APPROVAL VOTING, REPRESENTATION, & LIQUID DEMOCRACY

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(and on *disjoint* work of Luis Sánchez Fernández et al.)

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Alexander von Humboldt Stiftung/Foundation
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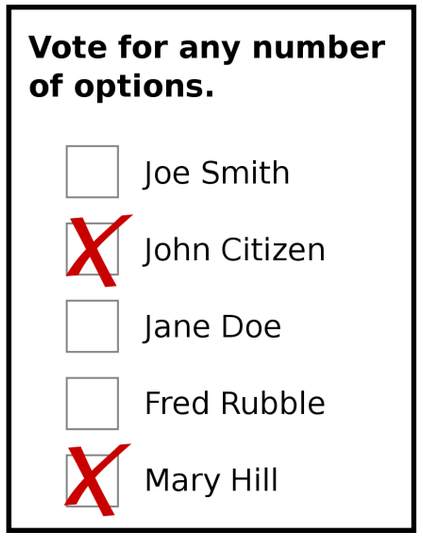
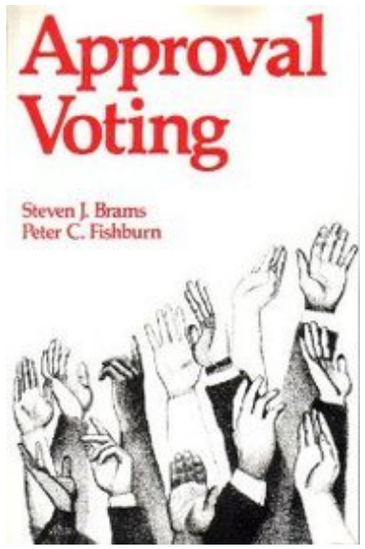
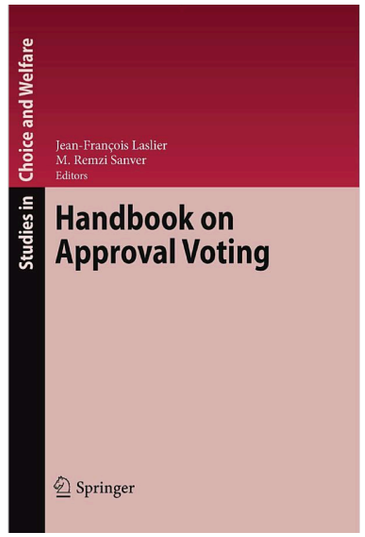
APPROVAL VOTING





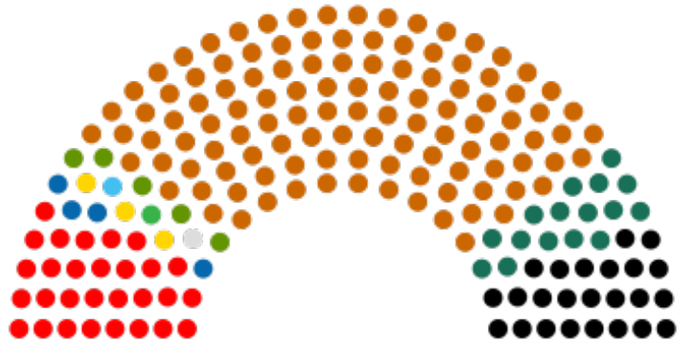


Official Ballot
Election for the United States House of Representatives
District One

Voting Instructions
1. You only have ONE vote.
2. Place an X in the box UNDER the party for whom you wish to vote.

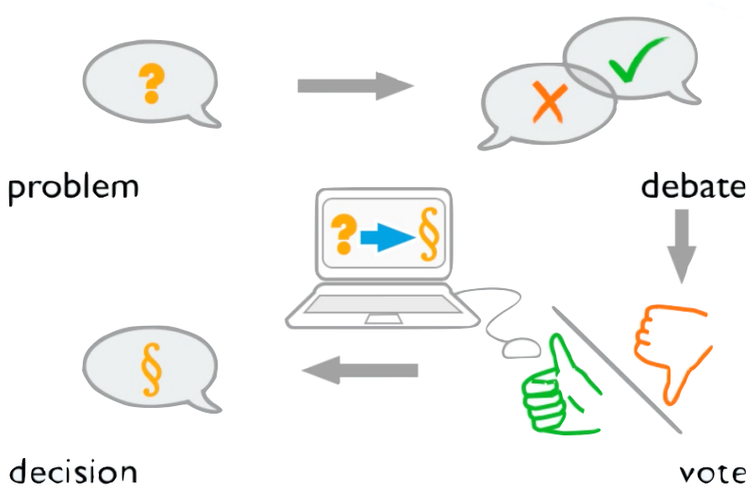
Democratic	Republican	Reform	Green	Independent Candidate
☐	☐	☐	☐	☐
1. Benjamin Foster	1. Wendy Berg	1. Steven Wong	1. Tom Wartenberg	1. Robert Moll
2. Sam Rosen-Amy	2. Steve Grolnic	2. Deborah Gorlin	2. Juan Hernandez	
3. Colin Volz	3. Sarah McClurg	3. Brad Crenshaw	3. Beata Panagopoules	
4. Benjamin Pike	4. Gerald Epstein	4. Daniel Czitrom	4. Alice Morey	
5. Megan Gentzler	5. Fran Deutsch	5. Meryl Fingrutd	5. Sarah Pringle	

