"Begin at the Beginning": Initial Conditions Matter for the Size Distribution of Firms
—*Preliminary and Incomplete*—

Rebecca Hellerstein and Miklós Koren*

June 20, 2006

Abstract

This paper quantifies the effects of firm size at entry on the distribution of firm sizes over time. The distribution of firm sizes is extremely skewed: A small number of firms account for a large share of output. Much of the literature on firm-size distributions has focused on the dynamics of individual firm sizes over time, deriving ergodic size distributions from models of firm growth and exit. Using *Compustat* data, we find that firm sizes are very persistent over time. The correlation of a firm's current size with its size 25 years ago is 0.71, which means that size depends more on initial conditions than on subsequent firm dynamics over 25 years. This finding indicates that firms fail to converge to an ergodic size distribution even over quite long horizons. We explore this fact using the *Compustat* data. We conclude with a simple theoretical model of optimal firm size on entry.

Keywords: Size distribution of firms; Firm entry and exit.

JEL classifications: L11, L16

*Federal Reserve Bank of New York. Corresponding author: Miklós Koren, International Research Group, Federal Reserve Bank of New York, 33 Liberty Street, New York, NY 10045; E-mail: `Miklos.Koren@ny.frb.org`. We thank Aaron Cooper for valuable research assistance. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or of the Federal Reserve System.

1 Introduction

This paper quantifies the effects of firm size at entry on the distribution of firm sizes over time. The distribution of firm sizes is extremely skewed: A small number of firms account for a large share of output. This has implications for our understanding of a range of macroeconomic and international-trade issues, such as the magnitude of business cycles (Gabaix, 2005) and the patterns of export-market participation (Bernard, Redding and Schott, 2005). Much of the literature on firm-size distributions has focused on the dynamics of individual firm sizes over time, deriving ergodic size distributions from models of firm entry, growth, and exit. This paper argues that pre-entry scale decisions may be more important than are subsequent firm dynamics in causing the observed skewness in firm-size distributions.

We begin by showing that firm sizes are very persistent over time using *Compustat data*. We find the correlation of a firm's current size with its size 25 years ago to be 0.71, which means that size depends more on initial conditions than on subsequent firm dynamics over 25 years. We then demonstrate that this finding means firms fail to converge to an ergodic size distribution over even quite long horizons. The existing literature on the causes of skewed firm size distributions may be mistaken in its (*de facto*) assumption of a Markovian model of firm growth *à la* Hopenhayn, 1992).

We quantify the importance of firm size at entry (initial sales) relative to the dynamics of firm size over time (variance in sales over time) for the cross-sectional distribution of firm sizes. We conclude with a simple non-Markovian model of the distribution of firm sizes in which an entrepreneur's pre-entry scale decision (which allows for shocks, both temporary and persistent, to profitability and entrepreneurial learning *à* la Jovanovic) determines the majority of their sales variance over time: Where firms start is where they end up. Our results are similar to those of Abbring and Campbell (2005), who show that Texas bars demonstrate non-Markovian history dependence – initial sales significantly contribute to forecasts of sales going forward – and to those of Pakes and Ericson (1998) who find that retail firms' sales exhibit non-Markovian growth over time.

2 Literature

The widespread appearance of skew distributions in various settings, particularly in the biological sciences, has received increased attention in recent years in economics. A number of recent papers examine the extreme skewness of the size distribution of firms.

Axtell (2001) finds that the log of the right tail probabilities of the log-size distribution, with firm size measured by employment, is on a virtual straight line with a slope of -1.059. This suggests that the distribution of firm size is well approximated by a Pareto distribution with right tail probabilities in the form of $S^{-\theta}$ where S is the size of the firm and $\theta = 1.05$. A number of authors argue that it is remarkable how well this distribution fits the log-size distribution of firms and some efforts have been made to connect it to data on firm growth, exit, and entry.[1]

A number of theoretical papers seek to explain observed firm size distributions by appealing to models of firm dynamics. For example, Klette and Kortum (2004) develop a model of R&D that produces a stable skewed firm-size distribution with firm growth rates independent of size (with growth dynamics that follow Gibrat's law).

We ask whether the recent focus on firm dynamics as the source of the observed skewness in firm size distributions has been misplaced. We suggest that pre-entry scale decisions may dominate firm dynamics in determining the distribution of firm sizes at any point in time.

3 The Markovian Model of Firm Size Distributions

This section presents a reduced form model of Markovian firm dynamics, in the flavor of Hopenhayn (1992). The model is general enough to serve as a benchmark that encompasses most existing Markovian models.

Firm size dynamics is first-order Markov: the current size of the firm is a sufficient statistic to describe the stochastic paths in the future. Firms are born with an exogenous size. They are hit by exogenous productivity shocks, which induces endogenous response in firm growth. Firms may choose to exit if they are unproductive. There is no aggregate uncertainty.

There are four key ingredients of the model: a function describing growth dynamics, a function describing exit rules, the entry size(s) of firms, and the mass of new entrants. We take the model as a reduced form since we do not discuss how the dynamics is derived from the primitives of the model (technologies, stochastic process of productivity, and the like).

Let $f(\Delta y_t | y_{t-1})$ denote the probability (density) that a firm will grow by Δy_t con-

[1] Gabaix (1999) discusses more generally how probability models give rise to power laws and the implications for economics.

ditional on being size y_{t-1} in year $t-1$. This distribution is conditional on the firm surviving until year t. Let $h(y_{t-1})$ denote the probability that a firm with size y_{t-1} in year $t-1$ will choose to exit in year t. The firm starts from size y_0 at time $t=0$, which is drawn from a distribution $G(y_0)$. Each year there are a measure μ of new, ex ante identical entrants. For ease of exposition, we assume that all distributions are continuous.

Let $\phi(y,t)$ denote the measure of firms of size y in year t. This firm size distribution evolves as follows:

$$\phi(y,t) = \mu g(y) - h(y)\phi(y,t-1) + \int_x f(y-x|x)\phi(x,t-1)dx. \tag{1}$$

There is a mass $\mu g(y)$ of firms that enter at exactly size y. A mass $h(y)\phi(y,t-1)$ of firms exits between $t-1$ and t. Each firm with size x in year $t-1$ becomes size y with probability $f(y-x|x)$, hence the measure of firms growing (or shrinking) to size y is $f(y-x|x)\phi(x,t-1)$.

