Voting Games: Trembling Hand Equilibria
Benevolence, managed by wisdom, limited by necessity

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Agenda

- Doodle Polls
    - Rational Theory
    - Human Anomaly
    - Statistics of the Anomaly
- Making the Rational Work Rationally
    - Benevolence (uncapped)
    - Limited by necessity (capped)
    - Managed by wisdom: Trembling Hand Equilibrium
- Further application of THE
    - Plurality Voting Games

Approval Polls

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Doodle Polls

Susan	✓	✓	✗	✗	✗
Londo	✓	✓	✗	✗	✗
Zathras	✗	✓	✓	✗	✗

Doodle Polls

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You					
Total:	2	3	1	0	0

Rational Theory

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You					
Total:	2	3	1	0	0

Rational Theory

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You	✔	✘	✘	✘	✔
Total:	2	3	1	0	0

Rational Theory: Approval's Non-Manipulative

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You	✔	✘	✘	✘	✔
Total:	3	3	1	0	1

Human Anomaly: Zou, Meir, Parkes (2015)

Susan	✓	✓	✗	✗	✗
Londo	✓	✓	✗	✗	✗
Zathras	✗	✓	✓	✗	✗
You					
Total:	2	3	1	0	0

Human Anomaly: Zou, Meir, Parkes (2015)

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You	✔	✘	✘	✘	✔
Total:	2	3	1	0	0

Human Anomaly: Zou, Meir, Parkes (2015)

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You	✔	✘	✔	✔	✔
Total:	2	3	1	0	0

Human Anomaly: Zou, Meir, Parkes (2015)

Susan	✔	✔	✘	✘	✘
Londo	✔	✔	✘	✘	✘
Zathras	✘	✔	✔	✘	✘
You	✔	✘	✔	✔	✔
Total:	3	3	2	1	1

Statistics: Zou, Meir, Parkes (2015)
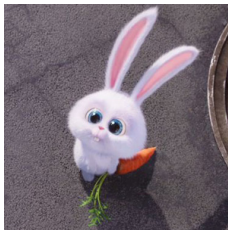
- Social Voting:
  - People wish to appear benevolent

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- Social Voting:
    - People wish to appear benevolent
    - Wisely cap it to avoid risk
- Approve your top choices, and several "safe" options
- Statistically correct wrt human data
- Can this behaviour be rational?

Doodle Poll Games (DPG): Approval + Social Bonus

- Assume that voters are rational
- Approval based voting
  - Lexigraphic or Random tie-breaking
- ϵ extra utility for each approved candidate
  - Never prevails over the original preference
  - Can be capped: bonus the first κ approved candidates

Uncapped Lexicographic Doodle Poll Game

Definition ($\exists$NEWIN)

Given a DPG with lexicographic tie-breaking, $\kappa = |C|$, and an alternative $w \in C$, is there a Nash Equilibrium with winner w?

Theorem

$\exists$NEWIN *can be solved in time polynomial in* $|V|$ *and* $|C|$.

Uncapped Randomised Doodle Poll Game

Definition (∃NE)

Does a given DPG with randomised tie-breaking and $\kappa = |C|$ possess an NE?

Definition (∃NE$_{\text{SINGLE}}$)

Given a DPG with randomised tie-breaking and $\kappa = |C|$, is there an NE with a single winner?

Definition (∃NE$_{\text{TIE}}$)

Given a DPG with randomised tie-breaking and $\kappa = |C|$, is there an NE with a non-singleton set of winners?

Uncapped Randomised Doodle Poll Game

Theorem

In games with dichotomous preferences, $\exists$NE, $\exists$NE$_{\text{SINGLE}}$ and $\exists$NE$_{\text{TIE}}$ are polynomial-time solvable.

Theorem

*$\exists$NE and $\exists$NE$_{\text{TIE}}$ are **NP**-hard for trichotomous preferences.*

Uncapped (Lex/Rand) DPG: P... NP... So what?

Lemma (Unchecked Benevolence has very few NEs)

*In dichotomous preferences the winner is approved by **all** voters*

Target Behaviour Not Replicated

Surely doesn't happen in practice in Doodle

Capped (Lex/Rand) DPG: Too many (weird) NEs

	Real Preference				
Susan(x4)	✓	✓	✗	✗	✗
Londo(x4)	✗	✓	✓	✗	✗
Total	4	8	4	0	0

	Equilibrium ($\kappa = 3$)				
Susan(x4)	✓	✗	✗	✓	✓
Londo(x4)	✗	✓	✓	✗	✓
Total	4	4	4	4	8

Capped (Lex/Rand) DPG: Too many (weird) NEs

Target Behaviour Not Replicated

Limited Benevolence is nearly meaningless

Trembling Hand Perfect Equilibrium in DPGs

Wisdom

To err is human.

