Common Agency with Moral Hazard and Asymmetrically Informed Principals*

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Abstract

In this paper, we analyze the equilibrium incentive schemes offered to an agent by two principals who can only observe correlated noisy signals of the one-dimensional action taken by the agent. We look at both cases when the two principals can or cannot cooperate in setting the terms of their incentive schemes. We show that minimizing the risk imposed on the agent may result in negative incentives being attached to the signal with the higher variation. We also find that under some conditions, the equilibrium effort level is a non-monotonic function of the correlation coefficient of the two signals. When comparing the power of the incentive schemes offered by the two principals, we show that the principal with the higher valuation of the agent's effort or the one observing a signal with smaller variance offers more powerful incentives to the agent. Finally, we give an example of overprovision of effort in the equilibrium with non-cooperating principals compared to the case of cooperating principals. This comes at the price of higher risk and welfare in former case is lower.

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1 Introduction

In this chapter we analyze a common agency game in which a risk-averse agent with constant absolute risk-aversion takes a one-dimensional action (chooses an effort level) that benefits two principals. Each principal only observes a noisy signal of the effort level chosen by the agent. The random components of these signals are drawn from different distributions and are correlated.

Many real-life examples fit this framework. An employee taking an economic decision may be responsible both to the marketing director and the financial director of the firm. Both of these directors are interested in the employee working hard, but they may observe different signals of his or her performance. Or, the effort level chosen by a property sales agent benefits both the seller and the potential buyers. However, the seller and the potential buyer may obtain different information about how hard the property sales agent works. A manager of a firm has to make both the owners and the creditors of the firm happy. The harder he or she works, the larger the benefit accrued by the other two.

A final example can be a retailer (agent) selling sport shoes on the behalf of two sport brand manufacturers (principals). Clearly, the harder he or she works, the larger the expected benefit of the two manufacturers. However, the two manufacturers cannot observe the effort exerted by the retailer, they can only observe the number of shoes sold as external shocks prevent a one-to-one mapping from the retailer's effort to the number of shoes sold. If the external shock is a fluctuation in the aggregate demand for sport shoes, the error terms in the signals observed by the two manufacturers will be positively correlated, as the number of shoes sold will tend to move together for any effort level of the agent. However, if the external shock is a fluctuation in the tastes for different brands, taking the aggregate demand fixed, an increase in the number of shoes sold from one brand will cannibalize the numbers of shoes sold from the other brand. In this case the error terms in the signals observed by the two manufacturers will be negatively correlated, as the number of shoes sold from the two brands move in opposite direction at fixed aggregate demand.

Some might argue that in these examples the activity of the agent can be decomposed into multiple tasks and that applying a multitasking approach would be more appropriate. We think that in many real life examples the truth lies somewhere in between these two extreme cases. It is true that some units of the effort exerted by the agent can be allocated to one task or another in a straightforward manner. However, other components of the agent's effort benefit both tasks. For example, in the retailing example above, longer opening hours, a cleaner shop or an overall friendly attitude towards the visitors would be hard to decompose into tasks benefiting particular principals exclusively.

Rather than exploring related multitasking issues, in this chapter we return to the fundamentals of common agency models with moral hazard and analyze how the structure of the principals' information set affects the equilibrium outcome of the common agency game. In particular, we are interested in how the correlation between the principals' signals about the agent's behaviour affects the optimal incentive schemes offered to the agent. To keep the analysis simple, we look at the case where the agent has a constant absolute risk-aversion utility function and focus on linear contracts in the spirit of Holmstrom and Milgrom (1987).

We analyze both cases when the principals can or cannot cooperate in setting the terms of their incentive schemes. Note that the case of cooperating principals amounts to the standard principal-agent problem with moral hazard, the only difference being that the principal can condition its optimal incentive scheme on two signals.

The second-best equilibrium outcome for the case with cooperating principals provides the following insights. First, when the two signals are strongly positively correlated, the principals can decrease the risk imposed on the agent by attaching negative incentives to the signal with the higher variance and offering incentives to work hard through the signal with the smaller variance. In all other cases, the incentives attached to both signals are positive.

Second, the second-best effort level is a non-monotonic, U-shaped function of the correlation coefficient of the two signals. In particular, for low values of the correlation coefficient, when the incentives attached to the two signals are both positive, an increase in the correlation coefficient increases the risk imposed on the agent, which in turn can be corrected by implementing a lower effort level in equilibrium. When on the contrary, the incentives attached to the signal with the higher variance are negative, which happens for large values of the correlation coefficient, an increase in the correlation coefficient decreases the risk imposed on the agent, which allows for a higher effort level to be implemented in equilibrium.

Third, balancing efficiency and optimal risk sharing between the principals and the agent leads to underprovision of effort in the second-best equilibrium compared to the first-best. The only exception are the cases when the signals are perfectly correlated and have different variances in case of positive correlation as in which case first-best can be implemented.

The analysis of the case when the principals do not cooperate offers further interesting insights. First, in a similar way to the second-best case, the principals' intention to minimize the agent's compensation for the risk incurred can lead to negative incentives in equilibrium.

Second, whenever the principals observe signals with the same variance, the one with the higher valuation of the agent's effort provides him with stronger incentives. When the principals enjoy the same benefit from the agent's effort, the one observing a signal with the lower variance will offer the agent stronger incentives. In the more general case, these two forces can be combined. In particular, the principal observing a signal with a slightly higher variance than the other principal, but having a much higher valuation for the agent's effort will provide him with stronger incentives.

Third, the slope of the optimal incentive scheme and the equilibrium effort level are a non-monotonic, U-shaped functions of the correlation of the signals whenever one of the principals is sufficiently superior compared to the other principal with respect to a combination of having higher valuation of the agent's effort and observing a signal with smaller variance. In all other cases, unlike in the case of cooperating principals, the relationship is monotonically decreasing.

Finally, we show that there is not always underprovision of effort in the third-best equilibrium compared to the second-best. In particular, with perfectly negatively correlated signals, the third-best effort level may exceed the second-best effort level, albeit at the cost of imposing higher risk on the agent. The aggregate welfare is lower in the third-best than in the second-best equilibrium.

1.1 Literature Review

This chapter belongs to the literature on common agency games in which the principals cannot observe the action taken by the agent. Bernheim and Whinston (1986b) were the first to analyze this class of games and show that no efficient equilibrium exists if the agent is risk averse. This is a similar result to the one in the standard one principal - one agent case. However, in the case of many principals, their lack of coordination results in additional losses of efficiency in equilibrium. The same authors show (see Bernheim and Whinston (1986a)) that the lack of coordination alone would not lead to inefficiencies under complete information.

The information structure in Bernheim and Whinston (1986b) is different from ours. In particular, in their model, each principal observes each element from a set of possible outcomes with some probability and the action chosen by the agent affects these probabilities. The conflict between principals arises from the differences in their sets of possible outcomes. In terms of our model, the principals in Bernheim and Whinston (1986b) observe uncorrelated signals of the agent's behaviour.

For a framework with correlated signals, one has to refer to the common agency literature with multitasking. Holmstrom and Milgrom (1988) formulate a model in which the agent has to perform two tasks that can be technologically connected and each task only benefits one of two principals. The agent's behaviour cannot be perfectly observed with only a signal available for the effort level chosen for each particular task. The error terms in these signals can be correlated. The authors perform welfare analysis in two different scenarios. In the first scenario, with disjoint observations, each principal can only contract on the signal related to the task she benefits from. In the second scenario, with joint observations, the two principals observe and can condition their contracts on both signals.

Dixit (1996) extends the model corresponding to the second scenario in Holmstrom and Milgrom (1988) to an arbitrary number of principals. In particular, by assuming that increasing the effort in one task causes substitution away from other tasks, he finds that in the non-cooperative equilibrium there is a loss of efficiency compared to the cooperative case. The reason for this is that the principals set negative incentives for the other principals' tasks in order to make the agents to exert more effort in the task they benefit from. In equilibrium, this causes a leakage of each principal's money to the other principals, weakening each principals incentives to offer the agent a powerful incentive scheme. The author also shows that if principals are only allowed to condition their incentive schemes on the signal related to their own task, the arising equilibrium incentive schemes are more powerful than in the unrestricted case where principals will compete with each other in providing incentives to the agent to work for them. In the limit, where the different components of the agent's effort become perfect substitutes, the resulting aggregate incentive scheme reproduces the first-best.

Note that even though the signals in these models are correlated, the authors concentrate more on the technological link between the tasks than on the correlation between the signals. Our focus is different as we abandon the multitask representation and rather analyze on informational externalities that arise in the simpler case of one task.

The framework of common agency with moral hazard has wideranging practical applications. Tirole (2003) looks at whether and when countries borrow too much or too little in the aggregate in a setting in which the government makes a policy choice that affects the wellbeing of domestic entrepreneurs and foreign investors. Bizer and De Marzo (1992) and Bisin and Guaitoli (2004) study externalities among contracts when agents borrow from competing financial intermediaries. Calzolari and Pavan (2005) examine the exchange of information between two sellers who contract sequentially with the same buyer. Finally, Tirole (1994) explores the potential of common agency with moral hazard in analyzing and designing efficient governmental institutions.

