Learning in Survey Expectations of Inflation

Krisztina Molnar* and Zoltan Reppa†
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Abstract

The main question of this paper is how private agents form their expectations. To answer this, we approximate survey expectations of inflation by constant gain learning algorithms. In particular we develop a Bayesian constant gain estimator, and show that this gives a much better approximation to survey expectations then simple constant gain estimators. We find that in more volatile economies private agents pay more attention to recent data when they form their inflationary expectations.Finally, when a regime change occurs private agents do not follow mechanically an adaptive rule, but understand it immediately that statistical relations between economic variables will change.

Introduction

Survey data are one of the tools commonly used to identify economic agents' expectations.[1] The main focus of economic research on survey expectations has been on inflationary expectations[2], and this is also the focus of our research.

To illustrate a typical path for survey data on inflation, Figure depicts the median forecast for one-year-ahead inflation (as measured by the Livingston survey[3]) as well as

*Norwegian School of Economics and Business Administration, Bergen, Norway; Email: krisztina.molnar@nhh.no

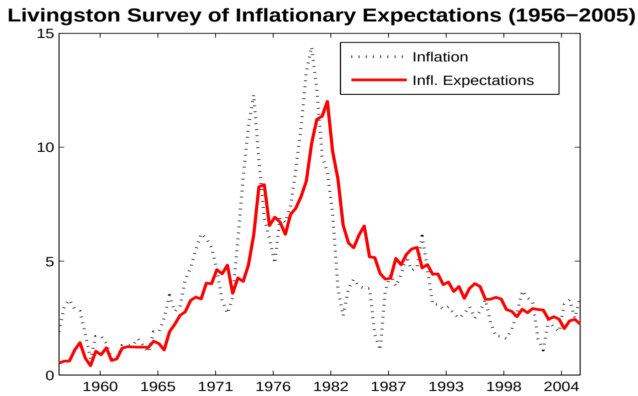
†Magyar Nemzeti Bank,Budapest, Hungary; Email: reppaz@mnb.hu.

[1] Survey evidence is subject to the caveat that survey respondents may not have incentives to provide accurate information. So survey expectations are at best a noisy measure of inflation expectations and at worst tell us nothing about actual inflation expectations. However it may boost confidence in the usefulness of survey expectations that they are helpful in modelling inflation and in predicting inflation, wages and interest rates. See for example Roberts (1995), Roberts (1997), Englander and Stone (1989)) and Mehra (2002).

[2] Relatively less research has been done about what type of expectation formation is a good approximation for other economic variables. On exchange rate expectations see for example Frankel and Froot (1987). They find that exchange rate expectations are non-rational and can be explained by both adaptive expectations, equilibrium (or regressive) expectations, and distributed-lag expectations.

[3] The Livingston survey is the oldest continuous survey of U.S. inflationary expectations, commenced in 1946 when Joseph Livingston began querying economists forecasts about several economic variables. The semi annual survey is published in June and December.

the inflation rate that eventually prevailed. Survey expectations are shifted one year ahead, thus the graph shows actual forecast errors. The sluggishness is clear. First, in times of generally rising inflation, such as the 1970s, expected inflation tends to underpredict realized inflation. In contrast, in times of falling inflation, such as the 1980s and 1990s, the forecasts appear to overpredict inflation. Second, the turning points in expected inflation consistently lag the turning points of actual inflation. These regularities suggests a strong adaptive or backwards-looking element in the formation of inflation expectations.



This sluggish behavior is indeed a general feature of survey inflationary expectations. Thomas (1999) documents sluggish turning points and underpredicting inflation for periods of upward-rarcheting inflation and overpredicting inflation for periods of decreasing inflation in three different survey measures of inflationary expectations: the Livingston survey, the Survey of Professional Forecasters[4] and the Michigan Survey of Households[5]. Mehra (2002) finds similar results for the Livingston and Michigan surveys of CPI expectations.

Recently several papers model expectations with learning algorithms and find a better fit of standard models then with rational expectations[6]. However empirical work on learning is relatively scarce.

One of the first papers to find that inflationary expectations are well approximated by adaptive expectations was Carlson and Parkin (1975). A recent paper by Orphanides and Williams (2004) shows that survey expectations in the US on inflation are well approximated by a constant gain learning algorithm. Orphanides and Williams (2004) assume private agents estimate a VAR of three variables, inflation rate, unemployment rate and the FED funds rate, allowing for discounting past data. To calibrate the constant gain parameter they

[4]This survey began in late 1968, but CPI forecasts are collected only from 1981. Participants are predominantly from the business sector, all make their living via forecasting.

[5]The Michigan survey was initiated in 1948 and involves a randomly selected sample of at least 500 households.

[6]See for example Milani (2005), Timmermann (1996).

examine how forecasts with different gain parameters fit the Survey of Professional Forecasters (SPF). For this they generated a time series of out of sample forecasts, in each quarter they reestimated a VAR using all historical data available in that quarter, and they allowed for discounting past data with geometrically declining weights. They found that gain parameters between 0.01 and 0.04 yielded forecasts closest to the Survey of Professional Forecasters. Milani (2005) estimates similar gain parameter 0.0187, with estimating the gain parameter together with other coefficients of a New Keynesian model with Bayesian estimation.

