Outside offers and bidding costs*

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July 22, 2006

Abstract

This paper provides a search theoretic model with endogenous job creation, and homogenous workers and firms. The model introduces bidding costs and allows the current employer to make a counteroffer with probability q when the worker receives an outside offer. In equilibrium, a higher level of ex-post competition (q) reduces the probability that an employed worker receives an outside offer. Therefore, a higher level of ex-post competition may decrease the expected income of the workers. In the extreme case when the competition is cutthroat ($q = 1$), no employed worker receives outside offers and each employed worker earns only the minimum wage.

In contrast to existing models, our model allows for wage dispersion even if *all* frictions (including bidding and search costs) converge to zero simultaneously. When bidding costs are small and ex-post competition is strong, a small change in parameter values may influence the equilibrium bidding, wage distribution and job creation substantially. Consequently, it is not only the overall level of market frictions that matters, but also their structure.

*I am grateful to Ronni Pavan, Előd Takáts and the participants of the Summer Workshop of the Hungarian Academy of Sciences for their useful suggestions. All remaining errors are mine.

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1 Introduction

An important question in labor economics is why similar workers earn different wages. Several studies (for references see Rogerson et. al. (2005)) show the significance of wage dispersion: only 30% of the observable wage differences can be explained by observable workers' characteristics like age, education, sex, race, etc. To address the issue of wage dispersion it is natural to study search theoretic models with homogenous agents. The vast majority of the literature concentrating on such models either assumes that when an employed worker obtains a second offer the ensuing bidding war drives the wage up to the marginal product of the worker (perfect competition) or alternatively, that the current employer does not make any counter offer at all (no bidding competition like in Burdett and Mortensen (1998)).

While these extreme cases are interesting possibilities, they are also unrealistic. On one hand, the current employer may not want to let its worker leave without trying to make a new offer to him or her. On the other hand, the assumption of perfect competition implies the non-existence of any frictions: first, for such a competition to arise it is vital that the wage offers are verifiable by all parties, an assumption whose validity may depend on the specific labor market in question. Second, a firm must find it costless to engage in a bidding war. Suppose that there is a small cost $c > 0$ of making an offer and that once an employed worker receives a new offer the wage is bid up to the level of marginal product. This means that once a worker is employed it is not profitable to bid for him or her, since the bidding costs cannot be recovered. Therefore, the current employer is in effect a monopolist and there is no reason why he should offer a wage higher than the worker's outside option or the (binding) minimum wage if it exists. As a result, too much competition ex-post leads to the elimination of effective competition by reducing the bidding activity dramatically.[1]

To study intermediate cases I introduce a parameter $q \in [0, 1]$ that captures the level of competition for an employed worker who receives a new offer: there is no such competition when $q = 0$, while the competition is perfect when $q = 1$.

[1] This result is somewhat similar to that of the sequential search model of Diamond (1971): any positive bidding cost undermines the market for employed workers if the ex-post competition is perfect once a competing offer arises.

To formalize this, I assume that q is the probability that the current employer can make a (costless) counter offer[2]. An offer to an already employed worker is called an outside offer and making an outside offer is also referred to as bidding for an employed worker. When $q = 0$ the current employer does not make a counter offer as in the paper of Burdett and Mortensen (1998). When $q = 1$ and an employed worker receives an outside offer, the ensuing competition drives the wage up to the marginal product.

To formally analyze the issue of bidding costs and employer competition I consider a continuous time non-directed search model where job (vacancy) creation is costly and is governed by free entry. Workers and firms are matched at a Poisson rate λ, and if matched, the firm may make an offer to the worker, which is not less than the minimum wage w_m, at a cost c. If the worker is already employed, then the current employer may make a counter offer with probability q. The wage offers (and the current wage) are unobserved by the rival firm, but it *is* observed whether the worker is currently employed or not. As standard in search models, each employment situation breaks up exogenously at a rate δ.

In equilibrium, a higher level of bidding competition (q) leads to a lower profit from making an outside offer. So, an outside offer is made only if q is not too high relative to the cost of bidding (c). More precisely, for small values of q ($q < \underline{q}$), firms always make outside offers to employed workers. For intermediate values ($q \in (\underline{q}, \overline{q})$) such offers are made with probability $p(q) \in (0, 1)$. For higher values of q outside offers are never made, and all workers are employed at the minimum wage. For some parameter configurations, the first regime does not occur ($p(0) < 1$), but the other two regimes always do ($\underline{q} < \overline{q} < 1$).

The level of job creation v is decreasing in q in the first regime ($q < \underline{q}$), increasing in the second regime, while it is constant in the third. It is intuitive why the level of job creation decreases in the first regime: a higher value of q makes job creation less profitable. In the second regime a higher value of q reduces bidding for employed workers (as measured by $p(q)$) making job creation more profitable. In the third regime, q is so high that an employed worker never receives offers and thus the exact value of q is unimportant.

[2] All of the results go through if there was a bidding cost for the current employer, but it was lower than c.

A higher level of ex-post competition (q) has a positive direct effect on employer competition and expected wages, but a negative indirect effect is present as well. First, a high level of competition reduces bidding for employed workers as was suggested above. Second, it can reduce job creation by making it less profitable when $q < \underline{q}$. Therefore, the total effect of a more competitive labor market on a worker's expected income is in general ambiguous. However, this ambiguity disappears if the competition is cutthroat ($q > \overline{q}$). In this case the workers are either unemployed or they work for the minimum wage, thus the workers are worst off if q is very high.

While bidding frictions might be low in real life, they can still affect the equilibrium outcome. The key observation is that both thresholds $\underline{q}$ and $\overline{q}$ converge to 1 as c converges to 0. This fact implies that if the bidding frictions vanish ($q \rightarrow 1$ and $c \rightarrow 0$), then the amount of bidding for employed workers crucially depends on the relative rate of convergence of c and q. If the convergence of q is not too quick ($q < \underline{q}(c)$ along the converging sequence), then there is sure bidding for employed workers in the limit. If $q > \overline{q}(c)$ along the sequence, then there are no outside offers, while if q is in the intermediate range, then outside offers are made with a probability strictly between 0 and 1.

The sensitivity of bidding for employed workers carries over to the wage distribution as well, thereby making the *structure* of bidding frictions (when they vanish in the limit) important for market outcomes. If q converges quickly ($q > \overline{q}(c)$ along the sequence), then the wage never exceeds the minimum wage. If q converges slowly ($q < \overline{q}(c)$ along the sequence), then the wage distribution of an employed worker who receives an outside offer converges in probability to the marginal product, since bidding competition is perfect in the limit. When q converges at an intermediate pace, the competition between firms has a medium strength even as $q \rightarrow 1$ (and $c \rightarrow 0$), leaving room for wage dispersion among workers who received an outside offer.