(PROPORTIONAL) REPRESENTATION



LiquidFeedback

DemocracyOS



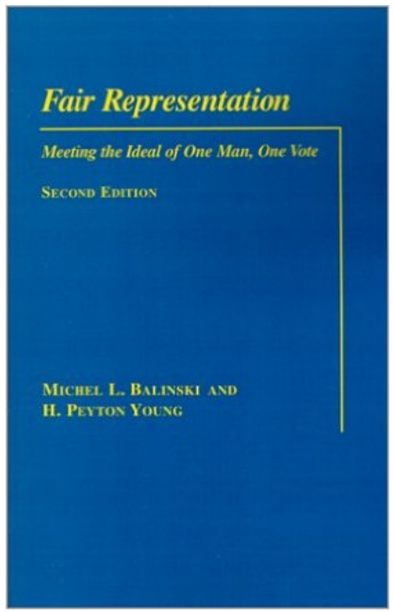
LIQUID DEMOCRACY

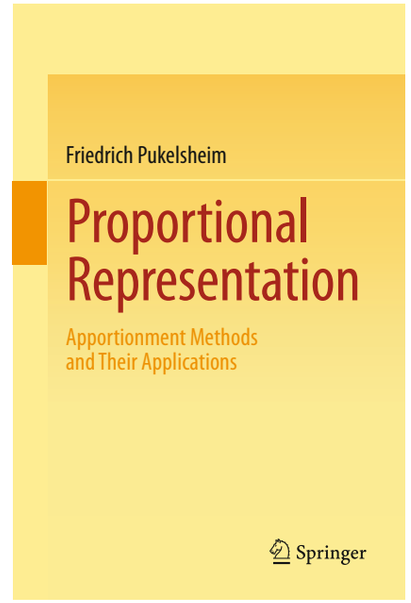
Closed List

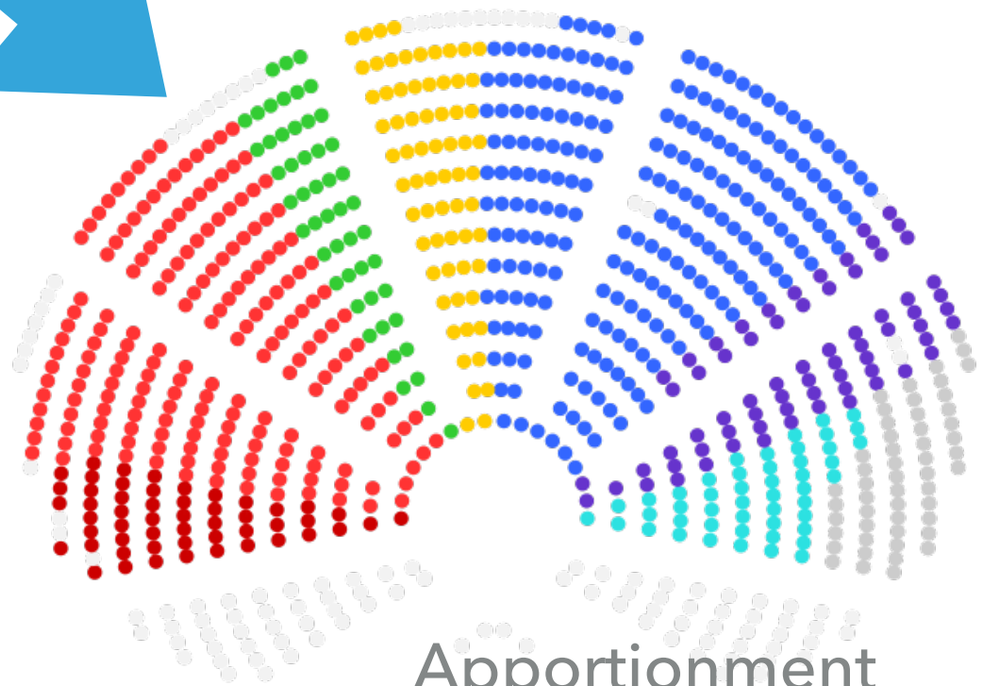
1 You vote for one list. The votes will determine the number of seats given to the list. Within each list, candidates will be elected according to the order that appears on the ballot, which has been decided by the party.

