

Inflation Rigidity and Monetary Policy Shocks¹

Julia Lendvai
University of Namur
Rempart de la Vierge 8, B-5000 Namur, Belgium
julia.lendvai@fundp.ac.be

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Abstract

In this paper we explore the influence of inflation rigidity on the monetary policy transmission mechanism in a model featuring the hybrid Phillips curve. We compare the New Keynesian Phillips curve and the hybrid Phillips curve for their contribution to reproducing stylized empirical facts about business cycles driven by monetary policy shocks. Variables' induced volatility and the dynamics of the impulse responses are analyzed. This includes the definition of a new persistence indicator based on the time profile of a variable's conditional variances as generated by monetary shocks.

Keywords: inflation inertia, monetary policy shocks, persistence indicator

1 Introduction

In recent years, macroeconomic research has been more and more focusing on the importance of monetary policy in the business cycle. A number of monetary business cycle models have been developed in this context. The distinguishing features of these so called New Keynesian models are the assumptions of imperfect competition and nominal rigidities which are specified on the basis of optimizing individual behavior. These assumptions account for linkages between nominal and real variables and have hence allowed researchers to study the real impact of monetary policy.

While the assumption of imperfect competition is in general modeled making use of the Dixit-Stiglitz specification of monopolistic competition, it has turned out to be less straightforward to place nominal rigidities on microeconomic foundations. There is still an open debate on how nominal rigidities can be specified in both a realistic and a tractable way so that at the same time the model is able to generate a behavior of aggregate economic variables compatible with what can be observed in the data.

Most New Keynesian models specify nominal rigidities based on the Calvo (1983) staggered price setting model. This specification, usually referred to as the New Keynesian Phillips Curve (NKPC), links the current inflation rate to the expected future inflation rate as well as to movements in the current output gap.

The NKPC is extensively used in literature; still, it has recently been subject to criticism, mainly on empirical grounds.

First, the NKPC implies purely forward looking inflation dynamics whereas the empirical rigidity of inflation is well documented. As argued by Fuhrer, Moore (1995), the NKPC also fails to reproduce the empirically observed dynamic link between inflation and the output gap.

In addition, the way price rigidity is specified influences the propagation of monetary shocks in the economy. However, as shown e.g. by Chari, Kehoe, McGrattan (2000), the Calvo model cannot, by itself, reproduce the observed persistence in business cycles generated by monetary shocks.

Finally, the issue of inflation rigidity also concerns the desirable conduct of monetary policy. In this aspect, the most obvious shortcoming of the NKPC is its inability to explain real costs of credible disinflationary policies¹.

These shortcomings have motivated the elaboration of specifications accounting for inflation rigidity. Gali, Gertler (1999, hereafter GG) have e.g. presented and estimated a structural specification of the *hybrid Phillips curve* which extended the NKPC to include lagged inflation based on the presence of backward looking firms². The authors assume a fraction of firms

¹For a discussion see e.g. Ball (1990), Fuhrer, Moore (1995), Gali, Gertler (1999), Walsh (1998).

²Other inflation rigidity frameworks have been suggested by Christiano, Eichenbaum, Evans (2001), Erceg, Levin (2001), Roberts (1997). Gali, Gertler, Lopez-Salido (2001)

in the economy, which, instead of optimizing rationally, readjust their prices according to a backward looking rule of thumb. This backward looking individual behavior allows them to account for inflation rigidity at the aggregate level. This is borne out by their empirical results. GG (1999) report indeed significant backward looking behavior in the determination of short run inflation dynamics³.

The hybrid Phillips curve has however met limited success in literature so far.

As for the description of short run inflation dynamics, the relative weight of backward and forward looking behavior is still an open debate. GG (1999) and Gali (2003) emphasize the predominance of forward looking behavior and claim hence that the pure forward looking NKPC provides a reasonably good description of inflation.

The hybrid Phillips curve has not been extensively taken over in the modeling of monetary business cycles either. Purely forward looking NKPC continues to dominate this literature⁴.

On the whole, the hybrid Phillips curve has gained most attention in studies investigating the optimal design of monetary policy⁵.