The rest of the chapter proceeds as follows. We introduce our model and briefly present the first-best outcome in Section 2. We analyze the equilibrium of the game with cooperating and non-cooperating principals in Section 3 and 4. Finally, Section 5 concludes. All the proofs are relegated to the Appendix.

2 The Model

An agent has the task to take a one-dimensional action (choose an effort level) on behalf of two principals. He receives payment w_i from principal i and it costs him $\frac{k}{2}\mu^2$ $(k > 0)$ to exert effort μ.[1] The agent is risk averse with constant absolute risk-aversion parameter $r > 0$ and his utility function can be written

[1] In the rest of the paper, the pronoun "she" is used in reference to the principals and pronoun "he" is used in reference for the agent.

as

$$U = 1 - e^{-r\left(w_1 + w_2 - \frac{k}{2}\mu^2\right)}. \tag{1}$$

We set the reservation utility of the agent to zero.

Each principal can only observe a noisy signal of the effort level chosen by the agent. In particular, if the agent exerts effort μ, principal i ($i = 1, 2$) observes signal $x_i = \mu + \varepsilon_i$, where ε_i is a normally distributed error term with mean 0. The covariance matrix of these two error terms can be written as

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{2}$$

where ρ is the correlation coefficient of the error terms. In the rest of the chapter, the term correlation refers to the correlation between the error terms ε_1 and ε_2.

Each principal offers a wage schedule to the agent conditional on the signal observed. In particular, principal i offers wage schedule $w_i(x_i)$ to the agent. The principals are risk neutral and the payoff of principal i can be written as

$$v^i = b_i x_i - w_i(x_i) \ , \quad i = 1, 2 \tag{3}$$

where $b_i > 0$.

As the agent has a CARA utility function that is additively separable in money and effort and the signals are drawn from a normal distribution, we can follow the tradition of Holmstrom and Milgrom (1987) and look at linear contracts of the form $w_i(x_i) = \alpha_i x_i + \beta_i$, or equivalently, $w_i(x_i) = \alpha_i(\mu + \varepsilon_i) + \beta_i$.[2] In this case the expected utility of the agent has the following form

$$U = e^{-r\left(\alpha_1(\mu+\varepsilon_1)+\beta_1+\alpha_2(\mu+\varepsilon_2)+\beta_2-\frac{k}{2}\mu^2\right)} \tag{4}$$

Observe that the agent receives an uncertain wage for any choice of effort. It is without loss of generality to look at the certainty equivalent of the agent upon choosing a given level of effort rather than work with the uncertain wage stream.[3] Technically, it is equal to payment Q such that $1 - e^{-rQ} = 1 -$

[2] Homstrom and Milgrom (1987) analyze a dynamic model in which the principal contracts repeatedly with a risk-averse agent with CARA utility function. They show that the optimal dynamic incentive scheme can be computed as if the agent were chosing the mean of a normal distribution only once and the principal were restricted to offering a linear contract. They show that in that setting the optimal contract offered to the agent is linear in the signal observed by the principal.

[3] By definition, the certainty equivalent is the certain payment, which makes the agent indifferent between it and the gamble offering the random payoff in the exponent of the RHS of equation (1).

$Ee^{-r\left(\alpha_1(\mu+\varepsilon_1)+\beta_1+\alpha_2(\mu+\varepsilon_2)+\beta_2-\frac{k}{2}\mu^2\right)}$, which implies that his certainty equivalent upon choosing action μ can be written as

$$CE = \alpha_1\mu + \alpha_2\mu + \beta - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \tag{5}$$

where $\beta = \beta_1 + \beta_2$. This is a convenient shortcut as β_1 and β_2 are not uniquely determined in equilibrium and the last term is the risk premium required by the agent for the uncertainty of his payment stream. It can be shown that this risk premium is always non-negative.

The interaction between the principals and the agent can be modelled as a two-stage game. In the first stage, the principals simultaneously offer a wage schedule to the agent, while in the second stage, the agent chooses an effort level taking the wage schedules offered by the principals as given.

To better understand our results in the subsequent sections, let us briefly review the first-best case of complete information and cooperating principals. In this case the principals' joint maximization problem can be written as

$$\max_{\mu, w_1(\mu), w_2(\mu)} \{[b_1\mu - \alpha_1\mu] + [b_2\mu - \alpha_2\mu]\}$$
$$s.t. \;\; \alpha_1\mu + \alpha_2\mu + \beta - \frac{k}{2}\mu^2 \geq 0 \tag{6}$$

By solving this optimization problem, one can derive that the first-best effort level is equal to

$$\mu^{FB} = \frac{b_1 + b_2}{k} \tag{7}$$

First-best welfare can be obtained by substituting this formula into the aggregate welfare function which can be written as

$$W = b_1\mu + b_2\mu - \frac{k}{2}\mu^2 \tag{8}$$

First-best social welfare is therefore equal to

$$W^{FB} = \frac{(b_1 + b_2)^2}{2k} \tag{9}$$

Note that individual incentive schemes are undetermined and the joint incentive scheme has the slope $\alpha^{FB} = \alpha_1^{FB} + \alpha_2^{FB} = b_1 + b_2$.

We now introduce asymmetric information into the model and derive the equilibrium of the game under the assumption of cooperating and non-cooperating principals.

3 Cooperating Principals

In this section we analyze the case when principals cooperate in designing their incentive schemes which means that they act as one principal

The joint optimization problem of the two principals, leading to the second-best equilibrium outcome, can be written as

$$\begin{aligned}
&\max_{\alpha_1,\alpha_2,\beta} \{[b_1\mu - \alpha_1\mu] + [b_2\mu - \alpha_2\mu] - \beta\} \\
&s.t.\ \ \mu = \arg\max_{\widetilde{\mu}} \{\alpha_1\widetilde{\mu} + \alpha_2\widetilde{\mu} + \beta - \frac{k}{2}\widetilde{\mu}^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2)\} \\
&\qquad \alpha_1\mu + \alpha_2\mu + \beta - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \geq 0
\end{aligned} \tag{10}$$

The solution to this optimization problem is given by the following theorem.

Theorem 1 *With cooperating principals, the common agency game has the following equilibrium outcome:*

(i) When the two signals are perfectly positively correlated ($\rho = 1$) and have the same variances ($\sigma_1 = \sigma_2 = \sigma$), then there is in fact only one signal. The incentive scheme is linked to this one signal and has the slope

$$\alpha^{SB} = \alpha_1^{SB} + \alpha_2^{SB} = \frac{b_1 + b_2}{1 + rk\sigma^2} \tag{11}$$

(ii) In all other cases, the incentive scheme can be linked to two signals and has the slopes

$$\alpha_i^{SB} = \frac{(b_1 + b_2)(\sigma_j^2 - \rho\sigma_1\sigma_2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2 + rk(1-\rho^2)\sigma_1^2\sigma_2^2} \quad . \tag{12}$$

The associated second-best equilibrium effort level can be written as

$$\mu^{SB} = \frac{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2 + rk(1-\rho^2)\sigma_1^2\sigma_2^2} \frac{b_1 + b_2}{k} \tag{13}$$

Proof. We do not offer a formal proof here. We only present an outline of it to better understand how the model works.

Note that the principals' joint optimization problem in (10) is in fact a standard representation of any principal-agent problem with moral hazard, where the first constraint is the incentive compatibility constraint of the agent, and the second constraint is his participation constraint. By solving the optimization problem of the agent, his optimal choice of effort is a function of the parameters of the incentive schemes offered by the two principals and can be written as

$$\mu = \frac{\alpha_1 + \alpha_2}{k} \tag{14}$$

This shows that only the sum of α_1 and α_2 matters for the agent's choice of effort.

By looking at the program in (10), it can be seen that the principals can extract all the surplus of the agent by setting β at the appropriate level. However, there is no specific rule for how they share the extracted surplus between themselves. As it does not affect the incentives provided to the agent, we are not interested in further details of this issue.

By eliminating the participation constraint of the agent, the principals' optimization problem can be rewritten as

$$\begin{aligned} &\max_{\alpha_1,\alpha_2} \left\{ (b_1 + b_2)\mu - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \right\} \\ &s.t. \qquad \mu = \frac{\alpha_1 + \alpha_2}{k} \end{aligned} \tag{15}$$

After substituting the agent's optimal effort choice into the principals' objective function, the optimization problem in (15) can be rewritten as

$$\max_{\alpha_1,\alpha_2} \left\{ (b_1 + b_2)\frac{\alpha_1 + \alpha_2}{k} - \frac{1}{2}\frac{(\alpha_1 + \alpha_2)^2}{k} - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \right\} \tag{16}$$

The first term in this expression stands for the joint benefit of the two principals from the agent choosing the level of effort as in (14), the second term is the cost of the agent associated to this effort level, while the third term is the risk-premium required by him for the uncertainty in his payments.