In this paper we examine whether in different environments people learn in a different way. We use survey expectations as a measure of private expectations, and approximate their expectation formation with learning. In the first section of the paper we follow the methodology of Orphanides and Williams (2004) and approximate expectations with constant gain learning. This methodolgy proves to be robust when there is a long enough dataset. In transition economies, like Hungary, reliable data can be found only after the transition period, therefore we have a short sample problem. Therefore in the second part of the paper we develop a constant gain Bayesian estimation analogous to constant gain learning, which gives more robust findings in short samples. Our first result is that constant gain Bayesian learning is a much better approximation of survey expectations, then simple constant gain learning. Our second result is that in more volatile environments people pay more attention to current data and discount more past data. Finally, when a change of the exchange rate regime takes place people understand it immediately that the econometric relationship between the exchange rate and other variables must change, and people start discounting past data heavily.

The first section describes the dataset. The second section summarizes results from constant gain estimations. The third section develops the Bayesian constant gain learning. The fourth section sums up results of the constant gain Bayesian estimation. Finally, the last section concludes.

1 Data

This paper compares empirical results for the US, Israel and Hungary.[7] These three cases are interesting to compare, since US is a stable economy, while Israel and Hungary are less stable, therefore we expect to find different expectation formation. Israel experienced a hyperinflation in the beginning of the 80's, after this the economy stabilized. In Hungary the economic environment is volatile from the nature of the transition itself and also because there was a regime change of the exchange rate system.

Examining expectation formation in transition economies is particularly interesting, since these economies faced a unique experiment. After the end on communism market economy started to develop and people had to learn about an environment that they have not experienced before, moreover this economic environment was also in a transition

[7]Our aim is to conduct the analysis on a wider dataset also.

which also depended on how agents behaved (ergo also on how agents formed their expectation).

> "...work on economic dynamics within parallel traditions in game theory, macroeconomics, and general equilibrium theory have given us theories of dynamics that have their best chance of applying when people are in *recurrent situations that they have experienced often before.* In Eastern Europe the transition is not like that: people there are confronted with unprecedented opportunities, new and ill defined rules, and daily struggle to determine the 'mechanism' that will eventually govern trade and production."
> Sargent (1993)

Besides, all transition economies faced several structural changes of monetary policy: first they adopted a fixed exchange rate regime and later switched to inflation targeting. Therefore it is also interesting to examine whether these structural changes influenced expectation formation: in theory one would expect agents to discount more past data in order to track better the structural changes.

Economic data is from the IFS database. All series have been seasonally adjusted. We used the treasury bill rate as interest rate.

For Israel we have survey data of inflationary expectations from 1997, from a survey conducted by the Bank of Israel. Figure 1 shows the evolution of survey forecasts compared to actual CPI inflation.

For the Hungarian estimations we use data from 1995, because of unreliability of earlier data. Survey expectations are forecasts of professional forecasters from the Reuters survey, available from the first quarter of 1997 until the second quarter of 2005 (see graph 2). Hungary experienced one big change in monetary policy during our sample period: in the 2nd quarter of 2001 Hungary abandoned the crawling peg exchange regime and switched to inflation targeting.[8]

For the US inflationary expectations we use the Survey of Professional Forecasters.

2 Constant gain Learning

In this section we follow the analysis of Orphanides and Williams (2004) and look into more detail how expectations change in different economic environments.

2.1 The methodology

We assumed that the participants in the survey formed their expectations through constant gain learning. We show in the Appendix that this is approximately the same as if they used VAR-s, but the equations of the VAR-s were estimated by weighted least

[8]Strictly speaking, from 2001 Hungary follows an inflation targeting with a very wide band ($\pm 15\%$) of the exchange rate.

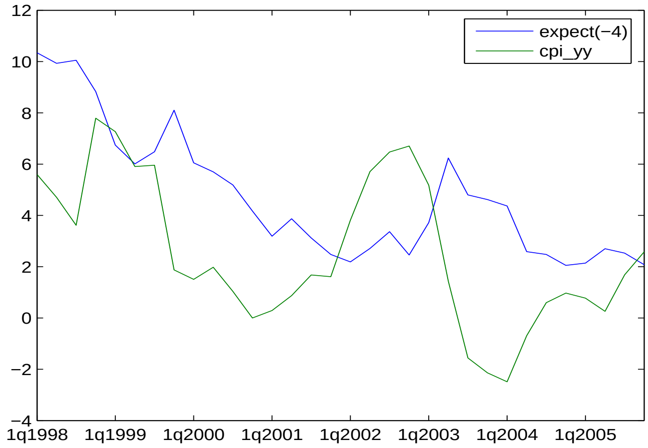


Figure 1: Survey expectations and inflation in Israel. Survey expectations are shifted, so the difference between the two lines reflect forecast errors.

