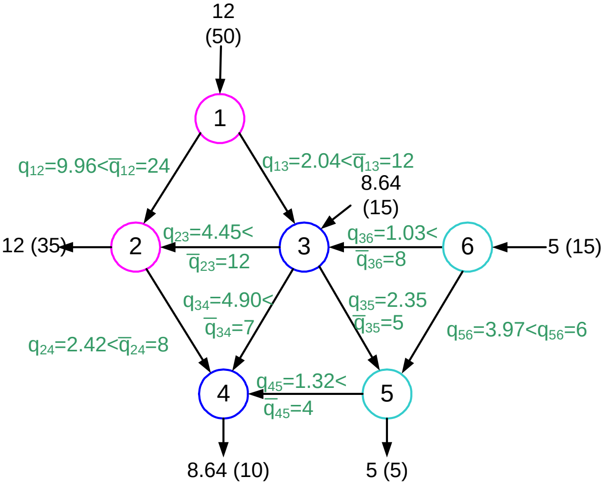
If one line fails, the network has to remain stable (with the same amounts of injected power, no line overloads may appear).

Constraints corresponding to stability issues may be included in the LP problem formulation of the rescheduling problem.

In nominal case: $A_{ineq} = s_Q^D B^D A^T B^+ \quad b_{ineq} = \bar{Q}$
B depends on the admittance values Y_{kl}
Let $\hat{B}_i$ correspond to the case where line i fails (which implies nullity of the corresponding Y_{kl})
Let the extended $\hat{A}_{ineq} = \hat{s}_Q^D \begin{pmatrix} B^D A^T B^+ \\ \hat{B}_i^D A^T \hat{B}_i^+ \end{pmatrix}$ where $\hat{s}_Q$ is the diagonal expansion of s_Q^D with the sign values corresponding to the initial flow directions, and
$\hat{b}_{ineq} = \begin{pmatrix} b_{ineq} \\ b_{ineq} \end{pmatrix}$
Similarly, the resulting $A_{ineq} =$ and b_{ineq} can be extended with blocks describing the fail of the remaining lines in a recursive way.

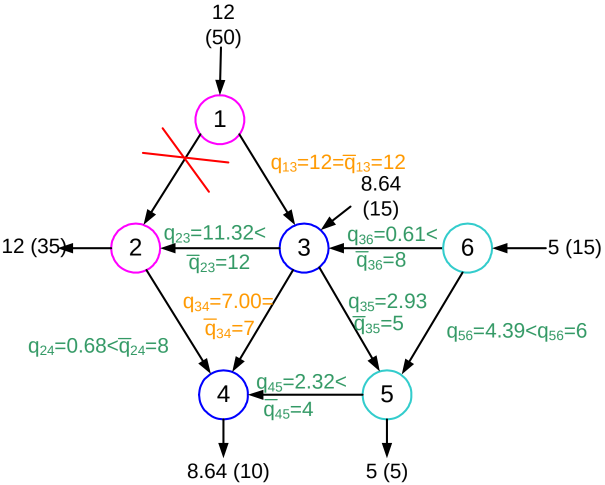
Stability issues, example

Example for { {1,2 },{3,4 },{5,6 } } - "safe" maximum of injected (or total transmitted) power:



Stability issues, example II

If line 1-2 fails:



Discussion

- The rescheduling problem (not supposing the possibility of building new lines) can be formulated as an LP problem, even in the case of stability requirements.
- If we suppose energy conservation equations for each coalition, the problem formulation leads to a PFF game
- Still neglected: Nonlinear losses of the network, different prices, and utility functions of generators and consumers, ect...