2 Hit "Confirm your choice"

United Left - Nordic Green Left	Freedom and Democracy	European People's Party	Conservatives and Reformists	Alliance of Liberals and Democrats	Greens - European Free Alliance	Progressive Alliance of Socialists and Democrats
1 Gabriele ZIMMER *Germany*	1 Nigel FARAGE *United Kingdom*	1 László SURJÁN *Hungary*	1 Robert STURDY *United Kingdom*	1 Jean-Luc BENNAHMIAS *France*	1 Amelia ANDERSDOTTER *Sweden*	1 Gianni PITTELLA *Italy*
2 Cornelia ERNST *Germany*	2 Zbigniew ZIOBRO *Poland*	2 Lambert van NISTELROOIJ *Netherlands*	2 Kay SWINBURNE *United Kingdom*	2 Andrew DUFF *United Kingdom*	2 Jean LAMBERT *United Kingdom*	2 Marek SIWIEC *Poland*
3 Sabine WILS *Germany*	3 Mara BIZZOTTO *Italy*	3 Sebastian Valentin BODU *Romania*	3 Valdemar TOMAŠEVSKI *Lithuania*	3 Cecilia WIKSTRÖM *Sweden*	3 Helga TRÜPEL *Germany*	3 Monika FLAŠÍKOVÁ BEŇOVÁ *Slovakia*
4 Vladimír REMEK *Czech Republic*	4 Jacek Olgierd KURSKI *Poland*	4 Thomas MANN *Germany*	4 Marina YANNAKOUDAKIS *United Kingdom*	4 Eduard-Raul HELLVIG *Romania*	4 Satu HASSI *Finland*	4 Ioan Mircea PAŞCU *Romania*
5 Sabine LÖSING *Germany*	5 Jaroslav PAŠKA *Slovakia*	5 József SZÁJER *Hungary*	5 Ryszard Antoni LEGUTKO *Poland*	5 Angelika WERTHMANN *Austria*	5 Christian ENGSTRÖM	5 Dolores GARCÍA-HIERRO CARABALLO *Spain*
6 Nikola VULJANIĆ *Croatia*	6 Paul NUTTALL *United Kingdom*	6 Barbara MATERA *Italy*	6 Michał Tomasz KAMIŃSKI *Poland*	6 Charles GOERENS *Luxembourg*	6 Carl SCHLYTER *Sweden*	6 Sergio Gaetano COFFERATI *Italy*
7 Kartika Tamara LIOTARD *Netherlands*	7 Jacek WŁOSOWICZ *Poland*	7 Horst SCHNELLHARDT *Germany*	7 Konrad SZYMAŃSKI *Poland*	7 Guy VERHOFSTADT *Belgium*	7 Eva JOLY *France*	7 Claudette ABELA BALDACCHINO *Malta*
8 Georgios TOUSSAS *Greece*	8 Frank VANHECKE *Belgium*	8 Marco SCURRIA *Italy*	8 Anthea McINTYRE *United Kingdom*	8 Giommaria UGGIAS *Italy*	8 Eva LICHTENBERGER *Austria*	8 Marino BALDINI *Croatia*
9 Charalampos ANGOURAKIS *Greece*	9 Roger HELMER *United Kingdom*	9 Roberta METSOLA *Malta*	9 Edvard KOŽUŠNÍK *Czech Republic*	9 Olle SCHMIDT *Sweden*	9 Philippe LAMBERTS *Belgium*	9 Spyros DANELLIS *Greece*
10 Marisa MATIAS *Portugal*	10 Bastiaan BELDER *Netherlands*	10 Theodor Dumitru STOLOJAN *Romania*	10 James ELLES *United Kingdom*	10 Jorgo CHATZIMARKAKIS *Germany*	10 Iñaki IRAZABALBEITIA FERNÁNDEZ *Spain*	10 Vilija BLINKEVIČIŪTĖ *Lithuania*







?

Apportionment methods:

- D'Hondt
- Hamilton
- Sainte-Laguë
- …

OUTLINE

- Representative committees
  - approval-based committee rules
  - representation axioms
    - computational problems
- Apportionment methods
- Representative rankings
- Liquid Democracy

SETTING

- set of candidates C
- set of voters $N = \{1, \dots, n\}$
- each voter i approves a subset of candidates $A_i \subseteq C$
- Goal: select a set W of $|W| = k$ winners (the committee)

WHAT'S WRONG WITH APPROVAL VOTING?