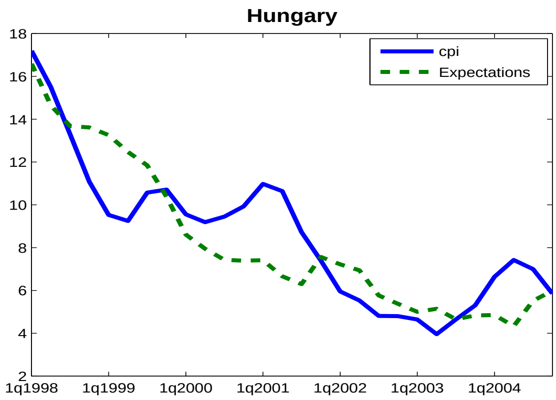


Figure 2: Survey expectations and inflation in Hungary. Survey expectations are shifted, so the difference between the two lines reflect forecast errors.

squares with exponentially declining weights; i.e. they minimize

$$\sum_{t=1}^{T} \alpha^{T-t} \left(y_t^i - x_t\beta\right)^2 = \sum_{t=1}^{T} \left(\alpha^{(T-t)/2} y_t^i - \alpha^{(T-t)/2} x_t\beta\right)^2, \text{ with } 0 < \alpha \leq 1. \tag{1}$$

Here y^i is the i^{th} variable in the VAR and x contains the lags of the variables. We determine α by a grid search method, which involves the following steps:

1. First we determine the time interval that is used to compare the survey results with the VAR forecasts: that is, we determine an interval $[T_1, T_2]$.

2. Then we chose α.
3. Next, we estimate the VAR-s minimizing (1), with the data available until each $[T_1 \leq T \leq T_2]$, and calculate one-year-ahead forecast of inflation.[9]
4. We the calculate the root mean squared difference (RMSD) between the VAR forecasts and the survey data.
5. We repeat steps 2) - 4) for different values of α, and chose the one with the minimal RMSD.

2.2 Empirical Results

For the US our first methodolgy found similar results to earlier papers, the calibrated gain parameter is very low. In this section we compare this result to the case of Israel.

2.2.1 Israel

Israel experienced a hyperinflation in the beginning of the 80's, do we conducted analysis both with using data from this period and without using hyperinflationary data.

Our main findings can be summarized as follows: (1) Agents use much higher discounting then in the US. (2) This holds true even if we use regressions where hyperinflationary data are not used. (3) Through time the extent of discounting past data decreased, which suggest agents believed more and more that they live an a stationary environment.

Table 2.2.1 summarizes our results for the best fitting regressions. Samples which start in 1984 include the hyperinflationary period, samples that start in 1992 do not.

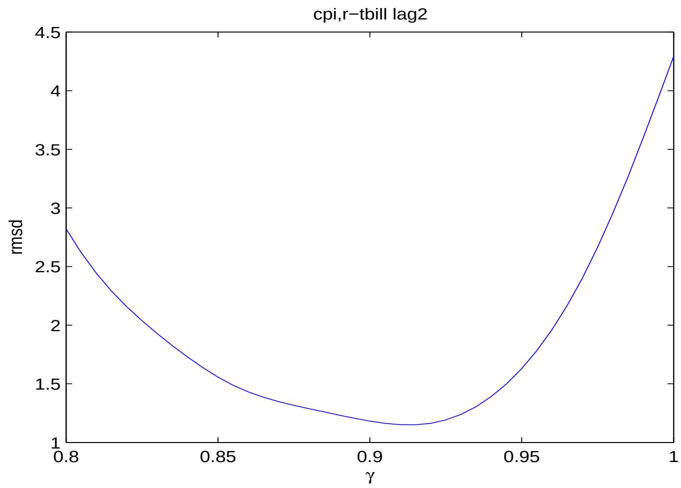
lags	Variables	rmsd	α	Sample
1	r-tbill	1.333004	0.875	2q1984
2		1.150591	0.915	
3		1.370329	0.89	
1	r-tbill,u	1.981976	0.88	1q1992
2		1.745203	0.99	
		2.084728	1	
1	r-tbill, gdp	1.483495	0.91	2q1984
2		1.46305	0.915	
3		1.750949	0.905	
1	u,gdp	1.495395	0.925	1q1992
		1.848548	0.945	
		1.979649	0.99	

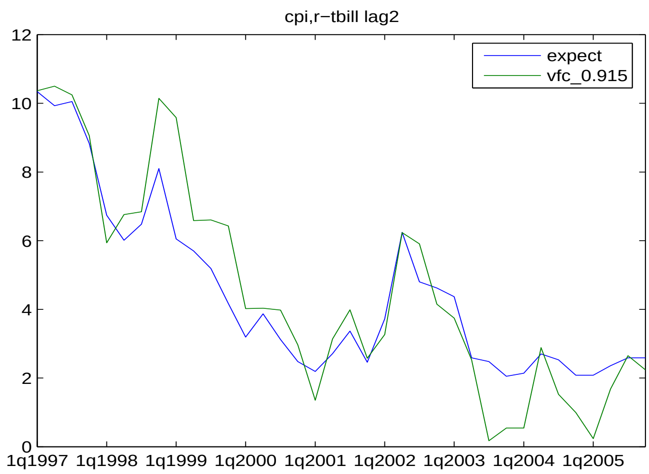
Our result is that discounting past data is much higher then in the US. The regression which approximates best survey expectations is a VAR with the treasury bill rate, with 2 lags and discounting past data with $\alpha = 0.915$, which roughly corresponds to a tracking

[9] In all cases the survey contains one-year-ahead forecasts of inflation (year-on-year index of the price level).