It is also interesting to study the equilibrium wage distribution when all market frictions, i.e. bidding and search frictions become small at the same time. Search frictions become small when either the exogenous probability of separation (δ) goes to zero or the arrival rate of offers (λ) goes to infinity and the cost of job creation (k) goes to zero. In contrast to previous findings, our model allows for wage dispersion even when *all* market frictions are small. This happens when $(q \to 1, c, \delta, k \to 0, \lambda \to \infty)$ in such a way that $q \in (\underline{q}(c, \delta, k, \lambda), \overline{q}(c, \delta, k, \lambda))$ along the sequence. If a worker receives an outside offer, then his wage increases, but does not jump to the level of his marginal product even as $c, \delta, k \to 0$ and $\lambda \to \infty$.

The layout of the rest of the paper is as follows: Section 2 describes the model and Section 3 solves for the equilibrium. Section 4 studies the expected wage of the workers, Section 5 analyzes the case of small market frictions and Section 6 concludes. Some of the proofs are in the Appendix.

2 Model

Consider a continuous time model where the mass of workers is normalized to 1 and the mass of vacancies is v. Since each firm has a constant returns to scale technology, the size of each firm is indeterminate and the number of vacancies is pinned down by aggregate considerations. Firms are free to create new vacancies at a flow cost of k implying that in equilibrium each vacancy has value 0. Each worker, employed or unemployed, finds offers according to a Poisson arrival rate λv and each firm meets a worker according to a Poisson arrival rate of $\lambda > 0$.[3] Each employment relationship breaks up exogenously according to a Poisson arrival rate $\delta > 0$.

Making an offer to a worker (both employed and unemployed) costs $c > 0$ for a firm. This bidding cost is in addition to the search costs that the firm has to incur to find potential candidates: one can think about c as the cost of putting together a contract. In most applications it is small even compared to k. However, even when the bidding cost is small it might have an important effect on equilibrium outcomes in certain cases.

When a firm meets an unemployed worker it makes an offer $w_u (\geq w_m)$.[4] If the worker accepts this offer, then he becomes employed with a wage w_u otherwise he stays unemployed. When a firm meets an employed worker it

[3] If the number of matches formed in a unit time length is $m(v) = \lambda v$ and each workers has the same chance of meeting a firm (and a similar condition holds for firms as well), then the above arrival rates readily arise.

[4] Making an offer to an unemployed worker is always optimal under our assumptions.

decides whether to make an offer; if it does, then the wage offered is denoted by $w_E (\geq w_m)$. After such an offer is made the current employer can make a counter offer with probability q at no cost, after which bidding ends and the worker chooses the best offer he has obtained. With probability $(1-q)$ the current employer cannot make a counter offer and therefore, the worker either accepts offer w_E and switches employer or stays with the old employer. When making an offer or a counter offer the current wage and the wage offer of the competing firm are not publicly observable (like in Burdett and Mortensen (1998)), but the employment status of the worker is observable as well as whether an outside offer has arisen.

The flow utility of the outside option for the workers and the firms are normalized to 0 and each match has productivity 1. The flow profit from a contract is $1-w$ for the firm and w for the worker where w is the wage paid. Each agent maximizes his expected discounted utility using discount rate $r > 0$. The following assumption is made, which is necessary and sufficient to bring about a positive level of job creation in equilibrium:

$$\lambda[(1-w_m) - (r+\delta)c] > k(r+\delta). \tag{1}$$

3 Analysis

The formal analysis below shows that there exists a unique symmetric stationary equilibrium for any parameter values. Depending on the parameters the equilibrium takes three different forms: a firm with a vacancy bids for an employed worker for sure, never or employs a mixed strategy in equilibrium. The next sections provide conditions under which each of them applies.

3.1 Equilibrium with sure outside offers

The worker may be in three possible states: unemployed (u), having received only one offer since being unemployed (s_1) and having received multiple offers since being unemployed (s_2). As we will see, in equilibrium an unemployed worker always accepts the wage offered. Therefore, the change in unemployment rate in time is

$$\dot{u} = \delta(1-u^o) - u^o \lambda v^o.$$

In the steady state $\dot{u} = 0$ or

$$u^o = \frac{\delta}{\delta + \lambda v^o}. \tag{2}$$

The same relationship holds between the job creation rate and the unemployment level in the other equilibrium types.

The second state occurs when a worker is employed but has received only one offer since he was unemployed. The law of motion is described as

$$\dot{s_1^o} = u^o \lambda v^o - (\lambda v^o + \delta) s_1^o = 0$$

or $s_1^o = u^o(1 - u^o)$. The probability of meeting multiple firms since being unemployed is $s_2^o = 1 - u^o - s_1^0 = (1 - u^o)^2$.

We describe some features of the equilibrium in the next Lemma:

Lemma 1 *An unemployed worker always receives (and accepts) an offer of* w_m*. A firm meeting an employed worker makes an offer* w *with positive density on* $[w_m, \overline{w}_E]$[5] *according to an atomless distribution. If the current employer can make a counter offer* w_E*, then he chooses on support* $[\underline{w}_E, \overline{w}_E]$ *without atoms, where* $\underline{w}_E > w_m$*.*

Proof. See the Appendix. ■

Similarly to the Burdett and Mortensen model and any first price auction where the type space is discrete the bidders randomize and the support of their strategies are intervals. The novelty is that when making a counter offer the current employer uses only high bids, because his situation is different from that of a competitor who bids for an employed worker, since the worker has obtained an additional offer (from the competing firm) and the current employer has to bid higher to retain the worker. The above Lemma is silent about the behavior of an employer who may make a counter offer and had already offered $w \geq \underline{w}_E$ to his worker. He could draw a new offer from $[\underline{w}_E, \overline{w}_E]$ or keep w, since they are both optimal to him. For the sake of simplicity I assume the latter one.[6]

[5] For simplicity I assume throughout that in case of a tie the worker chooses the offer that arrived later.

[6] This assumption is appealing especially if the incumbent has a very small but positive cost of making a counter offer.