In year 0, the distribution of firms is just

$$\phi(y,0) = g(y). \tag{2}$$

Given the functions f, g, and h, one can derive the evolution of the firm size distribution, $\{\phi(y,1),...,\phi(y,t)\}$, and an ergodic distribution (if exists),

$$\phi_\infty(y) = \lim_{t\to\infty} \phi(y,t).$$

An invariant distribution $\tilde{\phi}$ satisfies the functional equation

$$\tilde{\phi}(y) = \mu g(y) - h(y)\tilde{\phi}(y) + \int_x f(y-x|x)\tilde{\phi}(x)dx,$$

and under some conditions, it is unique and coincides with the ergodic distribution.

To illustrate how the properties of f, g, and h affect the evolution of the firm size distribution, let us consider some simple examples.

Example 1 (Random growth) *Suppose there is no entry or exit, $\mu = h = 0$, and growth is independent of current size, $f(x|y) = f(x)$. Then big firms are just as likely to grow as small firms, and small firms are just as likely to shrink as big firms. The size dispersion increases without bound, and there is no limiting distribution.*

More concretely, if $f(x)$ has mean μ and variance σ^2, then, by the law of large numbers, $(y_t - \mu t)/(\sqrt{t}\sigma)$ converges to a standard normal distribution as $t \to \infty$.

Example 2 (Mean reverting growth) *Suppose there is no entry or exit, $\mu = h = 0$, and growth is negatively related to current size. That is, if $x' < x$, then $f(y|x')$ stochastically dominates $f(y|x)$ in the first order. Then there exists a steady-state size distribution.*

More concretely, suppose y_t is $AR(1)$ with iid Gaussian disturbances,

$$\Delta y_t = -\rho y_{t-1} + u_t,$$

and y_0 is normally distributed with variance σ_0^2. Then y_t is normally distributed with variance

$$\mathrm{Var}(y_t) = \sigma_0^2(1-\rho)^{2t} + \sigma_u^2 \frac{1-(1-\rho)^t}{\rho}.$$

The steady-state variance is σ_u^2/ρ, independent of the initial size distribution.

Example 3 (No entry) *Suppose there is no entry, and f and h are such that an ergodic distribution exists. Then the initial size distribution of firms does not affect the ergodic distribution.*

We can summarize existing Markovian models of firm dynamics with what they state (or assume) about f, g, and h.

Model	Entry size	Growth rate	Exit rule
Gabaix	none	size independent for large firms	none
Luttmer	drawn from incumbents	size independent	small size
Klette–Kortum	small	size independent	size = 0
Rossi-Hansberg–Wright	same as incumbents	mean reverting	size independent

Table 1: Markovian Models of Firm Dynamics

4 Data

Our data come from *Standard & Poor's Compustat North America Industrial Annual* (NAIA) database. *Compustat* provides financial data covering publicly traded companies in the U.S. and Canada. Companies are added to the *Compustat* database when their stock begins trading on the *New York Stock Exchange, American Stock Exchange* or *NASDAQ*. Companies must meet additional *Securities and Exchange Commission* requirements in order to be included.

Compustat's *NAIA* database presents annual data from 1950 through 2004. This is a particularly appealing characteristic of the data, since it allows us to thoroughly study the long-term dynamics of the firm size distribution. Our analysis uses firm sales data (as opposed to the popular employment figures) as a measure for firm size. The correlation of employees with sales is .63. Since we are interested in the output share of large firms in the distribution, sales data makes intuitive sense. Because of the extreme skewness of the size distribution, we also find it convenient to examine the data using logs.

The sample changes dramatically over the course of its 50 year span. *Compustat* reports sales for 652 firms in 1950, increasing to over 7100 by 2004. Firms are added to the data three times, one group in 1950, another in 1966 and a third in 1985. The sample of firms reporting sales grows much more smoothly over time, with noticeable jumps in 1960, 1974 and 1995 (see Figure 1). The newly added firms are typically smaller, which induces downward jumps in the average size of firms over time (Figure 2). Grouping firms into cohorts based on entry year allows us to control for the volatility of the data due to such influxes.

The average lifespan of a firm is just over 12 years. There are, however, 4400 firms with at least 20 years of data (the number is 2200 for 30, 920 for 40, and 380 for 50 years). This enables us to study the long-run dynamics of a large set of firms.

While firms may enter the sample in a consistent manner, the nature of the *Compustat* database prevents us from observing the true "birth" of a firm; consequently, we proxy a firm's entry by its first year of reported sales. On the other hand, there are a multitude of reasons for firm exit. In *Compustat*'s *NAIA* dataset, firms may exit via bankruptcy (the most relevant for our discussion), merger and acquisition, going private, liquidation, or leveraged buyout. We control for exits other than bankruptcy in most cases by dropping those firms from the data entirely.

As an added perk, over 20 2-digit NAICS industries are represented, as well as many more sub-categories of these industries. Manufacturing is the most highly represented industry in the data, with Information, Financial Services, Mining and Utilities as other large industries. In 1990, for example, 25 percent of the firms were in Manufacturing, while 11 percent were in Financial Services and 7 percent were in Information.

One final limitation of the data is that we can only observe the size dynamics of the

