Career Concerns and Dynamic Arbitrage
Very preliminary

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Abstract

This paper analyzes the effect of career concerns on risky arbitrage. It presents an analytically tractable model where some fund managers can locate arbitrage opportunities, i.e., fundamentally very similar assets with a temporary difference in their price. Fund managers need operating capital from investors to exploit these opportunities. Investors judge the abilities of fund managers and decide whether to keep or fire them based on their past performance. Career concerns of fund managers distort their strategies which effect prices. In equilibrium, investors keep those arbitrageurs with higher probability who speculate on fast convergence of prices, even if the expected profit of this strategy is lower. As an effect, fund managers over invest in these strategies which increase the probability of liquidity crises: episodes with large price divergences and large aggregate losses of fund managers without any change in the fundamentals.

JEL classification: G10, G20, D5.

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1 Introduction

Trading strategies built on the price convergence of similar assets have been widely popular among hedge funds over the last decade. However, these strategies can lead to large losses when diverging prices force these hedge funds to unwind some of their positions. The spectacular near-collapse of the Long-Term Capital Management in 1998 is frequently cited as an example of this phenomenon.[1] In this paper, I will analyze the role of informational asymmetry between hedge funds and their investors

[1] For detailed analysis of the LTCM crisis see e.g. Edwards (1999), Loewenstein (2000), MacKenzie (2003). Although after the collapse of the LTCM many market participants made changes to their risk-management systems to avoid similar events, it is clear that financial markets are still prone to similar liquidity crises. A recent example is the turbulence in May 2005 connected to the price differential between General Motors stocks and bonds:

> "The big worry is that an LTCM-style disaster is occurring with hedge funds as they unwind GM debt/stock trade (a potential Dollars 100bn trade across the industry) at a loss, causing massive redemptions from convert arb funds, forcing them to unwind other trades, and so on, leading to a collapse of the debt markets and then all financial markets." (Financial Times, US Edition, May 23, 2005)

in the development of such episodes. In the model, investors hire or fire hedge fund managers after assessing their abilities by their past performance. Thus, fund managers will distort their strategies because of their career concerns. I show that the equilibrium price effect due to career concerns increase the probability of liquidity crises: episodes with large price divergences and large aggregate losses without any change in fundamentals.

I present an analytically tractable general equilibrium model of delegated risky arbitrage. The model is based on Kondor (2006). There are two main group of agents: fund managers and investors. Only some fund managers have the ability to locate windows of arbitrage opportunity. These fund managers are called arbitrageurs. In such a window, temporarily there is a gap between prices of two fundamentally identical assets traded in different markets. These windows are present only during a random time interval. After windows close, assets are traded at the same price. In effect, arbitrageurs bet on the time of the convergence of prices. When a window closes, another one opens involving a different pair of assets. Arbitrageurs need operating capital from investors to be able to exploit windows of arbitrage opportunity, because they have to collateralize their trades. For the operating capital, fund managers share the profit (or loss) with investors. The problem is that investors do not know which fund managers are arbitrageurs and which of them are quacks. Quacks cannot locate windows of arbitrage opportunity. They can only invest in negative expected-value gambles, but they can still mimic the performance of arbitrageurs in certain states. Investors observe the past performance of fund managers at the beginning of each window and decide which of them to hire, i.e., whom to provide with operating capital. The decision rule of investors will affect the strategies of arbitrageurs which in turn will determine the probability of liquidity crises.

I emphasize two main results. First, due to the informational asymmetry, investors will fire those arbitrageurs with higher probability who follow higher expected value strategies. Thus, in equilibrium investors' decision-rule provide implicit incentives to arbitrageurs *not to* maximize expected trading profit in a given window. This result is due to the fact that equilibrium expected profits of different strategies has to adjust in a way to make arbitrageurs indifferent between equilibrium strategies. Otherwise, dominated strategies would not be followed. The full expected benefit of following a particular strategy is the sum of the trading profit in the given window and future trading profits weighted by the chance that the arbitrageur can keep her job. It turns out that betting on early convergence gives a better chance of being successful, because short windows happen more frequently. Thus, following such strategy provide a better chance for the arbitrageur to signal her abilities and to keep her job. Arbitrageurs will keep choosing only this strategy over others as long as its expected profit is not reduced to equalize the full benefit of this strategy and other strategies. Interestingly, the resulting negative relationship between the *expected* profit of an arbitrageur and the chance that she gets capital for the next window as well, is perfectly consistent with the positive relationship between the *realized* profit of the arbitrageur and the chance of continuation. The latter one is the well documented positive capital flow-performance relationship of fund managers (see Chevalier and Ellison, 1997, Agarwal et al., 2004)

As a second main result, I show that career concerns of fund managers increase the probability of large liquidity crises. It is a result of the distorted implicit incentives provided by the decision rule of investors. Because betting on early time of convergence provides a better chance for the arbitrageur to keeping her job, in equilibrium more arbitrageurs will follow such strategy. In effect, arbitrageurs will overinvest in early stages of the window. Its effect is two fold. Because of career concerns, the gap is typically lower when the window is short and this is a high probability event. So in this sense, most of the time career concerns helps efficiency. However, time to time the window happens to be relatively long. In this case there will not be enough arbitrageurs with liquid capital who could stop the gap from a substantial increase. Thus, career concerns increase the magnitude of a liquidity crisis of given probability and – equivalently – increase the probability of a liquidity crisis of the same magnitude.

To my knowledge, this model is the first attempt to incorporate the agency problem of delegated portfolio management in a general equilibrium framework of risky arbitrage. Relatedly, this is also the first analytical model to show that career concerns increase the chance of liquidity crises. This model provides the natural bridge between two streams of the literature. First, it builds on the results of general equilibrium models of risky arbitrage (e.g., Gromb and Vayanos, 2002, Zigrand, 2004, Xiong, 2001, Kyle and Xiong, 2001, Basak and Croitoru, 2000). Second, it is naturally connected to models which analyze the effect of delegated portfolio management on traders' decisions and asset prices in general (e.g. Dow and Gorton, 1997, Allen and Gorton, 1993, Cuoco and Kaniel, 2001, Vayanos, 2003, Berk and Green, 2004, Gümbel, 2005, Dasgupta and Prat, 2005, 2006). None of these models consider the effect of career concerns on the activity of convergence traders and the relative prices of similar assets which is in our focus.

The closest paper to this work in spirit is the seminal paper of Shleifer and Vishny (1997) on limits of arbitrage. They also emphasize the effect of informational asymmetry between investors and arbitrageurs to arbitrageurs' activity and the price gap. They use a two period, representative agent framework where investors cut back the operating capital of arbitrageurs by an exogenously given fraction if arbitrageurs suffer trading losses. They show that due to this exogenous decision rule related to asymmetric information, the gap will always be higher than in the full information case. Furthermore, if the gap increases in the first period, investors cut back the operating capital of arbitrageurs more, so arbitrageurs can invest less in the second period exactly when the gap is higher and the market is more profitable. In contrast, in our model where arbitrageurs can follow heterogenous timing strategies, the picture is more differentiated. Most of the time asymmetric information induce arbitrageurs to keep the gap at a lower level, but for the price of larger liquidity crises in the small probability cases. Furthermore, the mechanism behind this result is not based on the direct effect of capital withdrawal of investors, but it is due to arbitrageurs' incentives to signal their abilities better by betting on fast convergence.

The structure of the rest of the paper is as follows. In the next section I introduce the model. In section 3, I present first the benchmark equilibrium without career concerns which is followed by the presentation of equilibrium with career concerns. In section 4, I compare the two equilibria and discuss the results. Finally, I conclude.

