

On Group Activity Selection

Andreas Darmann

University of Graz, Austria

Introduction

Agents have to choose among multiple activities, with differing preferences over activities

Consider, e.g., a workshop whose organizers have to arrange social activities (e.g., hike, bus trip, and table tennis competition) for the free afternoon based on agents' preferences

- each agent can participate in at most 1 activity
(due to cost reasons, or activities take place simultaneously)

First approach: elicit preferences over activities, divide agents into subgroups

Example

Use, e.g., plurality voting: Each agent names her favourite activity.
bus: 7 participants (high costs); table tennis: 48 p. (one table?!)

What if preferences depend on number of participants?

Introduction

- Idea: Exogenously add constraints on the group size.
E.g., bus: ≥ 20 participants, table tennis: $2 \leq \#participants \leq 8$?
- Problem: preferences on the group size may differ
E.g., senior faculty: bus trip with 10 people acceptable;
students: at least 25 people per bus trip

leads to a more fine-grained approach GASP:

- elicit agents' preferences over pairs "(activity, number of participants)", and allocate agents to activities on basis of this information
- in general, agents' preferences can be considered weak orders over all such pairs (we consider the case of strict orders in the first part of the talk)
- possibility of non-participation in any activity: *void activity* a_{\emptyset}

Example

Let $N = \{1, 2, 3, 4, 5, 6\}$ and $A^* = \{a, b, c\}$. Let P be given as follows:

1	2	3	4	5	6
(a, 6)	(c, 5)	(c, 6)	(c, 2)	(a, 1)	(b, 4)
(a, 5)	(a, 5)	(c, 5)	(b, 5)	(a, 5)	(a, 2)
(b, 4)	(a, 6)	(b, 4)	(a, 2)	(a, 4)	(c, 4)
(a, 2)	(a, 3)	(c, 3)	(a, 3)	(a, 3)	(c, 3)
(c, 3)	(b, 4)	a_{\emptyset}	(b, 4)	(c, 6)	(c, 2)
a_{\emptyset}	a_{\emptyset}	(b, 5)	a_{\emptyset}	(c, 5)	a_{\emptyset}
(b, 5)	(c, 1)	(a, 4)	(a, 6)	(c, 4)	(b, 1)
(b, 6)	(b, 6)	(b, 6)	(b, 6)	a_{\emptyset}	(a, 6)
(a, 2)	(c, 6)	(b, 3)	(a, 2)	(c, 2)	(c, 2)
(a, 1)	(b, 2)	(c, 1)	(a, 1)	(c, 1)	(a, 4)
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots

- Goal: find a “good” assignment of agents to activities, when agents preferences depend on the activity and the number of participants in the activity (GASP)
- minimum requirement: no agent should be assigned to an alternative ranked below the void activity

GASP can be seen as

a voting problem or a coalition formation problem

Aim of this work:

- introduce solution concepts for such a setting, and analyze computational complexity involved in finding a solution
- consider aspect of manipulability

Formal Model of GASP

Instance (N, A, P) of GASP

- Set of *agents* $N = \{1, \dots, n\}$
- Set of *activities* $A = A^* \cup \{a_\emptyset\}$, where $A^* = \{a_1, \dots, a_p\}$, and a_\emptyset is the *void activity*
- Set of *alternatives* $X = X^* \cup \{a_\emptyset\}$, where $X^* = A^* \times \{1, \dots, n\}$; alternative (a, k) , $a \in A^*$, is interpreted as “activity a with k participants”
- *Profile* P , which consists of n votes (one for each agent):
 $P = (V_1, \dots, V_n)$.
 - For $i \in N$, vote V_i is a weak order over X

Formal Model of o-GASP

Instance (N, A, P) of o-GASP

- Set of *agents* $N = \{1, \dots, n\}$
- Set of *activities* $A = A^* \cup \{a_\emptyset\}$, where $A^* = \{a_1, \dots, a_p\}$, and a_\emptyset is the *void activity*
- Set of *alternatives* $X = X^* \cup \{a_\emptyset\}$, where $X^* = A^* \times \{1, \dots, n\}$; alternative (a, k) , $a \in A^*$, is interpreted as “activity a with k participants”
- *Profile* P , which consists of n votes (one for each agent):
 $P = (V_1, \dots, V_n)$.
 - For $i \in N$, vote V_i is a **strict order** \succ_i over X

Basic Definitions

Solution to GASP:

Definition

An *assignment* for an instance (N, A, P) of GASP is a mapping $\pi : N \rightarrow A$.

