Multiwinner Election Rules: Axioms and Applications

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Based on my joint works with: Markus Brill, Edith Elkind, Piotr Faliszewski, Martin Lackner, Jean-François Laslier, Dominik Peters, Arkadii Slinko and Nimrod Talmon

Multiwinner Elections

Setting:

- Set of candidates $C = \{c_1, \ldots, c_m\}$,
- Collection of voters $V = (v_1, \ldots, v_n)$,
- Each voter has preferences over candidates.

Goal:

- Select K candidates that would satisfy the voters most.

Preferences as Rankings

$V_1:$  $\succ$  $\succ$  $\succ$  $\succ$ 

$V_2:$  $\succ$  $\succ$  $\succ$  $\succ$ 

$V_3:$  $\succ$  $\succ$  $\succ$  $\succ$ 

$V_4:$  $\succ$  $\succ$  $\succ$  $\succ$ 

$V_5:$  $\succ$  $\succ$  $\succ$  $\succ$ 

Preferences as Approval Ballots

$V_1:$

$V_2:$

$V_3:$

$V_4:$

$V_5:$

Multiwinner Elections: the Challenge

The Goal

Select K candidates that would satisfy the voters most.

Multiwinner Elections: the Challenge

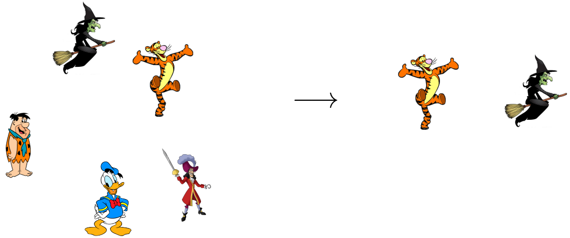
The Goal

Select K candidates that would satisfy the voters most.

The Problem

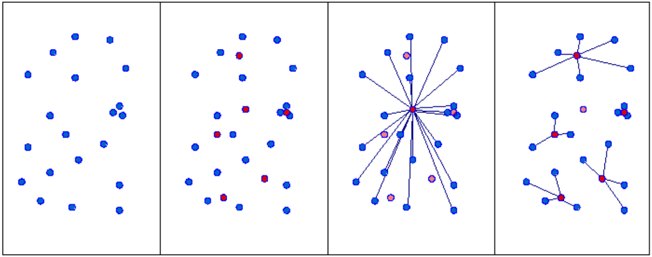
It is absolutely unclear how to do that.

Applications: Shortlisting



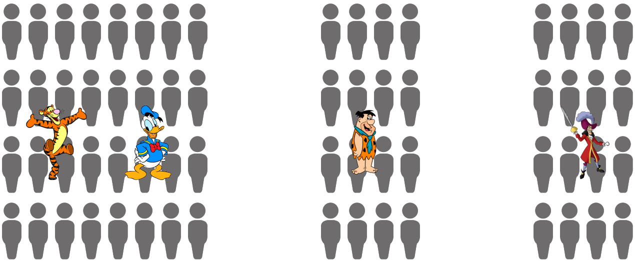
Goal: We want to select a collection of high-quality individuals (here, voters are, e.g., reviewers).

Applications: Facility location



Goal: Selecting locations for set of facilities (e.g., hospitals, fire stations, markets, etc.).

Applications: Selecting a Representative Body, e.g., a Parliament



Goal: We want to select a set of candidates that well represent the population.

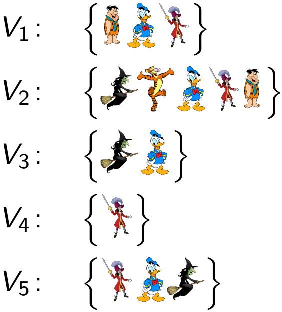
Multiwinner Elections: the Challenge

The Question

How should we select candidates for each of these applications?

Multiwinner Election Rules: Approval Voting

- A voter i approves a set of candidates A_i.
- The score of each candidate is the number of voter who approve him or her.
- The K candidates with the highest score form a winning committee.



Multiwinner Election Rules: PAV

- A voter i approves a set of candidates A_i.
- Satisfaction of voter i from committee S is $\sum_{i=1}^{|S \cap A_i|} \frac{1}{i}$.
- The committee with the highest total satisfaction wins.