In view of this context, it seems all the more surprising how little work has been devoted so far to studying the influence of inflation rigidity on the monetary policy transmission mechanism. In this paper, we therefore seek to explore the way and the extent inflation rigidity affects the propagation of monetary policy shocks in a monetary business cycle model. We also try to assess whether the assumption of inflation rigidity can significantly improve a model's performance in reproducing stylized empirical facts about monetary business cycles in comparison to a flexible inflation setting. This should yield further insights into whether the hybrid Phillips curve can, theoretically, come up for the NKPC's shortcomings. At the same time, a deeper understanding of the monetary policy transmission under inflation rigidity might provide better insights for subsequent normative monetary policy analyses.

With this aim, we present a simple closed economy model featuring the

present a variant of the Gali, Gertler (1999) model, Smets, Wouters (2003) modify the Christiano et al. (2001) model. Nelson (1997), Coenen, Wieland (2003) compare different inflation persistence specifications. Discussions can be found e.g. in Woodford (2003) and Walsh (1998).

³Gali, Gertler, Lopez-Salido (2001) report a similar finding for the euro area. Benigno, Lopez-Salido (2002) estimate inflation persistence in five major countries of the euro area and find evidence for inflation persistence in France, Italy, Spain and the Netherlands, whereas inflation appears to be forward looking in Germany. Similar estimates are reported in Coenen, Wieland (2003).

⁴Monetary business cycle models incorporating the hybrid Phillips curve are presented by Christiano et al. (2001) and Smets, Wouters (2003).

⁵See e.g. Angeloni, Coenen, Smets (2003), Benigno, Lopez-Salido (2002), Coenen (2003), Steinsson (2003).

hybrid Phillips curve as specified by GG (1999). In this model, we study how inflation rigidity modifies the economy's reaction to a monetary shock. We then analyze how these modifications contribute to generating real variability by a monetary shock in the model. We also discuss the effect of inflation rigidity on the dynamics of inflation and output responses. The analysis of the dynamics includes the definition of a persistence indicator allowing us to compare complex dynamic processes.

The main findings of our analysis are as follows. Inflation rigidity implies on impact a smaller nominal and hence a greater real reaction. Inflation rigidity increases the real volatility generated by monetary shocks while leaving the variability of inflation almost constant. As for the dynamics, it increases the persistence of the inflation response; it does however not necessarily lead to an increase in the output gap's persistence.

Relating the effects of backward looking behavior to the standard Calvo assumption of staggered price setting, we find that inflation rigidity can act as a substitute to the Calvo-type price rigidity in generating real variability by a monetary shock. The two assumptions accounting for nominal rigidities have similar implications for the model's reaction in the period of the shock. Inflation rigidity introduces qualitatively new features into the dynamics of output and prices. While these new dynamic features do not clearly improve the model's performance in reproducing the observed persistence in monetary business cycles, they may come closer to explaining the empirically observed dynamic link between inflation and output.

The remainder of the paper is organized as follows. Section 2 presents the model and its solution. Section 3 describes the model's response to a monetary shock and discusses the modifications in the transmission mechanism implied by inflation rigidity. Section 4 studies the implications of these modifications with regard to the variability and the dynamics of inflation and output. Section 5 concludes.

2 The Model

In this section, we shall lay out a simple dynamic general equilibrium model of a monetary economy with imperfect competition and nominal rigidities. The model presented below is closely related to the specification described by Jeanne (1998). Decision taking is decentralized between optimizing households and firms. Monetary policy is exogenous. In our model, the labor market is assumed to be perfectly competitive and firms' staggered price setting is specified following Gali, Gertler (1999, hereafter GG) instead of the NKPC. These modifications should enable us later to identify the effects of inflation rigidity on the model economy⁶.

⁶For a more complete discussion of the consumption side of the economy see Jeanne (1998).

2.1 Households

Infinitely-lived households are identical and their number is normalized to 1. Households are assumed to maximize their lifetime utility⁷

$$U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{\sigma+1}}{\sigma+1} + \theta_l \frac{(1-l_t)^{\vartheta+1}}{\vartheta+1} \right]$$

with $0 < \beta \leq 1$ and $\sigma, \vartheta < 0$. A household's labor hours in period t are denoted by l_t ; normalizing the total time at households' disposal to 1, $1 - l_t$ stands for a household's leisure time; c_t is a CES aggregator over the quantities of different goods consumed, $c_t(i)$:

$$c_t = \left(\int_0^1 c_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (1)$$

with $i \in [0, 1]$ and $\varepsilon > 1$.

