All about priorities

(no school choice under the presence of bad schools)

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October 19, 2011

School Choice

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School Choice: Boston versus Deferred Acceptance?

- BM and DA are by far the two most popular mechanisms
- ...although other mechanisms (TTC, Pseudomarkets) have been theoretically proposed
- A long-lasting debate between BM and DA
- However, we show that these two mechanisms may not differ that much in practice
- ...if there are bad schools (that everyone dislikes)
- ...and priority structure maps each student to one school
- Students' preferences may not matter

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Our preliminary contributions

- In big economies with bad schools and binary priority structure mapping each student to one school...
- Every mechanism containing a strategy for each student that ensures assignment to her priority-giving school has a NE whose outcome is an assignment completely driven by priorities
- This is the unique dominant strategy assignment in DA
- This is the unique-NE outcome of BM if sufficiently many/low-valued bad schools

Boston Mechanism (BM) and Deferred Acceptance (DA)

- Parents report a ranking over schools. Round by round assignment
- In each round we consider each not-removed student for her reported best school that has not rejected her yet
- With excess demand, schools reject some students according to priorities and lotteries
- Differences with respect to how **accepted** students are treated:
  - BM: they obtain their slots and do not go to further rounds (**definite acceptance**).
  - DA: they are reconsidered for that school in further rounds (**tentative acceptance**).

Example: Agents 1,2,3; Schools a,b,c

Pref.	Prio.	BM	DA
$a \succ_1 b \succ_1 c$	$1 pr_a 2$	Round 1 $1 \rightarrow a, 2 \nrightarrow a, 3 \rightarrow c$	Round 1 $1 \rightarrow a, 2 \nrightarrow a, 3 \rightarrow c$
$a \succ_2 c \succ_2 b$	$2 pr_c 3$	Round 2 $2 \nrightarrow c$	Round 2 $1 \rightarrow a, 2 \rightarrow c, 3 \nrightarrow c!$
$c \succ_3 b \succ_3 a$		Round 3 $2 \rightarrow b$ (put c first)	Round 3 $1 \rightarrow a, 2 \rightarrow c, 3 \rightarrow b$

The model

- Set G of J good schools j, plus one bad school w
- $\eta_w < 1$ capacity of bad school, $\frac{1-\eta_w}{J}$ capacity of each j
- Set $X = [0,1]$ of students x, (uniform measure λ)
- Preferences $v : X \rightarrow [0,1]^J \times \{v_w\}$, $v_w \leq 0$ ($\lambda(Indiff) = 0$)
- Priority structure $\pi : X \rightarrow G \cup \{w\}$
- $S_j = \{x \in X : \pi(x) = j\}$, $S_G = \bigcup_{j \in G} S_j$
- Assignment $\mu : X \rightarrow G \cup \{w\}$
- Random assignment $q : X \rightarrow \Delta^J$ (Q: all feasible rand. assig.)
- Expected utility $q(x) \cdot v(x)$

Mechanisms, equilibria and outcomes

- Pure strategy profile $\sigma : X \rightarrow \Pi(G \cup \{w\})$ (ranking)
- Σ: set of all σ
- Game $\Gamma_\pi : \Sigma \rightarrow Q$
- Standard notions of equilibria σ^* (NE, DSE...)
- An (NE-, DSE-) outcome is $\Gamma_\pi(\sigma^*)$
- *Completely driven by priorities* if collapses to $\mu = \pi$

General observation

Let Γ_π be such that each student $x \in S_G$ has a strategy $s(x)$ ensuring sure assignment to $\pi(x)$. Consider any $s(y)$ for each student $y \in X \backslash S_G$. Then s constitutes a NE.

- Finite economy: If every $x \in S_G$ but one (y) plays $s(x)$, this student will have chances at either $\pi(y)$ or w. She will optimally respond with $s(y)$
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Deferred Acceptance

Let DA_π denote the game described by the DA algorithm with priorities π and fair tie-breaking lotteries. Then the DSE σ^{DA} leads to an outcome $DA_\pi(\sigma^{DA})$ that is completely driven by π.

- Let $m : X \to [0,1]$ be a fair lottery outcome breaking ties in the increasing order, and $\mu_{\pi m}(\sigma^{DA})$ the corresponding DA assignment
- $\mu_{\pi m}(\sigma^{DA})$ respects strict priorities set by π and m (stable)
- $\mu_{\pi m}(x) \neq w \ \forall x \in S_G$ (they would have applied for $\pi(x)$ earlier) $\implies \mu_{\pi m}(x) = w \ \forall x \in X \backslash S_G$
- Let $X' = \{x \in S_G : \mu_{\pi m}(x) \neq \pi(x)\}$ have positive measure
- Stability violated: agents $x \in X \backslash S_G$ with $m(x) < \max_{y \in X'} m(y) = 1$

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Boston Mechanism

- Need additional structure
- Each $v_j(x)$ drawn independently from F (full support, no atoms)
- Symmetric NE (SNE) σ^*. For any $x, y \in S_G$ such that $v_{\pi(y)}(y) = v_{\pi(x)}x$ and there is a permutation $\rho : v(y) = \rho(v(x))$, we have $\sigma^*(y) = \rho(\sigma^*(x))$.
- Characterized by α^*, proportion of students with priority at a good school who put another school in first place.
- Let $\bar{\alpha}$ be the maximum α^*.
- The outcome under $\bar{\alpha}$ ex-ante Pareto-dominates the outcome under $\alpha^* = 0$.

Boston Mechanism

Fix F and J. If $\bar{\alpha} > 0$, it is decreasing in η_w and $-v_w$. $\bar{\alpha} = 0$ if either η_w or $-v_w$ are high enough.

- Let $\bar{\alpha} > 0$. No-priority supply of a good school: $\bar{\alpha}\frac{1-\eta_w}{J}$
- No-priority demand $(J-1)\bar{\alpha}\frac{1-\eta_w}{J}/(J-1) + \eta_w/J$. Acceptance probability $q(\bar{\alpha}) = \frac{\bar{\alpha}(1-\eta_w)}{\bar{\alpha}(1-\eta_w)+\eta_w}$
- Eq. $q(\bar{\alpha}) = \omega(\bar{\alpha})$, $\omega(\cdot)$: inverse of distribution of $\frac{v_{\pi(x)}(x)-v_w}{\max_{j \in G} v_j(x)-v_w}$
- $\Delta\eta_w$ moves $q(\cdot)$ down. $\Delta(-v_w)$ moves $\omega(\cdot)$ up.
- Reinforced by extra valuation for priority-giving schools

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Top-Trading Cycles

Let TTC_π denote the game described by the TTC algorithm with priorities π and fair tie-breaking lotteries. Then the DSE σ^{TTC} leads to an outcome $TTC_\pi(\sigma^{TTC})$ that ex-ante Pareto-dominates $\mu = \pi$.

- For any $m : X \to [0, 1]$ fair lottery outcome breaking ties in the increasing order, and being $\mu_{\pi m}(\sigma^{TTC})$ the corresponding TTC assignment, $\mu_{\pi m}(\sigma^{TTC})$ Pareto-dominates μ
- Plus it does not violate stability with this type of π
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The political economy of School Choice

- For years we have thought of the authority as a student–choice-maximizer subject to the respect for priorities
- But maybe priorities reflect authority's own objective function
- ...and (the mirage of) school choice is the constraint

Summary

- In big economies with bad schools and binary priority structure mapping each student to one school...
- DA leads to an assignment that fits the priority structure
- Also BM if the worst school is sufficiently big/bad
- TTC Pareto-dominates this assignment while keeping stability
- ..yet only DA and BM are widely used!