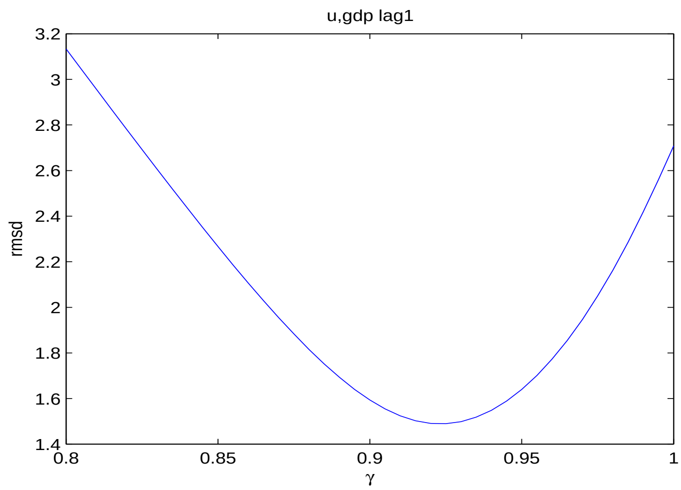
parameter 0.085. Figure 2.2.1 shows the root mean squared deviation of VAR forecasts from survey expectations for this VAR specification. $\alpha = 0.915$ gives the best approximation to surveys. Figure 2.2.1 shows the one year ahead VAR forecasts together with survey expectations, one can observe that this VAR captures quite well the dynamics in survey inflationary expectations.

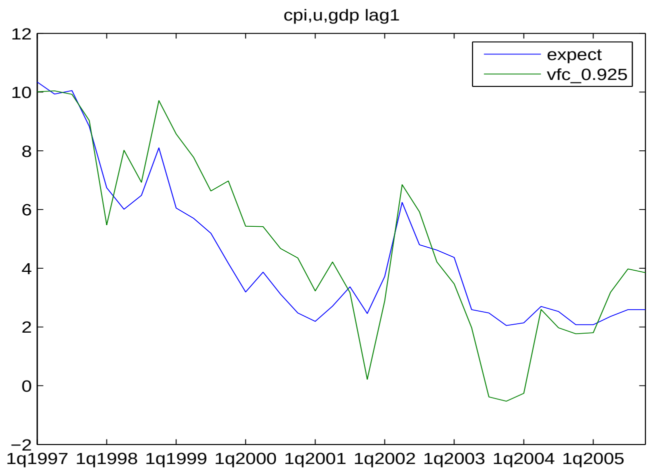
The calibrated degree of discounting remains high even if we consider different specifications and lags of the VAR, and discounting remain high even if we use a sample period which does not include hyperinflationary data. Figure 2.2.1 and 2.2.1 shows results for a VAR with 2 lags including the endogenous variables: CPI, GDP and unemployment rate.





It is interesting to examine how discounting changes through time. For this we conducted the same analysis with changing the sample of survey expectations: we assumed





agents (thus VARs) use all available data, but now we minimize the distance between VAR forecasts and a smaller sample of surveys.The best fitting VARs imply a high discounting, α varies between 0.86 and 0.93. An interesting result is that discounting decreases in time. As we approximate a set of surveys that include less and less data from the initial periods the calibrated measure of discounting decreases. This suggests that that expectations pay more and more attention to past data, agents believe more and more that the economic environment is not prone to structural changes. Figure 3 shows estimation results for a VAR with 1 lag including the endogenous variables: CPI, , treasury bill rate and the unemployment rate. We approximated expectations on the whole sample 1997-2005, then on a smaller sample 1998-2005 and so on. We can observe, that the calibrated α increases over time, which implies that the degree of discounting decreases over time.

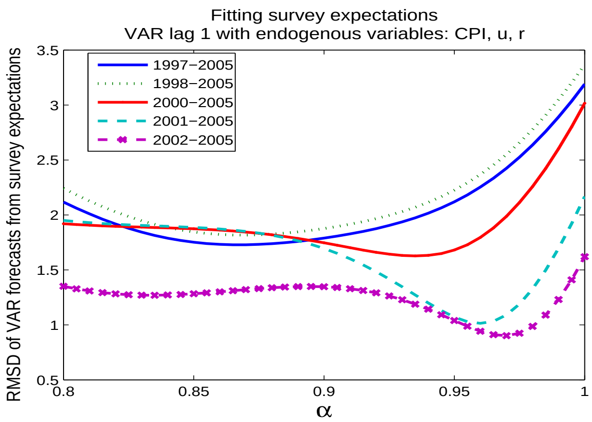


Figure 3: Root mean squared distance of survey expectations and VAR forecasts with different discounting, α. The minimum gives the value of α which provides the best approximation of survey forecasts. Results for VAR with one lag, endogenous variables are CPI,treasury bill rate, and the unemployment rate.