It is optimal for a competitor to make an offer of $w = w_m$ to the employed worker and attract the worker exactly when his current wage is w_m and the current employer cannot make a counter offer. This happens with probability $(1-q)t^o$ where t^o is the probability that the worker receives the minimum wage conditional on being employed, which is true if and only if the worker had only one offer out of unemployment, i.e. with probability

$$t^o = \frac{s_1^o}{1-u^o} = u^o(q).$$

Denote the value of employing the worker at wage w_m as $V_E^o = V_E^o(w_m)$. Then the utility of the competitor is $U_c(q) = (1-q)u^o V_E^o - c$ and in equilibrium

$$U_c(q) \geq 0, \tag{3}$$

because the competitor must find it worthwhile to make the offer.

To check condition (3) we write up the Bellman-equations describing the value function of the firm, where V is the profit from creating a vacancy. Free entry implies

$$0 = rV = -k + \lambda u^o(V_E^o - c) + \lambda(1-u^o)\{(1-q)u^o V_E^o - c\}. \tag{4}$$

Also,

$$rV_E^o = 1 - w_m + \delta(0 - V_E^o) + \lambda v^o(1-q)(0 - V_E^o) + \lambda v^o q(V_E^o(\overline{w}) - V_E^o). \tag{5}$$

In the last equation I used the fact that if a current employer with offer w_m cannot make a counter offer, then it always loses the worker when a new offer arises, since a new offer is greater than w_m with probability 1. When the firm can make a counter offer then it is optimal for him to jump to the highest wage offered in equilibrium, $\overline{w}_E$ and always keep the worker.

Next we calculate the value of $V_E^o(\overline{w}_E)$. When a competing firm makes an offer to an employed worker he is indifferent between making an offer w_m and $\overline{w}$. With an offer w_m he can hire the worker with probability $(1-q)u^o$, while with an offer $\overline{w}$ the firm can always hire the worker. Therefore,

$$V_E^o(\overline{w}_E) = u^o(1-q)V_E^o. \tag{6}$$

Using (5) and (6) yields

$$V_E^o = \frac{1 - w_m}{r + \delta + \lambda v^o - \lambda v^o q(1-q)u^o}. \quad (7)$$

From (4) it follows that

$$V_E^o = \frac{k + \lambda c}{\lambda u^o + \lambda(1-u^o)u^o(1-q)}. \quad (8)$$

Using the last two equations and that $u^o = \frac{\delta}{\delta + \lambda v^o}$ one obtains an equation with one unknown only:

$$[k+\lambda c][ru^o + \delta - (1-u^o)u^o \delta q(1-q)] = u^o[\lambda u^o + \lambda(1-u^o)u^o(1-q)](1-w_m). \quad (9)$$

Equation (9) has a solution on the $(0,1)$ interval, because at $u^o = 0$ the left hand side is greater than the right hand side, while at $u^o = 1$ the right hand side is greater under assumption (1). Using similar considerations one can show that (9) has a solution that is negative and one that is greater than 1. Since this is a third degree polynomial it follows that there is a unique solution such that $u^o \in [0,1]$.

Using the solution of (9) one can compute v^o and V_E^o after further substitutions. Finally after using (8), (3) becomes

$$u^o(1-q) \geq 1 - \frac{k(1-q)}{\lambda c}. \quad (10)$$

The Lemma below shows that this condition is equivalent to $q \leq \underline{q}(< 1)$ and that if $q \leq \underline{q}$ holds then u^o is increasing in q.

Lemma 2 *There exists a threshold $\underline{q}(< 1)$ such that an equilibrium with outside offers exists if and only if $q \leq \underline{q}$. If $q \leq \underline{q}$, then $\frac{\partial u^o}{\partial q} > 0$.*

Proof. See the Appendix. ■

The result that the unemployment rate is increasing in the level of ex post competition (q) is not surprising, since a higher level of q makes it less profitable to create a job. This result implies that the workers are not necessarily better off if the level of ex post competition increases. An analysis concerning the welfare of workers is provided in Section 4.

Finally, we study conditions under which $\underline{q} \geq 0$ holds, implying that for some values of q an equilibrium with (sure) outside offers exists:

Lemma 3 *There exists a* $\underline{c}$, *such that if* $c < \underline{c}$, *then* $\underline{q} > 0$. *There exist* $\underline{k}$, $\overline{\lambda}$ *such that if* $k \in (0, \underline{k})$ *or* $\lambda \geq \overline{\lambda}$, *then* $\underline{q} < 0$ *and thus an equilibrium with (sure) outside offers does not exist for any* q.

Proof. See the Appendix. ∎

If k is close to 0 or λ is very large, then the level of job creation is so high that the workers are employed and their wage is close to 1 almost always. Therefore, making an offer to employed workers is not profitable if there is a high cost of doing that. On the other hand, when c is close to 0 making such an offer is obviously profitable.

3.2 Equilibrium with no outside offers

We start with a result that applies in equilibrium:

Lemma 4 *An unemployed worker always receives (and accepts) an offer of* w_m.

Proof. Because the minimum wage constraint is binding the worker is better off accepting such an offer than rejecting it. Therefore, the only reason to offer a wage higher than that is to reduce turnover. But since an employed worker does not obtain an offer in this type of equilibrium, this consideration does not play a role and the result follows immediately. ∎

Then the wage of an employed worker is always w_m in equilibrium implying that

$$0 = rV = -k + \lambda u^n (V_E^n - c) \tag{11}$$

and

$$rV_E^n = 1 - w_m + \delta(V - V_E^n).$$

Then it follows that

$$V_E^n = \frac{1 - w_m}{r + \delta}. \tag{12}$$

To check whether not making an offer to an employed worker is optimal one needs to analyze what would happen after such an offer is made. In this case we must specify the out of equilibrium belief the current employer has about this offer. We assume that the current employer thinks that the competing firm has a low cost of making an offer and this belief is common knowledge. Using this assumption the arising equilibrium after such a deviation is such that the competitor mixes on $[w_m, \overline{w}]$ and the current employer mixes on $[\underline{w}_E, \overline{w}]$.[7] Then conditional on making an offer, it is optimal for the competitor to bid w_m and win if and only if the current employer cannot make a counter offer, which is with probability $1-q$.[8] Then making an offer of w_m yields a profit of

$$V_E^n(1-q) - c = \frac{(1-q)(1-w_m)}{r+\delta} - c.$$

So, the condition for having an equilibrium with no outside offers is

$$\frac{(1-q)(1-w_m)}{r+\delta} - c \leq 0, \tag{13}$$

or $q \geq \overline{q}$ where $\overline{q} \in (0,1)$ under assumption (1).

In order to obtain a positive level of job creation one needs $u^n < 1$ and thus (11) implies that

$$\frac{k}{\lambda(V_E^n - c)} < 1$$

or after substitution

$$\lambda > \frac{k(r+\delta)}{(1-w_m) - (r+\delta)c},$$

which holds by assumption (1).