- $\pi(i) = a_\emptyset$ means that agent i does not participate in any activity
- For $a \in A$, $\pi^a := \{i \in N \mid \pi(i) = a\}$
- For $i \in N$, $\pi_i := \{j \in N \mid \pi(j) = \pi(i)\}$

Minimum requirement: no agent should be assigned to an activity in a way such that she deems the corresponding pair “(activity, group size)” unacceptable

Definition

Given an instance (N, A, P) of GASP, an assignment $\pi : N \rightarrow A$ is

- *individually rational* if for every $a \in A^*$ and every agent $i \in \pi^a$ it holds that $(a, |\pi^a|) \succsim_i a_\emptyset$.
- *maximum individually rational* if π is individually rational and $\#(\pi) \geq \#(\pi')$ for every individually rational assignment π' , where $\#(\pi) = |\{i \in N \mid \pi(i) \neq a_\emptyset\}|$

Special Cases

restrictions on agents' preferences that may simplify the problem of finding a good assignment

Definition

Consider an instance (N, A, P) of GASP. We say that the preferences of agent i are

- *increasing (INC)* if for all $a \in A^*$, $(a, k)_i \succsim_i (a, k - 1)$ holds for each $k \in \{2, \dots, n\}$.
- *decreasing (DEC)* if for all $a \in A^*$, $(a, k - 1) \succsim_i (a, k)$ holds for each $k \in \{2, \dots, n\}$.

We say that instance (N, A, P) has increasing/decreasing preferences, if each $i \in N$ has increasing/decreasing preferences.

Example

Let $A^* = \{a, b\}$ and $n = 9$. Consider agent i with vote V_i given by

$(a, 9) \succ_i (a, 8) \succ_i (b, 9) \succ_i (a, 7) \succ_i (b, 8) \succ_i a_\emptyset \succ_i (b, 7) \succ_i (b, 6) \succ_i (a, 6) \dots$

Related Work

Group activity selection problem [joint w. E. Elkind, S. Kurz, J. Lang, J. Schauer, G. Woeginger; 2012]:

- model of GASP introduced
- approval-scenario a-GASP:
 - indifference between two alternatives an agent prefers to a_0
 - focus laid on *maximum individually rational assignments* and stability notions (also w.r.t. increasing/decreasing preferences)

Group activity selection from ordinal preferences [D., 2015]:

- o-GASP introduced
- computational complexity of finding stable assignments respectively maximizing k -approval scores

This talk follows up these works in two ways:

- Applying different solution concepts for o-GASP
- Considering aspects of manipulability in finding maximum individually rational assignments

Related Work

Hedonic games [Banerjee et al., 2001; Bogomolnaia & Jackson, 2002]:

- each agent i has preferences over the subsets of agents containing i
- GASP can be embedded in that framework
- hardness results for anonymous and non-anonymous hedonic games w.r.t. stability notions such as Nash, core, (contractual) individual stability known [Ballester, 2004]
- Finding a Pareto optimal solution is NP-hard for both non-anonymous and anonymous hedonic games [Aziz et al., 2013]

Solution Concepts for o-GASP

Different approaches to find a “good” outcome:

- 1 Borda scores
- 2 Condorcet criterion
- 3 Pareto optimality

Approval and Borda scores

$f(\pi) := \sum_{i \in N} f_i(\pi(i), |\pi_i|)$ with $f_i : X \rightarrow \mathbb{R}_0^+$. The value $f(\pi)$ is called *score of π* .

- In *approval scores*, for $i \in N$, let $f_i(x) = 1$ if $x \succ_i a_\emptyset$ and $f_i(x) = 0$ otherwise.
k-approval scores, $k \in \mathbb{N}$, correspond to approval scores in the case that $|\{x \in X : x \succ_i a_\emptyset\}| = k$ holds for all $i \in N$.
- In *Borda scores*, for $i \in N$ we have $f_i(x) = |\{x' \in X : x \succ_i x'\}|$.

