Energy transmission networks and cooperative game theory

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Outline

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1. Energy transmission networks - physical basis and model
2. Cooperative game theory - fundamental theoretical concepts
3. $1 \bigcap 2$

Energy transmission networks I

Graph: Buses ($\equiv$ nodes) connected by transmission lines ($\equiv$ edges)

- Buses: generators or/and consumers - (from physical point of view) can be characterized by quantity of consumed/generated power
- Transmission lines - can be characterized by admittance (susceptance), and capacity constraints

$$Z = R + jX \qquad Y = Z^{-1} = \frac{1}{R + jX} = \frac{R}{R^2 + X^2} + j\frac{-X}{R^2 + X^2} = G + j\mathbf{B}$$

Z - impedance, R - resistance, X - reactance, **Y - admittance**, G -conductance, **B - susceptance**

Energy transmission networks II

Voltage at bus i: sinusoidal waveform: $v_i(t) = V_i \sin(\omega t + \theta_i)$

V_i - magintude, $\omega = 2\pi f$ - frequency in rad/s, θ_i - phase angle

Some nodes are connected with transmission lines through which power can flow. Line connecting i and j: $Y_{ij} = Y_{ji}$

(real) power flow from i to j:

$$q_{ij} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j)$$

sign convention: $q_{ij} > 0$ if the power flows from i to j. $q_{ij} = -q_{ji}$

Energy conservation: The net power q_i injected into the network at bus i:

$$q_i = \sum_{j=1}^{n} q_{ij}$$

Energy transmission networks III

Wlg let us assume $V_i \equiv 1 \rightsquigarrow$

$$q_i = \sum_{j=1}^{n} Y_{ij} \sin(\theta_i - \theta_j)$$

n-1 independent equations $(q_1 + ... + q_n = 0) \rightsquigarrow \theta_n \doteq 0$. Individual line flows:

$$q_{ij} = Y_{ij} \sin(\theta_i - \theta_j)$$

Assuming that $(\theta_i - \theta_j)$ is small $\rightsquigarrow \sin(x) \rightarrow x$ "DC load flow model"

$\rightsquigarrow$ *Given power injections and power consumptions at each bus (node), the phase angles* θ_i *can be determined by solving a system of linear equations. From the phase angle differences, the line flows can be determined.*

Energy transmission networks IV

Matrix form of equations:

$$AQ = P$$

A - Node-branch incidence matrix, Q - power flow vector ($[q_{12}...]$),
P - power injection vector ($[q_1, q_2...]$)

$$B(Y)\Theta = P$$

B(Y) - susceptance matrix whose elements are $B_{kl} = -Y_{kl}$ for the off-diagonal terms and

$$B_{kk} = \sum_{l \neq k \in \Omega} B_{kl}$$

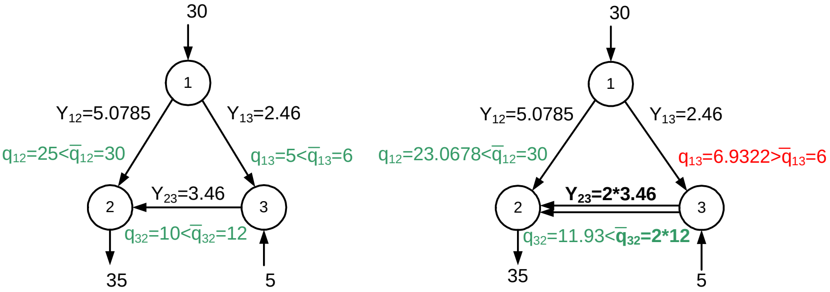
Θ - vector of nodal voltage angles. Ω -actual column

$$|Q| = |B^D A^T \Theta| < \bar{Q}$$

$|\bar{Q}|$ branch power flow limit vector, B^D - diagonal matrix with $B^D_{kk} = Y_{ij}$