- Approval Voting (AV):
  select the k candidates with the highest approval score
  - variant: block voting (every voter approves k candidates)
- Problem: "dictatorship of the majority"
  - 51% vote $\{c_1, c_2, \ldots, c_k\} \Rightarrow W = \{c_1, c_2, \ldots, c_k\}$
  - toy example: 4 candidates c_1, c_2, c_3, c_4, $n = 9$ voters, $k = |W| = 3$
    - 5 x $\{c_1, c_2, c_3\}$
    - 4 x $\{c_4\}$

APPROVAL-BASED COMMITTEE RULES

- Minimax-AV: minimize the maximal (Hamming) distance to ballots
  [Brams, Kilgour & Sanver 2007]

Toy example (k=3)
- 5 x $\{c_1, c_2, c_3\}$
- 4 x $\{c_4\}$

- Satisfaction-AV: maximize total voter satisfaction $\sum_{i \in N} sat(i)$, where $sat(i) = |A_i \cap W| / |A_i|$ [Brams & Kilgour 2014]
  - equivalent to *equal and even cumulative voting*

- Greedy-AV: sequentially pick candidates that are approved by most *non-represented* voters

THIELE'S METHOD

- Proportional Approval Voting (PAV):
  maximize total score $\sum_{i \in N} score(i,W)$, where
  $score(i,W) = 1 + 1/2 + \ldots + 1/|W \cap A_i|$

[Thiele 1895, Simmons 2001]

- sequential version:
  select candidates greedily, reduce weights of already represented voters
- generalizations: replace (1, 1/2, 1/3, ...) with arbitrary weights

Toy example (k=3)

- 5 x $\{c_1, c_2, c_3\}$
- 4 x $\{c_4\}$

PAV scores

- $\{c_1, c_2, c_3\}$: 9.17
- $\{c_1, c_2, c_4\}$: 11.5

JUSTIFIED REPRESENTATION

- Intuition: each group of n/k voters "deserves" one representative
  - group needs to be "cohesive"
- Similarly: for each $\ell \in \{0,1,\ldots,k\}$, each "cohesive" group of $\ell \cdot$ n/k voters "deserves" ℓ representatives

REPRESENTATION AXIOMS

- EJR: extended justified representation

$$\forall X \subseteq N : |X| \geq \ell \frac{n}{k} \text{ and } |\cap_{i \in X} A_i| \geq \ell \Rightarrow (\exists i \in X : |W \cap A_i| \geq \ell)$$

- PJR: proportional justified representation

$$\forall X \subseteq N : |X| \geq \ell \frac{n}{k} \text{ and } |\cap_{i \in X} A_i| \geq \ell \Rightarrow |W \cap (\cup_{i \in X} A_i)| \geq \ell$$

- JR: justified representation

$$\forall X \subseteq N : |X| \geq \frac{n}{k} \text{ and } |\cap_{i \in X} A_i| \geq 1 \Rightarrow |W \cap (\cup_{i \in X} A_i)| \geq 1$$

REPRESENTATION AXIOMS

- EJR

PAV (Thiele's method)
- not for the sequential version!
- not for any other weight vector!

Dans le genre «proportionnel» la fonction de satisfaction a la forme $f(n) = 1 + \frac{1}{2} + \ldots + \frac{1}{n}$. La victoire du deuxième

- PJR
- JR

Greedy-AV

Many rules fail JR:

AV, Minimax-AV, Satisfaction-AV, …

COMPUTATIONAL COMPLEXITY

- PAV satisfies EJR, but is NP-hard to compute [Skowron et al. 2015]

COMPUTATIONAL WORK ON PAV

- checking PAV is coW[1]-hard w.r.t. k [Aziz et al. 2015]
- FPT algorithms [Elkind & Lackner 2015, Skowron 2015]
- restricted domains [Peters 2016]
- approximation [Skowron et al. 2015]

- **Open:** are there polynomial-time rules satisfying EJR?
  - checking if a committee provides EJR is coNP-complete