2 A simple model of arbitrage and career concerns

There are three main groups of agents in this model: local traders, fund managers and investors. Differences in the demand functions of local traders in different markets create arbitrage opportunities. Fund managers attempt to locate and exploit arbitrage opportunities. Investors hire fund managers, provide them with operating capital and fire them if they are not satisfied with their performance. All agents are small and take prices as given. Time is continuous and infinite. Now, I present each group in detail.

2.1 Local traders and a window of arbitrage opportunity

Each arbitrage opportunity is based on two risky assets with the same cash flows: the A-asset and the B-asset. The only difference between them that they are traded in different local markets: the A-market and the B-market respectively. Local traders are divided in two subgroups. A-traders trade only in island A, while B traders trade only in island B. Local traders are identical across the two markets, except that they are subject to asymmetric and temporary demand shocks. Thus, there is an interval of random length when the demand curves of local traders across the two markets differ. This is the window of arbitrage opportunity. If local markets cleared separately, there would be a gap g^* between the prices of the two assets. In each time point the window closes, i.e., the asymmetry disappears, with the equal probability δ given that the window is still open at that time point. Consequently, $\tilde{t}$, the random closing time of the window is distributed exponentially.

Arbitrageurs, introduced in the next part, can reduce the size of the gap at time t during the window by buying $x(t)$ units of the cheap asset and selling $x(t)$ units of the expensive asset. Preferences and optimal decision of local traders is summarized in the inverse demand function that arbitrageurs face,

$$g(t) = f(\overline{x}(t)),$$

which shows that if the window is still open and arbitrageurs take opposite positions of the aggregate size of $\overline{x}(t)$ than the gap between the asset prices is reduced to $g(t)$. Note, that $g(t)$ stands for the size of the gap conditional on $t < \tilde{t}$, i.e., on the window being still open. This characterizes the unconditional price pattern as well, as the unconditional size of the gap is

$$\tilde{g}(t) = \left\{ \begin{array}{ccc} g(t) & w.p. & e^{-\delta t} \\ 0 & w.p. & 1 - e^{-\delta t} \end{array} \right\}.$$

In terms of the function $f(\cdot)$, the autarchy price gap, the gap when arbitrageurs are inactive is defined by

$$g(t) = g^* = f(0).$$

Thus, if arbitrageurs are not present the gap is constant during the window and disappears when the window closes. It is a one-sided bet for any entering arbitrageur. I further assume that $f(\cdot)$ is continuous, monotonically decreasing and there is a position $\overline{x}^{\max} > 0$ that

$$0 = f\left(\overline{x}^{\max}\right).$$

If arbitrageurs keep selling more of the expensive asset and buying more of the cheap asset, the gap can be eliminated.

Our modelled world is full of similar A and B market pairs. Thus, similar windows of arbitrage opportunities are keep popping up and closing. In particular, I assume that in each time point there is exactly one open window with the same structure as the previous ones. As a matter of notation, the time, t, shows the time since the beginning of the given window. Values of variables in different windows will be denoted by the subscript s for window number s, but only when distinction among variables in different windows is necessary.

2.2 Investors: the supply of capital

There is an infinite measure of potential investors. Each of them has a unit of capital to invest, but none of them has the expertise to locate windows of arbitrage opportunities. This is why they hire fund managers. Each investor can hire only one fund manager at the beginning of a given window. Investors do not have the possibility to fire or hire investors during the windows.[2] They can only reconsider their decision when the window closes. Investors choose to hire the fund manager which looks the most able of those who are available. To judge this, they have access to the performance record of fund managers during the last window if the fund manager was hired then. The assumption that investors do not keep records of the performance of arbitrageurs during several windows is made for simplicity. Thus, an investor who hired a fund manager in window $s-1$ and observed the performance of hired fund mangers, π_{s-1}, can choose an action, $\sigma_s(\pi_{s-1})$, of three possibilities in window s. She either keeps her existing fund manager ($\sigma_s(\pi_{s-1}) = K$) or hires the best available one ($\sigma_s(\pi_{s-1}) = H$) or stays out of the market ($\sigma_s(\pi_{s-1}) = O$). If an investor stayed out of the market in window $s-1$, she can choose only $\sigma_s(\pi_{s-1}) = H, O$.

Investors are risk-neutral. The contract between fund managers and investors is exogenously fixed: fund managers keep a γ share of their gross profit during their employment while $(1-\gamma)$ share goes to the investors.[3] Hence, there is no mismatch between the explicit incentives of fund managers and the preferences of investors. I make this assumption to be able to concentrate on the distortions from the implicit incentives: career concerns of fund managers.

Each investors face with a participation constraint. A potential investor hires a fund manager only

[2] The assumption that investors cannot reconsider their decision of providing capital for a fund manager anytime is consistent with the fact that the investors of hedge funds can usually withdraw their capital only after an initial lock-up period and after advance notice (see Agarwal et al., 2004).

[3] Unlike mutual fund managers, hedge funds do get a share of their profit as an incentive fee. This feature is consistent with our assumption of proportional profit sharing, even if real world hedge fund contracts are more complicated (see Agarwal et al., 2004).

if she can find one with gross profit π_s in window s who is available and

$$(1-\gamma)\,\Sigma_{s=s^+}^{\infty} \Pr\left(\bigcap_{u=s^+}^{s} \sigma_u\left(\pi_{u-1}\right)=K|\pi_s\right) E\left(\pi_s|\pi_{s^+-1}\right)\geq 1. \tag{1}$$

The left hand side shows the investor's expected share of future profit if she hires the fund manager. The term in the bracket is the probability that the fund manager is kept until the future window of s. This term shows that the investor takes into account the option value of keeping the fund manager. Investors will participate only if they at least break even in expectation. Thus, the supply of aggregate capital is infinite if the expected rate of return or gross profit is larger than 1 while it is 0 if it is lower than 1.

2.3 Fund managers

There is an infinite measure of potential fund managers. Only those become active fund managers who are hired by investors. Each fund manager can be hired by a single investor at the beginning of a window.

Only a fixed proportion, $\alpha \in (0,1)$, of the active fund managers can locate the windows of arbitrage opportunity. I will call these fund managers *arbitrageurs*. Arbitrageurs instantly find the new open window when the previous one has closed. Arbitrageurs need operating capital because they are required to fully collateralize their potential losses. For example, if their largest potential loss on an invested unit in the next time instant is $\dot{g}(t)$, to take a position of the size $x(t)=\frac{v(t)}{\dot{g}(t)}$, an arbitrageur has to be able to present $v(t)$ cash, i.e., deposit $v(t)$ on a margin account as the maximal possible loss on each unit is $\dot{g}(t)$. This assumption can also be regarded as the formalization of endogenous margin requirements or VaR constraints. Putting it another way, the assumption requires that the marked-to-market value of arbitrageurs portfolios together with their capital always has to be non-negative.