Households begin each period by trading on the financial market to reallocate their portfolio of cash, bonds and shares. Their initial money holdings are constituted by wages and dividend earnings paid at the end of the previous period. In addition, households receive lump sum transfers from the government. The financial trade is subject to the portfolio reallocation constraint. After this, households supply labor and production takes place followed by the trade on the goods market. Goods trade is subject to the CIA constraint:

$$P_t c_t \leq m_t^h + P_t W_t (l_t - L_t), \quad (2)$$

where P_t is an index of different goods' prices, m_t^h stands for the individual money holdings, W_t is the real wage and L_t denotes the aggregate labor supply⁸.

If a shock hits the economy, this happens before the opening of the financial market.

The *solution* to the households' problem can be summarized by the following five optimality conditions.

⁷Throughout the entire paper, notations are as follows. The current level of an aggregate variable in period t is denoted by X_t . The current individual levels are denoted by lower case letters x_t . The steady state level of a variable is denoted by letters without time index X respectively x for aggregate and individual levels. Lower case letters with tilde will denote the variables' percentage deviation from their steady state level, i.e. $\tilde{x}_t \equiv \frac{X_t - X}{X}$. For the nominal interest rate, the inflation rate and the money supply growth rate $\tilde{x}_t \equiv \frac{X_t - X}{1 + X}$.

⁸The second term on the right hand side represents an in-kind compensation for hours worked in excess of the aggregate labor supply. This modification of the standard CIA constraint has been introduced by Jeanne (1998) in order to abstract from the inflation tax effect on labor supply. The correction term disappears in equilibrium. For a more detailed discussion see Jeanne (1998) p. 1014.

First, households' demand for a good i can be expressed as:

$$c_t(i) = \left(\frac{P_t}{P_t(i)} \right)^\varepsilon c_t, \quad (3)$$

from which the aggregate price index is $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$, with $P_t(i)$ standing for the price of good i ⁹.

Second, the labor supply on the competitive labor market is given by:

$$\frac{\theta_l (1 - l_t^s)^\vartheta}{c_t^\sigma} = W_t. \quad (4)$$

Third, the intertemporal path of a household's aggregate consumption must satisfy:

$$c_t^\sigma = \beta E_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} c_{t+1}^\sigma \right], \quad (5)$$

where i_t stands for the riskless nominal interest rate; the inflation rate π_{t+1} is the rate of change of the aggregate price level between the periods t and $t + 1$: $1 + \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. This relationship is usually referred to as the consumption Euler equation.

Fourth, the share price of a firm i , $Q_t(i)$, can be expressed from the households' optimizing behavior as:

$$Q_t(i) = \frac{1}{\phi_t} \sum_{s=0}^{\infty} \beta^{s+1} \phi_{t+s} D_{t+s}(i) \quad (6)$$

where $D_{t+s}(i)$ denotes the nominal dividend on a share paid by firm i at the end of period $t + s$; ϕ_t is the marginal value of a currency unit for a household in period t , which will be equal across households in equilibrium.

Finally, the CIA constraint is binding optimally, which yields a household's money demand:

$$P_t c_t = m_t^h + P_t W_t (l_t - L_t). \quad (7)$$

2.2 Firms

There is a continuum of firms in the economy, each of which produces a differentiated good, $Y_t(i)$, using labor as the only input. The production technology is given by:

$$Y_t(i) = L_t(i),$$

where $L_t(i)$ is the labor employed by firm $i \in [0, 1]$.

⁹For a detailed discussion see e.g. Obstfeld, Rogoff (1999) Ch. 4.1.1 pp. 226-228.

On the aggregate level, the following relationship applies:

$$L_t^d \cong zY_t,$$

with $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ and total labor demand $L_t^d \equiv \int_0^1 L_t(i) di$ and z standing for a constant scalar¹⁰.