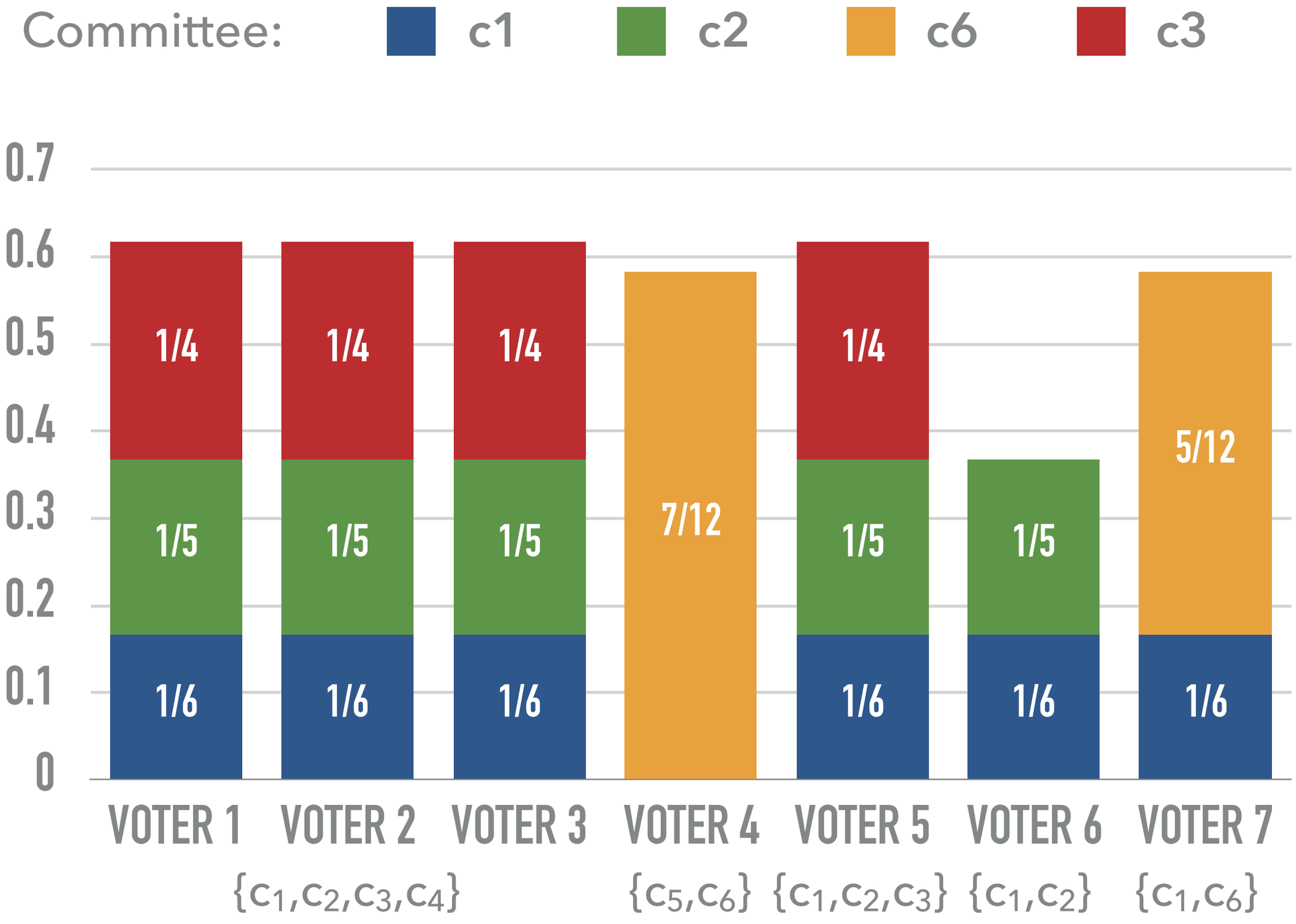
PHRAGMÉN'S RULE

- Proposed by Swedish mathematician Lars Edvard Phragmén (1863–1937)
- Sequential "load balancing" procedure:
  - at each step, a candidate c is chosen and one unit of "load" is distributed among the approvers of c
- goal: make distribution of loads as "even" as possible (i.e., maximal load as small as possible)

EDVARD PHRAGMÉN

PHRAGMÉN'S RULE: EXAMPLE

- 6 candidates $c_1, c_2, c_3, c_4, c_5, c_6$
- $k = |W| = 4$
- $n = 7$ voters:
  - $3 \times \{c_1, c_2, c_3, c_4\}$
  - $1 \times \{c_5, c_6\}$
  - $1 \times \{c_1, c_2, c_3\}$
  - $1 \times \{c_1, c_2\}$
  - $1 \times \{c_1, c_6\}$


Committee:
c1
c2
c6
c3
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1/4
1/5
1/6
7/12
5/12
VOTER 1
VOTER 2
VOTER 3
VOTER 4
VOTER 5
VOTER 6
VOTER 7
{c1,c2,c3,c4}
{c5,c6}
{c1,c2,c3}
{c1,c2}
{c1,c6}

PHRAGMÉN'S RULE: RESULTS

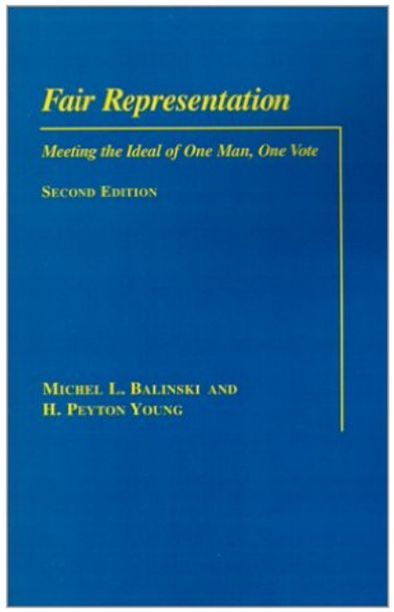
- Phragmén's rule satisfies PJR
  - thus: PJR committee can be found efficiently!
  - if k divides n, *GreedyMonroe-AV* satisfies PJR as well
- Phragmén's rule fails EJR
- Open problems:
  - complexity of finding an EJR committee
  - complexity of checking whether a committee satisfies PJR