Example

May show properties similar to Braess's paradox: Adding extra capacity to a network, can in some cases reduce overall performance



phase angles: [-2.0325 2.8902 0] and [-2.8180 1.7243 0]

n person cooperative games I

TU (transferable utility) games are defined by

- Set of players: $N = \{1, ..., n\}$
- Characteristic function: $v : \mathcal{N} \to \mathbb{R}$ where $\mathcal{N} = \mathcal{P}\{N\}$, $|\mathcal{N}| = 2^n$

subsets of N: **coalitions**, N: Grand coalition

$v(\emptyset) = 0$

Superadditivity: $v(S \bigcup T) \geq v(S) + v(T) \quad (\forall S, T \subseteq N)(S \bigcap T = \emptyset)$

Essential game: The superadditivity inequality is strict

n person cooperative games II

Assume that the grand coalition is formed.
The division of the joint payoff $v(N)$, represented by the payoff vector $\bar{x} = (x_1, x_2, .., x_n)$ is not evident. A payoff vector has to satisfy

$$v(N) = \sum_{i=1}^{n} x_i \qquad (\textit{Group rationality})$$

$$x_i \geq v(i) \quad \forall i \in N \qquad (\textit{Individual rationality})$$

to be a reasonable candidate for a solution

The Core

Group rationality, individual rationality, and coalitional rationality:

$$\sum_{i \in S} x_i \geq v(S) \quad \forall S \in \mathcal{N}$$

Example ($v(1) = v(2) = v(3) = 0$):

$$\begin{aligned} x(1) \geq 0 \ , \ x(2) &\geq 0 \ , \ x(1) \geq 0 \\ x(12) \geq v(12) \ , \ x(23) &\geq v(23) \ , \ x(13) \geq v(13) \\ x(123) &\geq v(123) \\ x(1) + x(2) + x(3) &= v(123) \end{aligned}$$

If eg. $v(12) = 90$, $v(13) = 80$, $v(23) = 70$, $v(123) = 135$, the core (restricted to coalition (123)) is a triangle (simplex) in the payoff space bounded by (66,55,15), (65,25,45), (35,55,45).

Network expansion games

Let us assume

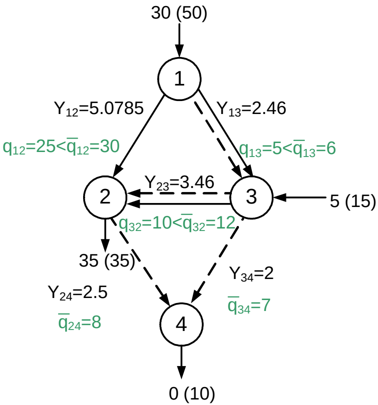
- A power transition network with n nodes (generators and consumers), with a given basic configuration
- The values of actual consumption and demand for the consumers and actual generation and maximum capacity for the generators
- A possible set of line additions with admittances, flow constraints and construction costs

Network expansion games II

Let us assume furthermore a possible coalition

- A line can be added only between two nodes which are in the coalition
- Only generators in the coalition can be rescheduled
- Only the consumption of a consumer in the coalition can be rescheduled
- Value for a given coalition can be determined based on the quantity of the energy sold (generators), or the difference between consumed and demanded energy. (e.g. in the simplest case the added value may be equal to these quantities)

Expansion game example I



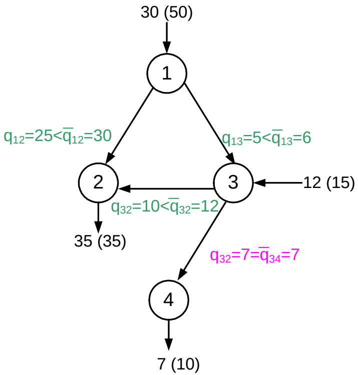
v(1)=30, v(2)=0, v(3)=5, v(4)=-10

Line	cost	admittance	flow constraint
3-4	6	2	7
2-4	2	2.5	8
1-3	1	2.46	6 (12 after exp)
2-3	1	3.46	12 (24 after exp)