Because of the nature of the arbitrage opportunity, in effect arbitrageurs have a timing problem. If they invest too much before the time of the convergence and the gap temporarily widens, they might lose their operating capital and be out of the market without making any profit. To simplify the analysis and to highlight the timing nature of the decision of arbitrageurs, their available strategies are restricted. Each arbitrageur can invest only in a single time-point during a window when she takes a maximal position. If she chooses time u, $x(t)=0$ for $t<u$ and $t>u$ and, if the maximal loss at time u is $\dot{g}(u)$, then $x(u)=\frac{1}{\dot{g}(u)}$. Effectively, arbitrageurs bet only on the time of the convergence. However, I allow for mixed strategies. Each arbitrageur's strategy is described by a density function $\{\mu(t)\}_{t=0}^{\infty}$ where $\int_{t_1}^{t_1+\Delta}\mu(t)\,dt$ is the probability that this arbitrageur takes a maximal position at a time $t\in[t_1,t_1+\Delta]$. Consequently, the expected gross profit of an arbitrageur in window s is

$$E\left(\pi_s\right)=\int_0^{\infty}\delta e^{-\delta t}\mu(t)\left(\frac{g(t)}{\dot{g}(t)}+1-\int_0^{t}\mu(u)\,du\right)dt$$

where $\delta e^{-\delta t}$ is the probability that the window closes at t, $\frac{g(t)}{\dot{g}(t)}$ is his net profit and the term $1-\int_0^t \mu(u)\,du$ is the expected capital which is not spent by time t. The full expected gross profit of an arbitrageur at the beginning of window s consists of the γ fraction of the sum of $E(\pi_s)$ and expected profits from future windows weighted by the probability that she will be hired then.

The rest of fund managers, a fraction of $1-\alpha$, cannot locate any arbitrage opportunity. I will call these fund managers *quacks*. If a quack is hired, he can only invest in a negative value gamble which leads to 0 net profit with probability β, and to a loss of all the unit capital with probability $(1-\beta)$ by the end of the window. Being hired and making this investment gives him a private benefit of B where

$$B + \beta\gamma v_0 > \gamma v_0 > 0.$$

Thus, even a quack prefers to be hired and if he is hired, he prefers to be active and to invest in the negative value gamble which he finds. The idea of the private benefit is that playing the role of an active fund manager during the window and pooling with the unlucky arbitrageurs when the window is closed feels good for a quack.

3 Equilibrium

As the effects of career concerns of arbitrageurs is in the focus of my analysis, I proceed as follows. First, I present the equilibrium concept. Then, as a benchmark case, I present the equilibrium without career concerns. Here, arbitrageurs maximize their expected profit in each window separately without considering the effect of their decision on their career prospects. This case coincides to the equilibrium of the model presented in Kondor (2006). Then I present the equilibrium with career concerns. In section 4, I discuss the properties of the equilibrium with career concerns and compare two cases.

3.1 Equilibrium concept

Our equilibrium concept will combine elements of the Rational Expectation Equilibrium and the Perfect Bayesian Equilibrium. We need this combination because our agents are price-takers, the framework is dynamic and fund managers have asymmetric information about their own type in the same time.

The equilibrium will be characterized for each window $s = 1, 2...$ by the conditional gap path $\{g_s(t)\}_{t=0}^{\infty}$, the aggregate strategy profile of arbitrageurs $\{\overline{\mu}_s(t)\}_{t=0}^{\infty}$, the aggregate measure of arbitrageurs who are hired Γ_s and the beliefs of investors about the type of the available fund managers given by $\Pr(A|\pi_{s-1})$, the probability of the given fund manager being an arbitrageur given her performance in the last window if she was hired.

In equilibrium for each window s the following conditions hold.

1. For given beliefs $\Pr(A|\pi_{s-1})$ and gap $\{g_s(t)\}_{t=0}^{\infty}$

(a) hired arbitrageurs' strategy is optimal in each t and

(b) at the beginning of each window, investors hire and fire fund managers optimally.

2. The given prices $\{g_s(t)\}_{t=0}^{\infty}$ and the implied expected rate of return of hiring an available fund manager

   (a) clears the arbitrage market in each t, i.e., $f\left(\frac{\overline{\mu}_s(t)}{\dot{g}_s(t)}\right) = g_s(t)$,

   (b) clears the capital market, i.e., fund managers hire exactly Γ_s measure of arbitrageurs.

3. Beliefs, $\Pr(A|\pi_{s-1})$, are consistent with the optimal strategies of fund managers and Bayes' Rule.

To simplify notation, I will omit the subscript s when distinction between variables in different windows is not necessary.

3.2 Equilibrium without career concerns

For this part only, let us suppose that fund managers' reemployment is independent of their performance. In particular, let us suppose that they are rehired in the new window with a fixed probability of $q \in [0, 1)$. For reasons specified below, the parameters of the model are restricted to satisfy

$$\frac{1-q}{1-\gamma} > (1-\alpha)\beta - \alpha. \tag{2}$$

As arbitrageurs receive a fixed proportion of the profit, without career concerns each hired arbitrageur maximize her expected profit during each window and solve the following problem

$$\begin{aligned} J(v(0)) &= \max_{\mu(t)} \int_0^\infty \delta e^{-\delta t} \mu(t) \left(\frac{g(t)}{\dot{g}(t)} + v(t) \right) dt \\ s.t. \qquad \dot{v}(t) &= -\mu(t), \quad v(0) = 1 \end{aligned} \tag{3}$$

where $v(t)$ is the expected capital not lost by period t. The solutions of this problem are given by the differential equations

$$\delta \frac{g^b(t)}{\dot{g}^b(t)} = J^b(v(t)) \tag{4}$$

$$\dot{J}^b(v(t)) = \delta J^b(v(t)) - \delta \tag{5}$$

where the superscript b denotes the equilibrium values in this benchmark case with no career concerns. The system gives the general solution of

$$J^b(v(t)) = J'^b(v(t))\,1 = 1 + c^b e^{\delta t} \tag{6}$$

$$g^b(t) = g_\infty \frac{c^b e^{\delta t}}{1 + c^b e^{\delta t}} \tag{7}$$

where I used the fact that $J^b(v(t))$ is linear, c^b is an arbitrary positive constant at this point while g_∞ is the limit of the path $\{g^b(t)\}_{t=0}^{\infty}$ as t increases without bound. Remember, that $g^b(t)$ is the size of the gap conditional on $t < \tilde{t}$, i.e., the gap if the window is still open. Along this path (7) arbitrageurs are indifferent when to make their investments. As they are indifferent, they are happy to follow any mixed strategy $\{\mu^b(t)\}_{t=0}^{\infty}$. In equilibrium the individual densities $\{\mu^b(t)\}_{t=0}^{\infty}$ are indeterminate, but the aggregate measure of arbitrageurs who choose to invest at each t, $\{\overline{\mu}^b(t)\}_{t=0}^{\infty}$, has to support the equilibrium gap path (7), i.e., has to be consistent with market clearing conditions

$$g^b(t) = f\left(\frac{\overline{\mu}^b(t)}{\dot{g}^b(t)}\right)$$

for all t. Thus a given c^b and g_∞ pins down the conditional gap path which determines $\{\overline{\mu}^b(t)\}_{t=0}^{\infty}$. This in turn gives the aggregate measure of arbitrageurs who are hired, Γ^b, by the condition

$$\int_0^\infty \overline{\mu}^b(t)\,dt = \int_0^\infty f^{-1}\left(g^b(t)\right)\dot{g}^b(t)\,dt = \Gamma^b,$$

as individual densities has to integrate to one. The expected gross profit made by each hired arbitrageur in each window is given by the value function at $t = 0$ by definition. From (6), it is simply

$$J^b(v(0)) = J'^b(v(0))\,1 = 1 + c^b.$$

As the investment is one unit, c^b is the expected net profit produced by a hired arbitrageur. Thus, the expected share of profit of an investor who hires a fund manager of unknown type is

$$(1-\gamma)\frac{1}{1-q}\left(\alpha\left(1+c^b\right)+(1-\alpha)\beta\right)$$

, because the fund manager is a quack with probability $(1-\alpha)$ who produces a gross return of β and each fund manager is rehired with probability q regardless of her performance Given the participation constraint (1) and the free entry of investors, the equilibrium rate of c^b is

$$c^b = \frac{1}{\alpha}\left(\frac{1-q}{1-\gamma} - (1-\alpha)\beta - \alpha\right).$$

Condition (2) serves the only purpose to ensure that $c^b > 0$.