The first order conditions associated to this optimization problem with respect to α_i $(i = 1, 2)$ can be written as

$$\frac{b_1 + b_2}{k} - \frac{\alpha_1 + \alpha_2}{k} - r\alpha_i\sigma_i^2 - r\rho\alpha_j\sigma_1\sigma_2 = 0 \qquad i = 1, 2; \qquad i \neq j \tag{17}$$

Observe that the sum of the first two terms of these equations are the same. Therefore, the sum of third and fourth terms must also be the same. So we have

$$\sigma_1(\sigma_1 - \rho\sigma_2)\alpha_1 = \sigma_2(\sigma_2 - \rho\sigma_1)\alpha_2 \tag{18}$$

One has to distinguish between two cases when looking at this equation. In the first case, when $\sigma_1 - \rho\sigma_2 = \sigma_2 - \rho\sigma_1 = 0$, which holds for $\sigma_1 = \sigma_2$ and $\rho = 1$, the two sides of equation (18) are zero and α_1 and α_2 are undetermined. In this case the two signals are identical, therefore the optimal incentive scheme is linked to one signal only and its slope α is determined by equation (17).

In all the other cases, equation (18) can be used to determine the ratio of α_1 and α_2, which is equal to

$$\frac{\alpha_1^{SB}}{\alpha_2^{SB}} = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - \rho\sigma_1\sigma_2} \tag{19}$$

This condition, together with (14), provides the following insights. First, the agent's optimal choice of effort only depends on the sum of α_1 and α_2. Second, the relative magnitude of α_1 and α_2 are set as in (19) to minimize the risk-premium to be paid to the agent for any effort level given by (14). This suggests that the optimization process of the principals can be decomposed into two steps. In the first step, the principals use the rule in (19) to determine the ratio of α_1 and α_2 that minimizes the risk-premium required by the agent for any given effort level in (14), and second, they choose the optimal level of effort, taking into account the associated minimum risk-premium.

The equilibrium levels of α_1^{SB}, α_2^{SB} and μ^{SB} in Theorem 1 can be obtained by solving the system of equations in (17). ■

A close examination of the results in Theorem 1 offers some interesting insights. Our first corollary compares the slopes of the optimal incentive scheme and determines their sign.

Corollary 1 *The slopes α_i^{SB} $(i = 1, 2)$ of the second-best equilibrium incentive scheme offered by the two principals have the following features:*

(i) $\alpha_j^{SB} > \alpha_i^{SB}$ whenever $\sigma_j < \sigma_i$;

(ii) $\alpha_i^{SB} < 0$ whenever $\frac{\sigma_j}{\sigma i} < \rho \leq 1$.

Proof. By simple algebra. ■

The intuition behind these results is the following. Assume that the principals link the same payment to both signals. As it can be seen from (5), the payment linked to the signal with the higher variance imposes a higher risk on the agent. In this case, the principals can decrease the agent's risk and still implement the same effort level by decreasing the payment linked to the signal with the higher variance by one unit and increase the payment linked to the signal with the smaller variance by the same unit. Some more adjustments may follow until the agent's risk is minimized. This happens when the ratio of the payments related to the two signals becomes equal to the expression in (19). As a result, a higher payment is linked to the signal with the lower variance.[4]

Following the same logic, it can be seen that the larger the difference in the variances of the two signals the larger the difference in the incentives linked to the two signals. According to point (ii) of Corollary 1, it is possible that for large values of the correlation between the two signals, the principals attach negative incentives to the signal with the larger variance in order to balance the strong incentives linked to the signal with the lower variance. In this case, incentives to exert effort are provided through the payments linked to the signal with the smaller variance, while the payments linked to the signal with the larger variance, going from the agent to the principals, are used to hedge the agent's risk. The agent accepts this type of incentive scheme because she is willing to give up some payments in exchange for lower risk.

The results in Theorem 1 can also be used for welfare analysis. The aggregate welfare under asymmetric information and linear contracts can be written as

$$W = b_1\mu + b_2\mu - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \tag{20}$$

Note that the first best welfare level can be obtained by maximizing the sum of the first three terms of this function with respect to μ. Since the risk premium required by the agent is always non-negative, a necessary condition for the first-best welfare to be implemented is that the risk-premium required by the agent is equal to zero. The following lemma identifies the necessary and sufficient conditions for this.

Lemma 1 *The risk premium required by the agent is zero if and only if one of the following two conditions hold:*

[4] Note that linking incentives only to the signal with the lower variance is not optimal either, as the agent's risk can be decreased by linking one unit of payment to the signal with the higher variance and decreasing the payment linked to the signal with the lower variance by the same unit.

(i) $\rho = -1$ *and*

$$\alpha_1\sigma_1 = \alpha_2\sigma_2 \tag{21}$$

(ii) $\rho = +1$ *and*

$$\alpha_1\sigma_1 = -\alpha_2\sigma_2 \tag{22}$$

Proof. See Appendix. ■

In these two cases the signals observed by the principals are perfectly correlated and the contracts are set in such a way that they hedge all the risk imposed on the agent. Note that the risk imposed on the agent can only be completely reduced when the signals are perfectly correlated. The reason for this is that with perfectly correlated signals, unless $\rho = +1$ and $\sigma_1 = \sigma_2$ at the same time, the principals can perfectly infer the effort level chosen by the agent and are able to fully eliminate the agent's uncertainty. This is not the case when the signals are not perfectly correlated as in that case some uncertainty regarding the agent's effort choice always persists.

The following corollary of Theorem 1 compares aggregate welfare in the first-best and second-best cases.

Corollary 2 *Unless* $\rho = -1$ *or* $\rho = 1$ *and* $\sigma_1 \neq \sigma_2$*, when first-best is implemented in equilibrium, there is underprovision of effort and lower aggregate welfare in the second-best equilibrium compared to the first-best.*

Proof. By simple algebra. ■

The results in Corollary 2 can be easier understood by the following argument. If $\rho = -1$, equality (19) simplifies to condition (21). This proves that the agent's risk drops to zero and he does not require any risk premium. To show that the first-best effort level can be implemented in equilibrium, note that if $\rho = -1$, for any $x_1 = \mu + \varepsilon$, we have $x_2 = \mu + a\varepsilon$, where $a = -\sigma_2/\sigma_1$. Clearly, in this case $\mu = \frac{ax_1 - x_2}{a-1}$ can be determined exactly and therefore, the principals implement the first-best effort level in equilibrium. With the first-best effort level implemented in equilibrium and no risk premium required by the agent, the first-best total welfare is achieved. The case of $\rho = 1$ can be discussed along similar lines, the only difference being that the variances of the two signals observed by the two principals must be different, otherwise we are in the special case of Theorem 1.

The economic intuition behind this result is that when the signals are perfectly correlated, unless $\rho = +1$ and $\sigma_1 = \sigma_2$, the principals can infer exactly the effort level chosen by the agent and, therefore, can design the incentive scheme in such a way, that the agent's risk is reduced to zero. Note, that with no risk borne by the agent, the first-best effort level can be implemented.

In all the other cases, the two signals are not sufficient to perfectly identify the effort level chosen by the agent and, therefore, his risk cannot be reduced to zero. With positive risk on the agent, the principals have to give up productive efficiency in order to move towards efficient risk sharing with the agent. As a result, aggregate second-best welfare is lower than first-best welfare.

Theorem 1 can also be used to derive the effects of a change in the correlation coefficient ρ on the second-best equilibrium effort level. The following proposition summarizes our findings.

Proposition 1 *The second-best equilibrium effort level chosen by the agent is a non-monotonic function of the correlation coefficient ρ. The second-best equilibrium effort level μ^{SB} is a decreasing function of ρ if and only if $\rho < \min\left\{\frac{\sigma_1}{\sigma_2}, \frac{\sigma_2}{\sigma_1}\right\}$.*

Proof. We only present a short outline of the proof here (for formal proof see Appendix) in order to offer some support for our results. We first have to refer to the two-step optimization technique of the principals described above, according to which they first use equation (19) to determine the ratio of the parameters α_1 and α_2 that minimizes the agent's risk-premium for any given effort level, and second, they determine the optimal effort level to be chosen by the agent.