2.2.1 Staggered Price Setting with Backward Lookingness

We shall follow Calvo (1983) in assuming that in any given period, each firm readjusts its price to innovations with probability $1 - \xi$, or equally, each firm keeps its price fixed with probability ξ . This probability is common across firms and constant over time. The time between two price readjustments for an individual firm follows hence a geometric distribution. The expected time between two price readjustments is therefore¹¹ $(1 - \xi) \sum_{k=1}^{\infty} \xi^{k-1} k = \frac{1}{1-\xi}$.

Following Gali, Gertler (1999), we assume two types of firms: there is a fraction $1 - \omega$ of firms that readjust their prices in a forward looking way to maximize their share value. In contrast, the other fraction ω is assumed to follow a backward looking rule of thumb when having the possibility to readjust.

We shall hence distinguish between *fixed* and *newly set* prices in each period, where the newly set prices may be set in either a *forward looking* or a *backward looking* manner.

Denoting an index of fixed prices¹² P_t^{fix} and an index of newly set prices P_t^* , the aggregate price index can be expressed as:

$$P_t = [\xi(P_t^{fix})^{1-\varepsilon} + (1 - \xi)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \quad (8)$$

The newly set price index can itself be described as a weighted average of

¹⁰The exact relationship between aggregate labor demand and aggregate output would be $L_t^d = \int_0^1 Y_t(i) di = Y_t Z_t$, with $Z_t = \int_0^1 \frac{Y_t(i)}{Y_t} di$. However, as shown in Gali, Monacelli (2002) Appendix 3 for instance, the percent deviations of Z_t around its steady state are of second order. For the purpose of the following first order approximation of the model's solution, it is hence sufficient here to consider Z_t constant.

¹¹As opposed to time dependent models, where the probability of readjustment is fixed and given exogenously, this probability might be explained endogenously by the state of the economy. For a description and a comparison see e.g. Dotsey, King, Wolman (1999). It is generally argued that the difference in outcomes is negligible for moderate inflation rates. See GG (2003).

¹²Precisely, the index of fixed prices can be expressed as $P_t^{fix} \equiv \left(\frac{1}{\xi} \int_{fixed\ in\ t} P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ and the newly set prices as $P_t^* \equiv \left(\frac{1}{1-\xi} \int_{adj\ in\ t} P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$.

backward looking prices¹³ P_t^b , and forward looking prices, P_t^f :

$$P_t^* = [\omega(P_t^b)^{1-\varepsilon} + (1-\omega)(P_t^f)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \quad (9)$$

Following Yun (1996) we shall assume that *fixed prices* are updated by the target rate of inflation π , i.e. $P_t^{fix}(i) = P_{t-1}(i)(1+\pi)$.

Forward looking firms set their price to maximize their share value as given by equation (6). Knowing the demand for a firm's product as expressed by equation (3), assuming firms pay out their total profits to the households in form of dividends each period, and taking into account the symmetry of forward looking firms in equilibrium, any forward looking firm readjusting its price in period t will set P_t^f to maximize:

$$E_t \sum_{k=0}^{\infty} \phi_{t+k} (\beta\xi)^k \left[P_t^f (1+\pi)^k - P_{t+k} W_{t+k} \right] \left(\frac{P_{t+k}}{P_t^f (1+\pi)^k} \right)^\varepsilon Y_{t+k}. \quad (10)$$

Equation (10) shows that the price set at t influences the firm's profits and hence its dividends as long as it is not allowed to reoptimize, the probability of which is ξ^k with k denoting the number of successive fixed pricing periods. The marginal value of a currency unit to the households is treated as exogenous by the firm.

The FOC of the reoptimization is:

$$E_t \sum_{k=0}^{\infty} \phi_{t+k} (\beta\xi)^k Y_{t+k}(i) \left[P_t^f (1+\pi)^k - \frac{\varepsilon}{\varepsilon-1} P_{t+k} W_{t+k} \right] = 0. \quad (11)$$

Note that this relation reduces to the standard constant mark-up pricing rule of a flexible price environment, when $\xi = 0$.