Expansion game example II

v(12)=30+0, v(13)=30+5 v(23)=0+5 v(14)=30-10 v(24)=35-10

v(34): line 3-4 is built, 3 is rescheduled to 12

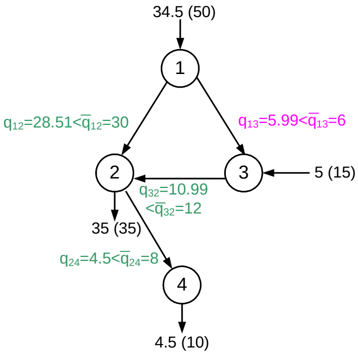


v(34)=12-3-6=3 (by 3, by 4, and construction cost) a possible feasible imputation is: ={10,-7}

Expansion game example III

v(123)=30+5

v(124): line 2-4 is built, 1 is rescheduled to 34.5



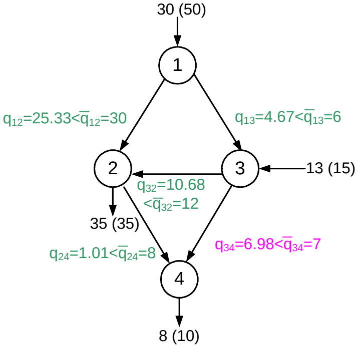
v(124)=34.5-5.5-2=27 possible imputation: {31,1,-5}

Expansion game example IV

v(134)$\leftrightsquigarrow$v(34): basically the same - line 3-4 can be built, line 1-3 does not matter, 3-4 will be congested v(134)=33

v(234):

- If only line 3-4 is built $\rightarrow$ v(34)
- If lines 2-4 and 3-4 are built, 3 is rescheduled to 13

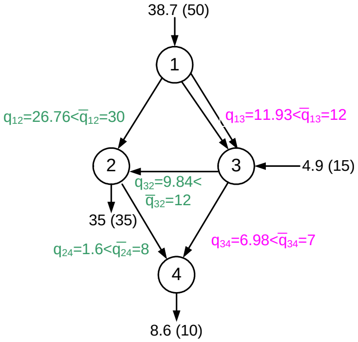


Expansion game example V

v(234)=13-2-6-2=3 (by 3, by 4, and construction costs)= v(34)

- If lines 2-3 2-4 and 3-4 are built, line 1-3 is overloaded (as in net calculation example)

v(1234): Lines 1-3, 2-4 and 3-4 are built, 1 and 3 rescheduled



v(1234):38.7+4.9-1.4-1-2-6=33.2 possible imputation: x={30,0,5,-1.8 }

Expansion game example VI

- imputation: x={30,0,5,-1.8 } is in the core:
  $x(1) = 30 \geq v(1) = 30, x(2) = 0 \geq v(2) = 0, x(3) = 5 \geq v(3) = 5,$
  $x(4) = -1.8 \geq v(4) = -10$
  $x(12) = 30 \geq v(12) = 30, x(13) = 35 \geq v(13) = 35,$
  $x(23) = 5 \geq v(23) = 5, x(14) = 28.2 > v(14) = 20,$x(34)=3.2 > v(34)=3
  $x(123) = 35 \geq v(123) = 35, x(124) = 28.2 > v(124) = 27,$
  $x(134) = 33.2 \geq v(134) = 33,$
  $x(234) = 3.2 \geq v(234) = 3,$
  $x(1234) = v(1234) = 33.2$

Summary

- Energy transmission network expansion problem may be put in a cooperative game theoretic framework in a straightforward way
- Determination of the characteristic value for a given coalition requires optimization: Constrained mixed integer problem (maximize the value via rescheduling of generation and consumption, subject to line flow constraints - addition of new lines correspond to integer variables)
- Details (e.g. how the values of consumed/generated energy determine the value of v) are subject to future considerations