Designing Benefit–Effort Rules: Welfare vs. Redistribution

András Simonovits

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Motivation

- 3 special problems, with different details
- - flexible retirement benefits
- - health insurance
- - optimal income taxation
- common “generalization”, imperfect
- adverse selection, welfare max. vs. redistribution

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Contents

Elements

- $n \geq 2$ types, $i = 1, 2, \ldots, n$
- observable effort e_i
- reward r_i
- utility $v_i = u_i(e_i, r_i)$, $\Downarrow, \Uparrow$
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Elements, cont.

- Frequency $f_i > 0$, $\sum_{i=1}^{n} f_i = 1$
- System: zero expected balance:

$$Z = \sum_{i=1}^{n} z_i f_i = 0.$$

- Degree of redistribution

$$D^2 z = \sum_{i=1}^{n} z_i^2 f_i.$$

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- The utilitarian social welfare function

$$V = \sum_{i=1}^{n} v_i f_i.$$

- Social objective function

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where $\delta \geq 0$ is the penalty coefficient

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First-best optimum: FB

- *First-best optimum*: full information and obedience: $(e_i^*, r_i^*)_i$,
- **Theorerem 1.** Necessary condition

$$\frac{\partial u_i}{\partial e_i} - (2\delta g_i + \lambda)\frac{\partial g_i}{\partial e_i} = 0 \tag{1}$$

and $i = 1, 2, \ldots, n$

$$\frac{\partial u_i}{\partial r_i} - (2\delta g_i + \lambda)\frac{\partial g_i}{\partial r_i} = 0 \tag{2}$$

for a suitable scalar λ.

- **Proof.** Lagrange multiplier

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Second best: SB

- Second best: type i cannot be observed, but his effort is observable
- Government proposes a menu $(\hat{e}_i, \hat{r}_i)_i$ such that type i choose "his" pair.
- Incentive compatibility (IC) conditions

$$u_i(e_i, r_i) \geq u_i(e_j, r_j), \qquad j \neq i, \qquad i = 1, \ldots, n.$$

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SB-2

- Under special conditions, $n(n-1)$ IC inequalities can be replaced by $n-1$ IC adjacent equalities:

$$u_{i+1}(e_{i+1}, r_{i+1}) = u_{i+1}(e_i, r_i), \qquad i = 1, 2, \ldots, n-1.$$

- **Theorerem 2.** Necessary condition

$$f_i \frac{\partial u_i}{\partial e_i} - (2\delta g_i + \lambda) f_i \frac{\partial g_i}{\partial e_i} - \mu_i \frac{\partial u_i}{\partial e_i} + \mu_{i+1} \frac{\partial u_{i+1}}{\partial e_i} = 0 \qquad (3)$$

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- Then the corresponding IC equation

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- a control equation if—following Mirrlees (1971)—the utility v_i is the state variable and the reward r_i is the control variable:

$$v_{i+1} = v_i + s(r_i), \qquad i = 1, 2, \ldots, n-1,$$

- **Conjecture.** $n(n-1)$ IC> $\Rightarrow$ $n-1$ IC=

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- $v_i = u_i(e_i, r_i) \Rightarrow e_i = G_i(v_i, r_i)$
- **Theorem 3.**

$$f_i - (2\delta G_i + \lambda) f_i \frac{\partial G_i}{\partial v_i} - \mu_i + \mu_{i+1} = 0 \qquad (5)$$

and $\qquad i = 2, \ldots, n$

$$-(2\delta G_i + \lambda) f_i \frac{\partial G_i}{\partial r_i} - \mu_{i+1} \frac{\partial s_i}{\partial r_i} = 0 \qquad (6)$$

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Flexible retirement

- ESŐ–SIMONOVITS–TÓTH (2007):
- Problem: how to punish/reward early/late retirement?
- Population with min. and max adult life expectancies integers S and T
- Type i is characterized by LEXP $t_i = i + S - 1$, $i = 1, 2, \ldots, n$, $T = n + S - 1$

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Retirement-2

- effort: retirement age
- reward: per period benefit (life annuity!)
- unit earnings and contribution rate τ
- balance (of lifetime net contributions)

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- lifetime utility

$$v_i = u_i(e_i, r_i) = \zeta e_i + w(r_i)(t_i - e_i),$$

- where per-period utilities $w(r_i) =$ of receiving pension r_i,
- $\varepsilon =$ labor disutility,
- $\zeta = w(1-\tau) - \varepsilon =$ worker's utility
- Special condition holds:

$$\zeta e_i + w(r_i)(t_i + 1 - e_i) = \zeta e_i + w(r_i)(t_i - e_i) + w(r_i),$$

- Special situation, because of collinearity of v_i and g_i

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Health insurance

- ROTHSCHILD–STIGLITZ (1976)
- Problem: How to force low-risk types to reveal their types? To charge deductions
- uniform wealth w
- uniform danger (sickness) with damage 1
- different risks $p_i = p_1 + (i-1)\pi$, $i = 1, \dots, n$
- partial insurance $0 < r_i \le 1$ with deductible $1 - r_i$
- premium e_i
- utility with final wealth x, $u(x)$

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- balance

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- lottery:
  $v_i = u_i(e_i, r_i) = (1 - p_i)u(w - e_i) + p_i u(w - e_i - 1 + r_i)$
- special but the additivity condition does not hold
- neutral: $z_i = e_i - p_i r_i = 0$

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Health 3

- First-best: full insurance $r_i = 1$ and $e_i = p_i \Rightarrow$ underreport risk
- Second-best: partial insurance for low risk: $r_1 < 1 = r_n$
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