Assume $\sigma_1 < \sigma_2$ and look first at the case when $-1 < \rho < \frac{\sigma_1}{\sigma_2}$. It can be seen from (12) that $\alpha_2^{SB} > 0$ in this case. Take the value of the correlation coefficient to be equal to $\rho = \rho_0$. The second-best equilibrium variables of the model are given by (12) and (13) with the value of ρ set at ρ_0. Let us now consider a change in ρ, from the value of ρ_0 to ρ_1 ($\rho_1 < \frac{\sigma_1}{\sigma_2}$). By using the Envelope Theorem it can be shown that an increase in the value of ρ increases the minimum risk-premium associated with the effort level $\mu_0^{SB} = \mu^{SB}|_{\rho=\rho_0}$. In this way, the original balance between efficiency and risk-sharing in the relationship between the principals and the agent is no longer optimal as too much risk is borne by the agent. This imbalance can be corrected by making the agent exert less effort, which means that in the new equilibrium we have $\mu_1^{SB} = \mu^{SB}|_{\rho=\rho_1} < \mu_0^{SB}$. This is exactly the negative relationship between μ^{SB} and ρ stated in Proposition 1. The intuition behind the case when $\frac{\sigma_1}{\sigma_2} < \rho < 1$ is identical, except that α_2^{SB} is negative in this case and an increase in ρ decreases the minimum risk-premium required by the agent, and therefore, the optimal balance between efficiency and risk sharing is restored by a higher effort level. ■

To understand the economic intuition behind Proposition 1, consider the case of strong negative correlation. The incentives attached to both signals are positive in this case as stated by Corollary 1. The principals know , that even though the signals contain certain errors, these errors tend to balance each other and the incentives attached to the two signals only impose a small risk on the agent. Because of opposite type of errors, the agent can use the payments received from the two principals to hedge his risk. Clearly, the stronger the negative correlation, the lower the risk and the higher the effort level that can be implemented in equilibrium. In the extreme case of perfect negative correlation, the risk imposed on the agent can be reduced to zero, which allows for the highest effort level to be implemented (see Corollary 2).

When the correlation between the two signals is increased from the large negative values, the errors in the two signals balance each other to a lesser extent, increasing the agent's risk. The increased risk imposed on the agent can be handled by implementing a lower effort level in equilibrium. This negative relationship between the correlation of the signals and the equilibrium effort level persists as long as the principals attach positive incentives to both signals, i.e. as long as $\rho < \min\left\{\frac{\sigma_1}{\sigma_2}, \frac{\sigma_2}{\sigma_1}\right\}$.

When the correlation between the signals is so high (and positive) that the principals attach negative incentives to the signal with the higher variance, i.e. when $\rho > \min\left\{\frac{\sigma_1}{\sigma_2}, \frac{\sigma_2}{\sigma_1}\right\}$, an increase in the correlation means that the errors in the two signals are cumulative. However, the closer the correlation coefficient to $+1$, the greater the chance to hedge the risk through negative incentives linked to the signal with the higher variance. However, the possibility of hedging the risk imposed on the agent decreases, and therefore, a higher effort level can be implemented in equilibrium. In fact, the higher the correlation between the signals, the larger fraction of the risk imposed on the agent can be hedged by attaching negative incentives to the signal with the higher variance, and the higher the effort level that can be implemented in equilibrium. In the extreme case of perfect positive correlation, the risk imposed on the agent can be reduced to zero, which allows for the highest effort level to be implemented (see Corollary 2). This positive relationship between the correlation of the signals and the equilibrium effort level persists as long as the principals attach incentives of different signs to the two signals, i.e. as long as $\rho > \min\left\{\frac{\sigma_1}{\sigma_2}, \frac{\sigma_2}{\sigma_1}\right\}$.

This result completes our analysis of the case when principals are allowed to cooperatively set the terms of their incentive schemes.

4 Non-Cooperating Principals

In this section we relax the assumption of cooperating principals and look at the case where principals cannot cooperate in providing incentives to the agent. The equilibrium of this common agency game can be defined as follows. An equilibrium is a triplet including the effort level chosen by the agent and the two linear incentive schemes offered by the two principals, such that: (i) the effort level chosen by the agent maximizes his expected utility taking the incentive schemes offered by the two principals as given, and (ii) the incentive scheme provided by each principal offers her the highest expected payoff taking the incentive scheme provided by the other principal and the agent's optimal effort choice rule as given.[5]

To solve for the equilibrium of the game, we solve each principal's optimization problem, taking the incentive scheme provided by the other principal as given. So, for each $i = 1, 2$ we have to solve the following optimization problem:

$$\begin{aligned}
&\max_{\alpha_i, \beta_i} \{(b_i - \alpha_i)\mu - \beta_i\} \\
&s.t. \quad \mu = \arg\max_{\widetilde{\mu}} \left\{ (\alpha_1 + \alpha_2)\widetilde{\mu} + \beta - \frac{k}{2}\widetilde{\mu}^2 - \frac{1}{2} r(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \right\} \\
&\qquad\quad (\alpha_1 + \alpha_2)\mu + \beta - \frac{k}{2}\mu^2 - \frac{1}{2} r(\alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) \geq 0 \qquad (23)
\end{aligned}$$

where again, we used the certainty equivalent representation of the agent's utility and the shortcut β for $\beta_1 + \beta_2$. As before, the first constraint in this optimization

[5] There might be other equilibria when principals do not offer linear contracts. However, Holmstrom and Milgrom (1988) prove that if one principal offers linear contracts, it is optimal for the other principal to offer linear contracts too.

problem is the incentive compatibility constraint of the agent, while the second constraint is his participation constraint. By looking at the program in (23), it can be seen that the principals can extract all the surplus of the agent by setting β at the appropriate level. As before, there is no explicit rule of how to share the extracted surplus between themselves. Clearly, this gives rise to a multiplicity of equilibria with identical qualitative features that differ only in the sharing rule of the surplus between the two principals.[6]

The solution to the optimization problem (23) is given by the following theorem.

Theorem 2 *The slope of the equilibrium incentive scheme offered by principal i is equal to*

$$\alpha_i^{TB} = \frac{b_i - \rho r k b_j \sigma_1 \sigma_2 + r k b_i \sigma_j^2}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2) - \rho^2 r^2 k^2 \sigma_1^2 \sigma_2^2}, \quad i = 1, 2; \quad j \neq i \quad . \tag{24}$$

The effort level chosen by the agent in the equilibrium can be written as

$$\mu^{TB} = \frac{1 - \rho r k \sigma_1 \sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2) - \rho^2 r^2 k^2 \sigma_1^2 \sigma_2^2} \frac{b_1 + b_2}{k} \tag{25}$$

Proof. By solving the maximization problem in (23) using simple algebra. ■

To explore the formulas presented in Theorem 2, let us first simplify the optimization problem in (23) by solving the agent's utility maximization problem and making his participation constraint binding. The simplified optimization problem of principal i can be written as

$$\max_{\alpha_i} \left\{ (b_i + \alpha_j)\mu - \frac{k}{2}\mu^2 - \frac{r}{2}(\alpha_1^2 \sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2) + \beta_2 \right\}$$
$$s.t. \quad \mu = \frac{\alpha_1 + \alpha_2}{k} \tag{26}$$

In particular, it can be seen from this formulation that the interaction between principal j and the agent affects principal i's optimization problem through both the incentive compatibility and the participation constraints of the agent. In particular, a higher α_j increases - through the incentive compatibility constraint of the agent - the marginal and total cost of each particular

[6] The reader interested in some possible characterization of the sharing of the rent extracted from the agent between the two principals should refer to Grosman and Helpman (1994).

unit of effort that principal i can implement by varying α_i. A change in α_j has two effects on principal i's optimization problem through the agent's participation constraint. First, an increase in α_j increases the payment that the agent receives for any given unit of effort, therefore the higher α_j the lower the amount of money that principal i has to pay the agent for any particular unit of μ. This decrease of the unit costs of each unit of effort for principal i is equivalent to an increase in her marginal benefit for any unit of μ.[7] Second, by looking at the optimization problem in (26), it can also be seen that a change in α_j also affects the risk-premium required by the agent for his uncertain payment stream. The sign of this effect is uncertain as the correlation coefficient ρ can take both positive and negative values.

In other words, the interaction between principal j and the agent imposes three externalities on principal i's optimization problem. First, a positive externality arises as it decreases principal i's cost for every unit of effort. Second, there is a negative externality as it increases the cost of implementing additional units of effort by varying α_i, and third, there is an externality of ex-ante unknown sign as it affects the risk-premium required by the agent.

To make the effect of these three externalities more transparent, substitute the agent's optimal effort into the maximand in (26). The associated first order condition can be written as

$$\frac{b_i + \alpha_j}{k} - \frac{\alpha_i + \alpha_j}{k} - r\sigma_i(\alpha_i\sigma_i + \rho\alpha_j\sigma_j) = 0 \qquad i,j = 1,2; \quad i \neq j \tag{27}$$

It can be seen from equation (27) that the first two effects of the presence of principal j cancel out and it is only the third externality that has a real impact.[8] In particular, when α_j takes positive values, a positive correlation increases the marginal cost of varying α_i (negative externality), whereas a negative correlation decreases this marginal cost (positive externality). When, on the contrary, α_j takes negative values, a positive correlation decreases the marginal cost of varying α_i (positive externality) whereas a negative correlation increases it (negative externality).

[7] This formulation reflects the idea of Bernheim and Whinston (1986b) of how to look at one particular principal's optimization problem in a common agency setting: "a principal can always compose his offer in two steps: he first undoes the offers of the other principals, and then decides upon some aggregate offer" (pp. 927 ibid).

[8] Note that the fact that the first two effects cancel out is not a general feature. In fact, it is a consequence of our choices of linear contracts and quadratic cost function.

