Feedback Effects of Rating Downgrades

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Abstract

This paper addresses whether credit rating downgrades feed back on the asset value of the downgraded companies, causing real losses. To investigate this issue we construct a structural credit risk model incorporating ratings and the feedback loss. To estimate the parameters of the model we develop a maximum likelihood estimator using time series of equity prices and credit ratings. Implementing the model on a sample of US public firms downgraded from investment grade to junk, we find strong support for the existence of feedback losses. First, estimated feedback losses are significant for a third of our sample with the cross-sectional averages of the feedback loss around 7-15 %. Second, the behavior of estimated asset volatilities around downgrades in real data is consistent with the predictions of our model. We observe a hump-shaped pattern of estimated asset volatilities when feedback is ignored. Using the feedback model, the hump-shaped pattern disappears. These findings suggest that ignoring feedback can lead to the appearance of changing asset volatility even when the real volatility is constant. Last, accounting for feedback helps in asset volatility prediction.

Keywords: Credit ratings, distress costs, maximum likelihood, option pricing, credit risk.

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1 Introduction

Credit ratings play an important role in modern financial markets. Corporate executives seem to keep their eyes on credit ratings. As an example, in their survey of CFO's, Graham and Harvey (1993) find that credit ratings come out as the second most important factor in deciding the level of debt. But what exactly is the economic rationale behind this focus? Are ratings simply an easy-to-understand summary measure of creditworthiness without any economic consequences? Or do rating changes themselves impose real costs on the companies in terms of higher costs of financing, forcing fire sales of assets or reducing access to credit?

In this paper we try to shed light on this question. In particular we set out to use the behavior of equity prices around downgrades to learn whether rating downgrades are likely to cause real economic losses to the downgraded companies. The main insight of the paper is that feedback losses can be thought of as a short position in a barrier option with a payoff triggered by the rating downgrade. Thus, one can think of the equity as a portfolio of a European call (no feedback losses) and the short position in the barrier option. This implies that if downgrade probabilities are somewhat predictable (tied to asset values), feedback losses make the equity of a company approaching downgrade riskier than the standard leverage effect would suggest.

To see why this is the case, consider the example of an investment grade company close to being downgraded to junk. Each fundamental negative (positive) shock to the asset value of the company has the secondary effect of increasing (decreasing) the probability that the company is downgraded. If downgrades lead to losses, this translates to a further downward (upward) push on equity prices. This effect gets stronger as the company gets closer to downgrade and disappears after downgrade, as the uncertainty about the rating change is resolved. This paper essentially uses this equity volatility pattern around downgrades to ascertain whether observed equity data indicates the presence of feedback losses.

We use the logic outlined above on two levels. First, we construct a full-fledged structural credit risk model incorporating credit ratings and the feedback from ratings to asset values. Our framework is in the tradition of Merton (1974) where equity is modelled as an option on the firm value. The model is set in discrete time and the firm continues until some future time T, when it is liquidated and the proceeds are divided between equityholders and bondholders. As in Merton (1974), the firm has zero-coupon debt outstanding, maturing at T. We add credit ratings to the model by postulating a rating assignment mechanism where a firm gets downgraded as its fundamental value falls below some point. In our rating assignment model we allow both for the fact that rating agencies base their decisions on longer-horizon averages of the financial performance of firms and for some noise surrounding the assignment process. Crucially, in this model, rating changes can feed back on the fundamental value in the form of a proportional loss. Thus ratings can have real effects in addition to being summary statistics of the firms' creditworthiness.[1] When feedback is switched off, our model reduces to Merton's model. We also need a way to estimate the parameters of the model based on equity and ratings data for a given firm. To derive a maximum likelihood (ML) estimator for the model we adopt the transformed data approach of Duan (1994).[2] Implementing the ML estimation of the model on a sample of 168 downgraded US public firms, real equity and ratings data are found to be consistent with our theory. We find significant feedback effects for a third of our sample which is much more than chance would suggest. The mean feedback loss in the whole sample is around 7 % of the fundamental value. Among the firms with a positive estimate of feedback loss, the mean loss is around 15 % and the median is around 10%.

Second, we try to conduct more robust tests of feedback losses, less dependent on the specific model. In particular, the discussion before suggests that if one ignores the feedback effect, asset volatility estimates will likely increase before downgrades and decrease afterwards. For this, we use the canonical model of Merton (1974) without the feedback effect to estimate asset volatilities. In line with the presence of feedback losses, we indeed find a significant hump-shaped asset volatility pattern around downgrades when we use Merton's model. Furthermore there is no such pattern when the feedback effect is incorporated. In addition, we investigate whether allowing for feedback helps asset volatility prediction out-of-sample. We find this to be the case. In short, the overall evidence strongly suggests that rating downgrades will cause economic losses.

Knowing whether rating downgrades themselves push the company toward financial hardship is interesting from several perspectives. First, if rating downgrades do imply real costs to the company, then these costs should be taken into account when a decision is made on capital structure. Second, recently there has been a tendency among regulators to use agency ratings more extensively in setting the risk capital of financial institutions.[3] If a feedback effect exists, these reforms are likely to strengthen the feedback effect further. As this may increase the number of firms in distress, this should not be ignored when discussing the costs and benefits of regulatory reforms. Effects of feedback can be particularly painful because there tend to be more downgrades during recessions (for empirical evidence see Altman and Saunders (2001)). Thus, regulatory reforms may have an unintended procyclical

[1] In our model, due to the path-dependency introduced by the feedback and the rating assignment policy, equity is basically a complex barrier option that cannot be priced analytically. We employ Markov chain techniques based on state-discretization to get the price numerically.

[2] The transformed-data MLE method of Duan (1994) has been applied in credit risk analysis by Ericsson and Reneby (2004a,b), Wong and Choi (2004) and Duan, Gauthier, Simonato, and Zaanoun (2003). A unique feature of our model is that we need to deal with a vector-to-vector transformation between our state variable vector and the observed sample, we cannot separate the transformations in different time periods. This is simply due to the use of a moving-average type rating statistic.

[3] A recent example of this is the Basel II. framework for capital adequacy of internationally active banks (see BIS (2004)).

consequence.

Our results are in line with the findings of two recent related papers. Vassalou and Xing (2003) implement Merton's model using equity prices on a sample of downgraded firms following a variant of the KMV (2002) methodology.[4] While this is not the focus of their paper, they document that asset volatilities of downgraded firms increase before downgrades and decrease afterwards. Similarly, we find the same hump-shaped pattern for asset volatilities when feedback is ignored in estimation. Moreover we find that this pattern disappears once we take feedback into account in estimation. Vassalou and Xing (2003) interpret their findings by attributing a disciplinary effect to rating downgrades. However, our results suggest that this phenomenon can simply be induced by the feedback effect. If the feedback effect is ignored in the structural model, asset volatility will appear to increase before downgrades and decrease afterwards even though the underlying risk of the firm stays put.

In a recent paper Kisgen (2004) documents that the capital structure decisions of firms seem to be affected by the possibility of future rating changes. Firms with a higher probability of a rating change tend to issue relatively less debt as opposed to equity. This is exactly what one expects if rating changes impose costs on firms and managers try to decrease the probability of these costs.[5]

There are numerous studies examining the behavior of stock prices around rating changes. In summary this literature uncovers a negative stock price reaction up to and including the day of downgrade. Goh and Ederington (1993) find negative stock price changes in the period preceding downgrades and on the days of downgrades for firms that has been downgraded because of the deterioration of their financial prospect. Goh and Ederington (1999) reach similar conclusions and document that stock price reactions are stronger for downgrades from investment to junk grades and between junk grades than for those within the investment grades. Holthausen and Leftwich (1986) find analogous results for downgrades and show significant increases (decreases) in stock prices for additions to the Standard and Poor's Credit Watch List indicating potential upgrades (downgrades). However to our knowledge we are the first to theoretically incorporate ratings into a structural credit risk model and estimate the resulting model.

The paper proceeds as follows. Before we turn to our full-fledged model, Section 2 outlines the basic intuition behind our results using an illustrative example. In Section 3 we present our structural credit risk model with credit ratings and feedback due to rating changes. Section 4 shows the likelihood function for the model

[4] In the context of Merton's model, the KMV estimates are identical to maximum likelihood estimates developed in Duan (1994). Unlike the MLE method, however, the KMV algorithm is silent about the distributional properties of the estimates and thus ill-suited for statistical inference. For more on this see Duan, Gauthier, and Simonato (2004).

[5] Kisgen (2004) motivates this observed capital structure behavior with discrete costs of rating changes. We refer the reader to his work for a thorough review of various potential reasons for the feedback effect.

using time series of equity and ratings data. Section 5 runs a Monte-Carlo exercise examining sample selection and the performance of a test investigating the null hypothesis of no feedback. In Section 6 we estimate the model on real data and find support for the existence of the feedback effect. Appendix A contains details of our Markov chain implementation used to price equity and Appendix B presents the derivation of the likelihood function.

2 An illustrative example

Before we build and estimate a structural credit risk model rich enough to be taken to real data, we illustrate in this section the basic insight of the paper using a simple example. In particular we show that feedback losses push up equity volatility before downgrades. Our structural credit risk model in essence uses this extra equity volatility to identify feedback losses. We also show that if one then tries to back out asset volatility ignoring the feedback effect, one finds an apparently increasing volatility trend before downgrades even when the true asset volatility does not change. After the downgrade these effects disappear.