It is instructive to rearrange this condition so as to express the percentage deviation of forward looking prices from the steady state as¹⁴:

$$\tilde{p}_t^f = \tilde{w}_t + \tilde{p}_t + E_t \sum_{k=1}^{\infty} (\beta\xi)^k \tilde{\pi}_{t+k} + E_t \sum_{k=1}^{\infty} (\beta\xi)^k (\tilde{w}_{t+k} - \tilde{w}_{t+k-1}). \quad (12)$$

This relation shows, that forward looking prices are set higher than the current nominal marginal cost when agents are expecting fast increasing prices for the future and/or if they expect the real marginal cost of production to increase. This behavior is what Christiano et al. (2001) call *frontloading*. Firms know that they might not be allowed to reoptimize their prices for a number of periods; the price they set today will then influence

¹³The precise expressions are $P_t^f = \left(\frac{1}{1-\omega} \int_f P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ and $P_t^b = \left(\frac{1}{\omega} \int_b P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$.
¹⁴See Christiano et al. (2001).

their future profits, too. Anticipating this, forward looking firms set their prices today to maximize their current and future expected profits. In addition to the current marginal cost, they thus need to take into account the future expected evolution of nominal marginal cost as well. This in turn depends on the future expected inflation rate and on the future expected changes of the real wage in our setting¹⁵.

Finally, *backward looking firms* are assumed to readjust their price according to the following rule of thumb:

$$P_t^b(i) = P_{t-1}^*(1 + \pi_{t-1}). \quad (13)$$

This shows, that a backward looking firm sets its price to the average of the newly set prices in the previous period updated by the previous period inflation rate of the aggregate price level¹⁶.

Equations (8), (9), (11) and (13) imply the following loglinearized relationship¹⁷:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t(\tilde{\pi}_{t+1}) + \lambda_w \tilde{w}_t \quad (14)$$

where $\gamma_b \equiv \frac{\omega}{\varphi}$, $\gamma_f \equiv \frac{\beta\xi}{\varphi}$, $\lambda_w \equiv \frac{(1-\omega)(1-\xi)(1-\beta\xi)}{\varphi}$ with $\varphi = \xi + \omega[1 - \xi(1 - \beta)]$.

This relationship is what GG (1999) call the *hybrid Phillips curve*.

2.2.2 Inflation Rigidity

To avoid confusion in the following exposition, it seems useful to define inflation rigidity at this place.

Inflation will be said to be sticky when it is not purely forward looking, i.e. in the case when past inflation has some influence on the current rate of inflation. In terms of the hybrid Phillips curve, inflation is hence purely forward looking (fully flexible), when the coefficient of lagged inflation, γ_b is 0. In contrast, inflation is backward looking (sticky or sluggish) when $\gamma_b > 0$.

It is important to point out that in the standard NKPC formulation, while prices are sticky, the inflation rate itself is fully flexible. Inflation rigidity is introduced by the presence of backward looking firms in the GG (1999) specification of the hybrid Phillips curve¹⁸. To see why, note that

¹⁵Note, that frontloading is not the only source of the forward looking price's deviation from the flexible price, i.e. the price level prevailing without rigidities. Forward looking price deviates from the flexible price also because the current nominal wage's path under sticky prices is different from its path under flexible prices.

¹⁶Although not very realistic, as discussed in GG (1999, p.13) this assumption has two appealing features: first, it implies no permanent deviations between the rule of thumb and the optimal behavior; second, P_t^b only depends on information known up to the period $t - 1$ but implicitly incorporates information about the future at the same time.

¹⁷For a derivation of this relationship see e.g. GGL (2001), Appendix A.

¹⁸Alternative explanations of inflation inertia are presented e.g. by Roberts (1997) who derives inflation rigidity from non-rational expectations and by Erceg, Levine (2001), who explain it with imperfect credibility.

any deviation of the inflation rate from its steady state is implied by the evolution of the newly set prices only¹⁹. Newly set prices are in turn purely forward looking in the NKPC setting while in the GG model, the newly set price index is an average of forward looking and of backward looking prices.

The greater the fraction of backward looking firms, ω , the more sluggish inflation will be. Setting $\omega = 0$ implies $\gamma_b = 0$, the hybrid Phillips curve (14) hence reduces to the standard NKPC in this case. A rise in ω implies a rise in the coefficient of the lagged inflation γ_b .