3 Bayesian constant gain learning

Since for short samples the results of the constant gain estimations were very sensitive to the specification of the VAR and the sample used to compare the forecasts[10], we decided to experiment with Bayesian techniques. The motivation was that the sensitivity is most likely the consequence of the transitory nature of the environment and the lack of data, and if this is realized by the forecasters, their forecasts will probably depend on some knowledge not related to the data. Notice that this implies that we assume the agents to be more knowledgeable than simple VAR forecasters.

3.1 Recursive formulas for the Bayesian estimate

In order to define a Bayesian version of constant gain learning we first have to derive a recursive formulation of the standard Bayesian estimates. We do this in the analytically simplest case when both the prior of the system parameters and the distribution of the data are normal, and the error precision (inverse of the covariance) matrix is Wishart.

More precisely, if the prior for β is $N(\bar{m}, \bar{M})$ and the other conditions are fulfilled then posterior will be $N(m^*, M^*)$ with

$$m^* = \left(\bar{M}^{-1} + X'X\right)^{-1} \left(\bar{M}^{-1}\bar{m} + X'y\right), \tag{2}$$

$$M^* = \left(\bar{M}^{-1} + X'X\right)^{-1}. \tag{3}$$

[10] This holds also true for estimations for Czech republic and Poland. The estimated optimal discounting parameter with simple VARs is sensitive to the specification of the VAR and the the sample size of surveys that we use in the analysis.

Equation (2) can be manipulated to get

$$\begin{aligned} m^* &= \left(\bar{M}^{-1} + X'X\right)^{-1} \left(\bar{M}^{-1}\bar{m} + X'y\right) = \\ &= \left(I + \bar{M}X'X\right)^{-1} \left(\bar{m} + \bar{M}X'y\right) = \left(I + \bar{M}X'X\right)^{-1} \left(\bar{m} + \bar{M}X'X\beta_{OLS}\right). \end{aligned} \tag{4}$$

In the standard OLS case we have that $X'X = \sum_{t=1}^{T} x_t' x_t = TR_T$. Also, let us introduce $M = \bar{M}^{-1}$, analogously to the fact that R_T is the precision of the OLS estimate. If we use the posterior from period $T-1$ as prior in period T, then we get the following four recursions for the Bayesian estimate:

$$R_T = R_{T-1} + T^{-1}\left(x_T' x_T - R_{T-1}\right), \tag{5}$$

$$\beta_T = \beta_{T-1} + T^{-1} R_T^{-1} x_T' \left(y_T - x_T \beta_{T-1}\right), \tag{6}$$

$$M_T = M_{T-1} + TR_T, \tag{7}$$

$$m_T = \left(I + TM_{T-1}^{-1} R_T\right)^{-1} \left(m_{T-1} + TM_{T-1}^{-1} R_T \beta_T\right). \tag{8}$$

3.2 Constant gain prior updating

The most straightforward choice is to replace all appearances of T^{-1} with a constant gain parameter γ, which gives

$$R_T = R_{T-1} + \gamma\left(x_T' x_T - R_{T-1}\right), \tag{9}$$

$$\beta_T = \beta_{T-1} + \gamma R_T^{-1} x_T' \left(y_T - x_T \beta_{T-1}\right), \tag{10}$$

$$M_T = M_{T-1} + \gamma^{-1} R_T, \tag{11}$$

$$m_T = \left(I + \gamma^{-1} M_{T-1}^{-1} R_T\right)^{-1} \left(m_{T-1} + \gamma^{-1} M_{T-1}^{-1} R_T \beta_T\right). \tag{12}$$

Equations (11)-(12) can also be derived from the "discounting the data" interpretation of the constant gain recursions as presented in the Appendix.

However, as can been seen on Figure 4, this will result in very slow learning.[11] To see this, one should examine the weight that is put on new information in (12). This weight is given by $w_T = \gamma^{-1} M_{T-1}^{-1} R_T$. Using also (11), we see that

$$\begin{aligned} w_T &= \gamma^{-1}\left(M_0 + \gamma^{-1}\sum_{t=1}^{T-1} R_t\right)^{-1} R_T = \left(\gamma R_T^{-1} M_0 + R_T^{-1}\sum_{t=1}^{T-1} R_t\right)^{-1} = \\ &= T^{-1}\left(\gamma T^{-1} R_T^{-1} M_0 + R_T^{-1} T^{-1}\sum_{t=1}^{T-1} R_t\right)^{-1}. \end{aligned} \tag{13}$$

If R_T converges to some R^* – as is the case with the constant gain recursions –, then so does $T^{-1}\sum_{t=1}^{T-1} R_t$, and therefore $w_T \approx T^{-1} I$. The relative weight on new information decreases linearly, just like in the case of least-squares learning.