The framework we use to illustrate these ideas is the canonical model of Black and Scholes (1973). Under the pricing measure, the asset value free of financial costs is assumed to follow a geometric brownian motion with a drift equal to the riskfree rate r.

$$\frac{dV_t}{V_t} = rdt + \sigma dW$$

To model ratings, we assume that there are 2 rating categories, investment and junk. The company is downgraded to junk the first time its asset value drops below a downgrade barrier c_d. Once the company gets into the junk category it is assumed to stay there forever. To model feedback losses, we assume that if the firm is downgraded, J fraction of the asset value is lost. Also assume that the company has debt F maturing at T. Denote the running minimum of V_t by $K_t = \min_{s \le t} V_s$. The payoff of the equity at T, S_T, can be written as

$$S_T = 1_{K_T > c_d} \max(V_T - F, 0) + 1_{K_T \le c_d} \max(V_T(1 - J) - F, 0)$$

Thus equity is the sum of 2 barrier options on the firm value, a down-and-out call and a down-and-in call. Importantly, because of the feedback effect, the terms of the two options are not identical. In our simple setup, standard results can be used to price these options in closed form,(see Rubinstein and Reiner (1991)). Denote the resulting equity pricing function by s^{FB}

$$S = s^{FB}(V; \sigma, r, T, F, c_d)$$

The upper panel of Figure 1 depicts equity values as a function of the asset value for a case with and without feedback. The parameters used to generate the graph are $\sigma = 0.15, r = 0.05, T = 5, F = 1, c_d = 1$. The case $J = 0$ is the case without feedback, i.e. the original Merton model. The case $J = 0.1$ is the case when downgrades have real effects. One can see on the graph that when the asset value is high, the equity values in the two cases are virtually identical. This is the consequence of the fact that the probability of a downgrade is practically 0. However, as the asset value gets closer to the downgrade barrier c_d, the two lines begin to diverge. Thus, the equity function is steeper when $J = 0.1$. The reason for this is that in the presence of feedback, a downward move in the asset value has the secondary effect of increasing the probability of a downgrade and hence of a feedback loss. Once the asset value gets below the downgrade barrier, this extra effect disappears and the equity function becomes flatter.

A straightforward application of Ito's lemma provides the following expression for the instantaneous volatility of equity $\sigma^S = \frac{\partial s^{FB}}{\partial V} \frac{V}{S} \sigma$. From this expression it becomes clear that the feedback effect increases the instantaneous equity volatility through two channels. First, by increasing the sensitivity of the equity price to the asset value and thus increasing the delta of equity, $\frac{\partial s^{FB}}{\partial V}$. Second, it decreases the equity value, S. The middle panel of Figure 1 shows the resulting equity volatilities for the cases with and without feedback. One can see that when $J = 0$, equity volatility increases smoothly as the asset value decreases showing the well-known leverage effect. When $J = 0.1$, the situation is drastically different. Feedback losses push up equity volatility, and the closer the company gets to downgrade, the bigger this increase becomes. Also, there is a downward jump in equity volatility at downgrade. The reason for this is that downgrade is not an uncertain event any more, thus an important source of equity volatility is switched off. In our example the introduction of a 10 % feedback loss doubles equity volatility for asset values close to the downgrade boundary.

The bottom panel of Figure 1 indicates what is likely to happen if feedback is present but is not properly taken into account. The asset volatility is backed out from the equity value and the equity volatility using the method of Jones, Mason, and Rosenfeld (1984) and Ronn and Verma (1986). This consists of solving the two nonlinear equations connecting the asset value and the asset volatility to the equity value and the equity volatility.[6] The model used to back out the asset volatility ignores feedback in both cases. When $J = 0$, the theoretical model employed is the correct one, therefore we get back the real asset volatility, $\sigma = 0.15$. However when $J = 0.1$, the model we use to obtain the asset volatility is wrong. As it omits the feedback effect, it shifts all the extra equity volatility due to feedback into the asset volatility before downgrades. This explains the increasing pattern of asset volatilities before downgrades. Again, one can observe dramatic consequences. When the asset value is close to the downgrade barrier, ignoring feedback leads to

[6] This procedure is valid in this illustrative example because we assume that the equity volatility is known. In practice, one of course does not observe instantaneous equity volatility thus the method is not valid any more.

the illusion of an asset volatility that is more than three times bigger than the true value. One can also see that the model misspecification disappears once the asset value gets below the downgrade barrier. This happens because after the downgrade, equity becomes a simple call option and the equity pricing function used to back out the asset value becomes the right one.

While the example described above tells the basic intuition behind our results, it is too simple to analyze real ratings and equity data. In particular one needs to take into account two features of credit ratings. First, agencies base their decisions on longer horizon averages of financial variables, not simply on the last realized value. Second, the rating policy is not completely transparent to the market, so one needs to allow for some noise surrounding the rating assignment process. In addition, the illustrative model sidesteps an important theoretical issue related to the risk-neutral asset value dynamics. In particular, as the asset value is assumed to be a traded asset, in the presence of feedback the risk-neutral dynamics of the asset value prior to the downgrade should reflect the loss-potential. In other words, the risk-neutral drift should be greater than r to compensate for the possibility of losses due to downgrade. In the following section we introduce a discrete-time model accounting for all of these issues.

3 The Model

This section outlines our model and the pricing theory. In this paper we concentrate on the real effects of credit rating downgrades, thus we try to keep everything else standard. We set up a structural credit risk model in the tradition of Merton (1974) where default happens when the value of the company falls below its debt at some future time point. The novelty of the model is that we also introduce credit ratings into the structural framework. In the model, credit ratings are determined by the financial situation of the company through a rating assignment mechanism that we will postulate later. However, as our focus is on the real effects of rating changes, ratings are not merely summary statistics of the financial health of the company and they feed back onto it. A downgrade may for instance force asset sales at knocked-down prices.

Our first state variable, V_t, is the fundamental value of the firm at t. While the fundamental value is not traded before T, equity, a contingent claim on the fundamental value is assumed to be traded. At T the company is liquidated for V_T. One can think of our firm as one which reinvests all of its earnings up to T.[7]

We want to formalize the link between the asset value of the company and credit rating changes. Thus, we also need to model how credit ratings are assigned. To do

[7] One simple way to interpret V_t, is to assume that at T, the company can be liquidated at some known earnings multiple m. Then, V_t can be defined as $V_t = E_t m$ where E_t is the earning of the company at t. Then, V_t directly corresponds to the earnings of the company at t, and at T, the company is liquidated for $V_T = E_T m$.

this we introduce two other state variables. One is the rating of the firm at t, R_t, taking discrete values between $1, ..., K$. where K denotes the highest grade.

One can argue that ratings are partly determined by the financial situation of the firm, i.e., V_t. Thus the simple way to go about a rating assignment policy would be to link ratings to the current value of the fundamental value process, V_t. However, rating agencies typically base their decisions on longer term averages of the firm's performance. To implement our model on real data, we need to allow for this fact. Thus we introduce a new state variable, a rating statistic M_t that summarizes the history of the fundamental asset value process. One can for example think of M_t as a moving average of past V_t's.

Thus, at $t < T$ the state of our system is determined by three variables: V_t, R_t and M_t. In the following we specify how the system moves from t to $t+1$.

Moving forward from t to $t+1$, the first variable to be determined is the new rating R_{t+1}. At this stage, all we need to assume is that it depends on the past value of the rating statistics, M_t, the past rating R_t and some rating noise, ξ_{t+1} which accounts for the uncertainty surrounding the rating assignment process. Using a function $g(.)$ to denote the rating assignment mechanism, we have

$$R_{t+1} = g(M_t, R_t, \xi_{t+1}) \tag{1}$$

Having determined the new value of the rating, we can specify the new fundamental value, V_{t+1}. This depends both on an exogenous shock to the fundamental value process ε_{t+1} and on the rating history. In particular we assume the following relationship

$$V_{t+1} = V_t e^{\mu - \frac{\sigma^2}{2} + \sigma \varepsilon_{t+1}} (1 - J(R_t, R_{t+1})) \tag{2}$$

The first part of the innovation, due to ε_{t+1}, can be interpreted as the change to the real business prospect of the company. This part is standard and a similar assumption is used, for instance, in Goldstein, Ju, and Leland (2001). The second part, $(1 - J(R_t, R_{t+1}))$, models the feedback from rating changes to the fundamental value process. We assume a jump that is proportional to the fundamental value and depends on ratings at t and $t+1$. The percentage feedback loss is $J(R_t, R_{t+1})$. The introduction of feedback is new to this paper and purports to model real effects of rating changes. A downgrade, for instance, may make the financing of a company more expensive because it may harden liquidity constraints or it may force the company to sell assets at knocked-down prices.

The rating statistic M_{t+1} is determined by its past value M_t and the new cashflow V_{t+1}

$$M_{t+1} = f(V_{t+1}, M_t) \tag{3}$$

The specific recursive relationship will be stated later. To fix intuition one can for now think of M_{t+1} as a moving-average type statistic.

The bankruptcy procedure is similar to Merton (1974). The firm is alive until some time T, when it is liquidated as an all-equity firm for V_T. The firm has debt maturing at T with a face value of F. Given limited liability, the payoff at T to the equity holders is $\max(V_T - F, 0)$.

We assume that the continuously compounded risk free interest rate is r. ε_{t+1} and ξ_{t+1} are assumed to be standard normal variables, independent of each other and across time. Let $\mathcal{F}_t$ denote the information set up to t; that is $\mathcal{F}_t = \sigma(V_0, M_0, \varepsilon_s, \xi_s, 0 < s \leq t)$.

To price contingent claims in the model, assume that the logarithm of the stochastic discount factor (SDF) and the innovation process $(\varepsilon_{t+1}, \xi_{t+1})$ follow a joint multivariate normal distribution conditional on $\mathcal{F}_t$. Following Lemma A.1 and A.2 in the proof of Theorem 2.1 of Duan (1995), one can show that under this restriction on the innovations and the SDF, the change of measure will induce a location shift in the innovation process, equal to the risk premia. Assuming constant risk premia, we have that under the pricing measure Q

$$R_{t+1} = g(M_t, R_t, \xi^*_{t+1} - \lambda_R) \tag{4}$$

$$\ln V_{t+1} = \ln V_t + \mu - \lambda_V \sigma - \frac{\sigma^2}{2} + \sigma \varepsilon^*_{t+1} + \ln((1 - J(R_t, R_{t+1}))) \tag{5}$$

where ε^*_{t+1} and ξ^*_{t+1} are standard normal variables under Q and they are independent of each other and across time. λ_V and λ_R are the corresponding risk premia. Since V_t is not a traded asset, one cannot do away with λ_V, a result that is less specific than Duan (1995).[8]

As usual, the price of the equity at t, S_t, is its conditional average payoff under Q discounted by the riskfree rate

$$S_t = e^{-r(T-t)} E_t^Q \left[\max(V_T - F, 0)\right] \tag{6}$$

In general (6) cannot be computed in closed form as the rating feedback in (5) introduces a complex path dependence to the system. Numerical methods need to be used to compute the equity price.