The only undetermined variable is the level of g_∞, the limit of the conditional gap path. In Kondor (2006), I argued that in any equilibrium which is robust to the small perturbation of the inclusion of arbitrarily small holding costs, $m > 0$, g_∞ must be g^*. The reason is that in this perturbed version, in any other case arbitrageurs in the aggregate will pay a non-diminishing cost for arbitrarily long time which is inconsistent with their limited capital. Formally, the aggregate capital of arbitrageurs to support a gap path $g(t)$ if $m > 0$,

$$\int_0^\infty f^{-1}(g(t))(\dot{g}(t)+m)\,dt,$$

converges to a finite level only if $g(t) \to g^*$. Here, I use the same equilibrium selection mechanism which leaves us with our equilibrium summarized in the next theorem.

Theorem 1 *If fund managers do not have career concerns, because they are rehired with a constant probability q regardless of their performance, the unique robust equilibrium is given by the monotonically increasing conditional gap path*

$$g^b(t) = g^* \frac{c^b e^{\delta t}}{1 + c^b e^{\delta t}}$$

and aggregate measure of arbitrageurs investing at t

$$\overline{\mu}^b(t) = f^{-1}\left(g^b(t)\right)\dot{g}^b(t)$$

for all $t < \tilde{t}$, *where*

$$c^b = \frac{1}{\alpha}\left(\frac{1-q}{1-\gamma} - (1-\alpha)\beta - \alpha\right).$$

Proof. The proof is a simplified version of the proof of the next theorem with straightforward modifications, so it is omitted. ■

The solid curve on Figure 1 shows the conditional gap path, $\left\{g^b(t)\right\}_{t=0}^{\infty}$, in the equilibrium without career concerns. The path is monotonically increasing. It shows that each time when the window remains open, the gap increases and the average arbitrageur suffer losses. The intuition behind this result is given by the indifference condition which determines the equilibrium gap path. Arbitrageurs have to be indifferent when they invest. The increasing conditional gap path implies higher reward for those arbitrageurs who are betting on a latter time of convergence. This higher reward if they are successful, compensate them for the larger risk that the window closes earlier and they miss out on the arbitrage opportunity. In Kondor (2006), I emphasized this result. I highlighted the sharp contrast with the autarchy case. When arbitrageurs are not present, the gap can never widen and the first arbitrageur could make a safe bet if she did not effect prices. By the price effect of their trades, arbitrageurs create their own losses even if their strategies are individually optimal. When the window happens to be long, the aggregate loss of arbitrageurs is large and the gap is high. This event is a liquidity crisis similar to the LTCM-crisis in September 1998. Although the fundamentals are unchanged, the level of liquid capital of arbitrageurs shrinks and the average arbitrageur liquidates her position and suffers losses.

Although the conditional gap path will be increasing in the equilibrium with career concerns as well, the equilibrium will change in a systematic way. These effects of career concerns is in the focus

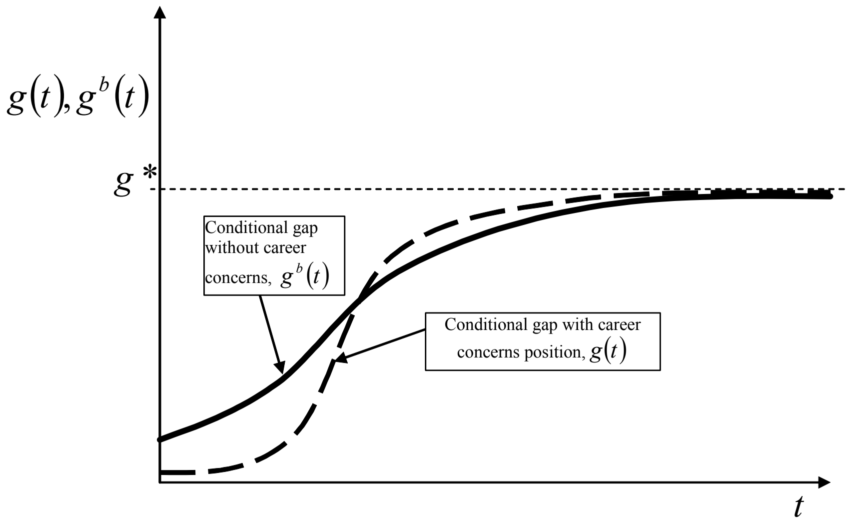

Figure 1: The curves show the conditional gap paths, $\{g(t)\}_{t=0}^{\infty}, \{g^b(t)\}_{t=0}^{\infty}$, in the benchmark equilibrium (solid line) and when career concerns are present (dashed line). The curves are always monotonically increasing and approach to g^*. They cross exactly once as depicted, if the difference in expected returns between the two equlibria, $c - c^b$, is sufficiently small.

of our analysis.

3.3 Equilibrium with career concerns

Now, we return to our original set up with career concerns.

In the candidate equilibrium, all possible profit levels are observed in some states of the nature in each window. For any time t, there is a positive measure of arbitrageurs who bet on convergence at that given t. Furthermore, some fund managers always realize 0 net profit while others lose their whole unit of capital. As none of the possible strategies of fund managers would lead to any other outcome, we do not have to consider out-of-equilibrium beliefs.

In this equilibrium, investors keep those arbitrageurs who manage to guess the time of the convergence right and fire all the others and the quacks. Thus, an arbitrageur maximizes the expected profit over her full time of employment and solves the following problem in each window when she is hired

$$\begin{aligned} J(v(0)) &= \max_{\mu(t)} \int_0^\infty \delta e^{-\delta t} \mu(t) \left(\frac{g(t)}{\dot{g}(t)} + v(t) + J(v(0)) \right) dt \\ s.t. \qquad \dot{v}(t) &= -\mu(t), \quad v(0) = 1. \end{aligned} \tag{8}$$

The difference between problems (3) and (8) is the presence of $J(v(0))$ in the brackets in (8). It shows that if the window closes at time t and the arbitrageur bet on the convergence at t, then she not only gets her share of the monetary profit, $\frac{g(t)}{\dot{g}(t)}$, and her remaining capital $v(t)$, but also gets the right to participate again in the next window. Note, that the value function $J(v(0))$ is the same in the present window (left hand side of (8)) and in the next window (right hand side of (8)). As we will see, the reason is that the profitability of each window is the same in this equilibrium.

We proceed similarly to the benchmark case. The development of the marginal value function, $J'(v_0)$ of (8) is given by the equation

$$\dot{J}'(v(t)) = \delta J'(v(t)) - \delta \tag{9}$$

which has the solution of

$$J'(v(t)) = 1 + ce^{\delta t}.$$

Thus, the expected net rate of return is c as the expected profit in the arbitrage sector with one unit of capital is given by

$$J(v(0)) = J'(v(0))\,1 = 1 + c. \tag{10}$$

Note, that c is the expected rate of return of an arbitrageur during the full span of her employment. This is in contrast to c^b which is the expected rate of return in each window in the benchmark case.