[11] Both Figures 4 and 5 show simulation results for the simplest model of "learning a constant" with $\gamma = \delta = 0.1$. The initial values of the recursions were $\beta_0 = m_0 = 1$ and $M_0 = 10^{-4}$. The standard deviation of the data was 0.25.

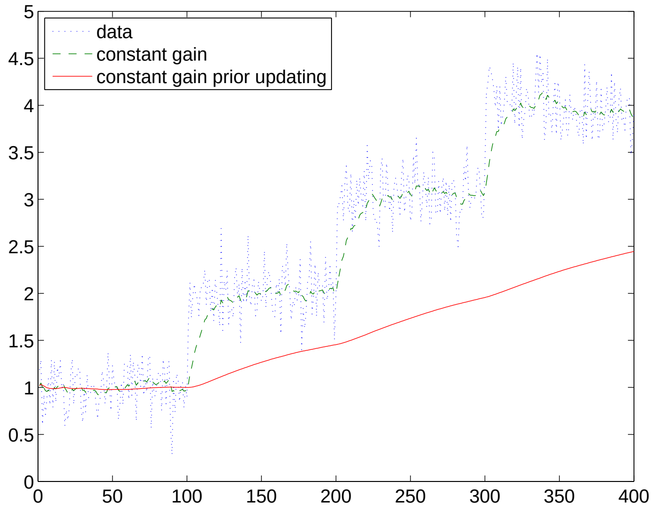


Figure 4: learning speed with constant gain prior updating

3.3 Exponential prior updating

Since the introduction of the Bayesian updating equations destroyed the fast speed of constant gain learning, we now try to find a weighting sequence α_T so that the recursions

$$
\begin{aligned}
R_T &= R_{T-1} + \gamma \left(x'_T x_T - R_{T-1} \right), & (14)\\
\beta_T &= \beta_{T-1} + \gamma R_T^{-1} x'_T \left(y_T - x_T \beta_{T-1} \right), & (15)\\
M_T &= M_{T-1} + \alpha_T R_T, & (16)\\
m_T &= \left(I + \alpha_T M_{T-1}^{-1} R_T \right)^{-1} \left(m_{T-1} + \alpha_T M_{T-1}^{-1} R_T \beta_T \right), & (17)
\end{aligned}
$$

will result in (at least asymptotically) a constant weight on new information. Analogously to equation (13), this weight is given by

$$
w_T = \alpha_T \left(M_0 + \sum_{t=1}^{T-1} \alpha_t R_t \right)^{-1} R_T = \left(\alpha_T^{-1} R_T^{-1} M_0 + \alpha_T^{-1} R_T^{-1} \sum_{t=1}^{T-1} \alpha_t R_t \right)^{-1}. \tag{18}
$$

From this and from the convergence of R_T we see that sufficient conditions to ensure that w_T converges to a constant matrix are $\alpha_T \to \infty$ and $\alpha_T^{-1} \sum_1^{T-1} \alpha_t \to \delta$ with some constant δ. Using equation (30) of the Appendix, we see that such a sequence is $\alpha_t = (1+\delta)^t$ with $0 < \delta$.

A drawback of this choice is that the learning rule if not time-independent, but if we are willing to use the asymptotic weight – which is $\delta^{-1} I$ – in finite sample as well, we get

$$
\begin{aligned}
R_T &= R_{T-1} + \gamma \left(x'_T x_T - R_{T-1} \right), & (19)\\
\beta_T &= \beta_{T-1} + \gamma R_T^{-1} x'_T \left(y_T - x_T \beta_{T-1} \right), & (20)\\
m_T &= (1+\delta)^{-1} \left(m_{T-1} + \delta \beta_T \right). & (21)
\end{aligned}
$$

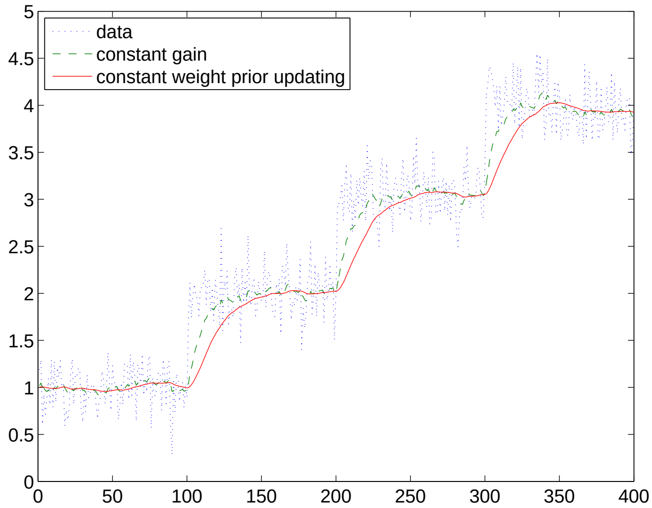


Figure 5: learning speed with constant gain prior updating

With this simplifying choice we do not need to keep track of the covariance matrices M_T, and the "Bayesian" estimates m_T will reduce to exponentially weighted averages of past and current β_T-s. Figure 5 shows that the learning speed is close to the speed of constant gain learning, while the parameters are remarkably smoother.