$V_1:$

$V_2:$

$V_3:$

$V_4:$

$V_5:$

$S =$ 

$$
\begin{aligned}
\mathrm{satisfaction}(V_1) &= 1 \\
\mathrm{satisfaction}(V_2) &= 1 + {}^{1}/_{2} + {}^{1}/_{3} \\
\mathrm{satisfaction}(V_3) &= 1 + {}^{1}/_{2} \\
\mathrm{satisfaction}(V_4) &= 0 \\
\mathrm{satisfaction}(V_5) &= 1 + {}^{1}/_{2} \\
\hline
\text{total satisfaction} &= 5\,{}^{5}/_{6}
\end{aligned}
$$

Multiwinner Election Rules: Chamberlin–Courant

- A voter i approves a set of candidates A_i.
- Voter i approves committee S if $S \cap A_i \neq \emptyset$.
- The committee approved by most voters wins.

$S = \{$  $\}$

V_1: $\{$  $\}$

V_2: $\{$  $\}$

V_3: $\{$  $\}$

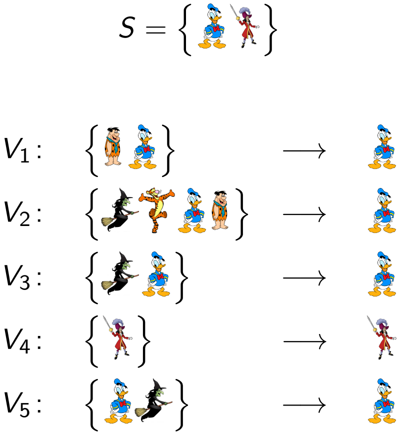
V_4: $\{$  $\}$

V_5: $\{$  $\}$

$$\mathrm{satisfaction}(V_1) = 1$$
$$\mathrm{satisfaction}(V_2) = 1$$
$$\mathrm{satisfaction}(V_3) = 1$$
$$\mathrm{satisfaction}(V_4) = 1$$
$$\mathrm{satisfaction}(V_5) = 1$$

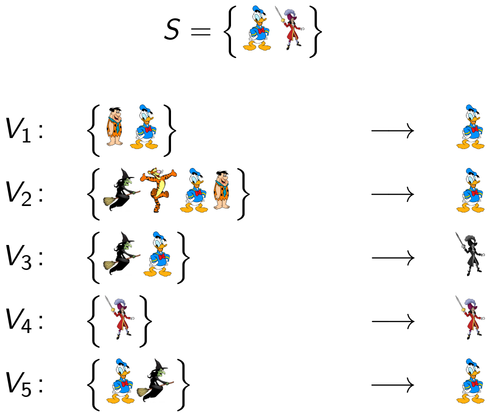
$$\text{total satisfaction} = 5$$

Under the Chamberlin–Courant rule each voter has a single representative within a committee. We wish to have a committee for which as many voters as possible approve their representatives.



Multiwinner Election Rules: Monroe

The same as Chamberlin–Courant but we require each committee member to represent at most $\lceil n/K \rceil$ voters.



total satisfaction $= 4$

Multiwinner Elections: the Challenge

The First Problem

It is unclear how to select K candidates that would satisfy the voters most.

But also ...

The First Problem

It is unclear how to select K candidates that would satisfy the voters most.

But also ...

The Second Problem

It is unclear what the aforementioned rules really do.

What do Multiwinner Rules do?: the Apportionment (Party-List) Profiles

Assume we can partition:

- the set of candidates C into pairwise disjoint sets $C_1, \ldots, C_p$, and
- the set of voters V into pairwise disjoint groups $V_1, \ldots, V_p$.

so that:

- the voters from V_i approve the candidates from C_i.

The Apportionment (Party-List) Profiles: PAV

50 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \emptyset$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \emptyset$$

$$\mathrm{score}(c_1) = 50$$
$$\mathrm{score}(c_{11}) = 30$$
$$\mathrm{score}(c_{21}) = 10$$
$$\mathrm{score}(c_{31}) = 10$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1\}$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1\}$$

$$\mathrm{score}(c_2) = {}^{50}/_{2} = 25$$
$$\mathrm{score}(c_{11}) = 30$$
$$\mathrm{score}(c_{21}) = 10$$
$$\mathrm{score}(c_{31}) = 10$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	$\mathbf{c_{11}}$	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1, c_{11}\}$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	$\mathbf{c_{11}}$	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1, c_{11}\}$$