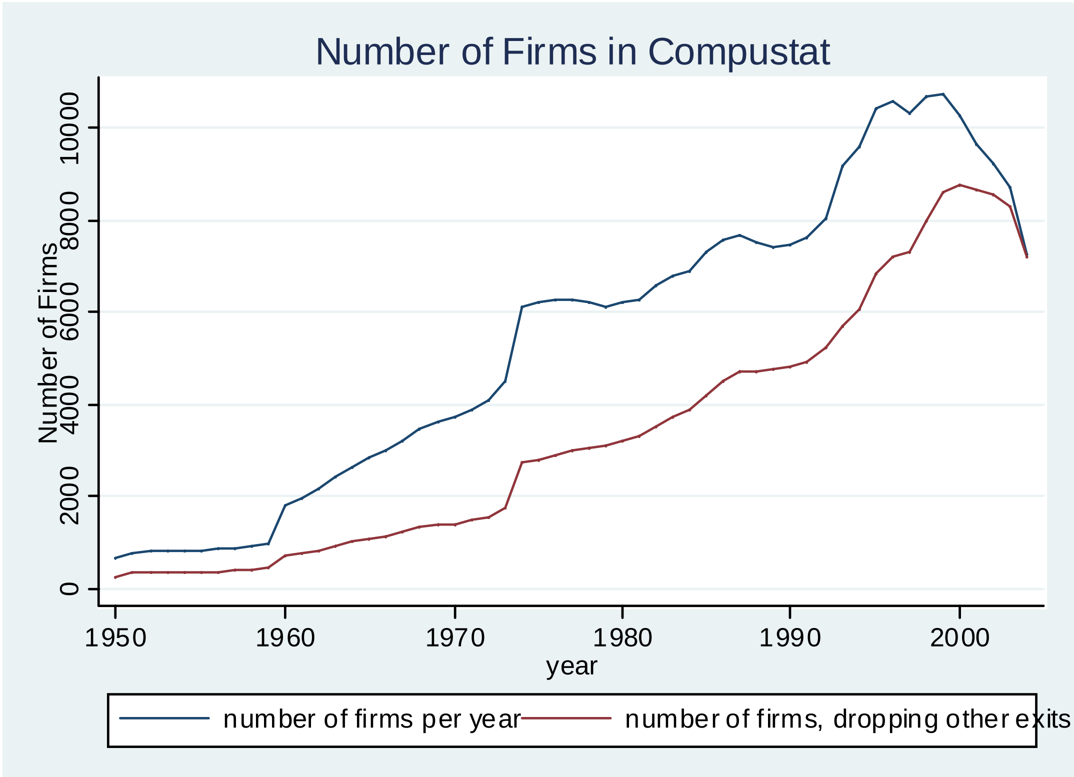

Figure 1:

largest firms in the economy. However, we have reason to believe that this will not limit our analysis, particularly as it pertains to the largest firms in the size distribution.

5 Stylized Facts on Firm Size Distributions

Fact 1 *The size distribution of firms is extremely skewed.*

The top 5 percent of firms sell about 50 percent of output. The distribution of firm size is well approximated by a lognormal distribution. The cross-sectional standard deviation of log sales in 1999 is 2.69. This means that the 75th percentile firm sells about 30 times as much as the 25th percentile. (The 90-10 range is about 760.) Figures 3–5 display the histogram of firm sales, log firm sales, and the Lorenz curve of firms sales, respectively.

Fact 2 *The size dispersion is much higher within industries than across industries.*

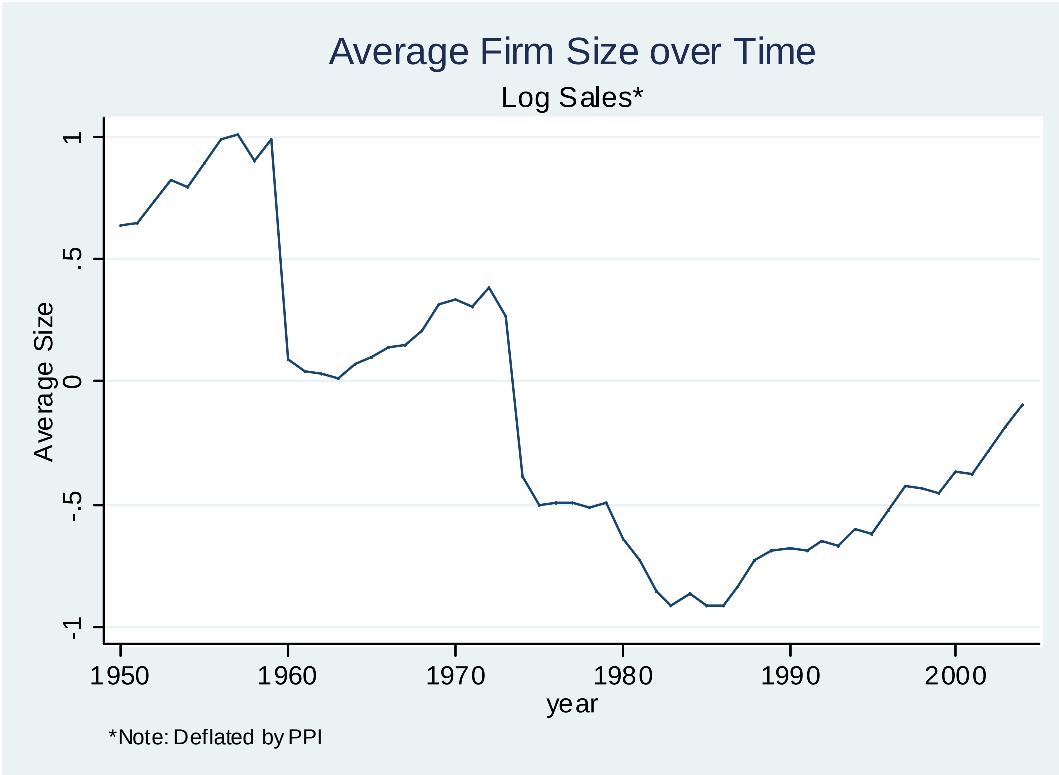

Figure 2:

Industry fixed effects (4-digit NAICS) explain only 25 percent of the variation in firm size. Broader sectors (23 2-digit NAICS categories) account for only 9 percent. This indicates that the vast heterogeneity of firms sizes is not primarily due to technological or market structure differences across industries, as all industries exhibit huge size variation.

Fact 3 *For a given cohort, the size distribution is relatively stable over time for earlier cohorts, and increases for later cohorts.*

We proxy firm entry by using the first recorded sales data, and then group firms in 5 year cohorts according to firm entry year. We analyze cohorts to make sure that the dynamics of the size distribution is not driven by new firms being added to the dataset. We use the standard deviation of log sales to examine the dispersion of the size distribution over time. Figure 6 shows the evolution of dispersion for all 5-year cohorts between 1950 and 1970. The 1950 and 1955 cohorts are relatively stable, whereas later cohorts exhibit increasing dispersion.