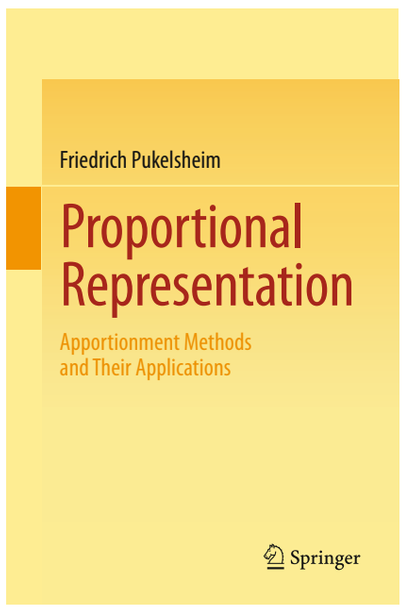
Example 5. *Consider the following instance with* $n = 24$, $k = 12$, *and* $C = \{a, b, c_1, c_2, \ldots, c_{12}\}$.

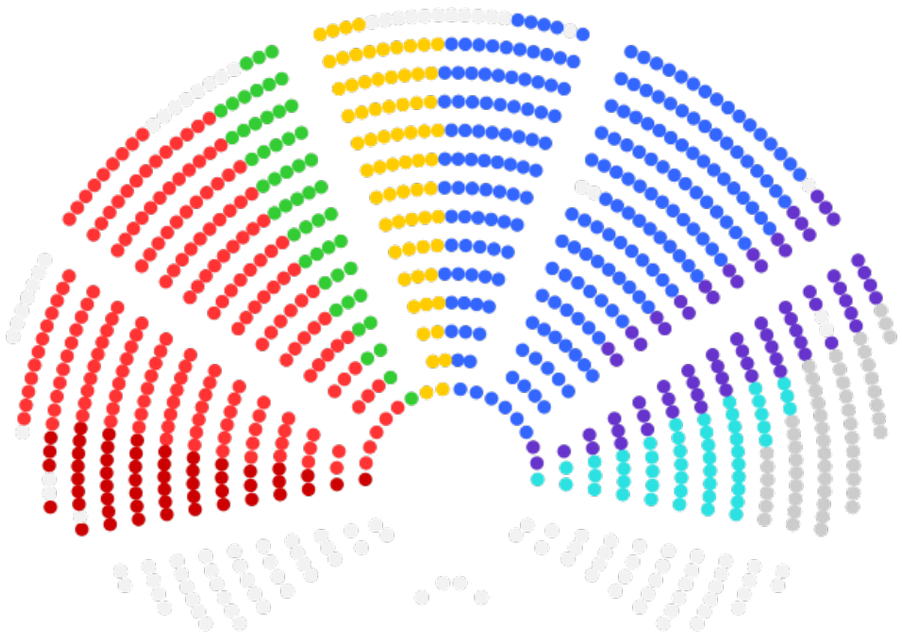
$2 \times \{a, b, c_1\}$	$6 \times \{c_1, c_2, \ldots, c_{12}\}$
$2 \times \{a, b, c_2\}$	$5 \times \{c_2, c_3, \ldots, c_{12}\}$
	$9 \times \{c_3, c_4, \ldots, c_{12}\}$

APPORTIONMENT

- Apportionment is a special case of committee selection
  - voting for a party = approving all candidates of that party
- Every committee rule induces an apportionment method







Committee rules:

PAV
(Thiele's method)

Phragmén's method
(sequential version)

Monroe-AV
(optimal version of
GreedyMonroe-AV)

Phragmén's method
(variance-version)

Apportionment Methods:

D'Hondt method
(aka Jefferson method)