After the substituting (10), problem (8) implies the equation

$$\delta\left(\frac{g(t)}{\dot{g}(t)} + (1+c)\right) = 1 + ce^{\delta t} \tag{11}$$

for the development of $g(t)$ with the general solution of

$$g(t) = g_\infty \left(\frac{ce^{\delta t}}{1 + ce^{\delta t} - \delta(1+c)}\right)^{\frac{1}{1-\delta(1+c)}} \tag{12}$$

where g_∞ is the limit of the conditional gap path $g(t)$.

Just as in the benchmark case, along this conditional gap path, arbitrageurs are indifferent when to invest. The individual mixed strategies $\{\mu(t)\}_{t=0}^{\infty}$ are indeterminate but the aggregate measure of arbitrageurs who are choosing to bet on the convergence of the gap at each t, $\{\overline{\mu}(t)\}_{t=0}^{\infty}$ is given by the market clearing condition

$$g(t) = f\left(\frac{\overline{\mu}(t)}{\dot{g}(t)}\right). \tag{13}$$

Note, that arbitrageurs are indifferent between available strategies only after taking into account the implications of different strategies on their career prospects. As we will see, this fact will be critical for the results.

Now let us turn for the problem of investors. First, we check which fund manager they hire if they hire any. Then we turn to investors' participation decision.

Given that quacks always invest in the gamble they find and given the aggregate strategy of arbitrageurs, $\{\overline{\mu}(t)\}_{t=0}^{\infty}$ investors can update their probability assessment about the type of their fund manager by Bayes Rule. Each investor will choose a fund manager who is an arbitrageur with the largest probability among the available ones. It is clear that investors never hire a fund manager instantly who was fired by an other investor. The reason is that all investors have the same information, so if one of them decides against keeping a fund manager, all of them would do the same. Thus, we only have to check how investors who were active in the last window decide between keeping their existing fund managers or hiring an unexperienced one.

An active fund manager will experience one of the following three cases. Her fund manager either made a profit or broke even or lost her whole unit of capital. Only arbitrageurs can make a positive net profit, so the probability that the fund manager making positive profit is an arbitrageur is 1. As an unexperienced fund manager is an arbitrageur only with probability α, any investor will keep her fund manager who made positive net profit. The probability that a fund manager is an arbitrageur if she brakes even and the window closes at $\tilde{t}$ is

$$\Pr(A|\pi_{s-1}=1)=\frac{\Pr(\pi_{s-1}=1|A)\Pr(A)}{\Pr(\pi_{s-1}=1|A)\Pr(A)+\Pr(\pi_{s-1}=1|Q)\Pr(Q)}=\frac{\left(1-\frac{1}{\Gamma}\int_0^{\tilde{t}}\overline{\mu}(t)\,dt\right)\alpha}{\left(1-\frac{1}{\Gamma}\int_0^{\tilde{t}}\overline{\mu}(t)\,dt\right)\alpha+\beta(1-\alpha)}$$

where $\Gamma=\int_0^{\infty}\overline{\mu}(t)\,dt$ is the measure of all arbitrageurs on the market and A and Q are the events that the currently hired fund manager is an arbitrageur and a quack respectively. The term in the brackets in the nominator is the proportion of arbitrageurs who bet on a time of convergence $t>\tilde{t}$. Similarly, the probability that a fund manager is an arbitrageur if she loses her whole unit of capital and the window closes at $\tilde{t}$ is

$$\Pr(A|\pi_{s-1}=0)=\frac{\Pr(\pi_{s-1}=0|A)\Pr(A)}{\Pr(\pi_{s-1}=0|A)\Pr(A)+\Pr(\pi_{s-1}=0|Q)\Pr(Q)}=\frac{\alpha\frac{1}{\Gamma}\int_0^{\tilde{t}}\overline{\mu}(t)\,dt}{\alpha\frac{1}{\Gamma}\int_0^{\tilde{t}}\overline{\mu}(t)\,dt+(1-\beta)(1-\alpha)}.$$

The term $\frac{1}{\Gamma}\int_0^{\tilde{t}}\overline{\mu}(t)$ is the fraction of arbitrageurs who bet on convergence before time $\tilde{t}$. Note, that as $0<\int_0^{\tilde{t}}\overline{\mu}(t)<1$ both

$$\Pr(A|\pi_{s-1}=1),\Pr(A|\pi_{s-1}=0)<\alpha.$$

Consequently, if a fund manager cannot produce positive net profit, the investor will prefer to hire an unexperienced fund manager instead. Arbitrageurs can keep their job only as long as they guess the time of the convergence right.[4] This is consistent with problem (8) of arbitrageurs.

The participation decision of arbitrageurs will determine the supply of capital. An investor whose

[4]In this model, arbitrageurs are not able to build enough reputation which would keep them employed if they are unsuccesful in one window. This is clearly an artifact of our assumption that investors use only the fund managers' last-window performance to judge their type. Although it is a strong assumption, I will argue later that the intuition of the main results would go through without this assumption as well.

fund manager produced positive net return in window $s-1$ will decide to participate in the market if the inequality

$$1 \leq (1-\gamma)(1+c)\,. \tag{14}$$

holds. This is participation constraint (1) given the equilibrium strategy of investors and arbitrageurs. Investors who decided to hire an unexperienced fund manager face with the participation constraint of

$$1 \leq (1-\gamma)(\alpha(1+c)+(1-\alpha)\beta)\,. \tag{15}$$

The two constraints are different because unexperienced fund managers might turn out to be quacks. Observe, that (15) implies (14), but the opposite is not true. This is very intuitive. If any unexperienced fund manager is hired than successful arbitrageurs are hired for sure.

The aggregate demand for capital is implied by the decision-rule of fund managers. It has two parts. Quacks will provide a flat return of β regardless of the aggregate capital which they get. The demand for capital by the arbitrage sector is given by

$$\kappa^{A}(c) \equiv \int_{0}^{\infty} f^{-1}(g(t))\,\dot{g}(t)\,dt = \int_{0}^{\infty} \overline{\mu}(t)\,dt = \Gamma \tag{16}$$

where $g(t) = g_{\infty}\left(\frac{ce^{\delta t}}{1+ce^{\delta t}-\delta(1+c)}\right)^{\frac{1}{1-\delta(1+c)}}$. This equation can be seen as a capital demand function, because Γ is not only the measure of arbitrageurs who provide an expected gross return of $1+c$, but also the size of the aggregate capital of arbitrageurs corresponding to this return. The next lemma shows that there is a negative relationship between c and Γ. If more arbitrageurs enter, the rate of return on the arbitrage market goes down.

Lemma 1 *The aggregate capital demand function of the arbitrage sector,*

$$\kappa^{A}(c) \equiv \int_{0}^{\infty} f^{-1}(g(t))\,\frac{\partial g(t)}{\partial t}dt = \Gamma$$

where $g(t) = g_{\infty}\left(\frac{ce^{\delta t}}{1+ce^{\delta t}-\delta(1+c)}\right)^{\frac{1}{1-\delta(1+c)}}$ *is downward sloping, i.e.,* $\frac{\partial \kappa^{A}(c)}{\partial c} < 0$.

Proof. The proof is in the appendix. ■

We use the downward sloping capital demand function to argue that in equilibrium some unexperienced fund managers are always hired. Thus, the equilibrium rate of return on the arbitrage market, c, is

$$c = \frac{1}{\alpha}\left(\frac{1}{(1-\gamma)} - (1-\alpha)\beta - \alpha\right)$$

given by (15). This is shown in the next lemma.