$$\mathrm{score}(c_2) = {}^{50}\!/_{2} = 25$$
$$\mathrm{score}(c_{11}) = {}^{30}\!/_{2} = 15$$
$$\mathrm{score}(c_{21}) = 10$$
$$\mathrm{score}(c_{31}) = 10$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	$\mathbf{c_4}$	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	$\mathbf{c_{11}}$	$\mathbf{c_{12}}$	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1, c_2, c_3, c_4, c_{11}, c_{12}\}$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	$\mathbf{c_4}$	c_5	c_6	c_7	c_8	c_9	c_{10}
30 voters:	$\mathbf{c_{11}}$	$\mathbf{c_{12}}$	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1, c_2, c_3, c_4, c_{11}, c_{12}\}$$

$$\mathrm{score}(c_5) = {}^{50}/_{5} = 10$$
$$\mathrm{score}(c_{13}) = {}^{30}/_{3} = 10$$
$$\mathrm{score}(c_{21}) = 10$$
$$\mathrm{score}(c_{31}) = 10$$

The Apportionment (Party-List) Profiles: PAV

50 voters:	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$	$\mathbf{c_4}$	$\mathbf{c_5}$	c_6	c_7	c_8	c_9	c_{10}
30 voters:	$\mathbf{c_{11}}$	$\mathbf{c_{12}}$	$\mathbf{c_{13}}$	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	$\mathbf{c_{21}}$	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	$\mathbf{c_{31}}$	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$S = \{c_1, c_2, c_3, c_4, c_5 c_{11}, c_{12}, c_{13}, c_{21}, c_{31}\}$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad {}^{n}/_{K} = 10$$

$$S = \emptyset$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad {}^{n}/_{K} = 10$$

$$S = \{c_1\}$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	$\mathbf{c_2}$	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad {}^{n}/_{K} = 10$$

$$S = \{c_1, c_2\}$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	$\mathbf{c_2}$	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	$\mathbf{c_3}$	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	$\mathbf{c_4}$	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	$\mathbf{c_5}$	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad n/K = 10$$

$$S = \{c_1, c_2, c_3, c_4, c_5\}$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	$\mathbf{c_2}$	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	$\mathbf{c_3}$	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	$\mathbf{c_4}$	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	$\mathbf{c_5}$	c_6	c_7	c_8	c_9	c_{10}
10 voters:	$\mathbf{c_{11}}$	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{21}	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	c_{31}	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad {}^{n}\!/_{K} = 10$$

$$S = \{c_1, c_2, c_3, c_4, c_5, c_{11}\}$$

The Apportionment (Party-List) Profiles: Monroe

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	$\mathbf{c_2}$	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	$\mathbf{c_3}$	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	$\mathbf{c_4}$	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	c_1	c_2	c_3	c_4	$\mathbf{c_5}$	c_6	c_7	c_8	c_9	c_{10}
10 voters:	$\mathbf{c_{11}}$	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	$\mathbf{c_{12}}$	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	c_{11}	c_{12}	$\mathbf{c_{13}}$	c_{14}	c_{15}	c_{16}	c_{17}	c_{18}	c_{19}	c_{20}
10 voters:	$\mathbf{c_{21}}$	c_{22}	c_{23}	c_{24}	c_{25}	c_{26}	c_{27}	c_{28}	c_{29}	c_{30}
10 voters:	$\mathbf{c_{31}}$	c_{32}	c_{33}	c_{34}	c_{35}	c_{36}	c_{37}	c_{38}	c_{39}	c_{40}

$$K = 10 \qquad {}^{n}/_{K} = 10$$

$$S = \{c_1, c_2, c_3, c_4, c_5, c_{11}, c_{12}, c_{13}, c_{21}, c_{31}\}$$

The Apportionment (Party-List) Profiles: Chamberlin–Courant

10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
10 voters:	$\mathbf{c_1}$	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
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$$K = 10 \qquad {}^{n}/_{K} = 10$$

$$S = \{c_1, c_{11}, c_{21}, c_{31}, c_{32}, c_{33}, c_{34}, c_{35}, c_{36}, c_{37}\}$$

The Apportionment (Party-List) Profiles

PAV $\longrightarrow$	d'Hondt method
Monroe $\longrightarrow$	Hamilton method

The Apportionment (Party-List) Profiles

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Theorem

Proportional Approval Voting is the only approval-based rule that satisfies symmetry, consistency, continuity and proportionality.

See Martin Lackner's Poster!

An Axiomatic Characterization of Proportional Approval Voting (PAV)

Martin Lackner, Piotr Skowron
{martin.lackner,piotr.skowron}@cs.ox.ac.uk
University of Oxford, UK

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We want to choose 2 out 4 candidates $\{a, b, c, d\}$ given the approval preferences:

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Voters have a utility of 1 for one approved candidate in the committee, $1 + \frac{1}{2}$ for two candidates in the committee, $1 + \frac{1}{2} + \frac{1}{3}$ for 3 candidates, etc.
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Main Result

Axiomatic characterization of PAV

Proportional Approval Voting is the only approval-based committee rule that satisfies **symmetry**, **consistency**, **continuity** and **D'Hondt-proportionality**.