Understanding these externalities helps us to characterize the equilibrium incentive schemes. Our first corollary looks at the determinants and the sign of the slope of the third-best equilibrium incentive scheme.

Corollary 3 *The slope α_i^{TB} of the third-best equilibrium incentive scheme offered by principal i has the following features:*

(i) it is increasing in b_i;

(ii) it is decreasing in b_j for $\rho > 0$ and increasing in b_j for $\rho < 0$;

(iii) it is negative whenever

$$\frac{b_i}{b_j}\left(\frac{\sigma_j}{\sigma_i} + \frac{1}{rk\sigma_i\sigma_j}\right) < \rho \leq 1 \tag{28}$$

Proof. By simple algebra. ■

The intuition behind this corollary is the following. An increase in b_i increases the marginal benefit of principal i from the effort exerted by the agent, inducing her to implement a higher level of effort by increasing the optimal value α_i^{TB}. Clearly, she has to take into account the effect on the risk-premium required by the agent when increasing the value of α_i^{TB}.

Because of the same reason, an increase in b_j increases the slope α_j^{TB} of the optimal incentive scheme provided by principal j. If $\rho > 0$, this increases the marginal cost of varying α_i for principal i as the risk imposed on the agent increases - the negative externality identified above comes into play. Principal i's optimal answer is to reduce α_i. For $\rho < 0$, an increase in α_j^{TB} has exactly the opposite effect on principal i's optimal choice of α_i.

Condition (28) in Corollary 3 corresponds to the feature identified in case of cooperating principals in point (ii) of Corollary 1. The intuition behind this result is the following. When the correlation between the signals is negative, the two principals, even if they are not cooperating, can provide positive incentives to the agent as the errors in the two signals balance each other to some extent, and the risk imposed on the agent is of moderate concern. However, when the signals are strongly positively correlated, the errors in the two signals amplify each other and the risk imposed on the agent becomes of strong concern. In this case the principal with a low valuation of the agent's effort or with a signal with high variance may be better off by providing negative incentives to the agent.

She can do so, because she expects that the other principal provides the agent with strong incentives and her negative incentives will decrease the agent's risk.

This mechanism works in a similar way as in the case of cooperative principals. However, unlike in the case of cooperative principals, it is possible that none of the principals offers negative incentives to the agent for high positive levels of the correlation coefficient. This is the case for example, when $b_1 = b_2$, $rk = 1$, $\sigma_1 = 1$ and $\sigma_2 = 0.5$.

Our next corollary compares the power of the incentive schemes provided by the two principals.

Corollary 4 *Principal i offers stronger incentives to the agent than principal j, i.e. $\alpha_i^{TB} > \alpha_j^{TB}$, whenever*

$$\frac{b_i}{b_j} > 1 + \frac{\sigma_i^2 - \sigma_j^2}{1 + \rho rk\sigma_1\sigma_2 + \sigma_j^2} \tag{29}$$

This likely to be the case whenever principal i's valuation of the agent's effort is high and the variance of the signal she observes is low. In the special case, when $b_1 = b_2$, $\alpha_i^{TB} > \alpha_j^{TB}$ iff $\sigma_i < \sigma_j$, while for $\sigma_1 = \sigma_2$, $\alpha_i^{TB} > \alpha_j^{TB}$ iff $b_i > b_j$.

Proof. By simple manipulation of equation (24). ■

The intuition behind this corollary can be better understood by looking first at the two special cases. If the principals enjoy the same benefit from each particular choice of effort by the agent, i.e. $b_1 = b_2$, the principal observing a signal with lower variance has to worry less about the risk that she imposes on the agent and can offer him stronger incentives. Following similar logic, if the principals observe signals with the same variance, i.e. $\sigma_1 = \sigma_2$, they worry equally about the risk that they independently impose on the agent. In that case, the principal with the higher valuation of the agent's effort will provide him with stronger incentives.

In more general cases, when principals differ both in their valuations of the effort level chosen by the agent and in the variance of their signal, the two forces identified above work simultaneously. Clearly, when one principal has higher valuation of the agent's effort and observes a signal with smaller variance, the two forces work in the same direction and this principal will offer stronger incentives to the agent than the other principal. The more interesting case is the one in which one of the two principals, say principal i, has smaller valuation of the effort level chosen by the agent but observes a signal with smaller variance. In this case, the two forces identified above work in opposite directions and principal i will provide stronger incentives to the agent than principal j whenever inequality (29) holds. The same reasoning can be applied for the case when one of the principals enjoys a higher benefit from the agent's effort and observes a signal with the larger variance.

This result can be related to the comparison of the power of the incentives schemes connected to the two signals in Corollary 1. However, note that in that case it was only the joint benefit of the principals that mattered in equilibrium, therefore, these valuations did not have to be included in the condition identified in Corollary 1.

Next, we analyze how a change in the correlation coefficient ρ affects the third-best equilibrium values of α_1 and α_2. The following proposition summarizes our results.

Proposition 2 *A change in ρ has the following effect on the equilibrium value of the third-best equilibrium choice α_i^{TB} of principal i:*

(i) for $\rho < 0$, α_i^{TB} is a decreasing function of ρ;

(ii) for $\rho > 0$ and

$$\frac{b_i}{b_j} > \frac{1}{2}\left[\frac{1}{rk\sigma_1\sigma_2} + \frac{\sigma_i}{\sigma_j} + \frac{1}{\frac{1}{rk\sigma_1\sigma_2} + \frac{\sigma_j}{\sigma_i}}\right] \tag{30}$$

α_i^{TB} is an increasing function of ρ if

$$\frac{b_i}{b_j} > \frac{1}{2}\left[\frac{1}{\rho rk\sigma_1\sigma_2} + \frac{1}{\rho}\frac{\sigma_i}{\sigma_j} + \frac{1}{\frac{1}{\rho rk\sigma_1\sigma_2} + \frac{1}{\rho}\frac{\sigma_j}{\sigma_i}}\right] \tag{31}$$

and it is a decreasing function of ρ otherwise;

(iii) if inequality (30) does not hold, α_i^{TB} is a decreasing function of ρ for every $\rho > 0$.

Proof. See Appendix. ■

A change in the correlation coefficient affects the incentive schemes as well as the externalities imposed on each other by the two principals. To better understand the results in Proposition 2, we refer to the first order conditions in

(27), which can be used to derive the optimal choice of α_i as a function of the optimal choice of α_j by principal j as well as the parameters of the model in the following way:

$$\alpha_i = \frac{b_i - \rho r k \sigma_1 \sigma_2 \alpha_j}{1 + r k \sigma_i^2} \tag{32}$$

Observe that this is principal i's best response function for any value of the variable α_j chosen by principal j. In particular, for a positive correlation between signals ($\rho > 0$), an increase in the value of α_j by principal j induces principal i to lower the value of α_i. The reason for this is the following. An increase in α_j increases the risk premium required by the agent because $\rho > 0$. This destroys the optimal balance between efficiency and risk sharing in the relationship between principal i and the agent as the agent now has to bear too much risk. Principal i can restore the optimal balance between efficiency and risk sharing by reducing the value of α_i. In the case of negative correlation between signals, these mechanisms work the other way around, and principal i will increase the value of α_i if principal j increases the value of α_j.

It can be seen from equation (32) that a change in ρ affects the equilibrium value of α_i in two ways. First, there is a direct effect through ρ, which depends on the sign of α_j, and second, there is an indirect strategic effect through α_j, coming from the response of principal j to the increase in ρ. Formally, these two effects can be separated as follows:

$$\frac{\partial \alpha_i}{\partial \rho} = -\frac{r k \sigma_i \sigma_j}{1 + r k \sigma_i^2} \alpha_j - \rho \frac{r k \sigma_i \sigma_j}{1 + r k \sigma_i^2} \frac{\partial \alpha_j}{\partial \rho} \tag{33}$$

where the first term stands for the direct effect, while the second term stands for the strategic effect coming into play through principal j's adjustment of α_j following a change in ρ.