When the feedback effect is switched off, the model reduces to a discrete time version of Merton's model with a nontraded asset and (6) can be expressed in a closed form. In particular it is given by

$$S_t = BS(V_t e^{-(r-\mu+\lambda_V \sigma)(T-t)}, \sigma, T-t, r, F) \tag{7}$$

where $BS(.)$ denotes the Black and Scholes (1973) call option formula.

If V_t were a tradeable asset value process, then by the results of Duan (1995) we have the equality: $\mu - \lambda_V \sigma = r$. Thus the pricing of equity would be identical to Merton (1974); that is, $S_t = BS(V_t, \sigma, T-t, r, F)$.

[8] Lemma A.2, item (b) in the proof of Theorem 2.1 in Duan (1995) is not applicable in our case.

4 Maximum likelihood estimation of the model using equity and ratings data

In general we are interested in estimating the ratings model described above given a time series of equity prices and ratings data. To do this we use the maximum likelihood (ML) transformed data method of Duan (1994). By this method, the likelihood of the observation vector is equal to the likelihood of the implied state vector using the theoretical model multiplied by a Jacobian term that takes into account the transformation.

We first study estimation of Merton's model using an equity time series when the asset value, V_t is not traded. We show that while the asset drift, μ is not identified, all the other parameters can be estimated via working with a transformed asset value, denoted by H_t. This does not entail any loss of generality as the asset value is not directly observed anyway. Then, we argue that specifying the rating statistic, M_t in terms of H_t has a nice economic interpretation and simplifies estimation because we do not need to worry about the nuisance parameter μ. We continue by choosing a specific rating assignment rule and make some further simplifying assumptions to prepare for the implementation of the structural credit risk model with feedback. We complete the section by presenting the likelihood function.

4.1 Estimation in Merton's model with nontraded asset values and equity observations

When one tries to estimate Merton's model with equity observations, the asset drift is not identified if the asset value is not traded. Fortunately, all the other parameters can be estimated after a suitable change of variables.

Assume that we have a time series of equity observations $\{S_t, t = 0, ..., N\}$ generated by Merton's model with a nontraded asset value. Then we have a simple version of our model without feedback. That is, the evolution of the logarithmic asset value, $\ln V_t$, under the physical measure is

$$\ln V_{t+1} = \ln V_t + \mu - \frac{\sigma^2}{2} + \sigma\varepsilon_{t+1}$$

and the equity price can be written as

$$S_t = BS(V_t e^{-(r-\mu+\lambda_V\sigma)(T-t)}, \sigma, T-t, r, F) \tag{8}$$

where $BS(.)$ denotes the Black and Scholes (1973) call option formula. Thus, the unknown parameter vector is (μ, λ_V, σ). In the following we show that while μ is not identified given the equity time series, the other two parameters can be estimated.

Introduce the new variable

$$H_t = e^{-r(T-t)} E_t^Q [V_T] = V_t e^{-(r-\mu+\lambda_V\sigma)(T-t)} \tag{9}$$

Note that H_t would be equal to the value of the firm at t if the assets were traded. We can rewrite our system in terms of H_t instead of V_t.[9]

The dynamics of $\ln H_t$ under P is

$$\ln H_{t+1} = \ln H_t + r + \lambda_V \sigma - \frac{\sigma^2}{2} + \sigma \varepsilon_{t+1} \tag{10}$$

and the observed equity price can be related to the state variable H_t by

$$S_t = BS(H_t, \sigma, T - t, r, F) \tag{11}$$

This implies that the likelihood of the observed equity sample does not depend on μ. The reason is that neither the transformation in (11), nor the dynamics in (10) depends on μ. Thus μ is unidentified but one can proceed with the ML estimation of the system in (10) and (11) to get estimates of (λ_V, σ). after a simple reparametrization this in effect becomes identical to the estimation problem posed by Duan (1994).

In the next subsection we describe what restriction we need to put on the definition of the rating statistic M_t to ensure that the intuition from Merton's model carries over to our more complex model with feedback and rating observations.

4.2 Specification of the ratings assignment policy

In the feedback model the estimation task is more involved than that for Merton's model. The reason for this is twofold. First, the probability of the observed ratings may be influenced by μ even after we changed the state variable to H_t. Second, if the ratings do depend on μ in the new representation, then the equity function linking the state variable to the observed equity prices will also depend on μ through the feedback effect.

To exclude these two channels we assume that the rating agencies base their decisions on the time series of H_t's. This has a nice economic interpretation. In a world with no feedback and if the value of the firm was traded, it would be equal to H_t. In other words, H_t is the intrinsic value of the company. Thus, in our model, rating agencies try to use past intrinsic values in setting ratings, but they ignore the feedback effect. We assume the following simple form for the determination of the rating statistic M_t

$$\ln M_{t+1} = (1 - \alpha) \ln M_t + \alpha \ln H_{t+1} \tag{12}$$

This mechanism allows for the fact that rating agencies base their decisions on longer term average statistics of the financial health of the company. The parameter α measures the speed with which the agency incorporates new information into the

[9] For the purposes of showing that μ is not identified one can use an arbitrary shift that contains μ, e.g. $V_t e^{\mu(T-t)}$. We make the specific choice of H_t, because it has a nice economic interpretation and we later use it in our definition of the rating assignment policy.

ratings. This rule translates into a rating statistic, M_t, that is an exponentially weighted average of past values of H_t's.

In the following we specify the rating assignment function in (1) and make some further simplifying assumptions to prepare for the implementation of the model.

A1 There are 2 rating classes, interpreted as investment and junk grade. Let 2 denote the investment grade category. The rating agency is assumed to segment the two categories based on a leverage-type variable defined as

$$L_{t+1} = \frac{M_t}{F} \times e^{\rho \xi_{t+1}} \tag{13}$$

The parameter ρ controls the amount of noise in the rating assignment process.

A2 The analysis is restricted to investment-grade firms downgraded to junk, not the other way around. In particular a firm gets downgraded when L_{t+1} gets below some c_d. Once a firm is in the junk category, it stays there forever. We ignore upgrades because the empirical literature has shown much more significant changes in asset prices around downgrades than around upgrades.[10] Further, we assume that there is a fixed percentage feedback loss to the fundamental asset value when the firm is downgraded to junk; that is, $J(R_{t+1} = 1, R_t = 2) = J$.

A3 The rating noise is diversifiable so that it does not entail a risk premium, that is, $\lambda_R = 0$.

4.3 Likelihood function

Now we are in the position to derive the likelihood function using the transformed data method of Duan (1994). The unknown parameter vector is denoted by θ. Our state variable vector constitute of the time series of $(\ln H_t, R_t)$ while our observation vector is the time series of (S_t, R_t). The dynamics of our state variable H_t under the real measure P is

$$\ln H_{t+1} = \ln H_t + r + \lambda_V \sigma - \frac{\sigma^2}{2} + \sigma \varepsilon_{t+1} + \ln((1 - J(R_t, R_{t+1}))) \tag{14}$$

The ratings are set according to the rule described in the previous subsection, and the rating statistics are written in the recursive form in (12).

Expanding the state variables from $\{\ln H_t, R_t\}$ to $\{\ln H_t, \ln M_t, R_t\}$ makes the system Markovian and thus the observed equity price at t can be written as

$$S_t = s(\ln H_t, \ln M_t, R_t, \theta) = e^{-(T-t)r} E_t^Q \left[\max(H_T - F, 0)\right] \tag{15}$$

[10] See for instance Hull, Predescu, and White (2004) or Hand, Holthausen, and Leftwhich (1992).

Under the pricing measure Q,

$$\ln H_{t+1} = \ln H_t + r - \frac{\sigma^2}{2} + \sigma\varepsilon^*_{t+1} + \ln((1 - J(R_t, R_{t+1}))) \tag{16}$$

The rating statistic evolves according to (12) and the rating evolves according to

$$R_{t+1} = g(M_t, R_t, \xi^*_{t+1}) \tag{17}$$

In Appendix A, we present the numerical techniques to get the equity function $s(\ln H_t, \ln M_t, R_t, \theta)$ and its derivatives with respect to $\ln H_t$ and $\ln M_t$.

The transformed data method of Duan (1994) can be applied to obtain the likelihood function of the observed sample $\{S_t, R_t, t = 1, ..., N\}$ given $M_0, \ln H_0$ (see Appendix B)

$$\begin{aligned} &L(S_t, R_t, t = 1, ..., N \mid \ln H_0, M_0, \theta) \\ &= -\tfrac{1}{2}\ln(2\pi) - \ln(\sigma) - \tfrac{(\ln H^*_t - a_t)^2}{2\sigma^2} \\ &+ \textstyle\sum_{t=1}^N \ln\left[P(R_t \mid \ln M^*_{t-1}, R_{t-1}; \theta)\right] - \sum_{t=1}^N \ln\left[\tfrac{\partial S_t}{\partial \ln H_t}\right] \end{aligned} \tag{18}$$

where $a_t = \ln(1 - J(R_t, R_{t-1}) + \ln H^*_{t-1} + r + \sigma\lambda_{CF} - \frac{\sigma^2}{2}$; $\ln H^*_t$ is the time series of $\ln H_t$ implied by the equity sample at the parameter vector θ; and $\ln M^*_t$ are the implied statistics affecting the ratings. Note that the $\ln M^*_t$'s can be determined from past $\ln H^*_t$'s. $\frac{\partial S_t}{\partial \ln H_t}$ can be expressed using the partial derivatives of the equity pricing function

$$\frac{\partial S_t}{\partial \ln H_t} = \frac{\partial s}{\partial \ln H}(\ln H^*_t, \ln M^*_t, R_t) + \frac{\partial s}{\partial \ln M}(\ln H^*_t, \ln M^*_t, R_t)\frac{\partial \ln M_t}{\partial \ln H_t}(\ln H^*_t, \ln M^*_t, R_t) \tag{19}$$

5 A Monte-Carlo study on a simulated sample of downgraded firms

In the empirical application we use our ML estimator on a sample of firms that are downgraded from investment grade to junk. Because the likelihood function in (18) has not reflected the sample selection, it is important to check for systematic biases in the parameter estimates. We conduct a Monte-Carlo study using a simulated sample of downgraded firms.