Goal: find an individually rational assignment that maximizes the total score

Condorcet & Pareto optimal assignments

Definition

Given an instance (N, A, P) of o-GASP,

- we say that agent i *prefers* assignment π over assignment π' (denoted by $\pi \triangleright_i \pi'$), if $(\pi(i), |\pi_i|) \succ_i (\pi'(i), |\pi'_i|)$ holds.
- An assignment π is *IR-Condorcet*, if π is individually rational and for all individually rational assignments $\pi' \neq \pi$ we have $|\{i \in N : \pi \triangleright_i \pi'\}| > |\{i \in N : \pi' \triangleright_i \pi\}|$.
- π is *MIR-Condorcet*, if π is maximum individually rational and for all maximum individually rational assignments $\pi' \neq \pi$ we have $|\{i \in N : \pi \triangleright_i \pi'\}| > |\{i \in N : \pi' \triangleright_i \pi\}|$.
- an individually rational assignment π is *Pareto optimal* if there is no assignment π' such that there is no $i \in N$ with $\pi \triangleright_i \pi'$ and for at least one $j \in N$ we have $\pi' \triangleright_j \pi$.

Results: Approval and Borda scores

- k -approval [D., 2015]:

	general pref.	INC	DEC
$k = 1$	in P	in P	in P
$k \in \{2, 3\}$	NP-c		in P
$k \geq 4$	NP-c		NP-c

- Borda:

	general pref.	INC	DEC
Borda	NP-c	NP-c	NP-c

Results: Condorcet solution & Pareto optimality

	general pref.	INC	DEC
IR-CONDORCET-EXISTENCE	coNP-hard	coNP-hard	in P
MIR-CONDORCET-EXISTENCE	coNP-hard	coNP-hard	?
DETERMINE A PARETO OPT. ASS.	in P		

Easiness results

IR-Condorcet-Existence.

In the case of decreasing preferences, an assignment π is IR-Condorcet \Leftrightarrow
 $\pi(i) = a_i$ for top-ranked alternative $(a_i, 1)$ in \succ_i .

Easiness results

Pareto Optimality.

Simple observation: For each agent i with top-ranked alternative (a, k) , there is a Pareto optimal assignment π that assigns i to a such that $|\pi^a| = k$ (if there are at least k agents who prefer (a, k) to a_\emptyset).

Basic algorithmic idea:

- First, pick an arbitrary agent i with top-ranked alternative (a, k) and assign agent i to a together with $k - 1$ arbitrarily chosen other agents that prefer (a, k) to a_\emptyset .
- Try to find an individually rational assignment that has i , and in total k agents, assigned to a and is better for at least one agent while no one gets assigned to a worse-ranked alternative.

Easiness results

Pareto Optimality.

More generally, the algorithmic idea can be summarized as:

- Consider an individually rational assignment in which
 - (i) for some agents the activities they are assigned to and
 - (ii) for some activities the number of agents assigned to the activity have already been fixed.
- Pareto-improve the assignment, i.e., find an assignment that respects (i) and (ii) and is better for at least one agent while making no agent worse off (this improvement-step can be performed by solving a flow problem with lower and upper edge capacities).

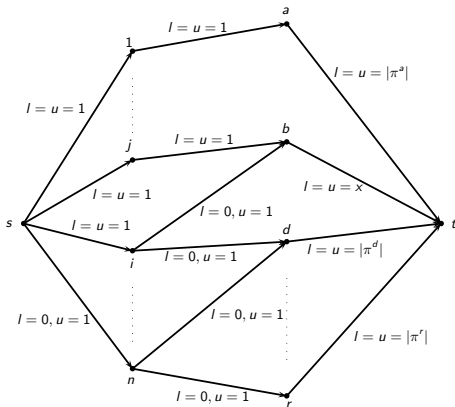
Easiness results

Pareto Optimality: *Improvement-step.*

Given an assignment π , agent j and alternative (b, x) , such that

- fixed: $\pi(1) = a$ and the number of agents assigned to a, d, r
- $\pi(i) = d$, $(b, x) \succ_i (d, |\pi^d|)$; $\pi(n) = a_\emptyset$, and $(d, |\pi^d|) \succ_n a_\emptyset$, $(r, |\pi^r|) \succ_n a_\emptyset$

Can we “Pareto-improve” by assigning j to b (with group size x)?



Manipulability in GASP: maximum individually rational assignments

Given instance $\mathcal{I} = (N, A, P)$ of GASP with set N of agents and set A of activities

- $\Pi(\mathcal{I})$ denotes set of maximum individually rational assignments in \mathcal{I} .
- $\mathcal{S}(N, A)$ denote set of all instances of GASP with agent-set N and activity-set A ,
- $\alpha(N, A) := \{\pi \mid \pi : N \rightarrow A\}$ is set of assignments of agents in N to activities in A

Definition

Given an instance $\mathcal{I} = (N, A, P)$ of GASP,

- The mapping $C : \mathcal{S}(N, A) \rightarrow 2^{\alpha(N, A)}$ with $C(\mathcal{I}) = \Pi(\mathcal{I})$ is called *mir-aggregation correspondence*.
- We call a function $f : \mathcal{S}(N, A) \rightarrow \alpha(N, A)$ with $f(\mathcal{I}) \in \Pi(\mathcal{I})$ *mir-aggregation function*.