Unfortunately, the term $\partial \alpha_j / \partial \rho$ in the indirect effect is an equilibrium variable itself and therefore it cannot be used to provide intuition for the overall effect of a change in ρ on α_i. Because of this, we, replace $\partial \alpha_j / \partial \rho$ in (33) with the corresponding expression that we have for $\partial \alpha_i / \partial \rho$ in the same equation. So we have

$$\frac{\partial \alpha_i}{\partial \rho} = -\frac{r k \sigma_i \sigma_j}{1 + r k \sigma_i^2} \alpha_j - \rho \frac{r k \sigma_i \sigma_j}{1 + r k \sigma_i^2} \left(-\frac{r k \sigma_i \sigma_j}{1 + r k \sigma_j^2} \alpha_i - \rho \frac{r k \sigma_i \sigma_j}{1 + r k \sigma_j^2} \frac{\partial \alpha_i}{\partial \rho} \right) \tag{34}$$

From this equation $\partial \alpha_i / \partial \rho$ can be determined as a function of α_i and α_j and the parameters of the model alone. In particular, we have

$$\frac{\partial \alpha_i}{\partial \rho} = A\left(-\alpha_j + \rho\frac{rk\sigma_i\sigma_j}{1+rk\sigma_j^2}\alpha_i\right) \tag{35}$$

where A is a function of the parameters of the model and is strictly positive.[9] It can be seen from (35) that for $\rho < 0$, $\partial\alpha_i/\partial\rho < 0$ as we also have $\alpha_i, \alpha_j > 0$ as shown in Corollary 3. If, on the contrary, $\rho > 0$, we have $\partial\alpha_i/\partial\rho > 0$, whenever $B\alpha_i > \alpha_j$, where $B = \rho rk\sigma_i\sigma_j/(1 + rk\sigma_j^2) > 0$. Note that this can never be the case if $\alpha_i < 0$. Therefore, $\alpha_i < 0$ implies $\partial\alpha_i/\partial\rho < 0$. As a result, the only case when α_i is increasing in ρ is when $\rho > 0$, $\alpha_i > 0$ and $\alpha_i/\alpha_j > 1/B$ which reduces to inequality (31) if we substitute in for the equilibrium values of α_i and α_j. This last condition means that the relative power α_i/α_j of the incentive schemes offered by principal i and principal j has to exceed a given threshold B. Based on the intuition behind the results in Corollary 4, this is likely to be the case when principal i's valuation of the agent's effort is high and the variance of the signal she observes is low. Observe that these are exactly the conditions for inequality (31) to hold.

To understand the intuition behind this result, note that in (34) we expressed $\partial\alpha_i/\partial\rho$ as a sum of a direct and an indirect effect, which in turn is also a sum of a direct and an indirect effect. However, this latter indirect effect is identical up to a parameter to the overall effect that we are interested in the first place, i.e. $\partial\alpha_i/\partial\rho$. Therefore, $\partial\alpha_i/\partial\rho$ can be rewritten as an additive function of the two direct effects separately incurred by the two principals when there is a small change in ρ. This argument is represented in equation (35). Note that the second direct effect has an extra coefficient as it comes in through the reoptimization of the other principal following a small change in ρ.

As it can be seen in equation (33), each direct effect is proportional to the slope of the other principal's optimal incentive scheme. The explanation for this is that the direct effects describe the change in the slope of a principal's optimal incentive scheme taking the slope of the other principal's optimal incentive scheme as fixed. This can also be seen from equation (32). For example, if α_i is high, the direct effect of a change in ρ on α_j will also be high in absolute value but with negative sign (follow equation (33) for α_j rather than α_i) and it

[9] A is equal to $1 - \rho^2 \frac{r^2k^2\sigma_i^2\sigma_j^2}{1+rk\sigma_i^2+rk\sigma_j^2+r^2k^2\sigma_i^2\sigma_j^2}$, which is a strictly positive number less than one.

is likely to dominate the strategic effect for α_j, leading to an overall negative value for $\partial\alpha_j/\partial\rho$. However, a negative value for $\partial\alpha_j/\partial\rho$ generates a positive strategic effect for $\partial\alpha_i/\partial\rho$, which can dominate the own direct effect of principal i if α_j is low, leading to a positive overall effect.

By taking into account our results in Corollary 4, which connects the relative magnitude of α_i and α_j with the parameters of the model, our previous discussion suggests that α_i^{TB} is likely to be increasing in ρ whenever b_i/b_j takes high values and σ_i/σ_j takes low values.

Note that the reason for the possible U-shape relationship between the correlation coefficient between the signals and the slopes of the incentive schemes is similar to that in case of cooperating principals. However, in case of non-cooperating principals it might be the case that the slopes of the incentive schemes never turn to become increasing functions of the correlation coefficient.

Clearly, a change in the correlation coefficient ρ affects the equilibrium effort level through its effect on the slope of the equilibrium incentive schemes. The following proposition identifies the effect of a change in ρ on the equilibrium effort level.

Proposition 3 *An increase in ρ has the following effect on the third-best equilibrium level of effort chosen by the agent:*

(i) for $\rho < 0$, μ^{TB} is a decreasing function of ρ;

(ii) for $\rho > 0$ and

$$\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2 > \frac{1}{2}\sigma_1\sigma_2\left[1+\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_1}{\sigma_2}\right)\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_2}{\sigma_1}\right)\right]-\frac{1}{rk} \tag{36}$$

μ^{TB} is a decreasing function of ρ if

$$\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2 > \frac{1}{2}\sigma_1\sigma_2\left[\rho+\frac{1}{\rho}\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_1}{\sigma_2}\right)\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_2}{\sigma_1}\right)\right]-\frac{1}{rk} \tag{37}$$

(iii) if inequality (36) does not hold, μ^{TB} is a decreasing function of ρ for every $\rho > 0$.

Proof. See Appendix. ■

According to (14), μ^{TB} is increasing in ρ whenever $\alpha_1^{TB}+\alpha_2^{TB}$ is increasing in ρ. However, this can only happen when α_1 (assuming $\sigma_1 \leq \sigma_2$) is increasing in ρ and it is increasing so strongly that it more than offsets the negative effect of an increase in ρ on α_2. As the RHS of inequality (36) is symmetric in σ_1 and σ_2, this can only be the case when b_i is larger (and much larger) than b_j for $\sigma_i < \sigma_j$. This is consistent with our findings in Proposition 2, where we required b_i/b_j to be high and σ_i/σ_j to be low. Note, that unlike in the case of cooperating principals, for some sets of the parameters of the model, no "U-turn" happens and the third-best equilibrium effort level is a decreasing function of the correlation coefficient for all values of ρ.

The final step of our analysis compares the second-best and third-best equilibrium effort levels.

Proposition 4 *When the error terms are negatively correlated, it might be the case that the third-best equilibrium effort level chosen by the agent exceeds the second-best equilibrium effort level. For example, this is the case is when $\rho = -1$ and*

$$(\sigma_j - \sigma_i)(b_i\sigma_i - b_j\sigma_j) > 0 \tag{38}$$

Proof. See Appendix. ■

The intuition behind this result is the following. By looking at equation (27), it can be seen that the marginal cost of principal i for varying α_i has two components: one incorporating the costs associated with the agent's cost function to exert effort and another one associated with the risk imposed on agent for the uncertainty of his payment stream. Observe that for $\rho = -1$ and $\alpha_i\sigma_i < \alpha_j\sigma_j$ the marginal cost component associated to the risk imposed on the agent is actually negative. This can happen when the positive externality imposed by the incentives provided by principal j is larger than principal i's own (when there is no other principal) marginal cost associated with the agent's risk. Inequality (38) identifies the condition for this to be the case. As a result, in the optimum, the marginal cost component $\frac{\alpha_i+\alpha_j}{k}$ of principal i associated to the effort function of the agent will be larger than her marginal benefit $\frac{b_i+\alpha_j}{k}$ from that effort level, which implies $\alpha_i^{TB} > b_i$. Since $\alpha_i\sigma_i < \alpha_j\sigma_j$ is a necessary condition for this to happen, it is not possible for $\alpha_i^{TB} > b_i$ and $\alpha_j^{TB} > b_j$ to happen at the same time as that would also require $\alpha_i\sigma_i > \alpha_j\sigma_j$. Therefore, if $\alpha_i^{TB} > b_i$ than α_j^{TB} has to be lower than b_j. However, Proposition 4 proves that whenever inequality (38) holds, the effect $\alpha_i^{TB} > b_i$ dominates the effect $\alpha_j^{TB} < b_j$ and as a result, we have $\alpha_i^{TB} + \alpha_j^{TB} > b_i + b_j$. Since $\alpha_i^{SB} + \alpha_j^{SB} = b_i + b_j$ by Corollary 2 if $\rho = -1$, we have that it is possible to have $\alpha_i^{TB} + \alpha_j^{TB} > \alpha_i^{SB} + \alpha_j^{SB}$, or equivalently $\mu^{TB} > \mu^{SB}$. Clearly, in this case the agent bears inefficiently little risk and therefore total welfare is lower in the third-best than in the second-best case.

Note that according to Corollary 2, first and second-best effort and welfare are the same for $\rho = -1$. If, in addition $\sigma_1 = \sigma_2$, in which case the inequality in (38) holds with equality, the second and third-best effort levels are also the same, which means that the third-best and first-best effort levels are identical too. Finally, if we have $b_1 = b_2$ in addition, the first-best welfare level can be implemented in the third-best equilibrium.

5 Conclusion

In this chapter we studied a common agency framework with moral hazard and an agent with constant absolute risk-aversion and quadratic cost function of effort. These assumptions allowed us to concentrate on linear contracts. We analyzed the equilibrium outcome in the two cases when the principals can or cannot cooperate in setting the terms of their incentive schemes.

In our analysis, we found that it is possible that one of the principals offers negative incentives to the agent when the signal observed by her has a large variance and the signals are strongly positively correlated. We also identified conditions for a certain principal to provide stronger incentives to the agent than the other principal.