At the same time, a larger fraction of backward looking producers implies a lower weight of the currently expected future inflation, γ_f , as well as a lower value of the coefficient of the current real wage λ_w . This is because only forward looking firms react contemporaneously to current market conditions. The importance of current variables in the determination of inflation dynamics is hence lower when the fraction of forward looking firms, $1 - \omega$ is smaller. In the limiting case, where all firms are backward looking, the inflation rate would not at all react to current revision of inflation expectations or to changes in the current real marginal cost. This limiting case will be ruled out, since the assumption that all firms follow a backward looking rule of thumb would be in contradiction with the optimizing foundations of our model. Moreover, with all firms being backward looking, a temporary monetary shock would have a permanent effect on the output gap while leaving the inflation rate unaffected which would also contradict empirical findings.

2.3 Monetary Policy

Money is injected into the economy by the government via lump sum transfers to households.

To keep things simple, monetary policy is assumed to be exogenous. The path of nominal money supply growth rate will be given by the AR(1) process:

$$\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t, \quad (15)$$

where $\tilde{\mu}_t$ denotes the percent deviation of the money supply growth rate from its steady state target value. The autocorrelation coefficient is denoted by $\rho_\mu \in [0, 1[$. The monetary shock ϵ_t follows an i.i.d. white noise process with a standard deviation of σ_e .

¹⁹It is easy to show, that with the independence of fixed prices probability across firms and over time, with the number of firms large enough, and taking into account the fixed pricing firms updating rule, the average of the fixed prices is $P_t^{fix} = (1 + \pi)P_{t-1}$. The fixed prices' average is hence determined by steady state and lagged variables only and does not react to any contemporaneous innovation.

2.4 Solution

After the clearing of the financial, labor, goods and money markets, the equilibrium processes of the nominal interest rate, the inflation rate and the output gap can be expressed by the following log-linearized equation system:

$$\sigma \tilde{y}_t = \tilde{i}_t - E_t \tilde{\pi}_{t+1} + \sigma \tilde{y}_{t+1}, \quad (16)$$

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t(\tilde{\pi}_{t+1}) + \lambda \tilde{y}_t, \quad (17)$$

$$\tilde{y}_t = \tilde{y}_{t-1} - \tilde{\pi}_t + \tilde{\mu}_t \quad (18)$$

where the money supply growth rate, $\tilde{\mu}_t$ follows the exogenous law of motion given by equation (15).

Equation (16) is a first order approximation of the consumption Euler equation (5) taking into account the goods market equilibrium condition. The hybrid Phillips curve (17) uses the linear correspondence between output and real wage, with²⁰ $\lambda \equiv \lambda_w \left(-\frac{Y}{1-Y} \vartheta - \sigma \right)$. Finally, equation (18) is a first order approximation of the money supply growth process in equilibrium.

The solution for this dynamic equation system can be found by the method of undetermined coefficients as described in McCallum (1983). With monetary policy defined as in equation (15), the solution for the nominal interest rate is recursive. This allows us to subsequently concentrate on the solutions of the inflation rate and/or the output gap only²¹.

These can be written as²²:

$$\tilde{y}_t = \nu_1 \tilde{y}_{t-1} + \nu_2 \tilde{y}_{t-2} + \nu_{\mu 1} \tilde{\mu}_t + \nu_{\mu 2} \tilde{\mu}_{t-1}, \quad (19)$$

respectively:

$$\tilde{\pi}_t = \kappa_1 \tilde{\pi}_{t-1} + \kappa_2 \tilde{\pi}_{t-2} + \kappa_{\mu 1} \tilde{\mu}_t + \kappa_{\mu 2} \tilde{\mu}_{t-1}. \quad (20)$$

²⁰Note that the parameter λ is positive since $\sigma, \vartheta < 0$ and because the specification of the utility function and the equilibrium conditions imply $0 < Y < 1$.

²¹Strictly speaking \tilde{y}_t is the deviation of output from its steady state level, whereas the output gap is usually defined as the deviation of output from the level that would prevail under flexible prices. However, with monetary shocks only, the flexible price output corresponds to its steady state level, which allows us to call \tilde{y}_t the output gap. See Galí (2003).