We estimate the model firm-by-firm. The parameter vector we estimate is $\theta = (\sigma, \lambda_V, c_d, J)$. All other parameters are treated as known. We constrain J to be positive. Throughout the paper we use the Hessian of the loglikelihood function to compute the sample information matrix.

To simulate data we choose values that are typical in our real sample of downgraded firms. The only exception is the risk premium parameter λ_V because sample selection makes its estimation unreliable. In simulation we simply set $\lambda_V = 1/3$.[11] For the asset volatility and the downgrade barrier we choose $\sigma = 0.15$ and $c_d = 1$ respectively. We repeat the exercise for several different values of the feedback loss. The values we investigate are $J = (0, 0.05, 0.1)$. We have two reasons to do this. First, this gives us a way to investigate the consequences of restricting J to be positive. Second, we can investigate the size and power of the one-sided t-test for the null hypothesis that J is 0. We set the face value of the debt to 1, the maturity of the debt to 5 years and the riskfree rate to $r = 0.05$. The rating parameters are $\rho = 0.05$ and $\alpha = 1/252$.

Similarly to the sample selection implicit in the real data, we simulate 252 days of daily equity data with the last day of each path being the day of the downgrade. We do this by beginning each path with $H_0 = 1.2, M_0 = 1.2$, choose the ones that are downgraded between days 253 and 504 and store 252 days of data ending with the day of the downgrade for each path. We keep 500 downgraded paths. Then, we perform ML estimation for each path and compute asymptotic standard errors.

When J is relatively low, the estimate of J is at the lower boundary, 0, for a significant fraction of the sample. For these paths, inverting the full Hessian to get the standard errors is inappropriate. Instead, we compute the standard errors for σ, λ_V and c_d using the corresponding 3×3 sub-Hessian matrix. These standard errors are then used to compute the empirical coverage rates in Table 1. For J, we compute the coverage rates conditional on having a positive estimate of J, i.e. we drop the paths where $J = 0$ and compute the coverage rates relative to the remaining sample. Table 1 provides the results of the Monte-Carlo exercise.

The first thing to notice is that the mean and median estimates are close to their true values with the exception of the risk premium parameter, λ_V. This suggests that while selecting a sample of downgraded firms introduces a downward bias to the estimate of the asset risk premium, none of the other parameters are affected by the sample selection.[12] The nominal coverage rates for σ are close to their theoretical values in all cases. For c_d, the results are still fairly good, though there seems to be some divergence from the theoretical predictions. For J, the quality of inference depends crucially on the value of J used to generate the data. When $J = 0.1$, empirical coverage rates are close to their theoretical values. However, as J decreases, the quality of inference is getting worse. The intuition is that for smaller values of J, the sampling distribution of J is truncated at 0. The more severe this truncation becomes, the worse the standard asymptotic normal distribution fares as an approximation to the sampling distribution.

Constraining J to be positive does not cause difficulties when we want to test the null hypothesis that $J = 0$ against the alternative that $J > 0$. The reason is

[11] Combining this value with our other parameter choices the intrinsic value grows at a rate of 10 %; i.e. $r + \lambda_V \sigma = 0.1$.

[12] This result is consistent with Duan, Gauthier, Simonato, and Zaanoun (2003) where survivorship bias only affects the estimate of the asset drift.

that under the null hypothesis we know the asymptotic distribution of the estimates of J that takes into account the nonnegativity constraint on J. In particular it is known to follow a mixture distribution with one half of its mass on 0 and the other half on a normal truncated at 0 (see for instance Gourieroux and Monfort (1995), Chapter 21). Thus testing our null amounts to a standard one-tailed t-test. Table 2 presents the Monte-Carlo evidence on the size and power of this test using the parameter values from before. In finite sample, the test seems to overreject slighly and has a very good power.

6 Empirical results

6.1 Data

To estimate our model we use a sample of US public firms with data on stock price, balance sheet and rating history. We match the CRSP daily stock database with the rating history data from Moodys and the balance sheet information from Compustat between 1970-2004. As a result, we have 1733 firms in the matched database. A firm is assumed to have a relevant observation at time t if

- We have the necessary Compustat data: By this we mean that the firm has at least one nonnegative asset value record (Data44) and equity record (Data59) with the asset value bigger than the equity, dated in calendar time with the end day of the calendar quarter in the interval [-250,-20] days relative to day t. If we have at least one such data point, we keep the last one in this interval.
- We have daily CRSP data on number of shares outstanding, stock price and holding period return.
- The day is in the span of the ratings history in the Moody's database. We use only the broader scales: C, Ca, Caa, B, Ba, Baa, A, Aa and Aaa

From this matched database, we choose a sample of firms downgraded from investment to junk grade for estimation purposes. A given firm is included in the sample if it has at least 504 observations in the 800 calendar days preceding the downgrades and it was in investment grade throughout this period. Our final sample of downgraded firm consists of 168 firms.

To estimate our model we also need to identify values of the debt and equity of the firm as well as the riskfree rate. To arrive at an estimate of the debt F for each day, we follow Brockman and Turtle (2003) and set the debt to Assets-Equity (Data44-Data59). We set the maturity of debt to 5 years. For any sample, days where the debt changed because of the change in the relevant Compustat quarter are dropped from the likelihood function.[13] We compute the equity value by computing

[13] Otherwise one would get jumps in the leverage unrelated to the volatility of the equity.

the market capitalization of the firm at the dates when the debt figure changes. Then we multiply this figure by the holding period returns to get the equity values between debt figure changes. Thus, we have accounted for stock splits and dividend distributions. We use the 1-Year Treasury Constant Maturity Rate series from the FED as the riskfree rate.

Table 3 presents some descriptive statistics of the downgraded sample. In terms of the timing of the downgrades, our sample is between 1976 and 2004, with roughly half of the downgrades before 2000.

6.2 Estimation results

To estimate the model for each firm in the downgraded sample, we use the 252 days up to and including the day of the downgrade. For each firm we estimate the parameter vector $\theta = (\sigma, \lambda_V, c_d, J)$. Before we can implement the model, we need to specify the initial value of the ratings statistics, M_0, and the rating parameters: ρ and α.

The initial value of the rating statistic M_0 is obtained by estimating Merton's model using only equity observations one year prior to downgrade.[14] This procedure yields a time series of estimated asset values. We then simply take M_0 to be the average of these.

We present results for various values of the rating parameters to ascertain that our results are not driven by ad-hoc assumptions.[15] The sets we investigate are: $(\rho = 0.05, \alpha = 1/252)$, $(\rho = 0.1, \alpha = 1/252)$ and $(\rho = 0.05, \alpha = 1/126)$. The results turn out to be qualitatively the same across different values of the rating parameters.

We are able to successfully optimize for 166 out of the 168 firms. Table 4 shows cross-sectional statistics of the estimates. In Panel A, the mean loss due to feedback is around 0.07, while the median is 0. This latter is due to the fact that we constrain J to be nonnegative and roughly one half of our sample has a positive estimate of feedback loss. To investigate the cross-sectional behavior of our estimates for the firms where feedback is likely to matter, we also look at cross-sectional statistics in the subsample with positive estimates of J. Table 5 shows that among the 77 firm in this subgroup, the mean estimate of the feedback loss is 0.153 while the median is 0.113.

There are two facts emerging from these results. First, there seems to be a fair amount of cross-sectional variation in the parameter estimates, underlining the need

[14] This is exactly the period preceding the sample period which is used to estimate the feedback model.

[15] One could in principle treat the rating parameters as unknowns and try to estimate them. However doing so is not practical. Unreported simulation evidence suggests that the firm-by-firm estimation is hindered by the fact that there are too many parameters to estimate relative to the data available. Some parameters could still potentially be estimated by pooling data across firms. However this would involve a hierarchical optimization which would be computationally very demanding.

for the firm-specific approach that we choose. Second, the number of firms with a significantly positive J at the 5% level amounts roughly to one third of the observed sample. This is much higher than what can be attributed to chance. More formally, we can compute a z-statistic for the null hypothesis of no feedback in any of the 166 firm.[16] This yields a z-statistics of $z = 16.2$, a highly significant rejection.

Our results up to now point towards the existence of significant feedback effects for a large number of firms. However one may argue that these estimates critically depend on the exact specification of the feedback model. In the following we try to conduct more robust tests supportive of the feedback effect. The basic rationale behind these tests is that a hump-shaped pattern in estimated asset volatilities will emerge if feedback is ignored. As described in Section 2, the intuition is that the feedback loss due to downgrade makes the equity of a company approaching downgrade riskier than the standard leverage effect would suggest. Further, once the uncertainty surrounding the downgrade is resolved, the extra risk ceases to affect equity prices. We test three implications of this reasoning. First, we check whether estimated asset volatilities increase before downgrades if we ignore feedback. Still ignoring feedback, we then investigate whether asset volatilities decrease once the downgrade is known. We also check to see whether the hump-shaped pattern disappears when feedback is accounted for in the year before downgrades. The third implication being examined is asset volatility prediction. In particular if feedback effects matter, ignoring feedback losses leads to poor asset volatility estimates. Thus asset volatility estimates with feedback should have better out-of-sample performance. In all of the tests we concentrate on firms where feedback is likely to matter based on the estimation of our structural credit risk model with feedback. In particular we conduct the tests on the subpopulation of firms with a positive estimate of the feedback loss, J.