Definition (THPE)

- Assume other players can misimplement their strategy
  - The error is symmetric
  - The error is i.i.d. over the candidates
- TH best response is in expectation over the errors of other.
- THPE strategies are jointly TH best response.

THPE in DPGs: Benevolence, managed by wisdom

Theorem

Given a voter $v \in V$ in a DPG with $\kappa << |C|$, lexicographic tie-breaking and dichotomous preference. It is possible to calculate a TH best response in time polynomial in $|V|$ and $|C|$.

Corollary

A THPE can be computed in polytime in $|V|$ and $|C|$.

We've got it!!

In a TH best response a voter approves all of his good candidates and a safe subset of his bad candidates.

Takehome Message

The pattern of human behaviour in DPGs is fully rational from the Trembling Hand Perfect Equiblibrium point of view.

Plurality Voting Games

- Plurality Voting: Voters give a point to 1 candidate
  - Randomized tie-breaking
- Standard model: ordinal utilities for lexicographic tie-breaking/cardinal utilities for randomized tie-breaking
- Hence, we assume that each voter assigns cardinal utilities to candidates.

Bad NE

	Real Preference				
Susan	✔	✘	✘	✘	✘
Londo(x3)	✘	✔	✘	✘	✘
Kosh(x3)	✘	✘	✔	✘	✘
Total	1	3	3	0	0
	Equilibrium				
Susan	✘	✘	✘	✘	✔
Londo(x3)	✘	✘	✘	✘	✔
Kosh(x3)	✘	✘	✘	✘	✔
Total	0	0	0	0	7

Equilibrium refinements

Need a tool to distinguish between bad and good equilibria:

- Additional assumptions on agents' utilities (lazy, truth-biased, etc.), OR
- Equilibria that satisfy additional conditions (e.g., strong equilibria).

Trembling hand (TH) perfect equilibrium

- Each voter assumes that other voters' hands my "tremble": with small (vanishing) probability they play a random strategy.
- Intuition: under this assumption, voters are more careful, as there is (even if a tiny) chance that their vote is pivotal.

Positive results

- TH best response can be computed in polynomial time.
- Characterisation of TH equilibria where all voters support the same candidate.
- A weak variant of Duverger's law holds.
- Sufficient condition for existence of TH equilibria.

Best Reply (1)

	Real Preference				
Susan(x4)	✓	✗	✗	✗	✗
Londo(x4)	✗	✗	✗	✗	✓
Kosh(x2)	✗	✓	✗	✗	✗
You	?	✗	✗	✗	?
Total	4	2	0	0	4

Best Reply (2)

			Real Preference		
Susan(x4)	✓	✗	✗	✗	✗
Londo(x3)	✗	✗	✗	✗	✓
Kosh(x2)	✗	✓	✗	✗	✗
You	?	?	✗	✗	?
Total	4	2	0	0	3

TH best response

Theorem: Given a voter $v \in V$ and a ballot profile $\mathbf{b}_{-v}$ of the remaining voters, we can find in polynomial time a TH best response of v to $\mathbf{b}_{-v}$.

- If $|W(\mathbf{b}_{-v})| > 1$ then v's best response to $\mathbf{b}_{-v}$ is to vote for his

  most preferred candidate in $W(\mathbf{b}_{-v})$.

- If $W(\mathbf{b}_{-v}) = \{w\}$ is a singleton, then v's best response to $\mathbf{b}_{-v}$

  is a subset of $\{w, c^0, c^1, c^2\}$ where c^i for $i = 0, 1, 2$ is v's most preferred candidate in the set of candidates with the score $s^* - i$ (where s^* is the runner-up score).

TH equilibrium existence

Theorem Suppose that for a pair of candidates a, c it holds that a strict majority of voters prefer a to c, yet at least three voters prefer c to a. Then there exists a TH equilibrium such that a is the winner, c is the runner-up, and a and c are the only candidates with positive scores.

- Large-scale elections are very likely to have at least one TH equilibrium.
- However, there exist small profiles with no TH equilibrium.

NE Existence

			Real Preference		
Susan(x4)	✓	✗	✗	✗	✗
Londo(x3)	✗	✗	✗	✗	✓
Kosh(x2)	✗	✓	✗	✗	✗
Total	4	2	0	0	3

Takehome Message(s)

The pattern of human behaviour in DPGs is fully rational from the Trembling Hand Perfect Equiblibrium point of view.

THPE effectively reduces the number of bad NEs

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Thank you...

- Edith Elkind
- Nicholas R. Jennings
- Maria Polukarov
- Zinovi Rabinovich