²²As discussed in McCallum (1983), a solution of a given equation system may be expressed in several forms corresponding to different initial conjectures. Uhlig (1999) suggests the form:

$$\begin{aligned} \tilde{y}_t &= \nu_y \tilde{y}_{t-1} + \nu_\pi \tilde{\pi}_{t-1} + \nu_\mu \tilde{\mu}_t \\ \tilde{\pi}_t &= \kappa_y \tilde{y}_{t-1} + \kappa_\pi \tilde{\pi}_{t-1} + \kappa_\mu \tilde{\mu}_t. \end{aligned}$$

These different forms can be shown to correspond to a same solution of the system.

The coefficients $\nu_1, \nu_2, \nu_{\mu 1}$ and $\nu_{\mu 2}$ as well as the coefficients $\kappa_1, \kappa_2, \kappa_{\mu 1}$ and $\kappa_{\mu 2}$ are functions of the parameters γ_f, γ_b and λ , and are hence implicit functions of the underlying structural parameters of the model. Under any plausible set of parameter values, the model has a unique stable solution. The derivation of the solution is described in the Appendix.

3 Transmission of a Monetary Policy Shock

Our aim is to explore in what way the assumption generating inflation inertia affects a monetary shock's impact onto the economy. As already pointed out, inflation rigidity is implied by the backward looking behavior of a fraction of producers. We shall therefore try to gain insight into the way backward looking behavior influences the transmission and the impact of a monetary shock in the economy. Formally, the exercise consists of analyzing the impact of a change in the fraction of backward looking firms, ω .

Since there exists no closed form solution to our model we shall proceed as follows. The model's structural parameters will be calibrated based on the results of existing literature. We shall then study the reaction of the model to a monetary shock under this baseline calibration for different levels of $\omega \in [0, 1[$, everything else unchanged.

3.1 Calibration

As in most NK models, one period equals a quarter of a year. Setting the subjective discount factor $\beta = 0.99$ hence implies an annual real interest rate of 4.04% in the steady state²³. The parameters of the utility function are set to $\sigma = \vartheta = -1$, which corresponds to a log-utility for both consumption and leisure²⁴. The elasticity of substitution between consumption and leisure, θ_l is set to imply a perfect competition steady state of labor equal to 0.33; that is, in the steady state a household is assumed to spend one third of its total disposable time on working. The elasticity of substitution between differentiated goods ε is set to 6, implying a steady state markup of 20%. This lies within the range of calibrations suggested in related literature²⁵. The steady state labor's share implied by this mark-up is equal to 0.833. The probability for a firm of not being able to reoptimize its price (hereafter probability of fixed price) ξ is set to 0.75. This implies an average price

²³This assumption is common in NK literature. See e.g. Walsh (1998), p.74.

²⁴For a discussion of this calibration see Gali (2003).

²⁵Gali, Monacelli (2002) set $\varepsilon = 6$ as well. Gali (2003) sets $\varepsilon = 11$. Christiano et al. (2001) have estimated $\varepsilon = 3$ for the US. Benigno, Lopez-Salido (2002) calibrate ε to imply a steady state mark-up of 20%. Gali, Gertler, Lopez-Salido (2001, p.1254) discuss the choice of markups in the empirically plausible range of 10-40%: they find that any choice within the interval yielded similar results.

duration of 1 year, which is in line with several empirical estimations²⁶. The benchmark value of the money growth rate's autoregression coefficient will be set to $\rho_\mu = 0.5$; this value corresponds to empirical estimates²⁷. The standard deviation of the money shock, σ_e will be normalized to 1 percent.

3.2 Impulse Responses

Figure 1 displays the impulse responses of the price level²⁸, inflation and output under the benchmark calibration for different levels of ω . The impulse responses have been simulated by means of a MATLAB algorithm we have written based on the solution given in equations (19) and (20). The solution has been evaluated under the baseline calibration. An unexpected 1 percent expansionary monetary shock takes place in the first period; the money growth autoregression coefficient is $\rho_\mu = 0.5$. The $\omega = 0$ case corresponds to the impulse responses with the NKPC. GG (1999) and Gali, Gertler, Lopez-Salido (2001, henceforth GGL) estimate values of ω in the interval of 0.2 - 0.4. The $\omega = 0.3$ case hence shows the reaction of our model corresponding to their estimations. Fuhrer-Moore (1995) presented a specification which would correspond to setting $\omega = 0.7$. This is also the value implied by the estimations of the Christiano et al. (2001) specification²⁹.