Our model reduces to Merton's model when feedback is switched off. So we simply use Merton's model to get asset volatility estimates when feedback is ignored.[17] For each firm, we get three estimates at different time points. For firm i, the asset volatility estimate using Merton's model one year prior to downgrade is denoted by $\hat{\sigma}_i^{(-1)}$. The corresponding estimates immediately before the downgrade and two years after are denoted by $\hat{\sigma}_i^{(0)}$ and $\hat{\sigma}_i^{(+2)}$ respectively. We leave a gap of one year after the downgrade to allow the uncertainty around the feedback to dissipate. The estimates from the feedback model immediately before the downgrade are denoted

[16]The z-statistic comes from the following argument: Denote the rejection index (0,1) for firm i as B_i. Let us fix significance level p for all firms. Then, under the null hypothesis, every B_i is distributed as a Bernoulli variable with parameter p. Assuming cross-sectional independence among these N variables, we have the following normal approximation: $\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(B_i - p) \sim N(0, p(1-p))$. From this we have the z-statistics $z = \frac{\frac{1}{N}\sum B_i - p}{\sqrt{\frac{p(1-p)}{N}}} \sim N(0,1)$.

[17]When feedback is ignored ratings do not materially influence asset volatility estimates. In this case we only use equity observations for estimation purposes.

by $\hat{\sigma}_i^{(FB,0)}$.[18]

We first test whether asset volatilities increase before downgrades. To investigate this hypothesis we take the ratio of the asset volatilities immediately before the downgrade and the year before that; i.e., $\frac{\hat{\sigma}_i^{(0)}}{\hat{\sigma}_i^{(-1)}}$. If estimated volatilities increase, this ratio should be higher than 1. The left column of Table 6 presents statistical tests based on the cross-sectional means and medians. The cross-sectional mean of the relative increase in the estimated volatility is about 30%, while the median is around 10%. Both the t-test on the mean and the Wilcoxon signed rank test on the median reject the null hypothesis of no increase of volatility. The right column of the same table suggests that when feedback is taken into account there is no longer an increase in volatility estimates. The cross-sectional means and medians of the ratio, $\frac{\hat{\sigma}_i^{(FB,0)}}{\hat{\sigma}_i^{(-1)}}$), are not significantly different from 1.

We now investigate whether asset volatilities subside after downgrades. To test this hypothesis, we choose firms with two years of data available after the downgrade that remained in the junk category throughout the sample period. there are 45,47 and 44 firms meeting these criteria, depending on the rating parameters chosen. Our focus now is on the ratio $\frac{\hat{\sigma}_i^{(0)}}{\hat{\sigma}_i^{(+2)}}$. Again, according to our reasoning, the ratio should be greater than 1 when feedback is ignored. Table 7 suggests that real data conforms to the prediction of our model. The left column shows cross sectional means and medians significantly greater than 1. In the right column, where feedback is taken into account, the means and medians are not significantly different from 1.

One may argue that higher levels of market volatility coincide with higher numbers of credit downgrades. In this case, by picking periods when firms are downgraded, we also pick periods with increased levels of volatility. This could be an alternative explanation to the hump-shaped volatility pattern exhibited by the data. To separate this from the feedback argument, we repeat our investigations of the volatility patterns around downgrades by controlling for changes in market volatility. In particular, instead of working with estimated asset volatilities for each firm, in each period we normalize by the concurrent standard deviation of the continuously compounded returns of the S&P 500 index. Table 8 and Table 9 show that controlling for market volatilities does not materially change our results. The volatilities of downgraded firms increase relative to the market before downgrades and decrease afterwards. As before, this pattern disappears once we control for feedback.

Our last test is to gauge the usefulness of our model in predicting asset volatilities. In particular we ask ourselves how well estimates from the model with and without feedback in the year preceding the downgrade predict future asset volatilities. The predictors with and without feedback are $\hat{\sigma}_i^{(FB,0)}$ and $\hat{\sigma}_i^{(0)}$ respectively.

[18]Ideally, we should also obtain estimates using the feedback model a year before the downgrade. However this is not informative because one is unlikely to observe a rating change from investment grade to junk in this sample period.

The quantity to be predicted is $\hat{\sigma}_i^{(+2)}$. We look at two measures of forecasting performance, absolute mean square errors and relative mean square errors.[19] Table 10 shows that the feedback model comfortably outperforms Merton's model based on both criteria.

7 Conclusions

If rating downgrades cause real losses to downgraded companies, their effect should be reflected in equity price movements before downgrades. In particular we suggest that the extra uncertainty due to the potential feedback from rating changes pushes up equity volatility in the period before downgrades more than the standard leverage effect would suggest.

To formalize this link between equity prices and feedback losses, we build a structural credit risk model by incorporating feedback to asset value from rating changes. Also, we develop a maximum likelihood estimator for the model parameters using equity and ratings time series data. Estimating the model on a sample of US public firms downgraded from investment to junk grade, we find support for the existence of feedback effects of rating downgrades. The mean loss due to downgrade in our sample is around 7 % of the fundamental firm value. Among the firms with a positive estimate of feedback, the mean loss is around 15 % and the median loss is around 10 %. As a further support of our model, we find the predicted hump-shaped pattern of asset volatilities around downgrades if feedback is ignored. Once we account for feedback, the hump-shaped pattern disappears. Disentangling fundamental asset volatility from the transitory extra risk due to feedback is also found to be important for forecasting purposes. Asset volatility estimates taking feedback into account perform better at forecasting asset volatility out-of-sample.

Overall, our results point towards the existence of a specific form of financial distress costs due to credit rating changes. Rating downgrades from investment to junk grade seem to impose real costs on the downgraded companies. These costs should be taken into account both in corporate financial decisions and in designing the regulatory framework governing financial markets.

[19] Given a prediction of asset volatility σ_i^{PR}, the absolute mean squared error is defined as $\frac{1}{N}\sum_{i=1}^{N}\left(\sigma_i^{PR} - \hat{\sigma}_i^{+2}\right)^2$ and the relative mean squared errors $\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_i^{PR} - \hat{\sigma}_i^{+2}}{\hat{\sigma}_i^{+2}}\right)^2$.

Appendix A: Implementation of the equity pricing function

Due to the feedback introduced by the rating downgrades, equity prices in the model are not available in closed form. In this appendix we present Markov Chain techniques to compute them. [20]

We approximate our model with a Markov chain and compute the prices in the resulting discrete system. If one chooses the node locations and the transition probabilities in the chain appropriately, the resulting Markov chain will converge to the original continuous-state system as the number of nodes goes to infinity. For proofs and further background on the Markov chain technique see Duan and Simonato (2001). The equity price computed from the Markov chain will also converge to the equity price implied by the theoretical model.

The state of our system at t is described by the the variables $(R_t, \ln H_t, \ln M_t)$. While the ratings, R_t are discrete in the original system, we need to discretize the theoretical model in $\ln H_t$ and $\ln M_t$.

In particular, assume that $\ln H_t$ can take up the discrete values

$$(h_1, ..., h_M)$$

and $\ln M_t$ takes values in the set

$$(m_1, ..., m_n)$$

For fixed M and N our Markov chain has $MN2$ different states. Let the $MN2 \times 1$ vector q_t denote the value of a contingent claim at t. I.e. a specific element of q_t corresponds to the value of the contingent claim in one state of the chain at t. Then, assuming no intermediate payoffs, and denoting by P the $MN2 \times MN2$ transition probability matrix of the Markov chain under the pricing measure, Q, the usual risk-neutral valuation formula takes the form of the following recursion

$$q_{t-1} = e^{-r} P q_t$$

[20]Of the two main candidates, Monte-Carlo and state discretization techniques we opt for the latter for two reasons. First, when we maximize our likelihood function, we use the equity function defined by the numerical technique within an optimizer. However, gradient based optimizers assume that the function being optimized is continuous and smooth. To ensure this, we need to have a numerical procedure for the computation of the equity prices that is smooth in the model parameters. However in our model the equity is a complex barrier option. In turn, it is known that Monte Carlo estimators of barrier options are not smooth due to the discontinuity introduced by the barriers. Thus in our application, using Monte Carlo would lead to a non smooth likelihood function. The second reason for the use of state discretization techniques is that these yield the equity function not only in one point in the state space, but on a whole grid. In our application, for a given call of the likelihood function, we need to evaluate the equity function at different values of the state variables, both because we need to invert this function from the observed equity to the underlying asset values and because we have multiple equity observations in the sample. Using a technique that provides values on a whole grid of the state space for given model parameters thus allows us to cut back on the computation time.

A naive implementation of the chain would involve specifying the whole transition probability matrix and then using this matrix for the backward calculation of any claim. This would have two problematic numerical consequences. First, even before starting the recursions, one would need to set up an $MN2 \times MN2$ transition probability matrix which can be quite time-consuming. Second, rolling the system one time step back would necessitate computations on the order of $MN2 \times MN2$. The numerical task however can be simplified by recognizing two specific features of our system.

First, the current value of the rating statistic, $\ln M_t$, and the future value of the financial health of the company, $\ln H_{t+1}$, determine the future value of the rating statistic $\ln M_{t+1}$ through the recursive relationship in (12). This means that for a given current node ($\ln H_t = h_j, \ln M_t = m_k, R_t = w$) we only have $M2$ continuation values instead of $MN2$. This is because conditional on the current state of the system a future value of $\ln H_{t+1}$ pins downs the future value of $\ln M_{t+1}$. As a consequence one time step back in the recursion only involves calculations on the order of $MN2 \times M2$ instead of $MN2 \times MN2$ workload of the naive implementation.

Second, while the rating evolution does depend on the current value of $\ln M_t$, conditional on the rating evolution, the transition of the system forward in independent of it. In other words, the forward movement of the system is described by the forward rating probabilities $P(R_{t+1} = v \mid R_t = w, \ln M_t = m_k)$ and the forward financial health movement transition probabilities $P(\ln H_{t+1} = h_i \mid \ln H_t = h_j, R_{t+1} = v, R_t = w)$. We have $4N$ rating probabilities and $MM4$ financial health transition probabilities. The sum of these is orders of magnitude less than the $MN2 \times MN2$ that would be necessary for the naive transition probability matrix.