Models

Fix committee size k and set of candidates C.

- **Approval-based committee (ABC) rules**: functions from approval profiles to weak orders of size-k committees
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Axioms

Symmetry. An ABC rule is symmetric if it is anonymous and neutral.

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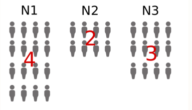
D'Hondt Proportionality



Consider a situation where voters belong to one of four parties $N_1, \ldots, N_4$ and parties have disjoint sets of candidates. We want to fill 10 committee seats.

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A committee counting function $f(x, y)$ gives the utility of an approval vote with x approved candidates in the committee and y approved candidates in total.
$f(x, y)$ defines a voting rule by maximizing the sum of utilities. Note that PAV is an example with $f(x, y) = \sum_{i=1}^{x} \frac{1}{i}$.

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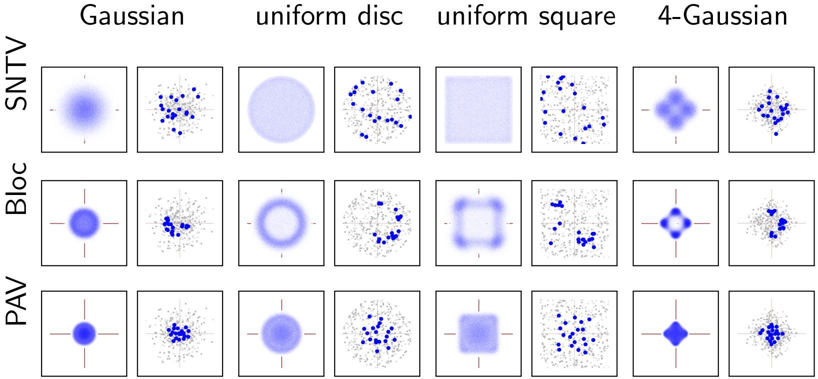
What do Voting Rules do?: Another Approach



See Edith Elkind's Poster!

What Do Multiwinner Voting Rules Do? An Experiment Over the Two-Dimensional Euclidean Domain

Edith Elkind†, Piotr Faliszewski‡, Jean-François Laslier*, Piotr Skowron†, Arkadii Slinko¶, Nimrod Talmon§
elkind@cs.ox.ac.uk, faliszew@agh.edu.pl, jean-francois.laslier@ens.fr, piotr.skowron@cs.ox.ac.uk ,
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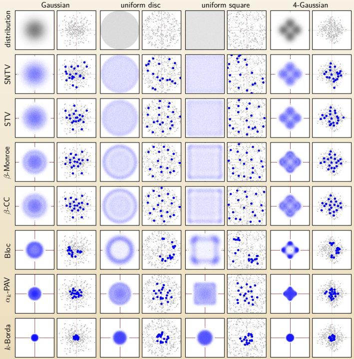
Experimental Setup

In each experiment, we sample 200 voters and 200 candidates according to one of the 4 distributions: Gaussian, uniform on disc, uniform on square, and a mix of four Gaussians. We then select a 20-member committee according to a given multiwinner rule. We repeat the experiment 10000 times for each voting rule. For each (distribution, voting rule) pair, we provide two images: a histogram (left) and a sample committee (right). The histograms show how often winners from a given location were selected; the higher color intensity corresponds to higher frequency. The first row shows the distributions themselves. Our results are useful for deciding which voting rules are suitable for popular applications of multiwinner voting, such as parliamentary elections, portfolio selection, or shortlisting (see paper).



How about shortlisting?

Committee Monotonicity

Assume that:

- for size K the rule selects a committee S,
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then it should hold that $S \subset S'$.

