MATCH-UP 2012

The Second Workshop on Matching Under Preferences

Further background information

Many practical situations give rise to large-scale matching problems involving sets of participants – for example pupils and schools, school-leavers and universities, applicants and positions – where some or all of the participants express preferences over the others. A large class of matching problems that involve preferences is the class of stable matching problems.

Such problems involve a set of agents, each of whom ranks a subset of the other agents in order of preference. The task is to find a *stable matching*, i.e., a set of pairs of acceptable agents such that no two agents could improve their assignment by becoming matched to each other. Since their introduction by Gale and Shapley in 1962, stable matching problems (and the classical stable marriage problem in particular) have attracted the attention of many researchers in the area of the design and analysis of algorithms as well as in other fields, such as discrete mathematics, game theory and economics. Important developments prior to the 1990s include the fundamental Gale-Shapley algorithm, algorithms for many-to-one hospitals-residents problems [12], and algorithms for non-bipartite stable roommates problems [18], as well as the identification, analyses and exploitation of lattice and poset structures underlying stable matchings [31, 21, 13, 22, 10]. The monographs of Gusfield and Irving [14] and Roth and Sotomayor [38] are comprehensive sources for aspects of these problems studied during that period.

Stable matching problems have important practical applications; the most famous example is the NRMP (National Resident Matching Program) in the US, for which Gale-Shapley type algorithms have been used for more than 50 years. Similar applications, to medical and other domains, also exist in Canada, Scotland, Japan, and many other countries. A variety of new problems arising from such applications, such as consideration of couples, have also been studied [33, 6].

Since the mid-1990s, the trend has been towards extensions and variants of the basic problem in several different directions. These have included different notions of stability for the case that preference lists may include ties [19, 24, 29], many-to-many versions [5, 32], game-theoretic analyses [7, 16, 17], dynamic systems and equilibrium analyses [3], and approximation algorithms for hard variants of stable matching [25, 15, 26, 27, 28, 23]. In addition, new research into matching problems where preferences are expressed on only side has been productive, including studies of so-called *Pareto optimal matchings* [1], *rank-maximal matchings* [20] and *popular matchings* [2, 35, 34]. Moreover, there has been a renewed interest in non-bipartite models (i.e., the Stable Roommates problem and its variants) because of important real-world applications involving kidney exchange in a number of countries [36, 37].

The stable marriage problem can be considered as a special cooperative game with non-transferable utilities. The corresponding game with transferable utilities is called assignment game, introduced by Shapley and Shubik [39], which can also be seen as the stable marriage problem with payments. Their relation and some common generalisations of these basic models have been studied [30, 11] from different aspects. In particular there are some recent works on sponsored search auctions [4, 8, 9] where such mixed models are analysed.

This workshop will mainly focus on these new developments, with keywords "matching" and "preferences". We are interested in not only on-going developments as above but also in future directions for the research community, e.g., towards more game-theoretic and financial aspects of the problems. We also hope that this meeting will provide an ideal opportunity to bring together researchers from the broad disciplines of discrete mathematics, computer science and economics.

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