As a result of these simplifications the backward recursion in the chain can be written as follows

$$
\begin{aligned}
& q_t(\ln H_t = h_j, \ln M_t = m_k, R_t = w) \\
= \; & e^{-r} \sum_{v=1}^{2} P(R_{t+1} = v \mid R_t = w, \ln H_t = h_j, \ln M_t = m_k) \times \\
& \sum_{i=1}^{M} P(\ln H_{t+1} = h_i \mid \ln H_t = h_j, R_{t+1} = v, R_t = w) \\
& q_{t+1}(\ln H_{t+1} = h_i, \ln M_{t+1}(\ln H_{t+1} = h_i, \ln M_t = m_k), R_{t+1} = v)
\end{aligned}
$$

Here $\ln M_{t+1}(\ln H_{t+1} = h_i, \ln M_t = m_k)$ is the rating statistic computed from the recursive form in (12). The value $q_{t+1}(\ln H_{t+1} = h_i, \ln M_{t+1}(\ln H_{t+1} = h_i, \ln M_t = m_k), R_{t+1} = v)$ is obtained by interpolating from values corresponding to the discrete grid $(m_1, ..., m_n)$.

Now let us specify the algorithm described above. The first question is how to determine the node values for $\ln H_t$, $(h_1, ..., h_M)$ and $\ln M_t$, $(m_1, ..., m_n)$. To ensure that the likelihood function is well-behaving, we need to keep the same grids for different values of the parameter vector θ. This means that there is not much scope to try to optimize the grids using knowledge of the parameter values. For $\ln H_t$ we simply set the grid to be uniform in $[\underline{h}, \overline{h}]$. For $\ln M_t$ we merge a uniform grid of n/2 between $(\underline{m}_1, \overline{m}_1)$ with a a finer inner grid of n/2 between $(\underline{m}_2, \overline{m}_2)$. This latter grid allows us to get better equity prices by producing a finer mesh in regions of $\ln M_t$ around the potential downgrade barriers.

The downgrade probabilities are

$$P(R_{t+1} = 1 \mid R_t = 2, \ln M_t = m_k) = \tilde{\Phi}\left(\frac{\ln(c_d) - m_k}{\rho}\right)$$

Here and elsewhere in the paper $\tilde{\Phi}$ denotes a truncated version of the standard normal distribution function whose density is triangular in each tail with probability p in the tails, allowing to set very low transition probabilities to 0. With p low enough, this speeds up the algorithm considerably without affecting the results. Throughout the paper we set: $p = 10^{-5}$.

The transition probabilities $P(\ln H_{t+1} = h_i \mid \ln H_t = h_j, R_{t+1} = v, R_t = w)$ would depend on the past ratings in general. For computational simplicity instead of dealing with the feedback effect in the transition probability matrices, we deal with them in the payoff function. I.e. defining $\ln H^-_{t+1}$ as the value before accounting for the feedback we can define a unique transition probability matrix by $P(\ln H^-_{t+1} = h_i \mid \ln H_t = h_j$ and then account for the rating changes in the payoff $q(\ln H^-_{t+1} = h_i, \ln M_{t+1}(\ln H_{t+1} = h_i, \ln M_t = m_k), R_{t+1} = v, R_t = w)$.

To get the transition densities we assume a discretization of $\ln H_{t+1}$ into bins with bin edges $(c_j, j = 0, ..., M)$

$$\begin{aligned} c_0 &= -\infty \\ c_j &= \frac{h_{j+1} + h_j}{2} j = 1, ..., M \end{aligned}$$

so we have

$$P(\ln H^-_{t+1} = h_i \mid \ln H_t = h_j) = \tilde{\Phi}\left(\frac{c_i - (h_j + r - \sigma^2/2)}{\sigma}\right) - \tilde{\Phi}\left(\frac{c_{i-1} - (h_j + r - \sigma^2/2)}{\sigma}\right)$$

Due to our assumption that the junk state is absorbing, the equity value in the junk state is equal to the no-feedback value which is a standard computation using the transition probabilities and the payoff at maturity.

To get the value in the investment grade at t we need the continuation value at $t + 1$ in case of downgrade. We have that

$$\begin{aligned} & q_{t+1}(\ln H^-_{t+1} = h_i, \ln M_{t+1}(\ln H_{t+1} = h_i, \ln M_t = m_k), R_{t+1} = 1, R_t = 2) \\ = \; & q_{t+1}(\ln H_{t+1} = h_i + \ln(1 - J), R_{t+1} = 1) \end{aligned}$$

where $q_{t+1}(\ln H_{t+1} = h_i + \ln(1 - J), R_{t+1} = 1)$ is computed by cubic interpolation in $\ln H_{t+1}$ from the grid of available no-feedback values.

Now all elements are known for rolling the system backwards. Matlab is used for all the computations. Further, sparse matrices are used to speed up the computations.

The Markov Chain recursion for a given set of parameters θ provides us with values of the equity for a grid of values $(\ln H, \ln M)$ for each rating. To compute the likelihood function we need to use this function to back out the implied $\ln H^*$'s from the observed equity and we need the derivatives of the function to compute the Jacobian. To ensure a well-behaved likelihood function all of these need to be smooth functions. To achieve this we fit bidimensional interpolating cubic splines on the grid of values obtained from the Markov chain and then consider the resulting spline as the equity function.[21] Inversion from the vector of observed equity prices to the vector of implied H_t^*'s is done by solving the system of nonlinear equations defined by the equity function using Newton's method. The derivatives of the equity function with respect to $\ln H$ and $\ln M$ are computed by differentiating the spline analytically.

Using a separate chain for each equity observations in a given data sample would be computationally prohibitive, so we use one chain for a given data sample. We assume the same maturity of debt for all equity observations in a given sample. We choose the length of time step in the Markov chain to be one month. For the calculations in the paper we choose $M = 200, N = 40$. The time frequency of the Markov Chain implementation is chosen to be 1 month throughout the paper.

Appendix B: Likelihood function

In our model the equity price S_t can be written as a function of the state of the system, $\{\ln H_t, \ln M_t, R_t\}$ at the model parameters, denoted here by θ. One can write

$$S_t = s(\ln H_t, \ln M_t, R_t, \theta, T - t, F, r)$$

For notational convenience we suppress these arguments from now on and work with the notation

$$S = s(\ln H, \ln M, R, \theta) \tag{20}$$

Then, given the observation vector $\{S_t, R_t, t = 0, ..., N\}$, M_0 and the parameter vector θ we can recursively deduce the vector $\{\ln H_t, t = 0, ..., N\}$.

- At $t = 0$ we have to invert the relationship $S_0 = s(\ln H, M_0, \theta)$ to get H_0.

[21] We use tensor product splines. We find that in the $\ln M$ direction extra care is needed in the spline interpolation because of the sudden change in the function value around the downgrade barrier. Thus we use shape-preserving splines in this direction.

- For $t > 0$ $\{\ln H_{t-1}, \ln M_{t-1}\}$ is already known. Then, conditional on $\mathcal{F}_{t-1}$, $\ln M_t$ can be written as a function of $\ln H_t$, $\ln M_t(\ln H)$. Then, we have a one-to-one mapping between $\ln H_t$ and S_t if the composite function $s(\ln H, \ln M_t(\ln H), \theta)$ is increasing in $\ln H$.

Write the transformation after 0 in vector form

$$\bar{S} = \bar{s}(\ln \bar{H}; \bar{R}, \theta)$$

where $\bar{S} = (S_1, ..., S_N)'$, $\ln \bar{H} = (\ln H_1, ..., \ln H_N)'$, $\bar{R} = (R_1, ..., R_N)$ and $\bar{s}$ is the vector-valued transformation.[22] Then, invertibility means that we can define the inverse $\bar{s}^{-1}$ by

$$\bar{S} = \bar{s}(\bar{s}^{-1}(\bar{S}; \bar{R}, \theta); \bar{R}, \theta)$$

Further, for a given parameter vector θ, denote the implied log financial health variable by $\ln \bar{H}^*$

$$\ln \bar{H}^* = \bar{s}^{-1}(\bar{S}; \bar{R}, \theta)$$

Then by the transformed data method the log likelihood function of the observed sample can be written as

$$L(S_t, R_t, t = 1, ..., N \mid \ln H_0, M_0, \theta) = L(\bar{S}, \bar{R}; \theta) = L^*(\ln \bar{H}^*, \bar{R}; \theta) + \ln |\det \mathcal{J}|$$

where L^* is the loglikelihood of the theoretical model taken at $\ln \bar{H}^*$ and $|\det \mathcal{J}|$ is the determinant of the Jacobian matrix of the transformation from $\bar{S}$ to $\ln \bar{H}^*$.

First, let us write out $L^*(\ln \bar{H}^*, \bar{R}; \theta)$. Let $\ln M_t^*$ be the time series of statistic affecting the ratings which is implied by the time series $\ln H_t^*$. The joint likelihood of the ratings process and the $\ln H_t^*$ process can be written as

$$P(\ln \bar{H}^*, \bar{R}; \theta) = \prod_{t=1}^{N} P(R_t, \ln H_t^* \mid \mathcal{F}_{t-1}; \theta)$$

Here the one-step-ahead probabilities are

$$P(R_t, \ln H_t^* \mid \mathcal{F}_{t-1}; \theta) = P(\ln H_t^* \mid \ln H_{t-1}^*, R_t, R_{t-1}; \theta) \times P(R_t \mid \ln M_{t-1}^*, R_{t-1}; \theta)$$

The logarithm of the first element on the left hand side of the previous expression is

$$\ln \left[P(\ln H_t^* \mid \ln H_{t-1}^*, R_t, R_{t-1}; \theta)\right] = -\frac{1}{2}\ln(2\pi) - \ln(\sigma) - \frac{(\ln H_t^* - a_t)^2}{2\sigma^2}$$

where $a_t = \ln(1 - J(R_t, R_{t-1}) + \ln H_{t-1}^* + r - \frac{\sigma^2}{2} + \sigma\lambda_{CF}$. Thus, the joint loglikelihood function of the theoretical model can then be written as

$$L^*(\ln \bar{H}^*, \bar{R}; \theta) = \sum_{t=1}^{N} \ln \left[P(R_t \mid \ln M_{t-1}^*, R_{t-1}; \theta)\right] + \sum_{t=1}^{N} \left[-\frac{1}{2}\ln(2\pi) - \ln(\sigma) - \frac{(\ln H_t^* - a_t)^2}{2\sigma^2}\right]$$

[22] We also implicitly condition on the initial values: $\ln H_0$ and $\ln M_0$. We suppress this for notational convenience.