Let me show next that the above two types of equilibria cannot coexist for any parameter values. To show this we compare (13) with the condition for an equilibrium with on the job wage offers, which is (3). First, note that by (7)

$$V_E^o < V_E^n = \frac{1-w_m}{r+\delta}.$$

Then

$$u^o(1-q)V_E^o - c \leq (1-q)V_E^o - c < (1-q)V_E^n - c.$$

Thus

$$(1-q)V_E^n - c \leq 0 \Longrightarrow u^o(1-q)V_E^o - c < 0,$$

[7] Since the cost of bidding does not depend on what bid is placed, we are in effect back to the results of Section 3.1 where we constructed an equilibrium with on the job offers. The main bite of the assumption on the out of equilibrium beliefs is that the current employer does not believe that the competitor has a much higher productivity than 1. (If it is believed that the competitor has only a slightly higher productivity than 1, then it is still optimal for the competitor to place a bid equal to w_m with positive density.)

[8] First, the the current wage is w_m in equilibrium, so such an offer from the competitor is sufficient to hire the worker away when there is no counter offer. Second, if the current employer can make a counter offer, then he makes one that is greater than w_m with probability 1.

which implies that if there is an equilibrium with no outside offers, then there is no equilibrium with outside offers and thus it is impossible that the two equilibria coexist for the same parameter values.

It is easy to show that for some parameter values neither of the above equilibria exists. In that case only a mixed strategy equilibrium may exist where a competitor is indifferent between making or not making an offer to an employed worker and he randomizes in equilibrium. The next Section analyzes this case formally.

3.3 Equilibrium with randomized bidding

Let us start with a useful Lemma:

Lemma 5 *An unemployed worker always receives (and accepts) an offer of* w_m*. A firm meeting an employed worker makes an offer* w *with positive density on* $[w_m, \overline{w}_E]$ *according to an atomless distribution. If the current employer can make a counter offer* w_E*, then he chooses on support* $[\underline{w}_E, \overline{w}_E]$ *without atoms, where* $\underline{w}_E > w_m$*.*

The proof follows the proof of Lemma 1 and is thus omitted. Let $p \in [0,1]$ denote the probability that a firm makes an offer to an already employed worker and let V_E^m be the value of the (optimal) strategy that offers w_m to an unemployed worker. The appropriate Bellman equations are written as follows:

$$0 = rV = -k + \lambda u^m (V_E^m - c) \tag{14}$$

and

$$rV_E^m = 1 - w_m + [\delta + \lambda v^m p(1-q)](0 - V_E^m) + \lambda v^m pq(V_E^m(\overline{w}_E) - V_E^m). \tag{15}$$

In the first equation we used the fact that making an offer to an employed worker yields zero expected profit, while in the second that making a counteroffer $\overline{w}_E$ is optimal.

The next result follows from the fact that a competing firm is indifferent between making and not making an offer to an employed worker:

Lemma 6 *The following results hold in equilibrium:*

$$\frac{(1-q)\delta}{\delta+\lambda v^m p}V_E^m = c. \tag{16}$$

and

$$V_E^m(\overline{w}_E) = \frac{1-\overline{w}_E}{r+\delta} = c. \tag{17}$$

Proof. See the Appendix. ■

Then (15), (16) and (17) imply that

$$(\delta+\lambda v^m p)(r+\delta+\lambda v^m p) - \frac{\delta(1-q)[(1-w_m)+\lambda v^m pqc]}{c} = 0, \tag{18}$$

which can be solved for $v^m p$ and then $u^m = \frac{\delta}{\delta+\lambda v^m}$, V_E^m, v^m and p can all be calculated using (14) and (16).

Corollary 7 *For all q such that a randomized offer equilibrium exists it holds that*

$$\frac{\partial(v^m p)}{\partial q} < 0. \tag{19}$$

Proof. See the Appendix. ■

After substituting into (15) the previous Lemma implies that $\frac{\partial V_E^m}{\partial q} > 0$ and then (14) implies that

$$\frac{\partial u^m}{\partial q} < 0.$$

Then by construction

$$\frac{\partial v^m}{\partial q} > 0$$

and then using (19) it follows that $\frac{\partial p}{\partial q} < 0$. If q increases, then the rate of job creation (v^m) goes up, because there is less bidding for employed workers and thus job creation is more profitable. But $\lambda v^m p$ decreases, which ensures that the competition for employed workers does not increase as q goes up.

3.4 Characterization of the different types of equilibrium

Assumption (1) implies that $\overline{q} > 0$. It is also obvious that $\underline{q} < \overline{q}$, since at $q = \underline{q}$ it holds that $p = 1$, while at $q = \overline{q}$ it holds that $p = 0$. Then on interval $(\underline{q}, \overline{q})$ only a randomized bidding equilibrium exists. It is clear that for $q > \overline{q}$ only a no outside offer equilibrium can exist. Similarly to the argument at the end of Section 3.2 it follows that for $q < \underline{q}$ an equilibrium with randomized bidding cannot exist. Then putting together these results one obtains the following result:

Corollary 8 *There is a unique symmetric stationary equilibrium:there exists* $\underline{q}, \overline{q}$*, such that* $\underline{q} < \overline{q} < 1$ *and*

i) an equilibrium with sure outside offers exists when $q \leq \underline{q}$ *and if* $\underline{q} \geq 0$ *then* $\frac{\partial u}{\partial q} > 0$ *for all* $q \leq \underline{q}$*,*

ii) a mixed strategy equilibrium exists when $q \in (\underline{q}, \overline{q})$ *and* $\frac{\partial u}{\partial q} < 0$, $\frac{\partial p}{\partial q} < 0$,

iii) and a no outside offer equilibrium exists when $q \geq \overline{q}$ *and* u *is constant throughout.*

4 Expected wages

4.1 The role of outside offers

In this Section we indicate some characteristics of the expected income of the workers in the steady state. If $q \geq \overline{q}$, then in equilibrium no outside offers are made and each worker receives wage w_m whenever he is employed. Then the expected income in the steady state (assigning a zero wage to unemployed workers) is

$$Ew^n = (1 - u^n)w_m.$$

In this case if the minimum wage is such that $w_m \leq 0$, then the minimum wage constraint is not binding, since a worker would not accept a negative wage knowing that he cannot obtain a positive one in the future. When $w_m = 0$ it follows that

$$Ew^n = 0,$$

implying that a very high level of ex post competition hurts the worker if no minimum wage requirement is present. In equilibrium it is not only job creation that is needed to drive the wages above the minimum wage, but also outside offers. When $q < \overline{q}$ then the employed workers obtain outside offers and they earn a wage above w_m, whenever they have had multiple offers since unemployment. Therefore, the workers are better off when q is lower, at least when the minimum wage is close enough to 0. When w_m is higher the comparison between the cases of $q < \overline{q}$ and $q \geq \overline{q}$ is ambiguous, but one can show that if the level of frictions is low (λ is high or k is low), then the workers are better off if $q < \overline{q}$.[9]