Lemma 2 *In equilibrium, (14) holds as a strict inequality and the participation constraint (15) is binding.*

Proof. The proof is in the appendix. ■

Because c does not change across windows, in all windows the expected profit of arbitrageurs is the same, just as we supposed in the derivation of the equilibrium strategies of arbitrageurs. Note, that

$$c - c^b = \frac{q}{\alpha(1-\gamma)} \geq 0. \tag{17}$$

The only undetermined variable is g_∞. Just as in the benchmark case, the equilibrium is required to be robust for the small perturbation of the presence of holding cost, so $g_\infty = g^*$.

I summarize the properties of the equilibrium in the following Theorem.

Theorem 2 *There is an equilibrium where in each window, the monotonically increasing conditional gap path is given by*

$$g(t) = g^* \left(\frac{ce^{\delta t}}{1 + ce^{\delta t} - \delta(1+c)} \right)^{\frac{1}{1-\delta(1+c)}} \tag{18}$$

where

$$c = \frac{1}{\alpha}\left(\frac{1}{(1-\gamma)} - (1-\alpha)\beta - \alpha \right). \tag{19}$$

Furthermore, the equilibrium aggregate strategies are given by

$$\overline{\mu}(t) = f^{-1}(g(t))\,\dot{g}(t) \tag{20}$$

for all $t < \tilde{t}$. The measure of arbitrageurs who are hired in each window is

$$\Gamma = \int_0^\infty \overline{\mu}(t)\,dt. \tag{21}$$

Beliefs of investors whose fund manager made a gross profit of π_{s-1} are

$$\Pr(A|\pi_{s-1}) = \left\{ \begin{array}{ccc} \frac{\frac{1}{\Gamma}\int_0^{\tilde{t}} \overline{\mu}(t)dt\alpha}{\frac{1}{\Gamma}\int_0^{\tilde{t}} \overline{\mu}(t)dt\alpha + (1-\beta)(1-\alpha)} & if & \pi_{s-1} = 0 \\ \frac{\left(1-\frac{1}{\Gamma}\int_0^{\tilde{t}} \overline{\mu}(t)dt\right)\alpha}{\left(1-\frac{1}{\Gamma}\int_0^{\tilde{t}} \overline{\mu}(t)dt\right)\alpha + \beta(1-\alpha)} & if & \pi_{s-1} = 1 \\ 1 & if & \pi_{s-1} > 1 \end{array} \right\}. \tag{22}$$

and investors keep their fund managers only if $\pi_{s-1} > 1$. Otherwise they will be indifferent whether to hire an unexperienced fund manager or stay out of the market. If the previous window closed at $\tilde{t}$, then $\frac{\Gamma - \tilde{\mu}(\tilde{t})}{\alpha}$ measure of arbitrageurs will decide to enter.

Proof. Proof is in the appendix. ■

The dashed line in Figure 1 shows the main properties of the equilibrium conditional gap path, $\{g(t)\}_{t=0}^{\infty}$. It is apparent that this gap path is also monotonically increasing, so arbitrageurs create losses in this case as well. However, the two paths do not coincide. In the case depicted in Figure 1, career concerns imply a lower gap in the early stages of the window and higher gap if the window lasts sufficiently long. The intuition behind this fact and the implications are discussed in the next part.

4 Discussion

Although the gap paths in the equilibria with and without career concerns looks similar, career concerns result in systematic distortions in individual strategies and the distribution of returns. I will concentrate on two important implications. First, I show that in equilibrium arbitrageurs who follow strategies with high expected profit are fired with larger probability. Thus, higher expected returns are penalized by the market for capital. Second, I show that if we compare the equilibrium with career concerns to the equilibrium with no career concerns but a similar expected rate of return, liquidity crises of the same magnitude happen more frequently when career concerns are present.

To facilitate the discussion, let us fix an arbitrary interval length $\Delta > 0$ and consider the case when arbitrageurs follow atomistic strategies defined as follows.

Definition 1 *An arbitrageur follows the t_1* ***atomistic strategy****, if she chooses a particular time t_1 and follows the mixed strategy where $\mu(t) = \frac{1}{\Delta}$ if $t \in [t_1, t_1 + \Delta]$ and $\mu(t) = 0$ otherwise.*

In this case each arbitrageur chooses a $\Delta-$interval and follows a mixed strategy with equal weights on each point of this interval. If the window closes before this interval, the arbitrageur will miss out on the arbitrage opportunity while if the window closes after $t_1 + \Delta$, she loses all of her capital. If the window closes within the interval, the arbitrageur makes positive net profit with positive probability. In particular, the expected profit in a window s of an arbitrageur following a t_1 atomistic strategy conditional on the closing time of the window $\tilde{t}$, $\pi_{s,t_1}(\tilde{t})$, is

$$\pi_{s,t_1}(\tilde{t}) \equiv E\left(\pi_s | \tilde{t}, t_1\right) = \left\{ \begin{array}{ccc} 1 & if & \tilde{t} < t_1 \\ \frac{1}{\Delta}\frac{g(\tilde{t})}{\dot{g}(\tilde{t})} + \frac{t_1+\Delta-\tilde{t}}{\Delta} & if & \tilde{t} \in [t_1, t_1+\Delta] \\ 0 & if & \tilde{t} > t_1 + \Delta \end{array} \right\}.$$

Note, that any mixed strategy $\mu(t)$ can be constructed from the building blocks of atomistic strategies, if Δ is small enough. Because individual strategies are undetermined in equilibrium, atomistic strategies can serve as equilibrium strategies. Thus, we do not lose any intuitive content by focusing on atomistic strategies only. Considering the distribution of the expected profit $\pi_{s,t_1}(\tilde{t})$ of atomistic strategies instead of more complicated mixed strategies or the return distribution of pure strategies also simplifies the discussion. It helps us to explicitly focus on the consequences of betting on the convergence in different time points without facing the difficulty of interpreting values of a density function instead of probabilities.

4.1 Career concerns and the labor market of fund managers

The distorting effect of career concerns is very apparent, if we consider the relationship between the expected profit of a strategy followed by an arbitrageur and her chance of keeping her job in equilibrium.

Just as the benchmark equilibrium, the equilibrium with career concerns is also determined by the indifference condition that each arbitrageur has to be indifferent when to invest. Choosing any time interval will provide the same expected profit. However, when career concerns are present arbitrageurs do not consider only the expected profit from the current window. They consider also the expected profit from future windows weighted by the probability that they will still be hired. For example, the expected profit from a t_1 atomistic strategy is

$$\left[1 - e^{-\delta t_1} + \int_{t_1}^{t_1+\Delta} \delta e^{-\delta t} \left(\frac{1}{\Delta}\frac{g\left(\tilde{t}\right)}{\dot{g}\left(\tilde{t}\right)} + \frac{t_1 + \Delta - \tilde{t}}{\Delta}\right) dt\right] + \left[\frac{e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}}{\Delta} J'\left(v_0\right)\right]$$

where the first term is the expected profit from the current window, while the second term is the expected profit from future windows. The term $\frac{e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}}{\Delta}$ is the probability that the arbitrageur can keep her job in the next window. As the sum is the same for any t_1, if one part increases the other part must decrease. Arbitrageurs trade off the current expected profit and the probability that they will be kept. This implies the following Proposition.

Proposition 1 *The larger the expected profit of a strategy in the current window, the larger the chance that an arbitrageur following this strategy will be fired.*

As the probability $\frac{e^{-\delta t_1} - e^{-(t_1+\Delta)\delta}}{\Delta}$ is decreasing in t_1, earlier strategies provide smaller expected profit and larger chance of survival for an arbitrageur than late strategies. An arbitrageur can signal her type easier with early strategies, because short windows are more frequent and she will be successful more often.