Manipulability in GASP: maximum individually rational assignments

Single-valued aggregation function:

Definition

An mir-aggregation function f is called *manipulable*, if there exist an instance $\mathcal{I} = (N, A, P)$, an agent $i \in N$ and a profile P' with $P|_{N \setminus \{i\}} = P'|_{N \setminus \{i\}}$ such that, with $f(\mathcal{I}) = \pi$, $\mathcal{I}' = (N, A, P')$ and $f(\mathcal{I}') = \pi'$,

$$\pi' \triangleright_i \pi$$

holds. f is called *strategyproof*, if f is not manipulable.

Manipulability in GASP: maximum individually rational assignments

Multi-valued aggregation correspondence: Consider Preference extensions.

E.g.,

Definition

The *maxi-max extension* is defined by: for $i \in N$ and $X, Y \in 2^\alpha$, $X \succsim_i^{\max} Y$ iff for $x \in \max_i X$, $y \in \max_i Y$, $(x \succeq_i y)$ holds.

Analogously, the *maxi-min extension* is defined by: for $i \in N$ and $X, Y \in 2^\alpha$, $X \succsim_i^{\min} Y$ iff for $x \in \min_i X$, $y \in \min_i Y$, $(x \succeq_i y)$ holds.

Manipulability in GASP: maximum individually rational assignments

Multi-valued aggregation correspondence: Consider Preference extensions.

E.g.,

Definition

Gärdenfors extension:

For $i \in N$ and $X, Y \in 2^{\alpha}$, $X \underset{i}{\sim}^G Y$ if one of the three following conditions is satisfied:

- 1 $X \subset Y$ and for all $x \in X, y \in Y \setminus X$ we have $x \succeq_i y$.
- 2 $Y \subset X$ and for all $x \in X \setminus Y, y \in Y$ we have $x \succeq_i y$.
- 3 neither $X \subset Y$ nor $Y \subset X$ and $(x \succeq_i y)$ for all $x \in X \setminus Y, y \in Y \setminus X$.

Manipulability in GASP: maximum individually rational assignments

Multi-valued aggregation correspondence C

Definition

Let ε be a preference extension. C is ε -manipulable if there exist an instance $\mathcal{I} = (N, A, P)$, an agent $i \in N$ and a profile P' with $P|_{N \setminus \{i\}} = P'|_{N \setminus \{i\}}$ such that, with $\mathcal{I}' = (N, A, P')$,

$$\Pi(\mathcal{I}') \succ_i^\varepsilon \Pi(\mathcal{I})$$

holds. C is ε -strategyproof, if C is not ε -manipulable.

Manipulability in GASP: Results

1 simple activity

- bad news: manipulable
- increasing preferences: strategyproof
- decreasing preferences:
 - every mir-aggregation function is manipulable
 - mir-correspondence C is maxi-min strategyproof, but Gärdenfors- & maxi-max-manipulable

Manipulability in GASP: Results

12 simple activities

- bad news: manipulable
- increasing preferences: ~~strategyproof~~ manipulable
- decreasing preferences:
 - every mir-aggregation function is manipulable
 - mir-correspondence C is maxi-min ~~strategyproof~~ manipulable, and Gärdenfors- & maxi-max-manipulable

Manipulability in GASP: Results

<i>preferences, activities</i>	<i>extension</i>		
	Gärdenfors	Maxi-max	Maxi-min
decreasing, 1 simple	man	man	sp
decreasing, 1 copyable	man	sp	man
decreasing, 2 simple	man	man	man
decreasing, 2 copyable	man	sp	man
increasing, 1 simple	sp	sp	sp
increasing, 1 copyable	man	sp	man
increasing, 2 simple/copyable	man	man	man

Table: Overview over the results regarding manipulability of correspondence C w.r.t. different preference extensions.

Conclusion and Outlook

We have

- obtained complexity results for different solution concepts in o-GASP
- considered manipulability in GASP

Future research (GASP):

- further solution concepts for o-GASP
- open complexity issues

Outlook

- Novel domains
- Concepts too strict?
- Approximation Algorithms/Hardness
- Fixed Parameter Tractability