4.2 The role of job creation

To study the role of job creation we focus on a simple case to analyze the combined effect of employer competition and a minimum wage regulation. We assume that there is no cost of bidding, $c = 0$ and compare the cases when $q = 0$ and $q = 1$. The firms are indifferent between making and not making an offer to an employed worker when $q = 1$, but they strictly prefer making the offer when $q < 1$. To abstract from the issue of whether bidding for employed workers occurs we assume that even when $q = 1$ the offer is made for sure.[10]

Since there is more job creation when the level of ex post competition is lower, $v^0 > v^1$, a result that follows from Lemma 2, it is not clear whether workers are better off with low or high level of ex-post competition: stronger ex-post competition ($q = 1$) has a positive direct effect on wages, but it also leads to a lower level job creation. Let w^* be the threshold level where all job creation activity stops, i.e. let $\lambda(1 - w^*) = k(r + \delta)$ and consider the following proposition:

Proposition 9 *There exists a threshold $\underline{w} < w^*$ such that if $w_m \in (\underline{w}, w^*)$, then $Ew^0 > Ew^1$. On the other hand, if r is small, then there exists a threshold $\overline{w}$ such that for all $w_m \leq \overline{w}$ it holds that $Ew^1 > Ew^0$.*

Proof. See the Appendix. ■

The intuition is the following: if the level of competition ($q = 1$) is high, then a high minimum wage ($w_m > \underline{w}$) depresses job creation so much, that the workers receive low expected wages in equilibrium. When the minimum wage is low the direct effect of stronger employer competition ($q = 1$ $vs.q = 0$) is decisive when comparing expected wage levels.

[9] The calculations are available from the author.

[10] In effect, we approximate the case of $q < 1$ and $c = 0$, by making the assumption that when $c = 0$ and $q = 1$ on the job offers are always made.

5 The case of small market frictions

Perhaps in most application the interesting case is when the market frictions are very small. First, we show the following result:

Proposition 10 *When c becomes arbitrarily small for any fixed $q < 1$ there is sure bidding for employed workers in equilibrium:*

$$\lim_{c \to 0} \underline{q} = \lim_{c \to 0} \overline{q} = 1.$$

Proof. Since $\overline{q} > \underline{q}$, it is sufficient to show that $\lim_{c \to 0} \underline{q} = 1$. Let $V_E(q)$ denote the value of employing a worker at wage w_m if the level of ex-post competition is q. By definition the incentive constraint (3) holds as an equality for $q = \underline{q}$ and thus

$$(1 - \underline{q})u(\underline{q})V_E(\underline{q}) = c. \tag{20}$$

Now, it is easy to show that $\lim_{c \to 0} u(\underline{q}), \lim_{c \to 0} V_E(\underline{q}) > 0$, which implies that $\lim_{c \to 0} \underline{q} = 1$. ■

This result is not very surprising, since as the bidding costs vanish the bidders have the incentive to bid for employed workers. The next result considers the case of strong ex-post competition together with the case of small bidding costs:

Theorem 11 *The level of job creation is such that*

$$\lim_{c \to 0} v(\overline{q}) - v(\underline{q}) > 0$$

and thus

$$\lim_{c \to 0} \frac{v(\overline{q}) - v(\underline{q})}{\overline{q} - \underline{q}} = \infty.$$

Proof. The first result is equivalent to $\lim_{c \to 0} u(\overline{q}) - u(\underline{q}) < 0$. We have already shown that for all c it holds that

$$u(\overline{q}) - u(\underline{q}) < 0$$

thus it follows that

$$\lim_{c \to 0} u(\overline{q}) - u(\underline{q}) \leq 0.$$

There, we only need to rule out that $\lim_{c\to 0} u(\overline{q}) - u(\underline{q}) = 0$. Using equations (11) and (14) one obtains that

$$\lim_{c\to 0} u(\overline{q}) - u(\underline{q}) = 0 \Longleftrightarrow \lim_{c\to 0} V_E(\overline{q}) - V_E(\underline{q}) = 0.$$

Note, that

$$V_E(\overline{q}) = V_E^n = \frac{1-w_m}{r+\delta}.$$

By (15), (17) and $p(\underline{q}) = 1$

$$V_E(\underline{q}) = \frac{(1-w_m) + \lambda v \underline{q} c}{r+\delta+\lambda v(\underline{q})}. \tag{21}$$

Then

$$\lim_{c\to 0} V_E(\overline{q}) - V_E(\underline{q}) = 0 \Leftrightarrow \frac{1-w_m}{r+\delta} = \lim_{c\to 0} \frac{(1-w_m)}{r+\delta+\lambda v(\underline{q})},$$

which does not hold because $\lim_{c\to 0} v(\underline{q}) > 0$ under assumption (1) as we have shown already. The second result follows from Proposition 10. ■

The above result highlights the non-robustness of equilibrium when both frictions vanish at the same time. If the friction that arises from costly bidding is negligible *relative to* the friction arising from the fact that the current employer might not be able to make a counter offer, then employed workers receive outside offers and their wage is above w_m if they obtained multiple offers since being unemployed. However, if the opposite is the case and the market becomes very competitive before bidding costs vanish, then employed workers never receive offers and their wage is always w_m, thus the competition is effectively eliminated. Even if frictions are small, it is not clear which is the more relevant case in a specific labor market and thus the *structure* of market frictions becomes crucial in the limit.

The structure of bidding frictions in the limiting case (i.e. when $c \to 0$ and $q \to 1$) influences the equilibrium wage distribution as well. For any c and q an unemployed worker obtains a wage of w_m only. If $q \geq \overline{q}(c)$ then outside offers do not arise and the wage of employed workers is also w_m; thus no equilibrium wage dispersion arises. If $c \to 0$ and $q \leq \underline{q}(c)$ for all c, then outside offers are always made. Moreover, if $q \to 1$ holds then the wage distribution of employed workers converges to their (common) marginal product in distribution. In this case the only form of wage dispersion in the limit is that workers with only one offer since being unemployed earn the minimum wage, while workers with multiple offers earn their level of productivity.