To compute the Jacobian, consider the following

$$\det \mathcal{J} = \det \left[\frac{\partial \bar{s}^{-1}}{\partial \bar{S}} \right] = \left(\det \left[\frac{\partial \bar{s}}{\partial \ln \bar{H}} \right] \right)^{-1} = \left(\prod_{t=1}^{N} \left[\frac{\partial S_t}{\partial \ln H_t} \right] \right)^{-1}$$

Here the first equality follows from the definition of the Jacobian, the second from the fact that the Jacobian of an inverse function is the inverse of the Jacobian of the function. The third equality follows from the fact that in our model the equity value at t, S_t depends only on past values of $\ln H_t$. As a consequence the Jacobian matrix is triangular. But the determinant of a triangular matrix is simply the product of the elements on the main diagonal. These elements can be further written as

$$\frac{\partial S_t}{\partial \ln H_t} = \frac{\partial s}{\partial \ln H}(\ln H_t^*, \ln M_t^*, R_t) + \frac{\partial s}{\partial \ln M}(\ln H_t^*, \ln M_t^*, R_t) \frac{\partial \ln M_t}{\partial \ln H_t}(\ln H_t^*, \ln M_t^*, R_t)$$

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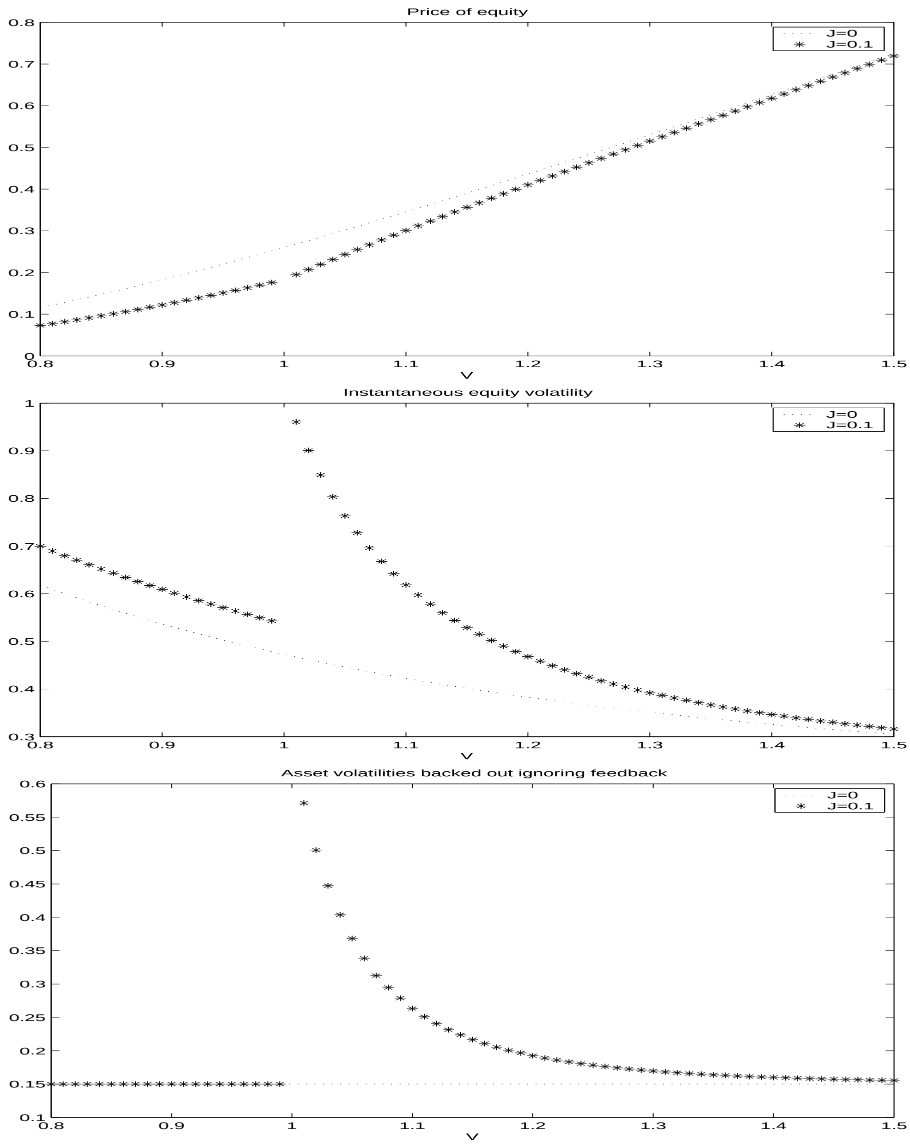
Figure 1: Illustrative example: Volatility behavior around downgrades due to feedback



This figure presents the consequences of feedback loss on volatility behavior around downgrades. In this illustrative example downgrade happens simply when the asset value falls below a barrier c_d. Working in the Black-Scholes model, in this case one can use standard analytical results to price equity. The parameters used to generate the plots are $\sigma = 0.15, c_d = 1, F = 1, T = 5, r = 0.05$. The upper panel shows equity prices, the middle one instantaneous equity volatilities as a function of the asset value. The bottom panel shows the asset volatilities that are computed from the equity value and the equity volatility ignoring the feedback loss. These are obtained by solving the two nonlinear equations connecting the asset value and asset volatility to equity and equity volatility.

Table 1: Monte Carlo Results on Samples of Downgraded Firms Simulated with Different Values of J

$J = 0$

	σ	λ_V	c_d	J
True Parameters	0.15	1/3	1	0
Mean	0.1459	-0.9727	1.0099	0.0081
Median	0.1460	-0.9468	1.0091	0.0000
10 Percentile	0.1326	-2.2021	0.9865	0.0000
90 Percentile	0.1604	0.0264	1.0351	0.0268
50 % coverage	0.4960	0.2180	0.4980	0.4834
75 % coverage	0.7160	0.4340	0.7300	0.7014
95 % coverage	0.9060	0.7900	0.9180	0.8578

$J = 0.05$

	σ	λ_V	c_d	J
True Parameters	0.15	1/3	1	0.05
Mean	0.1522	-0.9979	1.0034	0.0439
Median	0.1503	-0.9533	1.0040	0.0483
10 Percentile	0.1337	-2.0511	0.9709	0.0000
90 Percentile	0.1723	0.0151	1.0354	0.0698
50 % coverage	0.5380	0.2100	0.4720	0.6000
75 % coverage	0.7760	0.4160	0.7040	0.8273
95 % coverage	0.9440	0.7760	0.9300	0.9568

$J = 0.1$

	σ	λ_V	c_d	J
True Parameters	0.15	1/3	1	0.1
Mean	0.1533	-1.0026	1.0015	0.0925
Median	0.1507	-0.9596	1.0041	0.0981
10 percentile	0.1347	-2.0662	0.9616	0.0629
90 percentile	0.1759	0.0134	1.0361	0.1158
50 % coverage	0.5100	0.2080	0.4660	0.5396
75 % coverage	0.7580	0.4180	0.6860	0.7809
95 % coverage	0.9440	0.7720	0.9120	0.9635

This table presents the results of a Monte Carlo experiment investigating the behavior of our maximum likelihood estimator on samples of downgraded firms. The results are based on 500 downgraded paths, each one with 252 days of data, ending with the day of downgrade. The rating parameters are $\rho = 0.05, \alpha = 1/252$. The maturity is set to 5 years. The coverage rates are computed using asymptotic standard errors. For J, we drop the cases when $J = 0$ and compute the coverage rates on the remaining samples. For σ, λ_V and c_d we use all 500 paths to compute the coverage rates but use the corresponding 3×3 information matrix to compute the standard errors when the estimate of J is 0.

Table 2: Size and Power of the One-sided t-test on J

J Used for Simulation	0	0.05	0.1
Rejection rate at 5 % nominal size	0.0820	0.6400	0.8800
Rejection rate at 10 % nominal size	0.1120	0.7000	0.9200

This table presents the results of a Monte Carlo experiment investigating the size and power of a one sided t-test, testing the null hypothesis of no feedback. The results are based on 500 downgraded paths, each one with 252 days of data. The rating parameters are $\rho = 0.05, \alpha = 1/252$, the firm-specific parameters are $\sigma = 0.15, c_d = 1, \lambda_V = 1/3$. The maturity is set to 5 years. The t-statistic is computed using asymptotic standard errors.

Table 3: Descriptive statistics of the downgraded sample

Variable	Mean	Std.Dev	10 Percentile	90 Percentile
Firm size (millions of $)	6653	14398	489	13385
Leverage	0.70	0.16	0.48	0.92
Equity volatility	0.47	0.23	0.25	0.75
Equity mean return	-0.37	.71	-1.24	0.31

Firm size is defined as *Market Value of Equity+Total Liabilities*, leverage as $\frac{\textit{Total Liabilities}}{\textit{Market Value of Equity+Total Liabilities}}$. All values use the last available value before the downgrades.

Equity volatility is the standard deviation of past daily log holding period returns, while the mean equity return is the mean. Both are annualized and use 252 days of data ending with the day of downgrade.