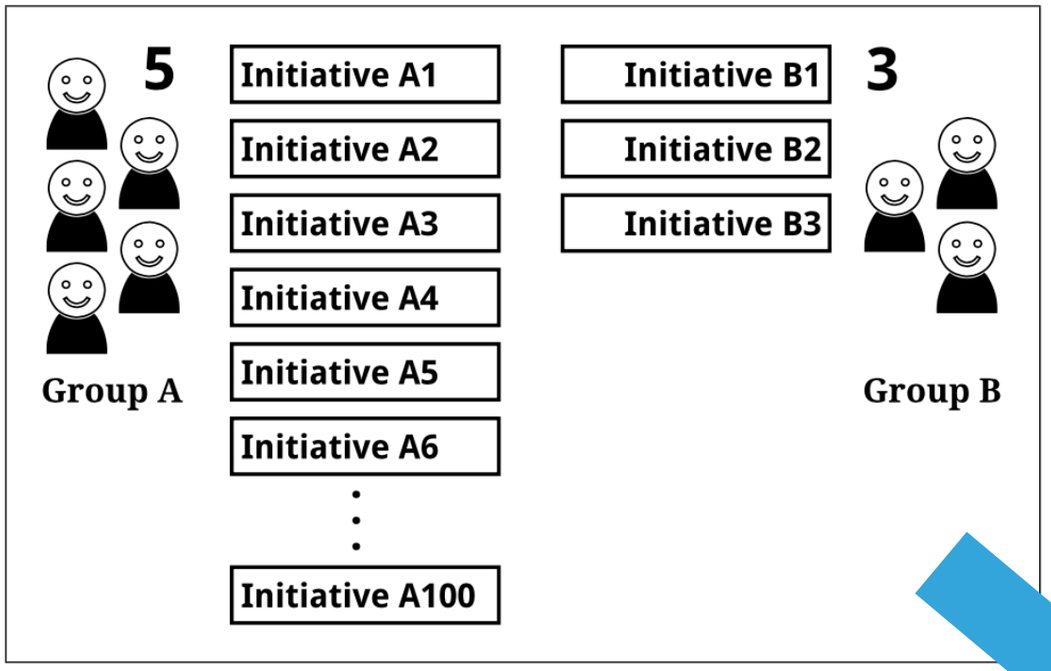
Hamilton method
(aka largest remainder method)

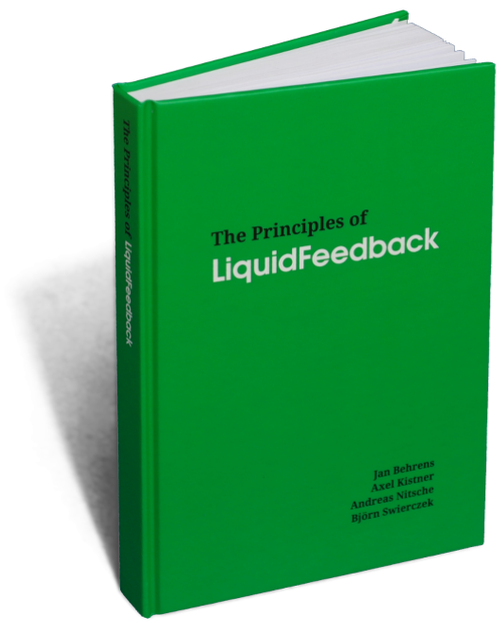
Sainte-Laguë method
(aka Webster method)

REPRESENTATIVE RANKINGS

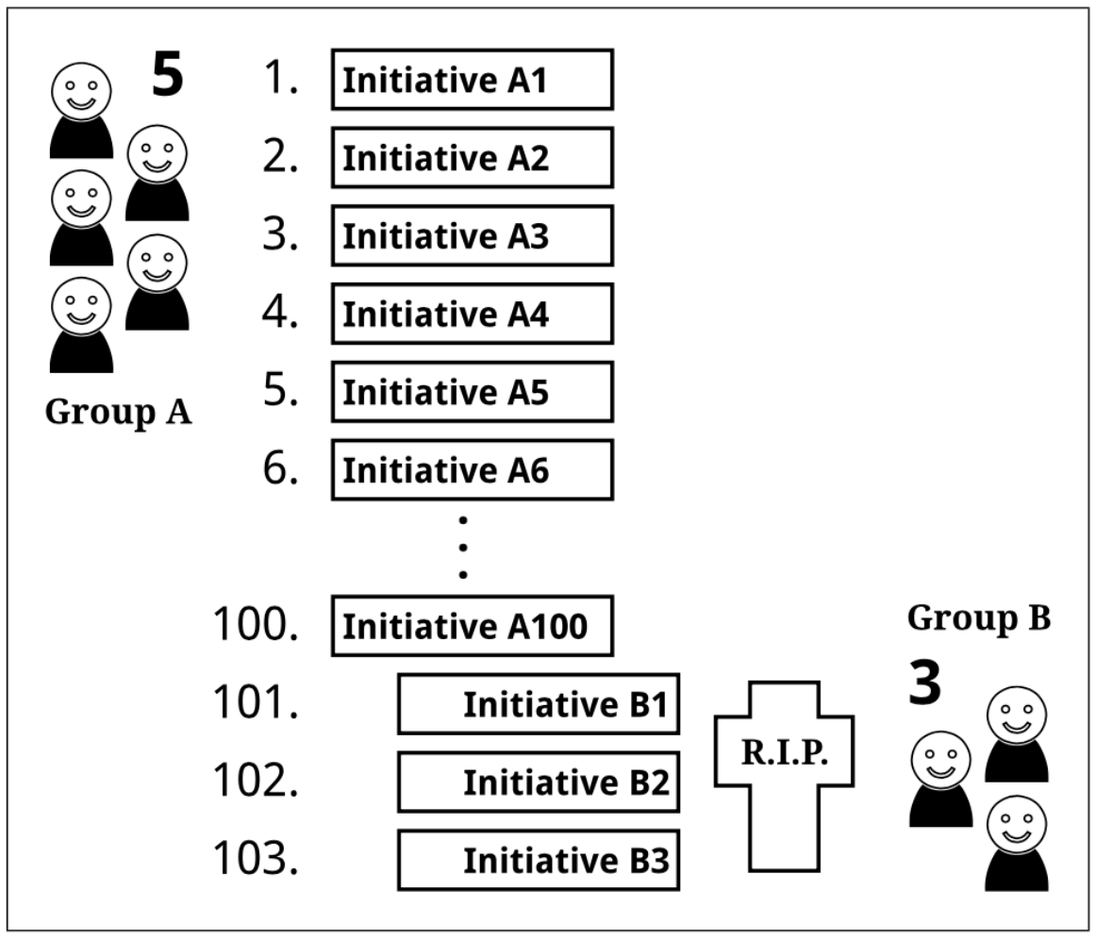
- Same input, but different output: ranking of candidates
- Sequential committee rules naturally produce rankings
- When is a ranking representative?