Wage dispersion arises in the limit (i.e. when $c \to 0$ and $q \to 1$) only when it holds along the sequence that $q \in (\underline{q}(c), \overline{q}(c))$. In this case the wage of those workers who obtained multiple offers is distributed on an interval just like in the model of Burdett-Mortensen (1998). The key is that outside offers are not always made but they are made sometimes. Consequently, the competition is not cutthroat (in which case workers with multiple offers would be paid their marginal product), but it is not entirely ineffective either (in which case workers are kept at the minimum wage level).

We have analyzed only the case when the bidding frictions were very small ($c \to 0$ and $q \to 1$), but it is interesting to know whether one can achieve wage dispersion when not only bidding frictions, but also the search frictions vanish ($k, \delta \to 0$ and $\lambda \to \infty$). The equilibrium analysis presented in Section 3 does not change as we let k, δ and λ converge. Note, that for all parameter values such that $c > 0$ it holds that $\underline{q} < \overline{q}$. As the search frictions vanish ($k, \delta \to 0$ and $\lambda \to \infty$) the unemployment rate converges to 0 and thus it is sufficient to concentrate on the wage distribution of employed workers. For $q = \overline{q}$ the employed workers always earn just the minimum wage, while if $q = \underline{q}$ and $c, k, \delta \to 0$ and $\lambda \to \infty$ then the employed workers earn their marginal product almost surely. If $q \in (\underline{q}, \overline{q})$, then the analysis of Section 3 implies that the wages are distributed on interval $[w_m, \overline{w}(q)]$, where $\overline{w}(q) > w_m$ for all $q < \overline{q}$ and $\lim_{c,k,\delta \to 0, \lambda \to \infty} \overline{w}(\underline{q}) = 1$. Note that

$$\lim_{c \to 0} \overline{q} = 1,$$

and if $\frac{\lambda c}{k} \to 0$ then

$$\lim \underline{q} = 1,$$

and thus a choice such that $q \in (\underline{q}, \overline{q})$ entails $q \to 1$.[11] This implies that when all frictions disappear ($c, k, \delta \to 0, \lambda \to \infty, q \to 1$) with $\frac{\lambda c}{k} \to 0$ and at the same time $q \in (\underline{q}(c, k, \delta, \lambda), \overline{q}(c, k, \delta, \lambda))$ is chosen appropriately then there is wage dispersion in the economy even in the frictionless limit.

[11] Suppose that $\frac{\lambda c}{k} \to 0$ and $\lim \underline{q} < 1$ holds. Since $q = \underline{q}$ if and only if (10) holds as an equation, it follows that the right hand side of that equation would converge to $-\infty$ and the two sides could not be equal.

Let us contrast this result with the Burdett and Mortensen (1998) model with observed employment status. In that model as frictions vanish ($\lambda \to \infty$ or $\delta \to 0$) the workers earn their marginal product almost surely in the limit.[12] The key is that in the BM model in a frictionless economy workers on average have received infinitely many offers already, so their wage must be high and any successful offer must be close the marginal product. In our model bidding costs prevent the workers from obtaining infinitely many outside offers (if $q > \underline{q}$), which makes it possible for firms to compete with offers less than the marginal product even in the frictionless limit.

6 Conclusion

This paper considers a search theoretic model where bidding costs and ex-post competition is introduced. Assuming that perfect competition takes place in an environment with homogenous workers and firms is a more restrictive assumption than it seems. Even if market frictions are small, job creation, wage levels and social welfare depend crucially on the *structure* of those frictions: if the cost of bidding is small, but large *relative to* the level of ex-post competition, then an employed worker never receives additional offers, which eliminates employer competition and holds the wage at the minimum wage level. In contrast to previous findings, this model allows for wage dispersion even when *all* frictions (i.e. both bidding and search frictions) converge to zero simultaneously. If q is in the intermediate range, then outside offers are made with a probability strictly between 0 and 1 and thus the competition between firms has a medium strength even as $q \to 1$ making room for wage dispersion. The paper also shows that increasing the level of ex-post competition may hurt workers by reducing job creation and bidding for employed workers. Even if job creation is high but there are few outside offers workers cannot earn much more than the minimum wage, thus competition for the *employed* workers is crucial to labor market outcomes.

7 Appendix

Proof of Lemma 1:

[12] Indeed, this is the case no matter whether the employment status is observed or not.

Proof. First, it follows from standard arguments that the support of the offers cannot have gaps, i.e. they form intervals. Second, the upper bound of the supports must be the same, since it is not profitable to propose more than what is necessary for winning. Suppose that $\underline{w}_E = w_m$ held. With such a counteroffer losing is guaranteed, because the outside offer w is greater than w_m with probability 1, since it was drawn from an atomless distribution. Therefore it cannot be optimal to propose such an offer and $\underline{w}_E > w_m$ holds.

Because the minimum wage constraint is binding the worker is better off accepting such an offer than rejecting it. Therefore, the only reason to offer a wage higher than that is to reduce turnover. We show that in equilibrium this concern is not sufficient to justify a wage $w \in (w_m, \overline{w}]$. For simplicity we only treat the case where $w \leq \underline{w}_E$, but a similar argument can be made for higher wage levels. Let $V_E^o(w)$ be the value of the firm from employing a worker at such a wage and let $G_E^o(w)$ be the steady state wage distribution of the (employed) workers. If a competing firm offers a wage $w \in [w_m, \underline{w}_E]$ he wins if the current wage is less than w *and* the current employer cannot make a counter offer. This happens with probability $G_E^o(w)(1-q)$. Since all such offers are optimal for a competing firm it holds that for all $w \in [w_m, \underline{w}_E]$

$$K_E = V_E^o(w) G_E^o(w)(1-q).$$

Since for all $w > w_m$ it holds that $G_E^o(w) > G_E^o(w_m)$ the last formula implies that $V_E^o(w) < V_E^o(w_m)$. But note that a firm that makes an offer w to an unemployed worker obtains him for sure and so offering wage w_m is more profitable when facing an unemployed worker. ■

Proof of Lemma 2:

Proof. Let us calculate this derivative using the implicit function theorem applied to (9):

$$\frac{\partial u^o}{\partial q} = \frac{\lambda(1-u)u \, \mathrm{V}_E^o}{\lambda \, \mathrm{V}_E^o(1-(1-q)u) + \lambda(1-u)(1-q)(\, \mathrm{V}_E^o + u^o \frac{\partial V_E^o}{\partial u^o}) + \lambda u^o \frac{\partial V_E^o}{\partial u^o}}.$$

Therefore, $\frac{\partial u^o}{\partial q} > 0$ holds if $\frac{\partial V_E^o}{\partial u^o} > 0$ holds. By formula (7) $\frac{\partial V_E^o}{\partial u^o}$ has the same sign as $-\frac{\partial v^o(1-q(1-q)u^o)}{\partial u^o}$ and

$$\frac{\partial v^o(1-q(1-q)u^o)}{\partial u^o} = -q(1-q)v^o + \frac{\partial v^o}{\partial u^o}(1-q(1-q)u^o) < 0.$$

Therefore $\frac{\partial V_E^o}{\partial u^o} > 0$ and $\frac{\partial u^o}{\partial q} > 0$ in the relevant region.