Table 4: ML estimation results on the full sample of US public firms downgraded from investment grade to junk

Panel A: $\rho = 0.05, \alpha = 1/252$

	σ	λ_V	c_d	J
Mean	0.200	-0.87	1.24	0.071
Median	0.185	-0.80	1.10	0
10 percentile	0.056	-2.48	0.73	0
90 percentile	0.329	0.68	1.86	0.208

Number of firms with positive J: 77

Number of firms with J significant at the 5% level: 54

Panel B: $\rho = 0.1, \alpha = 1/252$

	σ	λ_V	c_d	J
Mean	0.202	-0.83	1.10	0.067
Median	0.186	-0.79	0.97	0
10 percentile	0.059	-2.38	0.64	0
90 percentile	0.341		1.63	0.20

Number of firms with positive J: 79

Number of firms with J significant at the 5% level: 55

Panel C: $\rho = 0.05, \alpha = 1/126$

	σ	λ_V	c_d	J
Mean	0.200	-0.85	1.18	0.066
Median	0.184	-0.81	1.06	0
10 percentile	0.057	-2.36	0.73	0
90 percentile	0.344	0.68	1.71	0.203

Number of firms with positive J: 76

Number of firms with J significant at the 5% level: 59

This table presents results of ML estimation on the full sample of downgraded firms for different values of the rating parameters ρ, α. The results are for the 166 firms with successful successful optimizations. The maturity is set to 5 years.

Table 5: Estimation results for a subsample of US public firms downgraded from investment grade to junk with a positive estimate of J

Panel A: $\rho = 0.05, \alpha = 1/252, N = 77$

	σ	λ_V	c_d	J
Mean	0.168	-1.23	1.24	0.153
Median	0.141	-1.12	1.03	0.113
10 percentile	0.033	-3.30	0.76	0.010
90 percentile	0.322	0.63	1.93	0.302

Panel B: $\rho = 0.1, \alpha = 1/252, N = 79$

	σ	λ_V	c_d	J
Mean	0.18	-1.20	1.07	0.142
Median	0.184	-1.11	0.91	0.108
10 percentile	0.04	-3.00	0.65	0.010
90 percentile	0.35	0.59	1.67	0.300

Panel C: $\rho = 0.05, \alpha = 1/126, N = 76$

	σ	λ_V	c_d	J
Mean	0.168	-1.28	1.21	0.144
Median	0.142	-1.26	1.01	0.112
10 percentile	0.039	-3.25	0.76	0.012
90 percentile	0.346	0.59	1.86	0.319

This table presents results of ML estimation on subsamples of downgraded firms with a positive estimate of J for different values of the ratings parameters ρ, α. The maturity is set to 5 years.

Table 6: How asset volatility estimates behave before downgrades?

	Merton Model ($\frac{\hat{\sigma}_i^{(0)}}{\hat{\sigma}_i^{(-1)}}$)	Feedback Model ($\frac{\hat{\sigma}_i^{(FB,0)}}{\hat{\sigma}_i^{(-1)}}$)
$\rho = 0.05, \alpha = 1/252, N = 77$		
Mean	1.28	1.03
(p-value of H_0 : Mean = 1)	(0.00036)	(0.589)
Median	1.09	0.87
(p-value of H_0 : Median = 1)	(0.0028)	(0.56)
$\rho = 0.1, \alpha = 1/252, N = 79$		
Mean	1.29	1.06
(p-value of H_0 : Mean = 1)	(0.00028)	(0.33)
Median	1.11	0.94
(p-value of H_0 : Median = 1)	(0.0019)	(0.91)
$\rho = 0.05, \alpha = 1/126, N = 76$		
Mean	1.31	1.01
(p-value of H_0 : Mean = 1)	(0.00017)	(0.77)
Median	1.12	0.89
(p-value of H_0 : Median = 1)	(0.0013)	(0.272)

This table presents cross-sectional descriptive statistics and tests on the behavior of estimated asset volatilities before downgrades for various sets of the rating parameters ρ, α. The p-values on the cross-sectional mean come from a t-test, while the p-values on the cross-sectional medians are from a Wilcoxon signed-rank test. The maturity of debt is set to 5 years. All the results are on the subsample of firms with a positive estimate of J.

Table 7: How asset volatility estimates behave after downgrades?

	Merton Model ($\frac{\hat{\sigma}_i^{(0)}}{\hat{\sigma}_i^{(+2)}}$)	Feedback Model ($\frac{\hat{\sigma}_i^{(FB,0)}}{\hat{\sigma}_i^{(+2)}}$)
$\rho = 0.05, \alpha = 1/252, N = 45$		
Mean	1.34	1.04
(p-value of H_0 : Mean = 1)	(0.0038)	(0.56)
Median	1.08	0.92
(p-value of H_0 : Median = 1)	(0.02)	(0.81)
$\rho = 0.1, \alpha = 1/252, N = 47$		
Mean	1.34	1.07
(p-value of H_0 : Mean = 1)	(0.0027)	(0.31)
Median	1.08	0.918
(p-value of H_0 : Median = 1)	(0.018)	(0.67)
$\rho = 0.05, \alpha = 1/126, N = 44$		
Mean	1.28	0.947
(p-value of H_0 : Mean = 1)	(0.016)	(0.48)
Median	1.01	0.84
(p-value of H_0 : Median = 1)	(0.17)	(0.092)

This table presents cross-sectional descriptive statistics and tests on the behavior of estimated asset volatilities after downgrades for various sets of the rating parameters ρ, α. The p-values on the cross-sectional mean come from a t-test, while the p-values on the cross-sectional medians are from a Wilcoxon signed-rank test. The maturity of debt is set to 5 years. All the results are on the subsample of firms with a positive estimate of J.

Table 8: How asset volatility estimates behave before downgrades after controlling for changes in market volatility?

	Merton Model ($\frac{\hat{\sigma}_i^{(0)}/\hat{\sigma}_{SP}^{(0)}}{\hat{\sigma}_i^{(-1)}/\hat{\sigma}_{SP}^{(-1)}}$)	Feedback Model ($\frac{\hat{\sigma}_i^{(FB,0)}/\hat{\sigma}_{SP}^{(0)}}{\hat{\sigma}_i^{(-1)}/\hat{\sigma}_{SP}^{(-1)}}$)
$\rho = 0.05, \alpha = 1/252, N = 77$		
Mean	1.24	1.02
(p-value of H_0 : Mean = 1)	(0.0009)	(0.76)
Median	1.09	0.86
(p-value of H_0 : Median = 1)	(0.0075)	(0.144)
$\rho = 0.1, \alpha = 1/252, N = 79$		
Mean	1.25	1.04
(p-value of H_0 : Mean = 1)	(0.0006)	(0.51)
Median	1.09	0.91
(p-value of H_0 : Median = 1)	(0.004)	(0.30)
$\rho = 0.05, \alpha = 1/126, N = 76$		
Mean	1.28	1.011
(p-value of H_0 : Mean = 1)	(0.00022)	(0.86)
Median	1.11	0.88
(p-value of H_0 : Median = 1)	(0.0014)	(0.069)

This table presents cross-sectional descriptive statistics and tests on the behavior of estimated asset volatilities before downgrades for various sets of the rating parameters ρ, α. The tests control for changes in market volatility ($\hat{\sigma}_{SP}^{(0)}$ for instance is the standard deviation of log returns of the S&P 500 in the year before downgrades). The p-values on the cross-sectional mean come from a t-test, while the p-values on the cross-sectional medians are from a Wilcoxon signed-rank test. The maturity of debt is set to 5 years. All the results are on the subsample of firms with a positive estimate of J.

Table 9: How asset volatility estimates behave after downgrades, controlling for market volatilities?

	Merton Model $(\frac{\hat{\sigma}_i^{(0)}/\hat{\sigma}_{SP}^{(0)}}{\hat{\sigma}_i^{(+2)}/\hat{\sigma}_{SP}^{(+2)}})$	Feedback Model $(\frac{\hat{\sigma}_i^{(FB,0)}/\hat{\sigma}_{SP}^{(0)}}{\hat{\sigma}_i^{(+2)}/\hat{\sigma}_{SP}^{(+2)}})$
$\rho = 0.05, \alpha = 1/252, N = 45$		
Mean	1.38	1.06
(p-value of H_0 : Mean = 1)	(0.0045)	(0.45)
Median	1.04	0.88
(p-value of H_0 : Median = 1)	(0.04)	(0.89)
$\rho = 0.1, \alpha = 1/252, N = 47$		
Mean	1.33	1.05
(p-value of H_0 : Mean = 1)	(0.0072)	(0.50)
Median	1.04	0.89
(p-value of H_0 : Median = 1)	(0.061)	(0.94)
$\rho = 0.05, \alpha = 1/126, N = 44$		
Mean	1.31	0.97
(p-value of H_0 : Mean = 1)	(0.015)	(0.75)
Median	1.05	0.85
(p-value of H_0 : Median = 1)	(0.12)	(0.132)

This table presents cross-sectional descriptive statistics and tests on the behavior of estimated asset volatilities after downgrades for various sets of the rating parameters ρ, α. Here we control for changes in the market volatility ($\hat{\sigma}_{SP}^{(0)}$ for instance is the standard deviation of log returns of the S&P 500 in the year before downgrades). The p-values on the cross-sectional mean come from a t-test, while the p-values on the cross-sectional medians are from a Wilcoxon signed-rank test. The maturity of debt is set to 5 years. All the results are on the subsample of firms with a positive estimate of J.

Table 10: Asset Volatility Prediction

	Merton Model	Feedback Model
$\rho = 0.05, \alpha = 1/252, N = 45$		
Absolute MSE	0.0226	0.0171
Relative MSE	0.67	0.234
$\rho = 0.1, \alpha = 1/252, N = 47$		
Absolute MSE	0.0221	0.0166
Relative MSE	0.668	0.254
$\rho = 0.05, \alpha = 1/126, N = 44$		
Absolute MSE	0.0229	0.0189
Relative MSE	0.649	0.243

This table investigates how well asset volatilities estimated with and without feedback in the year preceding the downgrade predict asset volatility estimates in the year ending 2 years after the downgrade. The two criteria used are relative and absolute mean square error (MSE). All the results are on the subsample of firms with a positive estimate of J.