Note that the negative association between the expected profit of strategies followed by each arbitrageur an her career prospect is consistent with the positive relationship between her career prospect and realized profit. Arbitrageurs are kept if they make a positive net profit and fired otherwise. Thus, our model is consistent with the positive flow-performance relationship documented in the literature.[5]

As the intuition behind the result is fairly simple, it should generalize to markets with other structures. If some strategies are better signalling devices than others, the expected profit of these strategies will decrease in equilibrium. Strategies which are easier to ,,sell" to investors will be less profitable.

[5] Chevalier and Ellison (1997) document positive flow-performance relationship in the mutual fund industry, while Agarwal et al. (2004) presents similar results for the hedge fund industry.

4.2 Career concerns and the frequency of liquidity crises

In this subsection, I present the effect of career concerns on the size and frequency of liquidity crises. First I show how the relative size of gap $g(t)$ and $g^b(t)$ changes with the length of the window.

Proposition 2 *There is a critical level of* $\varepsilon^0 > 0$ *that*

1. *For* $c = c^b$, $g(t) < g^n(t)$ *for all* t *and* $\lim_{t\to\infty}\left(g^b(t) - g(t)\right) = 0$,
2. *if* $0 < c - c^b < \varepsilon^0$, *the curves* $g(t), g^b(t)$ *intersect exactly once in a point* t^* *and* $g(t) > g^b(t)$ *for all* $t > t^*$ *and* $g(t) < g^n(t)$ *for all* $t < t^*$ *and*
3. *if* $c - c^b > \varepsilon^0$, *then* $g(t) > g^n(t)$ *for all* t.

Proof. The proof is in the appendix. ■

The relative position of the conditional gap paths depends on the difference between the expected rate of return arbitrageurs face with and without career concerns, $c - c^b$.[6] The first part of the proposition refers to the case when the expected rate of return over one window with no career concerns, c^b, equals to the long-run expected rate of return with career concerns, c. In this case, the gap is always smaller with career concerns, but the difference goes to 0 as the length of the window increases. The average slope of the conditional gap path with career concerns is larger.

As c^b slightly decreases relative to c, the gap path $g^b(t)$ decreases in each point and the two curves intersect exactly once. In short windows the gap is smaller with career concerns, but with long windows $g(t)$ exceeds $g^b(t)$. This case is depicted on Figure 1.

The first two cases of Proposition 2 demonstrates that career concerns induce too many arbitrageurs to bet on short windows, because this strategy provides a better chance of survival. As a result, the gap is reduced for small (or in the first case, for all) t, but if the window happens to be long, the gap may increase drastically. As the following proposition shows, this result in the fact that for large crises, the magnitude of aggregate loss of arbitrageurs with career concerns stochastically dominates the same loss in the benchmark case. Crises of the same size happen more often with career concerns, and – equivalently – if we compare two events happening with the same probability, the crisis with career concerns will be larger.

Proposition 3 *If* $c - c^b < \varepsilon^0$ *there is a critical* L^* *that the aggregate loss of all arbitrageurs who did not choose to invest exactly in* $\tilde{t}$ *with career concerns stochastically dominates the same loss in the benchmark case in the first order sense given that this loss is larger than* L^*. *Formally,*

$$\Pr\left(\int_0^{\tilde{t}} \overline{\mu}(u)\,du \le L \middle| \int_0^{\tilde{t}} \overline{\mu}(u)\,du > L^*\right) \quad < \quad \Pr\left(\int_0^{\tilde{t}} \overline{\mu}^b(u)\,du \le L \middle| \int_0^{\tilde{t}} \overline{\mu}^b(u)\,du > L^*\right)$$

for all $L \in (L^*, \Gamma)$.

[6] As it is clear from equation (17), the difference $c - c^b$ depends critically on the exogenous probability that a fund manager keeps her job in the benchmark case, q. Although, it seems reasonable to compare cases with and without career concerns which are otherwise very similar, i.e., $c - c^b$ is small, there is no firm theoretical argument why we should choose any given q. Thus, I discuss the results for any q.

Proof. The aggregate loss of arbitrageurs at t if $t < \tilde{t}$ is

$$\int_0^t \overline{\mu}(u)\, du = \int_0^t f^{-1}(g(u))\, \dot{g}(u)\, du = F^{-1}(g(t)) - F^{-1}(g(0))$$

with career concerns and

$$\int_0^t \overline{\mu}^b(u)\, du = F^{-1}\left(g^b(t)\right) - F^{-1}\left(g^b(0)\right)$$

with no career concerns. As $\frac{\partial F^{-1}(g(t))}{\partial g(t)} = f^{-1}(g(t)) > 0$, losses are increasing in $g(t)$ and $g^b(t)$ and decreasing in $g(0)$ and $g^b(0)$ Both in the first and second cases of Proposition 2, $g^b(0) > g(0)$. As

$$\lim_{t\to\infty} g(t) = \lim_{t\to\infty} g^b(t) = g^*,$$

it is either true that for any L

$$\Pr\left(\int_0^{\tilde{t}} \overline{\mu}(u)\, du \leq L\right) < \Pr\left(\int_0^{\tilde{t}} \overline{\mu}^b(u)\, du \leq L\right)$$

or there must be a t^0 that for any $t > t^0$,

$$\int_0^t \overline{\mu}^b(u)\, du < \int_0^t \overline{\mu}(u)\, du$$

Defining

$$L^* = \int_0^{t^0} \overline{\mu}(u)\, du = \int_0^{t^0} \overline{\mu}^b(u)\, du$$

gives the result as the probability of longer windows is smaller. ■

In the third case of Proposition 2 shows that if the expected return in the benchmark case,c^b , is required to be very low, the gap will be lower without career concerns for any windows. This case does not provide clear predictions about the distribution of liquidity crises.

5 Conclusion

I presented a general equilibrium model of delegated risky arbitrage. In the model two types of fund managers compete for the capital of investors who are uninformed about the type of fund managers. Some fund managers can locate windows of arbitrage opportunities: pairs of fundamentally very similar assets traded temporarily at different prices. These are the arbitrageurs. They can make profit from betting on the time of convergence. Other fund managers cannot locate these windows but they can invest in gambles which mimic the outcome of arbitrageurs in certain states. Investors observe the performance of fund managers and update their beliefs about fund managers' type. Based on their beliefs they hire and fire arbitrageurs.

I highlighted two main results. In this set-up, arbitrageurs who speculate on the fast convergence of prices are fired with smaller probability, because they are successful more often. Consequently, they can signal their type more often. However, because of this advantage of this strategy, the competition of arbitrageurs drive down the corresponding expected profit. Thus, in equilibrium, there is a negative association between the career prospects of an arbitrageur and the expected profit of the strategy she follows. Relatedly, the second main result is that liquidity crises are typically more frequent when career concerns are present. The reason is that there are too many arbitrageurs are betting on the fast convergence of prices, so the arbitrage sector as a whole do not save enough liquidity for the rare events of long price discrepancy. Hence, if it happens, prices will diverge to a large extent and the aggregate loss of arbitrageurs will be large.

Although the presented model focuses on the complicated problem of delegated portfolio management in a general equilibrium set up, it is analytically very tractable. This is a unique feature in the similar literature. This gives the hope that the model will be applicable to further problems in the field. For example, in this model the aggregate capital of arbitrageurs is the same in each window regardless what happened in the past. In reality, we can observe that after a crisis, price gaps tend to remain high for a while. It is a task for further research to exploit the potential of this model to explain this phenomenon.