4 Empirical results

In this section we summarize the results from the Bayesian constant gain estimations.[12]

As we have seen in the previous section we have two free parameters to calibrate, δ governing the smoothness of the forecasts, when an average is taken between the Bayesian prior and the OLS estimate, and γ, which is the discounting in the OLS part of the estimates. Both a higher δ and a higher γ imply faster learning, in other words imply that private expectations pay more attention to recent data.[13]

Table 1 and 2 show the estimated optimal regressions and the calibrated free parameters. We can observe that in all cases the Bayesian constant gain (BCG) estimations (Table 1) give a better fit then the simple constant gain (CG) (Table 2) in terms of root mean square deviation of the survey data from the out of sample forecasts of the estimations. We would like to note that simple constant gain can be seen as a special case of Bayesian constant gain, for a very high δ. For this reason it is natural that BCG gives a better fit then CG. Therefore the important result is that BCG gives a much

[12] We estimated regressions using CPI, GDP, nominal interest rate, and exchange rate and up to 2 lags. The baseline estimation was with CPI, GDP and the nominal interest rate, and we checked whether including less variables improves the fit. It remains to examine robustness depending on varying the variables entering the VAR, for example including different types of interest rates.

[13] When δ and γ change in the opposite directions our preliminary analysis suggests that speed of learning depends on the exact values.

better fit then CG.

Result 1 *Bayesian constant gain gives a much better approximation to survey inflationary expectations then simple constant gain algorithms.*

Table 1: Best fitting Bayesian constant gain regressions

	Optimal δ	Optimal γ	RMSD	Model
USA	0.0147	5.84 e-7	0.0147	CPI, GDP, i, 2 lag
Israel	3.7655	2.27 e-8	0.8364	CPI, GDP, i, 2 lag
Hungary	0.3676	0.0042	1.0085	CPI, GDP, i, 2 lag

RMSD is the root mean square deviation of survey expectations and econometric forecasts of the best fitting model.

Table 2: Best fitting simple constant gain regressions

	Optimal γ	RMSD	Model
USA	0.0888	1.4318	CPI, GDP, i, 2 lag
	0.0624	1.1360	CPI, GDP, i, 1 lag
Israel	0.0002	1.7623	CPI, GDP, i, 2 lag
Hungary	0.0230	1.5693	CPI, GDP, i, 2 lag

RMSD is the root mean square deviation of survey expectations and econometric forecasts of the best fitting model.

Our second observation from Table 1 is that in Israel and Hungary private expectations pay more attention to recent data then in the US. This suggests that in more volatile environments it pays off to discount more past data and track recent changes in the economy by putting more weight on recent data.[14]

Result 2 *Bayesian constant gain gives a higher discounting of past data for Hungary and Israel then for the US.*

Table 3 shows the same result that we already found with constant gain estimations and discussed in the former chapter. When we use a survey sample of inflationary expectations that is further away from the hyperinflationary period discounting of past data decreases. This suggests that people believe more and more that they live in a more stable environment then before.

Finally Table 4 shows estimation results for Hungary, where the aim is to understand how people have changed their expectation formation before and after the change of the exchange rate regime from fixed to float in the first quarter of 2001.

When we try to fit only expectations after the regime change, 2q2001 we experience a surprising result: agents start heavily discounting past data. This suggests that agents

[14]It remains to examine this proposition on a wider set of countries. Also it remains to understand the exact role of δ and γ.

Table 3: Bayesian constant gain regressions for Israel on different samples

Initial period	Optimal δ	Optimal γ	RMSD
1q1992-	3.7655	2.27 e-8	0.8364
1q1998-	1.0600	1.15 e-8	0.7656
1q1999	0.0467	4.26 e-8	0.5946
1q2001	0.1886	9.46 e-008	0.7923

RMSD is the root mean square deviation of survey expectations and econometric forecasts.
The econometric model includes the variables CPI, GDP, treasury bill rate, with 2 lags.
We assume agents use all available data, but we calibrate
δ and γ to fit survey expectations from the initial period till the last survey data.

Table 4: Bayesian constant gain regressions of Hungary with exchange rate before and after the regime change.

Period	Optimal δ	Optimal γ	RMSD
1q1997-1q2001	0.32	0.08	1.04
2q2001-end	0.02	0.0006	0.52
1q1997-2q2001	0.32	0.0742	1.11
3q2001-end	0.37	0.69	0.36
1q1997-3q2001	0.3141	0.06	1.1
4q2001-end	0.029	0.75	0.32
1q1997-4q2001	0.31	0.058	1.087
1q2002-end	0.016	0.93	0.267
1q1997-1q2002	0.31	0.0587	1.06
2q2002-end	0.0227	0.8089	0.2505
1q1997-2q2005	0.3018	0.0484	1.09

RMSD is the root mean square deviation of survey expectations
and econometric forecasts of the best fitting model.
Regressions are with 1 lag, and include CPI, GDP, treasury bill rate, exchange rate.

perceived that abandoning the fixed exchange rate regime must change the relationship between the exchange rate and other economic variables, therefore believed less in the data before the regime change and started heavily discounting past data. Moreover, agents did abruptly change their perception and did not mechanically use their past estimations.[15]

Result 3 *After the monetary regime change to inflation targeting in Hungary survey expectations reflect that agents changed their perception about the role of the exchange rate very quickly: in particular they perceived a change in the structural relationship between exchange rate and inflation.*

[15] Regressions with 2 lags show a slightly worse fit to survey expectations.