At least for the older, more established cohorts, the data contradicts models assuming or implying Gibrat's law that states that growth rates are independent of size (Gabaix,

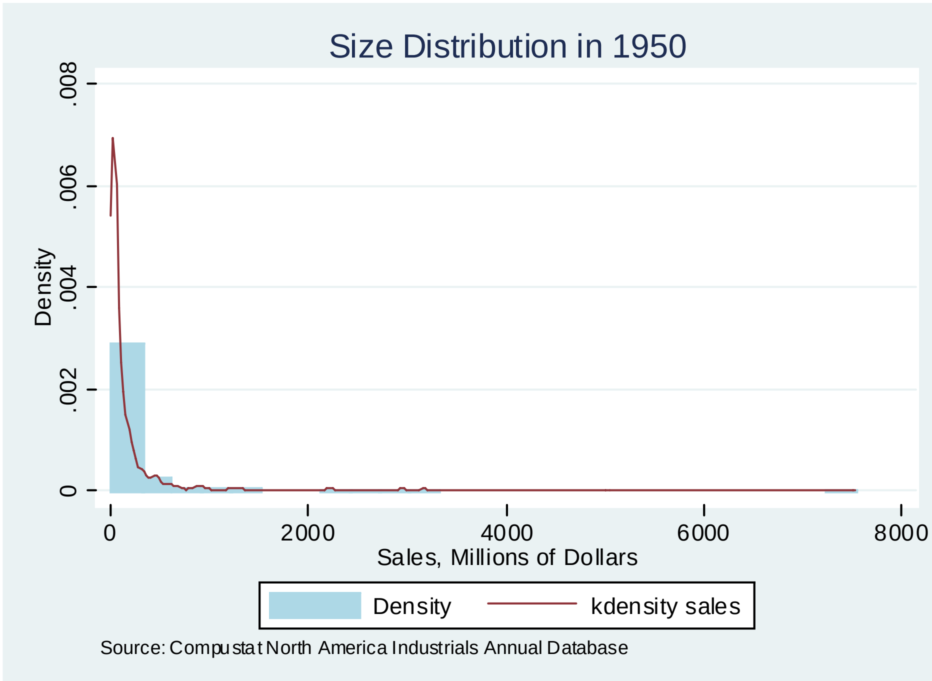

Figure 3:

1999; Luttmer, 2004; Klette and Kortum, 2004). These models would imply a continuous increase in the size dispersion of a given cohort of firms.

Fact 4 *Differences in firm growth contribute little to changes in the firm size distribution.*

Figure 7 presents the results of the following decomposition exercise:

$$\begin{aligned}
\mathrm{Var}(y_t) &= \mathrm{Var}(y_{t-1} + \Delta y_t|\text{survival}) \\
&= \mathrm{Var}(y_{t-1}) + [\mathrm{Var}(y_{t-1}|\text{survival}) - \mathrm{Var}(y_{t-1})] \\
&\quad + \mathrm{Var}(\Delta y_t|\text{survival}) + 2\,\mathrm{Cov}(\Delta y_t, y_{t-1}|\text{survival})
\end{aligned}$$

The cross-section variance (dispersion) of firm sizes depends on past dispersion, non-random exit (if smaller firms are more likely to exit, this compresses the distribution), the dispersion of growth rates, and their relationship to the firms' past size. In particular, if Gibrat's law holds (growth rates are independent of size), the covariance term is zero

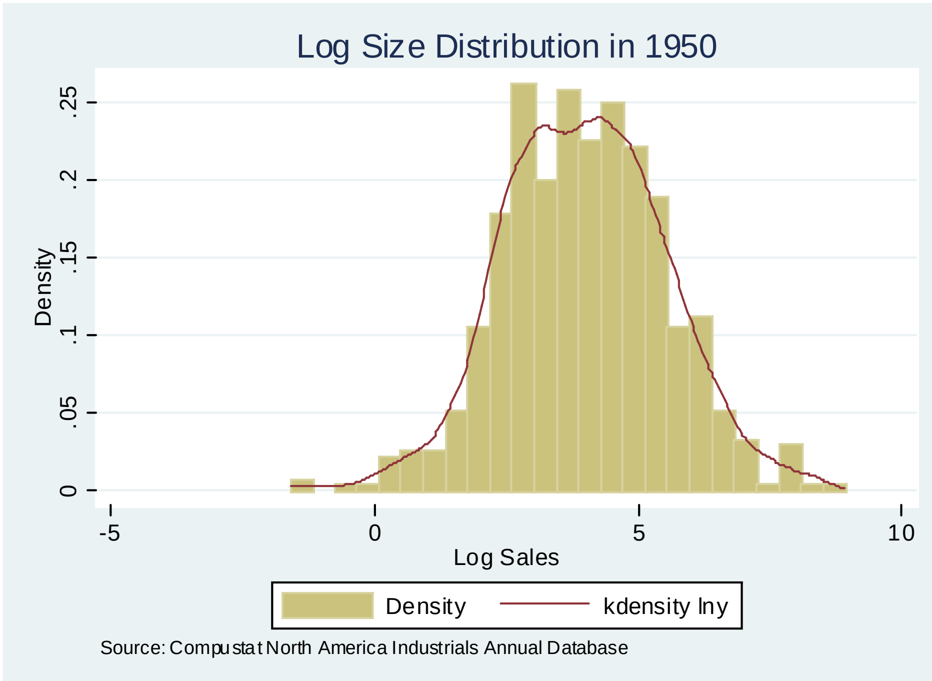

Figure 4:

and idiosyncratic growth dynamics increases size dispersion. Mean reverting models imply a negative covariance term.

This finding demonstrates that the size dispersion of a cohort is primarily determined by its 10-year-ago size dispersion, and is little affected by growth and exit throughout the past 10 years. This is in contrast with models that derive the ergodic size distribution from mean-reverting growth dynamics (Rossi-Hansberg and Wright, 2004).

What can we learn about the largest firms in the sample? Figure 8 tracks the shares of GDP of the biggest 5% of firms in the data over time. We select firms based on their size in their seventh year. These shares are very steady over time, although they vary between different cohorts, suggesting that big firms remain big once they are established.

Fact 5 *The position of top firms is remarkably stable over time.*

The total sales of the top 5 percent of firms in 1950 remained 18–30 percent of total GDP throughout the subsequent 40 years. For all entry cohorts, there is very little