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Appendix

Proof of Lemma 1. First note that

$$\int_0^\infty f^{-1}(g(t)) \frac{\partial g(t)}{\partial t} dt = F^{-1}(g^*) - F^{-1}(g_0)$$

where $F^{-1}(\cdot) = \int f^{-1}(g)\, dg$, because $\frac{\partial F^{-1}(g(t))}{\partial t} = f^{-1}(g(t)) \frac{\partial g(t)}{\partial t}$. Thus, we can write

$$\frac{\partial \kappa^A(c)}{\partial c} = \frac{\partial \kappa^A(g_0(c))}{\partial g_0} \frac{\partial g_0}{\partial c}.$$

Furthermore,

$$\frac{\partial \kappa^A(g_0(c))}{\partial g_0} = -\frac{\partial F^{-1}(g_0)}{\partial g_0} = -f^{-1}(g_0) < 0.$$

Also note, that $g_0 = g(0) = g^* \left(\frac{c}{1+c-\delta(1+c)}\right)^{\frac{1}{1-\delta(1+c)}}$. Hence,

$$\frac{\partial g_0}{\partial c} = g^* \left(\frac{c}{(1+c)(1-\delta)}\right)^{\frac{1}{1-\delta(1+c)}} \frac{\delta \left(\ln \left(\frac{c}{(1+c)(1-\delta)}\right)\right) c(1+c) + 1 - \delta(1+c)}{(-1+\delta+\delta c)^2 c(1+c)}.$$

The sign of the derivative depends on the term $\delta \left(\ln \left(\frac{c}{(1+c)(1-\delta)}\right)\right) c(1+c) + 1 - \delta(1+c)$. The minimum of this term for $c > 0$ is at $c = \frac{1-\delta}{\delta}$ when it is 0. Thus the term is always non negative and $\frac{\partial g_0}{\partial c} > 0$. ■

Proof of Lemma 2. At the beginning of the first window all investors can hire only unexperienced fund managers. Investors will participate in the market and hire a fund manager (15) holds. Because of the free entry of investors, the equilibrium c in the first window will be

$$c_1 = \frac{1}{\alpha}\left(\frac{1}{(1-\gamma)} - (1-\alpha)\beta - \alpha\right).$$

This determines a Γ_1 by (16). Let us suppose that the first window closed at $\tilde{t}$. In the second window, if (15) did not hold, the maximum measure of investors who would hire an investor would be $\mu\left(\tilde{t}\right)$, as this is the measure of investors who decide to keep their existing arbitrageur. However, because of lemma 1, $\Gamma_1 > \mu\left(\tilde{t}\right)$ implies that the equilibrium rate of return of arbitrageurs implied by (16) and consistent with a measure of arbitrageurs smaller or equal than $\mu\left(\tilde{t}\right)$ is larger than c_1. Thus, investors prefer to hire unexperienced fund managers, which is a contradiction. Consequently, the rate of return is equal to c_1 in the second window, and with the same argument in all subsequent windows as well. ■

Proof of Theorem 2. Because the main steps of the proof are discussed in the main text, here I only give the draft and specify those details which are not discussed in the main text.

It is clear that if beliefs of investors are such that they keep only those arbitrageurs who produce positive net returns than the problem of arbitrageurs is given by (8) with interior solutions given by the differential equations of (9) and (11). We have to check whether there are corner solutions. In a corner solution, there is a time T, that by $|T$ all arbitrageurs made her maximal investment in all states of the world, thus arbitrageurs cannot take positions. Hence, if such a period existed, $\overline{\mu}(t) = 0$ and $g(t) = g^*$ for $t \geq T$ as in autarchy. But observe, that the expected marginal profit in autarchy is infinity as investing one unit in T would give a trading profit of $\delta \frac{g^*}{g(T)}$ but $\dot{g}(T) = 0$. Which is a contradiction as it would imply that arbitrageurs would not save capital for a period with infinite marginal profit. Thus we have only interior solutions given by expression (12). As discussed in the main text, only $g_\infty = g^*$ gives a solution which is robust for the inclusion of arbitrarily small trading cost. It is also easy to check that (12) is monotonically increasing as

$$\frac{\partial g\left(t\right)}{\partial t}=g^{*}\delta\frac{\left(c\frac{e^{\delta t}}{1+ce^{\delta t}-\delta(1+c)}\right)^{\frac{1}{1-\delta(1+c)}}}{1+ce^{\delta t}-\delta\left(1+c\right)}>0.$$

Given the conditional gap path, the aggregate measure of arbitrageurs investing in each time t, $\{\overline{\mu}\left(t\right)\}_{t=0}^{\infty}$, is given by (20) and the equilibrium measure of arbitrageurs who enter the market, Γ, is (21). Quacks always invest their unit in the negative value gamble. Bayes Rule gives the equilibrium beliefs (22) which are consistent with the equilibrium strategies of arbitrageurs and quacks. With these equilibrium beliefs, investors keep arbitrageurs only if they make positive net profit which closes the argument. The last step is to determine c, which must be given by (19), because of Lemma 2. ■

For the proof of Proposition 2, first we have to prove the following lemma.

Lemma 3 *There is a threshold* $t^{0}=\max\left(0,\frac{1}{\delta}\ln\frac{\delta(1+c)}{\left(c-c^{b}\right)}\right)$ *that if* $t>(<)\,t^{0}$ *then* $\frac{g(t)}{\dot{g}(t)}>\frac{g^{b}(t)}{\dot{g}^{b}(t)}$.

Proof. The result is a straightforward consequence of equations (4) and (11). ■

Proof of Proposition2. Note, that Lemma 3 implies that if there is a t^{+} where $g\left(t^{+}\right)=g^{n}\left(t^{+}\right)$ and $\dot{g}\left(t^{+}\right)>\dot{g}^{n}\left(t^{+}\right)$ then $t^{+}<t^{0}$ where t^{0} is defined in 3. Similarly, if there is a t^{-} that $g\left(t^{-}\right)=g^{n}\left(t^{-}\right)$ and $\dot{g}\left(t^{-}\right)<\dot{g}^{n}\left(t^{-}\right)$ then $t^{-}>t^{0}$. Consequently, if there is an intersection where $g\left(t\right)$ crosses $g^{n}\left(t\right)$ from below, there cannot be an intersection at a smaller t where $g\left(t\right)$ crosses $g^{n}\left(t\right)$ from above.

An other implication of Lemma 3 is that if there is any t^{++} that $g^{n}\left(t^{++}\right)>g\left(t^{++}\right)$ there must be a $t^{+},t^{+}>t^{++}$, where $g\left(t\right)$ crosses $g^{n}\left(t\right)$ from below. The reason is that if there is no such t^{+} then $g^{n}\left(t\right)>g\left(t\right)$ for $t>t^{++}$ including all $t>t^{0}$. Thus, Lemma 3 implies $\dot{g}^{n}\left(t\right)>\dot{g}\left(t\right)$ for all $t>t^{0}$ and

$$g^{n}\left(t\right)-g\left(t\right)=g^{n}\left(t^{0}\right)-g\left(t^{0}\right)+\int_{t^{0}}^{t}\left(\dot{g}^{n}\left(u\right)-\dot{g}\left(u\right)\right)du$$

increases for all $t>t^{0}$. But this is in contradiction with $\lim_{t\to\infty}g\left(t\right)=\lim_{t\to\infty}g^{n}\left(t\right)=g^{*}$.

These two implications of Lemma 3 imply that if $g^{n}\left(0\right)>g\left(0\right)$ then there will be exactly one intersection where $g\left(t\right)$ crosses $g^{n}\left(t\right)$ from below and if $g^{n}\left(0\right)<g\left(0\right)$ there will not be any intersections. ■