PROTECTION OF MINORITIES IN LIQUID DEMOCRACIES



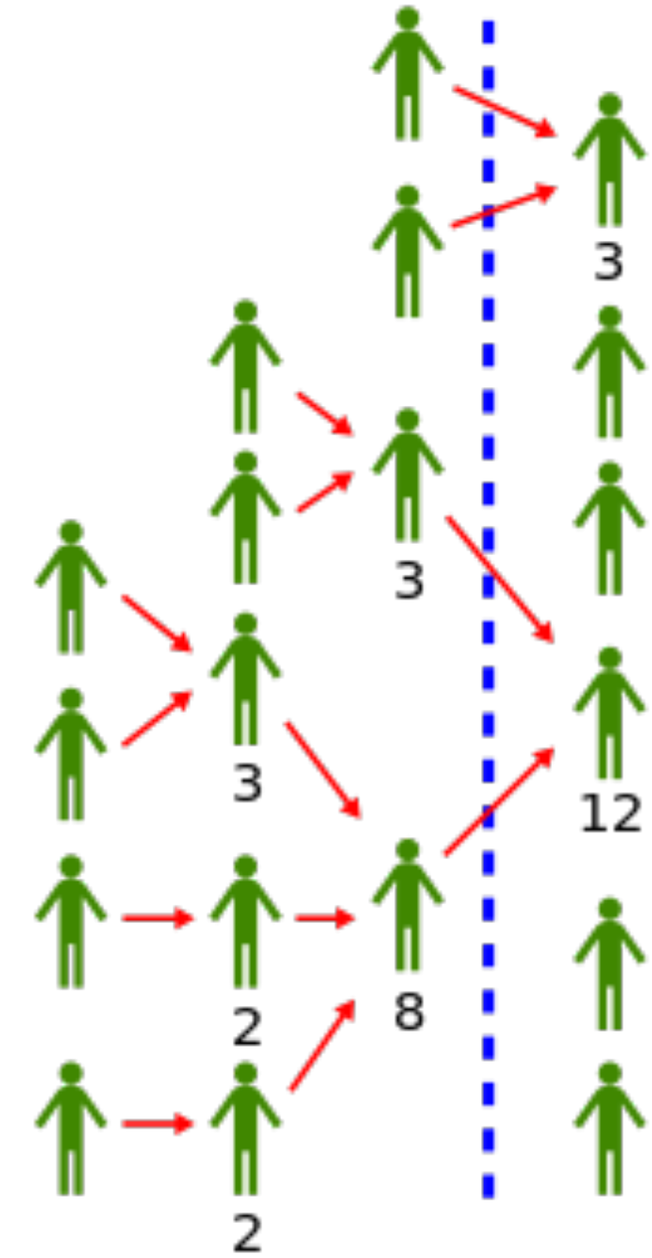


The Principles of LiquidFeedback (Behrens, Kistner, Nitsche, Swierczek)



OTHER RESEARCH PROBLEMS INSPIRED BY LIQUID DEMOCRACY

- Delegated voting
  - dealing with delegation cycles, abstentions, …
- Preference elicitation (e.g. participatory budgeting)
- Desirable properties (e.g. independence of clones)
- Strategic aspects



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