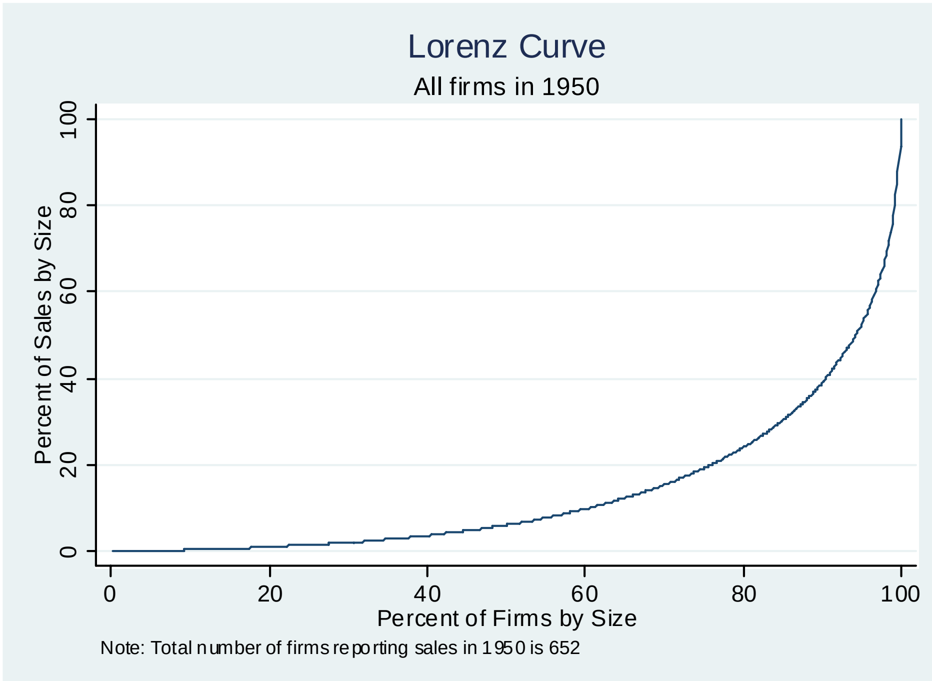

Figure 5:

tendency of the big firms to decline. Big firms are remarkably stable and are able to keep their relative positions in the economy for very long periods. Again, this indicates that year-to-year growth dynamics is relatively unimportant for the understanding of these firms. If there existed substantial mean reversion in size, one would expect big firms to shrink rapidly.

Fact 6 *Throughout the distribution, there is little turnover in firm sizes.*

To examine the persistence of size throughout the distribution, we estimate transition matrices for firm size. Firms are most likely to remain in their size bin in the next decade (with a probability of $0.62 - 0.79$). Based on the estimated transition matrix, we can calculate long-run transition probabilities to examine the long-run dynamics of firms. Firms starting in the lowest bin have only a 17 percent probability of ever moving to the top bin. Those that do reach the top bin do so in 152 years on average. Firms starting in the top bin have an expected lifetime of 247 years. However, 41 percent of these firms will go bankrupt before shrinking to the smallest bin.

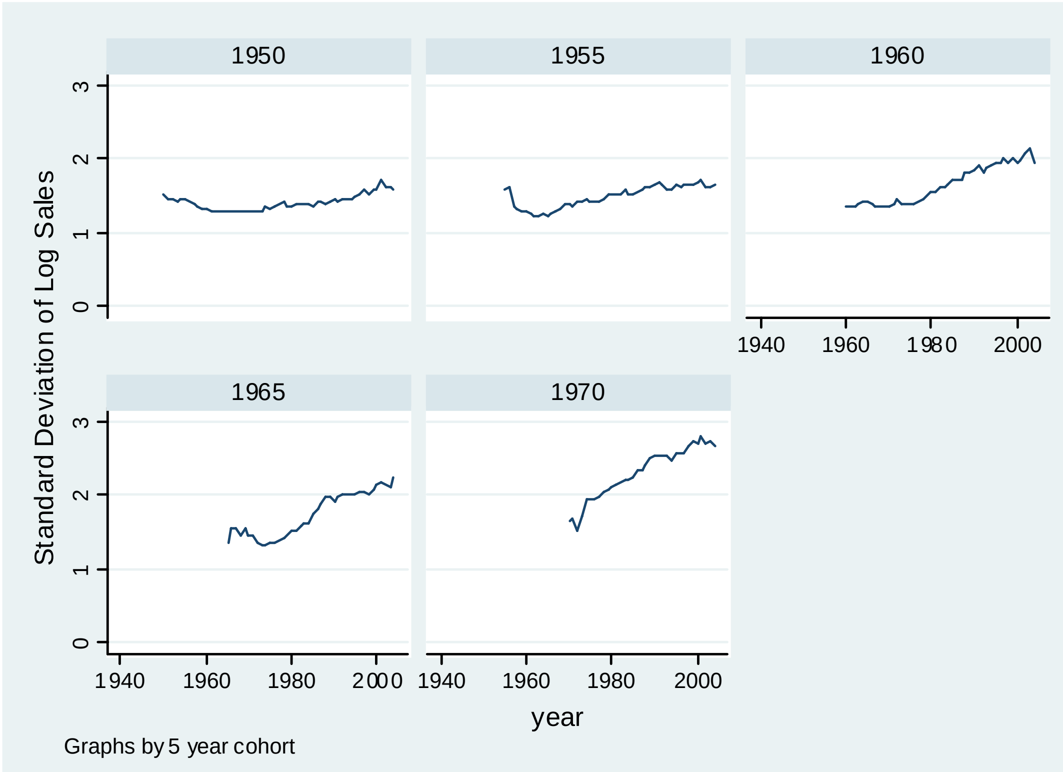

Figure 6: The Evolution of Within-Cohort Size Dispersion Over Time

Table 2: Transition Probabilities from 1980 to 1990

		1990					
		<10	**<100**	**<1000**	**>1000**	**Exit**	***Total***
	<10	0.7054	0.1752	0.0181	0.0000	0.1012	*662*
1980	**<100**	0.1861	0.6227	0.1207	0.0030	0.0674	*994*
	<1000	0.0098	0.1540	0.7269	0.0850	0.0242	*1,117*
	>1000	0.0039	0.0019	0.1965	0.7881	0.0096	*519*
	Total	*665*	*908*	*1,046*	*507*	*166*	***3,292***

Notes: Bins are based on sales levels in millions of dollars. Firms that exit for reasons other than bankruptcy are not included.