By investigating the effects of a change in the correlation coefficient between the signals, we obtained that the relationship between the correlation of the signals and the slopes of the equilibrium incentive schemes and output is not necessarily monotonic. In particular, both the slopes of the incentive schemes and the effort level are decreasing functions of the correlation coefficient when it takes negative or small positive values, and can switch to be increasing function of it for large positive values of the correlation coefficient.

We also compared the second-best and third-best equilibrium effort levels and found that there is not always an underprovision of effort in the third-best equilibrium. Instead, in case of strongly negatively correlated signals, the opposite may happen, and it can be an overprovision of effort in the third-best equilibrium compared to the second-best. Note that this comes at the expense of higher risk imposed on the agent and third-best total welfare stays lower than second-best total welfare.

Appendix

Proof of Lemma 1:

The risk-premium (less the factor $r/2$) required by the agent can be written as

$$R = \alpha_1^2\sigma_1^2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 + \alpha_2^2\sigma_2^2 \tag{A.1}$$

By using the inequality between the arithmetic and geometric means, we have

$$R \geq 2 \cdot |\alpha_1| \cdot \sigma_1 \cdot |\alpha_2| \cdot \sigma_2 + 2\rho\alpha_1\alpha_2\sigma_1\sigma_2 \tag{A.2}$$

The RHS of this equation is always non-negative as

$$|\alpha_1| \cdot |\alpha_2| \geq -\rho\alpha_1\alpha_2 \tag{A.3}$$

for $\rho \in [-1, 1]$. Therefore, the only case when the risk premium required by the agent is zero is when both inequalities above hold with equality. This is the case if and only if (i) $|\alpha_1| \cdot \sigma_1 = |\alpha_2| \cdot \sigma_2$ and (iia) $\rho = -1$ and $\alpha_1\alpha_2 \geq 0$ or (iib) $\rho = +1$ and $\alpha_1\alpha_2 \leq 0$. Rearranging these conditions give the conditions in Lemma 1.

Proof of Proposition 1:

Denote by A and B the terms $\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$ and $rk(1-\rho^2)\sigma_1^2\sigma_2^2$ in equation (13). By differentiating μ^{SB} with respect to ρ, it can be see that $\partial\mu^{TB}/\partial\rho \geq 0$, whenever

$$-2\sigma_1\sigma_2(A+B) - A(-2\sigma_1\sigma_2 - 2\rho rk\sigma_1^2\sigma_2^2) \geq 0 \tag{A.4}$$

or, equivalently

$$A\rho rk\sigma_1\sigma_2 \geq B \tag{A.5}$$

which can be rewritten as

$$(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)\rho \geq (1-\rho^2)\sigma_1\sigma_2 \tag{A.6}$$

or, equivalently, as

$$(\rho\sigma_2 - \sigma_1)(\sigma_2 - \rho\sigma_1) \geq 0 \tag{A.7}$$

Since we assumed that $\sigma_1 \leq \sigma_2$, the above inequality holds if and only if $\rho > \frac{\sigma_1}{\sigma_2}$.

Proof of Proposition 2:

Denote by F and G the numerator and the denominator of

$$\alpha_i^{TB} = \frac{b_i - \rho rkb_j\sigma_1\sigma_2 + rkb_i\sigma_j^2}{(1+rk\sigma_1^2)(1+rk\sigma_2^2) - \rho^2r^2k^2\sigma_1^2\sigma_2^2} \tag{A.8}$$

By differentiating it with respect to ρ, it can be seen that $\partial\alpha_i^{TB}/\partial\rho \geq 0$, whenever

$$-rkb_j\sigma_1\sigma_2G - F(-2\rho r^2k^2\sigma_1^2\sigma_2^2) > 0 \tag{A.9}$$

or, equivalently, whenever

$$2\rho rk\sigma_1\sigma_2F > b_jG \tag{A.10}$$

As $\alpha_i^{TB} = \frac{F}{G} > 0$ for $\rho < 0$, this inequality can never hold for $\rho < 0$, and therefore α_i^{TB} is a decreasing function of ρ for $\rho < 0$.

From (A.10) it can be seen that for $\rho > 0$, we have $\partial\alpha_i^{TB}/\partial\rho \geq 0$ whenever

$$2\rho rk\sigma_1\sigma_2[b_i - \rho rkb_j\sigma_1\sigma_2 + rkb_i\sigma_j^2] > b_j[(1+rk\sigma_1^2)(1+rk\sigma_2^2) - \rho^2r^2k^2\sigma_1^2\sigma_2^2] \tag{A.11}$$

or, equivalently

$$b_jr^2k^2\sigma_1^2\sigma_2^2\rho^2 - 2b_irk\sigma_1\sigma_2(1+rk\sigma_j^2)\rho + b_j(1+rk\sigma_1^2)(1+rk\sigma_2^2) < 0 \tag{A.12}$$

Define function $f(\rho)$ as

$$f(\rho) = b_jr^2k^2\sigma_1^2\sigma_2^2\rho^2 - 2b_irk\sigma_1\sigma_2(1+rk\sigma_j^2)\rho + b_j(1+rk\sigma_1^2)(1+rk\sigma_2^2) \tag{A.13}$$

Our task is to determine the conditions under which this function has an intersection with the horizontal axis in the $[0,1]$ interval.

A necessary condition is that the determinant of the function is positive, which is the case whenever

$$b_i^2(1+rk\sigma_j^2)^2 - b_j^2(1+rk\sigma_1^2)(1+rk\sigma_2^2) > 0 \tag{A.14}$$

or, equivalently, whenever

$$b_i^2(1+rk\sigma_j^2) > b_j^2(1+rk\sigma_i^2) \tag{A.15}$$

This condition assures that $f(\rho)$ crosses the horizontal axis at least once.

We also have that

$$f'(\rho = 0) = -2b_irk\sigma_1\sigma_2(1+rk\sigma_j^2) < 0 \tag{A.16}$$

which means that function $f(\rho)$ crosses the vertical axis from above. In this case, all intersection points with the horizontal axis must be at positive values.

A necessary condition for function $f(\rho)$ to cross the horizontal axis twice in interval $[0, 1]$ is that

$$f'(\rho = 1) = 2b_j r^2 k^2 \sigma_1^2 \sigma_2^2 - 2b_i r k \sigma_1 \sigma_2 (1 + rk\sigma_j^2) > 0 \tag{A.17}$$

or, equivalently

$$\frac{b_j}{b_i} > \frac{1 + rk\sigma_j^2}{rk\sigma_1\sigma_2} \tag{A.18}$$

However, from (A.15) we have that

$$\frac{1 + rk\sigma_j^2}{1 + rk\sigma_i^2} > \frac{b_j^2}{b_i^2} \tag{A.19}$$

so, inequalities (A.18) and (A.19) imply that

$$\frac{1 + rk\sigma_j^2}{1 + rk\sigma_i^2} > \frac{(1 + rk\sigma_j^2)^2}{r^2 k^2 \sigma_1^2 \sigma_2^2} \tag{A.20}$$

or, equivalently

$$r^2 k^2 \sigma_1^2 \sigma_2^2 > (1 + rk\sigma_i^2)(1 + rk\sigma_j^2) \tag{A.21}$$

must hold. However, as this last inequality can never hold, function $f(\rho)$ can never cross the horizontal axis twice in the interval $[0, 1]$. Even if inequality (A.15) holds, it can cross the horizontal axis at most once in the interval $[0, 1]$. For $f(\rho)$ to cross the horizontal axis at all in the interval $[0, 1]$ we must have

$$f(\rho = 1) = b_j r^2 k^2 \sigma_1^2 \sigma_2^2 - 2b_i r k \sigma_1 \sigma_2 (1 + rk\sigma_j^2) + b_j (1 + rk\sigma_1^2)(1 + rk\sigma_2^2) < 0 \tag{A.22}$$

which in turn holds if

$$\frac{b_j}{b_i} < 2\frac{rk\sigma_1\sigma_2(1 + rk\sigma_j^2)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2) + r^2 k^2 \sigma_1^2 \sigma_2^2} \tag{A.23}$$

So, we have that $f(\rho)$ crosses at least once the horizontal axis in the interval $[0, 1]$ whenever

$$\frac{1 + rk\sigma_j^2}{1 + rk\sigma_i^2} > \frac{b_j^2}{b_i^2} \tag{A.24}$$

and

$$2\frac{rk\sigma_1\sigma_2(1 + rk\sigma_j^2)}{(1 + rk\sigma_1^2)(1 + rk\sigma_2^2) + r^2 k^2 \sigma_1^2 \sigma_2^2} > \frac{b_j}{b_i} \tag{A.25}$$

both hold.