To prove the first claim, note that if $\frac{\partial u^o}{\partial q} < \frac{1}{(1-q)^2}$, then (10) is satisfied for if and only if q is small enough. Since $\frac{\partial V_E^o}{\partial u^o} > 0$ it follows that

$$\frac{\partial u^o}{\partial q} < \frac{\lambda(1-u)u\,\mathrm{V}_E^o}{\lambda\,\mathrm{V}_E^o(1-(1-q)u)+\lambda(1-u)(1-q)\,\mathrm{V}_E^o} = \frac{(1-u)u}{qu+(1-u)(2-q)}.$$

It holds that

$$\frac{(1-u)u}{qu+(1-u)(2-q)} < \frac{u}{2-q} < \frac{1}{(1-q)^2},$$

which concludes the proof. ■

Proof of Lemma 3:

Proof. Rewriting condition (10) yields

$$u^o \geq \frac{1}{1-q} - \frac{k}{\lambda c}. \tag{22}$$

If k is small enough, or λ is large enough then for all $q > 0$

$$\frac{1}{1-q} - \frac{k}{\lambda c} > 1$$

and thus $u^o > 1$ would need to hold, which is impossible. If c is small enough then

$$\frac{1}{1-q} - \frac{k}{\lambda c} < 0$$

and thus $u^o \geq 0$ needs to hold, which is obviously true. ■

Proof of Lemma 6:

Proof. If a firm is making an offer to an employed worker, then in equilibrium it is optimal to make an offer with the minimum wage. That offer is accepted by the worker if and only if the current firm cannot make a counter offer *and* the worker had only one offer out of unemployment, i.e. he is in state 1. What is the probability of an *employed* worker being in state 1 in a steady state equilibrium? First, a similar argument as in Section 3.1 implies that

$$u^m = \frac{\delta}{\delta + \lambda v^m}.$$

The probability of state 1 can be calculated by writing up the law of motion:

$$\dot{s_1} = -s_1(\delta + \lambda v^m p) + u^m \lambda v^m = 0.$$

Then

$$s_1 = \frac{u^m \lambda v^m}{\delta + \lambda v^m p} = \frac{\frac{\delta}{\delta + \lambda v^m} \lambda v^m}{\delta + \lambda v^m p} = \frac{\delta(1 - u^m)}{\delta + \lambda v^m p}.$$

Then the probability that an employed worker accepts an offer with the minimum wage is

$$(1-q)\Pr(s_1 \mid \text{being employed}) = (1-q)\frac{s_1}{1-u^m} = \frac{(1-q)\delta}{\delta + \lambda v^m p}.$$

Then the expected profit from making such and offer is $\frac{(1-q)\delta}{\delta+\lambda v^m p} V_E^m - c$ and the fact that such an offer yields a zero expected profit implies the first claim. Also, the expected profit from making an outside offer $\overline{w}_E$ is c, which implies the second result. ■

Proof of Corollary 7:

Proof. Equation (18) implies via the implicit function theorem that

$$\frac{\partial(\lambda v^m p)}{\partial q} = -\frac{\delta \frac{1-w_m}{c} - \lambda v^m p \delta c(1-2q)}{r + 2\delta + 2\lambda v^m p}. \tag{23}$$

Therefore, we need to show

$$\lambda v^m p(1-2q) < \frac{1-w_m}{c},$$

for which it is sufficient to prove that $\lambda v^m p < \frac{1-w_m}{c}$, which follows from using (18). To see this note that if $\lambda v^m p = 0$ then $q < \overline{q}$ implies that

$$A = (\delta + \lambda v^m p)(r + \delta + \lambda v^m p) - \frac{\delta(1-q)[(1-w_m) + \lambda v^m pqc]}{c} < 0.$$

Also, note that $\lambda v^m p = \frac{1-w_m}{c}$ implies that $A > 0$, so equation (18) has a unique positive solution and the root is indeed such that $\lambda v^m p < \frac{1-w_m}{c}$. ■

Proof of Proposition 9:

Proof. Since the outside offers are made for sure one can use the approach of Section 3.1. When $q = j$ denote the endogenous variables by placing a superscript j on them. Then (9) implies

$$(\delta + \lambda v^1)(r + \delta + \lambda v^1) = \frac{(1-w_m)\lambda\delta}{k} \tag{24}$$

and

$$k = \frac{(1-w_m)(\lambda\delta^2 + 2\lambda^2\delta v^0)}{(r + \delta + \lambda v^0)(\delta + \lambda v^0)^2}. \tag{25}$$

■

When $q = 1$ the expected income is the wage in the three different states weighted by the probability of the three states:

$$Ew^1 = u^1 * 0 + s_1^1 w_m + (1 - u^1 - s_1^1). \qquad (26)$$

In Lemma 12 we derive the expected income of a worker for the case when $q = 0$ (see formula (32)). If $w_m = w^*$, then $v^0 = v^1 = 0$ and therefore, $Ew^0 = Ew^1 = 0$. Then it is sufficient to show that decreasing w_m slightly has a higher effect on Ew^0 than on Ew^1, which would imply the first result. To show this result we first notice that for $i = 0, 1$

$$\frac{dEw^i}{dw_m} = \frac{\partial Ew^i}{\partial w_m} + \frac{\partial Ew^i}{\partial v^i}\frac{\partial v^i}{\partial w_m}.$$

Then (26) and (32) imply that[13]

$$\frac{\partial Ew^0}{\partial w_m} \mid_{w_m = w^*} = \frac{\partial Ew^1}{\partial w_m} \mid_{w_m = w^*} = 0,$$

because at such a high minimum wage the worker is always unemployed and thus $s_1^1 = s_1^0 = s_2^1 = s_2^0 = 0$. After some algebra and using the formulas for the expected welfare of the worker it follows that

$$\frac{\partial Ew^0}{\partial v^0} \mid_{v^0 = 0} = \frac{\partial Ew^1}{\partial v^1} \mid_{v^1 = 0} = \frac{w^*}{\delta}.$$

Then to show that

$$\frac{dEw^0}{dw_m} \mid_{w_m = w^*} < \frac{dEw^1}{dw_m} \mid_{w_m = w^*},$$

it is sufficient to show that

$$\frac{\partial v^0}{\partial w_m} \mid_{w_m = w^*} < \frac{\partial v^1}{\partial w_m} \mid_{w_m = w^*}$$

or that if w_m is close enough to w^*, then $v^0 > v^1$. This follows from the previous proposition, which concludes the proof of the first result.