This finding demonstrates that the lack of turnover carries over to other parts of the distribution. The model that firms enter small, grow, shrink and then die (e.g., Klette and Kortum, 2004) is not consistent with the data. Small firms are very unlikely to ever make it to the top. A big firm has a large chance of going bankrupt even before

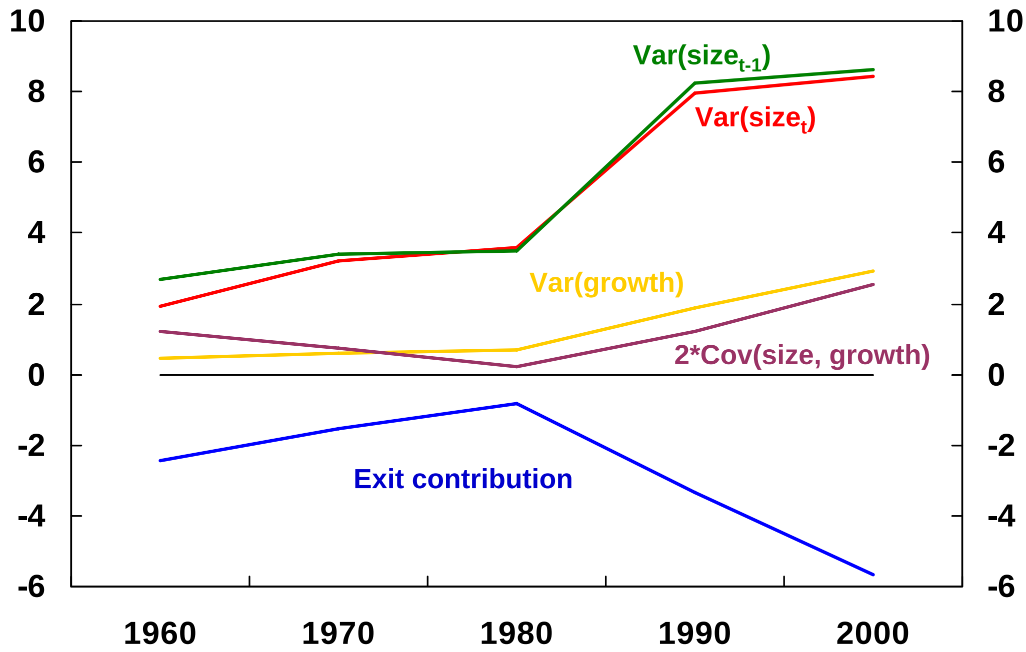

Figure 7: Decomposition of Size Dispersion

Notes: Firms exiting for reasons other than bankruptcy are not included.

shrinking substantially.

Fact 7 *Firm size dynamics is non-Markovian.*

We regress current size of the firm on lagged sizes and its size upon entering *Compustat*. Figure 10 reports the estimated coefficients and the 95 percent confidence intervals. Even at age 20, the initial size matters significantly.

In a more structural exercise, we estimate a fixed-effects $AR(p)$ model ($p = 1, 2, 3$),

$$y_{it} = \alpha_i + \sum_{k=1}^{p} \beta_k (y_{i,t-k} - \alpha_i) + u_{it}$$

using the Arellano and Bond (1991) estimator. In this model, the expected steady-state size of firm i is

$$\lim_{T\to\infty} E_t(y_{iT}) = \alpha_i.$$

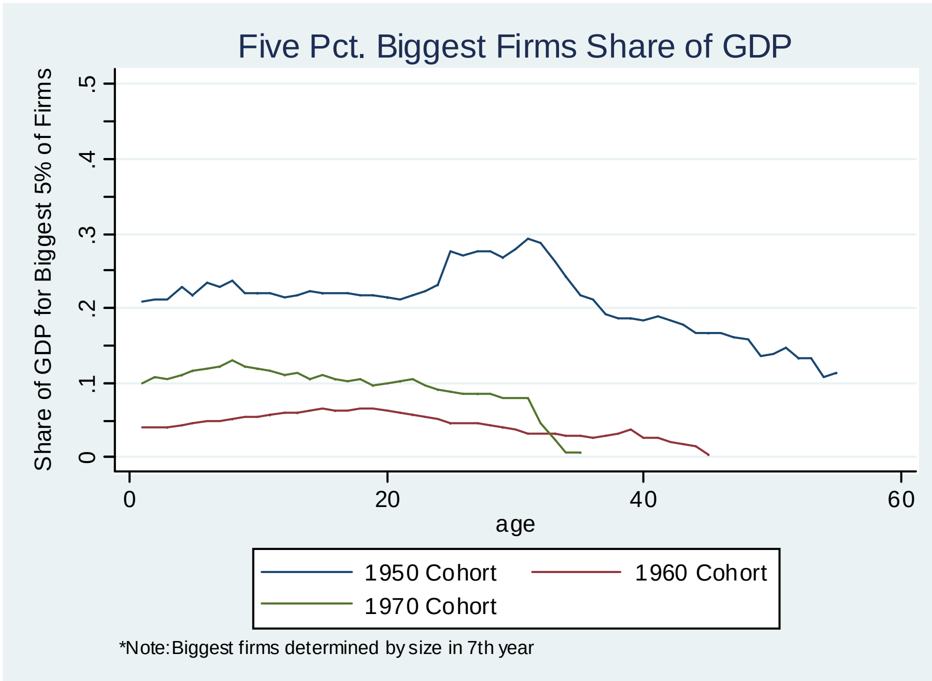

Figure 8: The Share of Large Firms over Time

As long as α_i varies across firms, their dynamics conditional on past sizes is different. That is, dynamics is not Markovian.

We find that the variance of fixed effects is 5.93, compared to the variance of firm size in 2004, 6.71. Overall, α_i explains 88 percent of the variation in firm sizes.

These exercises provide some evidence that firms may fail to converge to an ergodic size distribution at all. That is, not only is firm size persistent, but different firms tend to converge to different long-run sizes. Though admittedly indirect (*Compustat* only contains publicly listed firms), our evidence also points to a substantial heterogeneity of firm sizes upon entry.

The above stylized facts indicate that there are persistent firm-specific factors that determine the firm's long-run size. This is in sharp contrast with models where firms are ex ante homogeneous and all ex post heterogeneity in size is the outcome of idiosyncratic size dynamics ("luck").

Firm size is partly predetermined upon entry, and there are ex ante heterogeneities

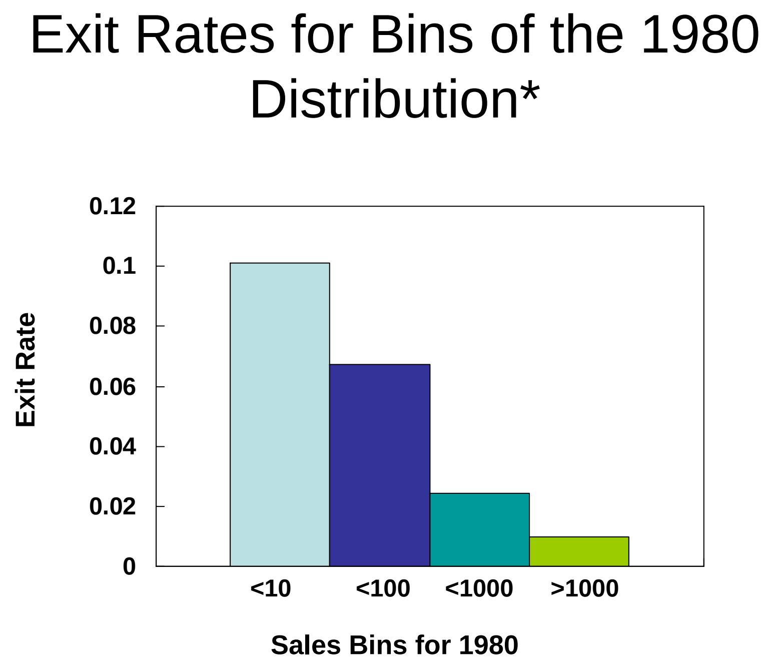

Figure 9:

across firms. Ex ante heterogeneity is an important feature of models such as the span of control model of Lucas (1978) (also see Atkeson and Kehoe, 2005 or the learning model of Jovanovic, 1982). However, these models tend to assume an exogenous distribution of firm types ("managerial ability," productivity etc.) Heterogeneity in the entry sizes of firms is entirely driven by the exogenous type heterogeneity.

To better understand what determines a firm's pre-entry scale decision, we build a model where potential entrants' optimal search, selection and matching affects the size distribution in an endogenous manner.

6 A Model of Optimal Firm Size

In this section we provide a simple model of firm size at entry and optimal firm size. We are particularly interested in why some firms choose to enter small while others choose to enter large. There are benefits as well as costs of being large. Bigger firms make

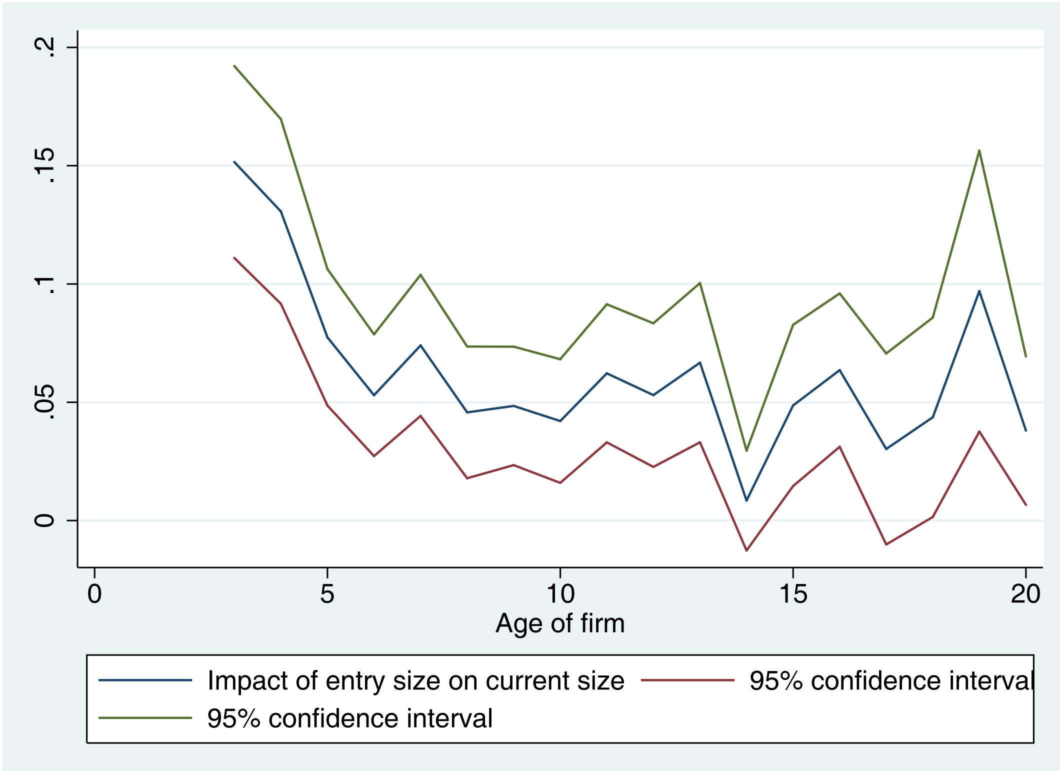

Figure 10: The Impact of Entry Size on Current Size

Notes: Reports regression coefficient of entry size and its 95% confidence interval based on heteroskedasticity-consistent standard errors. Regression controls include lagged size and year fixed effects.

higher revenues and command higher market power, which may enable them to charge higher markups. Both effects result in higher overall profits. However, starting larger firms may involve higher sunk costs and may be riskier than small-scale businesses. In what follows, we investigate these trade-offs.

Given that the cross-sectional size dispersion is dominated by initial conditions rather than subsequent dynamics (see Figure 7, for example), we start with a model that neglects dynamics altogether.

Firms require a capacity K_i and labor L_i to produce. Capacity can be thought of as capital equipment, knowledge capital, managerial abilities.

$$Q_i = F(K_i, L_i), \tag{3}$$

where $F()$ is a concave production function subject to constant returns to scale.

Capacity is a fixed factor (we will model accumulation later). Labor is hired in competitive markets at wage rate w. Output price is p. The per-period profit maximization problem of the firm is

$$\max_L pQ_i - wL, \tag{4}$$

$$\text{s.t. } Q_i = F(K_i, L_i). \tag{5}$$

Optimal employment is proportional to capacity,

$$L_i = lK_i, \tag{6}$$

where the factor of proportionality, l (labor intensity), is determined by real wage and is the same across firms,

$$F_L(1, l) = w/p.$$

Denoting $F(1, x)$ by $f(x)$, we can write output and profit as

$$Q_i = K_i f(l) \tag{7}$$

$$\pi_i = pQ_i - wL_i = K_i \left[pf(l) - wl\right] \equiv K_i \pi, \tag{8}$$

and concavity of F implies that profits are positive. Profits linearly increase in capacity.

What determines capacity? Obviously, capacity must be costly, otherwise all firms would choose the maximum capacity and no small firms would exist.

Firms are owned by managers. Each manager (indexed by k) has a scarce resource M_k (e.g., liquid assets, managerial ability) that he can devote to operating firms. If a manager operates a set of firms $\mathcal{I}_k$, their capacities must satisfy

$$\sum_{i \in \mathcal{I}_k} K_i \leq M_k.$$

This resource constraint explains why there are small firms: some managers lack the resources to start big firms.

However, it does not explain why there are big firms: a manager is indifferent between operating two firms of size K and one firm of size $2K$.