Observe that the LHS of (A.25) is larger than square of the LHS of (A.24) whenever

$$4\frac{r^2k^2\sigma_1^2\sigma_2^2(1+rk\sigma_j^2)^2}{[(1+rk\sigma_1^2)(1+rk\sigma_2^2)+r^2k^2\sigma_1^2\sigma_2^2]^2} > \frac{1+rk\sigma_j^2}{1+rk\sigma_i^2} \tag{A.26}$$

or, equivalently, whenever

$$4r^2k^2\sigma_1^2\sigma_2^2(1+rk\sigma_j^2)(1+rk\sigma_i^2) > [(1+rk\sigma_1^2)(1+rk\sigma_2^2)+r^2k^2\sigma_1^2\sigma_2^2]^2 \tag{A.27}$$

which can be never the case as it is equivalent to

$$0 > [(1+rk\sigma_1^2)(1+rk\sigma_2^2)-r^2k^2\sigma_1^2\sigma_2^2]^2 \tag{A.28}$$

So we have that $f(\rho)$ changes sign in the interval $[0,1]$ whenever

$$\frac{2rk\sigma_1\sigma_2(1+rk\sigma_j^2)}{(1+rk\sigma_1^2)(1+rk\sigma_2^2)+r^2k^2\sigma_1^2\sigma_2^2} > \frac{b_j}{b_i} \tag{A.29}$$

which is exactly the inequality in (30).

Inequality (31) can be derived directly form inequality (A.12).

Proof of Proposition 3:

Denote by F and G the numerator and the denominator of

$$\mu^{TB} = \frac{1-\rho rk\sigma_1\sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1+rk\sigma_1^2)(1+rk\sigma_2^2)-\rho^2r^2k^2\sigma_1^2\sigma_2^2}\frac{b_1+b_2}{k} \tag{A.30}$$

By differentiating it with respect to ρ, it can be seen that $\partial\mu^{TB}/\partial\rho \geq 0$, whenever

$$-rk\sigma_1\sigma_2 G - F(-2\rho r^2k^2\sigma_1^2\sigma_2^2) > 0 \tag{A.31}$$

or, equivalently, whenever

$$2\rho rk\sigma_1\sigma_2 F > G \tag{A.32}$$

As $\mu^{TB} = \frac{F}{G} > 0$ for $\rho < 0$, this inequality can never hold for $\rho < 0$, and therefore μ^{TB} is a decreasing function of ρ for $\rho < 0$.

From (A.32) it can be seen that for $\rho > 0$, we have $\partial\mu^{TB}/\partial\rho \geq 0$ whenever

$$2\rho rk\sigma_1\sigma_2\left[1-\rho rk\sigma_1\sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)\right] > (1+rk\sigma_1^2)(1+rk\sigma_2^2)-\rho^2r^2k^2\sigma_1^2\sigma_2^2 \tag{A.33}$$

or, equivalently

$$r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)\rho^2-2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]\rho+(b_1+b_2)(1+rk\sigma_1^2)(1+rk\sigma_2^2)<0 \tag{A.34}$$

Define function $f(\rho)$ as

$$f(\rho)=r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)\rho^2-2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]\rho+(b_1+b_2)(1+rk\sigma_1^2)(1+rk\sigma_2^2) \tag{A.35}$$

Our task is to determine the conditions under which this function has an intersection with the horizontal axis in the $[0,1]$ interval.

A necessary condition is that the determinant of the function is positive, which is the case whenever

$$\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]^2>(b_1+b_2)^2(1+rk\sigma_1^2)(1+rk\sigma_2^2) \tag{A.36}$$

This condition assures that $f(\rho)$ crosses the horizontal axis at least once.

We also have that

$$f'(\rho=0)=-2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]<0 \tag{A.37}$$

which means that function $f(\rho)$ crosses the vertical axis from above. In this case, all intersection points with the horizontal axis must be at positive values.

A necessary condition for function $f(\rho)$ to cross the horizontal axis twice in interval $[0,1]$ is that

$$f'(\rho=1)=2r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)-2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]>0 \tag{A.38}$$

or, equivalently

$$rk\sigma_1\sigma_2(b_1+b_2)>\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right] \tag{A.39}$$

By combining conditions (A.36) and (A.39), we must have

$$r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)^2>(b_1+b_2)^2(1+rk\sigma_1^2)(1+rk\sigma_2^2) \tag{A.40}$$

which can never be the case. Therefore, we have that even if inequality (A.36) holds, the function $f(\rho)$ can cross the horizontal axis at most once in the interval $[0,1]$. For $f(\rho)$ to cross the horizontal axis at all in the interval $[0,1]$ we must have

$$f(\rho=1)=r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)-2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]+(b_1+b_2)(1+rk\sigma_1^2)(1+rk\sigma_2^2)<0 \tag{A.41}$$

which in turn holds whenever

$$2rk\sigma_1\sigma_2\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right] > \left[r^2k^2\sigma_1^2\sigma_2^2(b_1+b_2)+(b_1+b_2)(1+rk\sigma_1^2)(1+rk\sigma_2^2)\right] \tag{A.42}$$

or, equivalently

$$\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right] > \frac{1}{2}(b_1+b_2)\left[rk\sigma_1\sigma_2+\frac{(1+rk\sigma_1^2)(1+rk\sigma_2^2)}{rk\sigma_1\sigma_2}\right] \tag{A.43}$$

So, we have that $f(\rho)$ crosses at least once the horizontal axis in the interval $[0,1]$ whenever

$$\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right]^2 > (b_1+b_2)^2(1+rk\sigma_1^2)(1+rk\sigma_2^2) \tag{A.44}$$

and

$$\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right] > \frac{1}{2}(b_1+b_2)\left[rk\sigma_1\sigma_2+\frac{(1+rk\sigma_1^2)(1+rk\sigma_2^2)}{rk\sigma_1\sigma_2}\right] \tag{A.45}$$

both hold.

Observe that the RHS of (A.44) is larger than square of the LHS of (A.45) whenever

$$(1+rk\sigma_1^2)(1+rk\sigma_2^2) > \frac{1}{4}\left[rk\sigma_1\sigma_2+\frac{(1+rk\sigma_1^2)(1+rk\sigma_2^2)}{rk\sigma_1\sigma_2}\right]^2 \tag{A.46}$$

which is never true as

$$\frac{1}{4}\left[rk\sigma_1\sigma_2-\frac{(1+rk\sigma_1^2)(1+rk\sigma_2^2)}{rk\sigma_1\sigma_2}\right]^2 > 0 \tag{A.47}$$

So we have that $f(\rho)$ changes sign in the interval $[0,1]$ whenever

$$\left[(b_1+b_2)+rk(b_2\sigma_1^2+b_1\sigma_2^2)\right] > \frac{1}{2}(b_1+b_2)\left[rk\sigma_1\sigma_2+\frac{(1+rk\sigma_1^2)(1+rk\sigma_2^2)}{rk\sigma_1\sigma_2}\right] \tag{A.48}$$

or, equivalently, whenever

$$\frac{b_2}{b_1+b_2}\sigma_1^2+\frac{b_1}{b_1+b_2}\sigma_2^2 > \frac{1}{2}\sigma_1\sigma_2\left[1+\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_1}{\sigma_2}\right)\left(\frac{1}{rk\sigma_1\sigma_2}+\frac{\sigma_2}{\sigma_1}\right)\right]-\frac{1}{rk} \tag{A.49}$$

Inequality (37) can be derived directly from inequality (A.34).

Proof of Proposition 4:

We have to investigate the following inequality:

$$\mu^{TB} \geq \mu^{SB} \tag{A.50}$$

Since

$$\mu^{SB} = \frac{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2 + rk(1-\rho^2)\sigma_1^2\sigma_2^2}\frac{b_1+b_2}{k} \tag{A.51}$$

and

$$\mu^{TB} = \frac{1 - \rho rk\sigma_1\sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1+rk\sigma_1^2)(1+rk\sigma_2^2) - \rho^2 r^2 k^2 \sigma_1^2\sigma_2^2}\frac{b_1+b_2}{k} \tag{A.52}$$

we are interested whether inequality

$$\frac{1 - \rho rk\sigma_1\sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{(1+rk\sigma_1^2)(1+rk\sigma_2^2) - \rho^2 r^2 k^2 \sigma_1^2\sigma_2^2} \geq \frac{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2 + rk(1-\rho^2)\sigma_1^2\sigma_2^2} \tag{A.53}$$

holds for any value of ρ in the interval $[-1, 1]$.

It can be seen that this is an inequality of degree 3 in ρ and therefore it is hard to evaluate. Therefore, we only prove that it may hold for the special case when $\rho = -1$.

For $\rho = -1$, inequality (A.53) can be rewritten as

$$\frac{1 + rk\sigma_1\sigma_2 + rk\left(\frac{b_2}{b_1+b_2}\sigma_1^2 + \frac{b_1}{b_1+b_2}\sigma_2^2\right)}{1 + rk\sigma_1^2 + rk\sigma_2^2} \geq 1 \tag{A.54}$$

or, equivalently

$$(\sigma_2 - \sigma_1)(b_1\sigma_1 - b_2\sigma_2) \geq 0 \tag{A.55}$$

This is exactly the condition in inequality (38).