Proof. To prove the second result let w_m^j $(j = 0, 1)$ be the greatest number such that if $w_m \leq w_m^j$ then the minimum wage constraint is not binding when $q = j$. If $w_m \leq \min(w_m^0, w_m^1)$ then one can solve the model assuming $r = 0$ and obtain

[13] In all formulas below we use the left hand derivatives at $w_m = w^*$.

$v^0 = v^1 = \frac{1}{k} - \frac{\delta}{\lambda}$.[14] One needs to compare expressions $(1-u^0)[G_E(w_m)w_m + \int_{w_m}^{\overline{w}_E} wG'_E(w)dw]$ and $s_1^1 w_m + (1-u^1-s_1^1)$ to rank Ew^0 and Ew^1. Since $v^0 = v^1$ it follows that $u^0 = u^1 = u$, $1-u^1-s_1^1 = (1-u)^2$ and thus it is sufficient to prove that $(1-u)^2 + u(1-u)w_m > (1-u)[uw_m + \int_{w_m}^{\overline{w}_E} wG'_E(w)dw]$[15] to obtain that $Ew^1 > Ew^0$. This simplifies to

$$(1-u) > \int_{w_m}^{\overline{w}_E} wG'_E(w)dw.$$

But

$$\int_{w_m}^{\overline{w}_E} wG'_E(w)dw < (1-u)\overline{w}_E,$$

because

$$\int_{w_m}^{\overline{w}_E} G'_E(w)dw = (1-u).$$

After using that $\overline{w}_E < 1$ one can conclude the result for the case when $r = 0$ and the case when r is small follows from continuity arguments. ■

Lemma 12 *The expected income when $q = 0$ can be written as*

$$Ew^0 = (1-u^0)[\overline{w}_E - \int_{w_m}^{\overline{w}_E} \frac{\delta}{\sqrt{\frac{1-w}{1-w_m}(\delta+\lambda v^0)(r+\delta+\lambda v^0) + \frac{r^2}{4}} - \frac{r}{2}} dw].$$

Proof. Let $F(w)$ denote the offer distribution made to an employed worker. It can be shown that F is continuous, strictly increasing and $F(w_m) = 0$. Let $G_E(w)$ denote the steady state distribution of the wage of an employed worker and let $T_E(w)$ denote the the probability that a given worker is employed and earns less than w. Then $T_E(w) = (1-u^0)G_E(w)$. The law of motion is

$$\dot{T}_E(w) = u^0\lambda v^0 - T_E(\delta + \lambda v^0(1-F)) = 0$$

or

$$T_E(w) = \frac{\lambda u^0 v^0}{\delta + \lambda v^0(1-F(w))}$$

and

$$G_E(w) = \frac{\delta}{\delta + \lambda v^0(1-F(w))}. \tag{27}$$

[14] The wage w_m^j is such that a worker is indifferent between acceping wage w_m^j or staying unemployed. The details of the calculations are available from the author.

[15] Here we used the fact that $G_e(w_m) = u$, which follows from (27) and $F_e(w_m) = 0$.

Let $V_E^0(w)$ be the value of employing a worker at wage w. Then

$$rV_E^0(w) = 1 - w + (\delta + \lambda v^0(1 - F(w))(0 - V_E^0(w))$$

or

$$V_E^0(w) = \frac{1 - w}{r + \delta + \lambda v^0(1 - F(w))}. \tag{28}$$

Since all wage offers to employed workers are equally profitable on interval $[w_m, \overline{w}_e]$ it follows that for all $w \in [w_m, \overline{w}_e]$

$$\widetilde{K}_E = V_E^0(w)G_E(w).$$

Therefore, for all $w \in [w_m, \overline{w}_e]$

$$\frac{1 - w}{r + \delta + \lambda v^0(1 - F(w)} \frac{\delta}{\delta + \lambda v^0(1 - F(w))} = \frac{1 - w_m}{r + \delta + \lambda v^0} \frac{\delta}{\delta + \lambda v^0} \tag{29}$$

and thus

$$\frac{1 - \overline{w}_E}{r + \delta} = \frac{1 - w_m}{r + \delta + \lambda v^0} \frac{\delta}{\delta + \lambda v^0}. \tag{30}$$

From these formulas, $\overline{w}_E$ can be calculated as well as F_e expressed as a function of w.

Finally, we calculate the average wage of a worker in the steady state, which is

$$\begin{aligned} Ew^0 = u^0 * 0 + (1 - u^0) \int_{w_m}^{\overline{w}_E} w dG_E(w) = \\ (1 - u^0)[G_E(w_m)w_m + \int_{w_m}^{\overline{w}_E} wG'_E(w)dw] = \\ = (1 - u^0)[\overline{w}_E - \int_{w_m}^{\overline{w}_E} G_E(w)dw]. \end{aligned} \tag{31}$$

Also, from (27)

$$\int_{w_m}^{\overline{w}_E} G_E(w)dw = \int_{w_m}^{\overline{w}_E} \frac{\delta}{\delta + \lambda v^0(1 - F(w))} dw$$

Therefore equation (29) implies that

$$\int_{w_m}^{\overline{w}_E} G_E(w)dw = \int_{w_m}^{\overline{w}_E} \frac{\delta}{\sqrt{\frac{1-w}{1-w_m}(\delta + \lambda v^0)(r + \delta + \lambda v^0) + \frac{r^2}{4}} - \frac{r}{2}} dw.$$

Therefore,

$$Ew^0 = (1 - u^0)[\overline{w}_E - \int_{w_m}^{\overline{w}_E} \frac{\delta}{\sqrt{\frac{1-w}{1-w_m}(\delta + \lambda v^0)(r + \delta + \lambda v^0) + \frac{r^2}{4}} - \frac{r}{2}} dw]. \tag{32}$$

■

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