6.1 Allocating Firms to Managers

Managers come up with business ideas. An idea is characterized by the capacity it requires, K_i. Each period the manager draws one idea at random from a continuous distribution $G(K)$. The function $G(\cdot)$ has bounded support $[K_L, K_H]$. The draws are independent from the amount of the manager's resources, across time and across managers.

If the idea is feasible given the manager's resources, $K_i \leq M_k$, the manager chooses whether to implement it and start a business or abandon it and wait for the next idea. If he chooses to implement the idea, he gains the present discounted value of profits $K_i\pi$, and starts the next period with fewer resources, $M_k - K_i$. If he decides to wait (or the idea is beyond his reach, $K_i > M_k$), he goes on to the next period with M_k resources. (Adding depreciation at a constant rate would not change the results.) The manager is risk neutral and discounts the future with discount factor β.

Given that there is no uncertainty about future profits, the lifetime value of a firm with capacity K_i is $K_i\pi/(1-\beta)$.

Consider a manager with resources M after he has drawn an idea K. The present discounted value of profits for this manager is

$$V(M,K) = \sum_{t=0}^{\infty} \beta^t \pi_t. \tag{9}$$

The Bellman equation characterizing the manager's search problem is

$$V(M,K) = \max\left\{K\frac{\pi}{1-\beta} + \beta EV(M-K,K'), \beta EV(M,K')\right\}, \tag{10}$$

with the constraint that the first option is only feasible if $M \geq K$. The next idea, K', is drawn from the distribution $G(K)$.

Obviously, if $M < K_L$, the manager cannot implement any ideas and his value is zero. Additionally, if $K = M$, the manager will implement the idea because waiting cannot bring higher profits. This pins down the value of $V(K,K)$,

$$V(K,K) = K\frac{\pi}{1-\beta}. \tag{11}$$

For a given M, "bigger" ideas are better, so $V_K \geq 0$.

The optimal waiting decision of the manager with resources M is characterized by a function $k(M) \in [K_L, M]$, such that he accepts all ideas with $K \geq k(M)$, and rejects all ideas below.

Proposition 4 *A manager with M amount of resources implements an idea of size K if and only if $k(M) \leq K \leq M$, where the function $k(M)$ is bounded between K_L and M, continuously differentiable, increasing, convex, and satisfies $k' > 1$.*

Proof. To be written. ■

6.2 Aggregation

There is a unit mass of managers, whose resources are distributed according to the distribution $H(M)$. The function $H(\cdot)$ is continuous and has a bounded support $[M_L, M_H]$.

We are interested in the size distribution of the first cohort of firms. This is obviously affected by the distribution of ideas. If all managers had resources $M > K_H$, all ideas would get implemented in the first period, and the size distribution of firms would just inherit the size distribution of ideas, G.

However, if some managers are constrained, they will only start smaller projects. This makes the firm size distribution more skewed, because there will be fewer big firms than big ideas, as some big ideas will hit managers without the resources to implement them.

A second effect is that resourceful managers may choose to ignore small ideas and wait for big ones. This makes the firm size distribution less skewed. At the extreme, if managers only implemented ideas completely exhausting their resources (although this is not an equilibrium for any $\beta < 1$), then the firm size distribution would inherit the resource distribution of managers, H.

In general, the firm size distribution will be a mixture distribution that can be characterized as follows.

Proposition 5 *The measure of firms with capacity x is*

$$m(x) = m_0 g(x) \left\{ H[k^{-1}(x)] - H(x) \right\}, \tag{12}$$

where m_0 is a positive constant such that

$$\int_{x=K_L}^{K_H} m(x)dx = 1.$$

Proof. To be written. ■

To illustrate how the firm size distribution can be more skewed than either of the underlying two distributions (ideas and resources), consider the following example.

Example 6 *Let G be Pareto with exponent γ_K, $G(x) = 1 - (x/K_L)^{-\gamma_K}$, and H be Pareto with exponent γ_M, $H(x) = 1 - (x/M_L)^{-\gamma_M}$. Suppose that managers are impatient enough to implement any feasible idea. Then the size distribution of firms is Pareto with exponent $\gamma_K + \gamma_M$.*

References

[1] Abbring, Jaap H. and Jeffrey R. Campbell. 2005. "A Firm's First Year," Working Paper, December.

[2] Arellano, Manuel and Stephen Bond. 1991. "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations," *Review of Economic Studies*, vol. 58, pp. 277-97.

[3] Atkeson, Andrew and Patrick J. Kehoe. 2005. "Modeling and Measuring Organization Capital," *Journal of Political Economy*. vol. 113, pp. 1026-53.

[4] Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes," *Science*, vol. 293 (September), pp. 1818-20.

[5] Bernard, Andrew B., Stephen Redding, and Peter K. Schott. 2005. "Importers, Exporters, and Multinationals: A Portrait of Firms in the U.S. that Trade Goods," Working paper. Dartmouth.

[6] Cabral, Luis M.B. and Jose Mata. 2003. "On the Evolution of the Firm Size Distribution: Facts and Theory," *The American Economic Review*, September, pp. 1075-1090.

[7] Gabaix, Xavier. 1999. "Zipf's Law for Cities: An Explanation," *Quarterly Journal of Economics*, vol. 114, pp. 739-67.

[8] Gabaix, Xavier. 2005. "The Granular Origins of Aggregate Fluctuations," Working paper. MIT.

[9] Hopenhayn, Hugo A. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, vol. 60, pp. 1127-50.

[10] Jovanovic, Boyan. 1982. "Selection and the Evolution of Industry," *Econometrica*, vol. 50, pp. 649-70.

[11] Klette, Tor Jakob and Samuel Kortum. 2004. "Innovating Firms and Aggregate Innovation," *Journal of Political Economy*.

[12] Lucas, Robert E. jr. 1978. "On the Size Distribution of Business Firms," *Bell Journal of Economics*, vol. 9, pp. 508-23.

[13] Luttmer, Erzo G. J. 2005. "Growth and the Size Distribution of Firms," Working paper. Minneapolis Fed.

[14] Pakes, Ariel and R. Ericson. 1998. "Empirical Implications of Alternative Models of Firm Dynamics," *Journal of Economic Theory*, vol. 79, pp. 1-45.

[15] Rossi-Hansberg, Esteban and Mark L.J. Wright. 2004. "Firm Size Dynamics in the Aggregate Economy," Working Paper.

[16] Sutton, John. 1997. "Gibrat's Legacy," *Journal of Economic Literature*, vol. 35 (March), pp. 40-59.