5 Conclusion

We have examined learning in survey expectations of inflationary expectations. We approximated survey expectations with constant gain learning, and also developed a Bayesian constant gain estimator. We have shown that the Bayesian constant gain is a better approximation to private expectations than simple constant gain algorithms used so far in the literature.

A value added in modelling private expectations with Bayesian constant gain might be that including an informative prior can give an information to private agents that is not present in the data. This way one can model private expectations to be more rational then simple backward looking expectations. Throughout the paper we assumed an uninformative prior, we find it interesting for future research to examine more thoroughly the role of an informative prior for our analysis.

Our results suggest that private expectations are not simply backward looking, and even more clever then constant gain learning algorithms. In particular after a regime change people understand the nature of the regime change immediately and understand that the econometric relationship between variables will be different in the new regime. It remains to investigate this finding on a wider set of countries to able to give a more robust statement.

6 Appendix: Learning and weighted leats squares

We want to see what the relationship between weighted least-squares estimation and learning is. To this end we derive the recursive formula corresponding to the estimation. More precisely, estimation means minimizing the sum

$$\sum_{t=1}^{T} \alpha_t \left(y_t - x_t\beta\right)^2 . \tag{22}$$

Here y_t is a scalar, x_t is a row vector, β is a column vector. The FOC is

$$0 = \sum_{t=1}^{T} \alpha_t x_t' \left(y_t - x_t\beta\right), \tag{23}$$

and therefore the estimate is

$$\beta_T = \left(\sum_{t=1}^{T} \alpha_t x_t' x_t\right)^{-1} \left(\sum_{t=1}^{T} \alpha_t x_t' y_t\right). \tag{24}$$

In this setting it is natural to define

$$A_T = \sum_{t=1}^{T} \alpha_t, \qquad R_T = \frac{\sum_{t=1}^{T} \alpha_t x_t' x_t}{A_T}. \tag{25}$$

Thus R_T is a weighted average of the $x_t' x_t$-s. From (25) we have the following recursion for R_T:

$$\begin{aligned} R_T &= \frac{\sum_{t=1}^{T-1} \alpha_t x_t' x_t + \alpha_T x_T' x_T}{A_T} = \frac{A_{T-1} R_{T-1} + \alpha_T x_T' x_T}{A_T} = \\ &= R_{T-1} + \frac{\left(A_{T-1} - A_T\right) R_{T-1} + \alpha_T x_T' x_T}{A_T} = R_{T-1} + \frac{\alpha_T}{A_T}\left(x_T' x_T - R_{T-1}\right). \end{aligned} \tag{26}$$

From (24) and (25) the recursion for β_T is the following:

$$\begin{aligned} \beta_T &= \left(A_T R_T\right)^{-1} \left(\sum_{t=1}^{T} \alpha_t x_t' y_t\right) = \left(A_T R_T\right)^{-1} \left(\sum_{t=1}^{T-1} \alpha_t x_t' y_t + \alpha_T x_T' y_T\right) = \\ &= \left(A_T R_T\right)^{-1} \left(A_{T-1} R_{T-1} \beta_{T-1} + \alpha_T x_T' y_T\right) = \\ &= \beta_{T-1} + \left(A_T R_T\right)^{-1} \left(\left[A_{T-1} R_{T-1} - A_T R_T\right] \beta_{T-1} + \alpha_T x_T' y_T\right) = \\ &= \beta_{T-1} + \left(A_T R_T\right)^{-1} \left(\alpha_T x_T' y_T - \alpha_T x_T' x_T \beta_{T-1}\right). \end{aligned} \tag{27}$$

So the two recursions are

$$R_T = R_{T-1} + \frac{\alpha_T}{A_T}\left(x_T' x_T - R_{T-1}\right), \tag{28}$$

$$\beta_T = \beta_{T-1} + \frac{\alpha_T}{A_T} R_T^{-1} x_T' \left(y_T - x_T \beta_{T-1}\right). \tag{29}$$

If we set $\alpha_t = 1$, then $\alpha_T / A_T = 1/T$, and we get least-squares learning. If $\alpha_t = \alpha^t$ with $\alpha > 1$, then

$$\frac{\alpha_T}{A_T} = \frac{\alpha^T}{\frac{1-\alpha^{T+1}}{1-\alpha}} = \frac{1-\alpha}{\alpha^{-T} - \alpha}. \tag{30}$$

If T is large, this is approximately $(\alpha - 1)/\alpha$, a constant.

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