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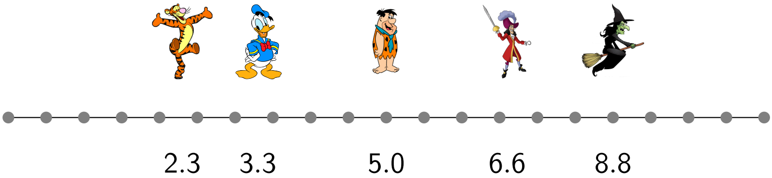

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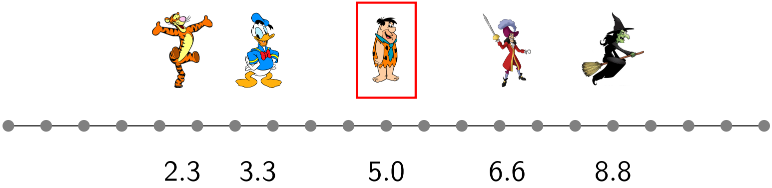

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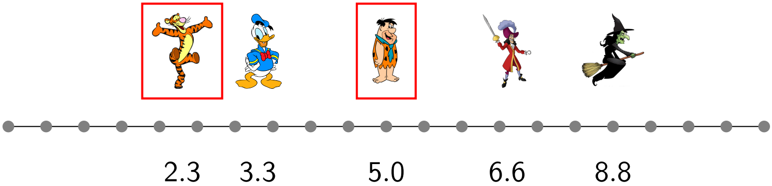

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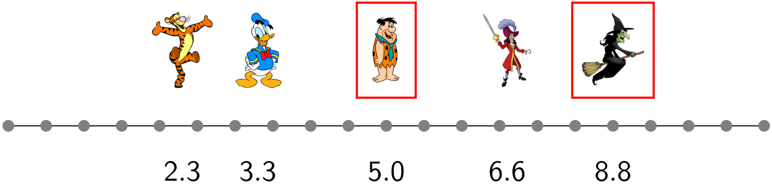

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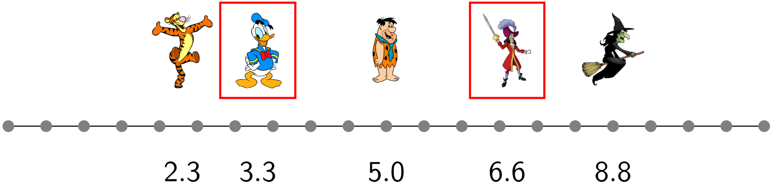

Proportional rankings

50 voters:	a_1	a_2	a_3	a_4	a_5	a_6	a_7	$\dots$	a_{100}
30 voters:	b_1	b_2	b_3	b_4	b_5	b_6	b_7	$\dots$	b_{100}
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κ-group representation

Let $\kappa(\alpha, \lambda)$ be a function from $((0,1] \cap \mathbb{Q}) \times \mathbb{N}$ to $\mathbb{N}$. A ranking r provides *κ-group representation (κ-GR) for profile P* if for all rational $\alpha \in (0,1]$, all $\lambda \in \mathbb{N}$, and all voter groups $N' \subseteq N$ that are (α, λ)-significant in P it holds that

$$\mathrm{avg}(N', r_{\leq \kappa(\alpha,\lambda)}) \geq \lambda.$$

A ranking rule f satisfies *κ-group representation (κ-GR)* if $f(P)$ provides κ-group representation for every profile P.

Proportional rankings

- α: the size of the group of voters N'.
- λ: the average number of alternatives approved by voters from N'.
- $\kappa(\alpha, \lambda)$: how far we need to go down the ranking to obtain the average number of approved candidates equal to λ.

Phragmén's rule $\longrightarrow$ $$\kappa(\alpha, \lambda) = \lceil \frac{5\lambda}{\alpha^2} + \frac{1}{\alpha} \rceil$$

Sequential PAV $\longrightarrow$ $$\kappa(\alpha, \lambda) = \lceil \frac{2(\lambda + 1)^2}{\alpha^2} \rceil$$

p-geometric rule $\longrightarrow$ $$\kappa(\alpha, \lambda) = \lceil \frac{p^{\lambda+1}}{\alpha(p-1)} \rceil$$

Conclusions

Analysis of the properties of rules allow us to:

- better understand the rules,
- better understand the properties,
- understand applicability of rules.

See the Posters!

An Axiomatic Characterization of Proportional Approval Voting (PAV)

Martin Lackner, Piotr Skowron
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Fix committee size k and set of candidates C.

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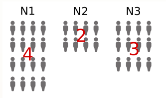
D'Hondt Proportionality



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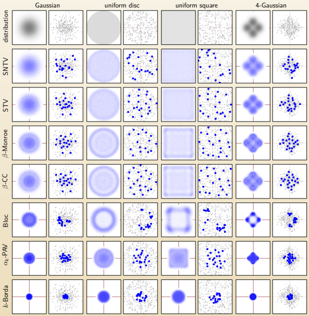

Questions?

Also, feel free to send any questions to:

`piotr.skowron@cs.ox.ac.uk`.