The graphs suggest that with a higher fraction of backward looking firms in the economy, the impact of a monetary shock on the price level and on inflation would be smaller, and the impact on output greater in the period of the shock. This impression is readily confirmed by evaluating coefficients $\nu_{\mu 1}$ and $\kappa_{\mu 1}$ in equations (19) resp. (20) as functions of ω . Indeed, $\nu_{\mu 1}$ and $\kappa_{\mu 1}$ capture directly the contemporaneous impact of a shock on output respectively inflation. The results are displayed in Figure 2.

The influence of backward looking firms on the persistence of the shock's effect is far less obvious. An inspection of Figure 1 indicates however that a high enough fraction of backward looking firms induces cyclical fluctuations of prices and output around their long run steady state.

The monetary shock is transmitted onto the real economy via the rigidities of firms' price setting. Firms that cannot readjust their prices in response to the monetary shock will benefit from a higher demand for their

²⁶See e.g. GG (1999) and Gali, Gertler, Lopez-Salido (2001). Christiano et al. (2001) find a somewhat lower value, but the standard deviation of their estimate is relatively high. See also discussions in Rotemberg, Woodford (1998) and Smets, Wouters (2003).

²⁷See e.g. Walsh (1998), Yun (1996), Gali (2003).

²⁸Note, that the steady state of any price index changes in response to a shock on the money supply growth rate. The following results are displayed and interpreted with respect to the initial steady state.

²⁹Setting $\omega = 0$ implies $\gamma_b = 0$, $\gamma_f = 0.99$, $\lambda = 0.12$. When $\omega = 0.3$, $\gamma_b = 0.286$, $\gamma_f = 0.709$, and $\lambda = 0.061$. Finally, $\omega = 0.7$ implies $\gamma_b = 0.485$, $\gamma_f = 0.514$, and $\lambda = 0.019$. Note that, while Christiano et al.'s (2001) specification implies values for γ_b and γ_f close to those implied by setting $\omega = 0.7$, their estimate of λ is much higher.

goods: the demand for a firm's good increases if the firm's price $\tilde{p}_t(i)$ is lower than the aggregate price level \tilde{p}_t . For the aggregate output index \tilde{y}_t to deviate from its steady state, however, the average price level set by all firms, \tilde{p}_t , needs to be different from the flexible price level, i.e. the level of prices that would prevail without rigidities. This will be the case in response to a monetary shock under any sticky price assumption.

3.3 Backward Looking Behavior: Discussion

The presence of backward looking firms changes the real impact of a monetary shock by modifying the path of the aggregate price level. This then changes the gap between \tilde{p}_t and the flexible price level and therefore modifies the path of output as well.

The changes in the \tilde{p}_t path when ω increases reflect the reaction of a complex system. Indeed, it is important to note that all deviations from the steady state of any price index in the model are the result of complex simultaneous interactions among all firms in the economy. Hence, any individual price reflects the past or current behavior of any other firm and also incorporates lagged or contemporaneous expectations about the future path of the entire economy. The differences implied by an increasing fraction of backward looking firms in the indexes of both fixed and newly set prices and hence in the aggregate price level are therefore hardly attributable to the behavior of one particular firm type or to the evolution of one special price category only.

Figure 3 shows for three different levels of ω the dynamic impulse responses of different price categories to a 1 percent monetary shock at $t = 1$ with a money supply growth autoregression coefficient $\rho_\mu = 0.5$ under the baseline calibration.

For the period of the shock, a higher level of ω implies the aggregate price index, \tilde{p}_t to react less. To see why, note that the economy is supposed to have been in a steady state up to the shock. In this case, backward looking firms behave like fixed price setting firms in the period of the shock: neither of these firms reacts contemporaneously to the shock. The contemporaneous change of the aggregate price level is hence implied by the change of a smaller fraction of forward looking prices, $1 - \omega$ only. This then implies a smaller impact of the shock on the aggregate price index³⁰.

Due to the similar behavior of newly set backward looking prices and fixed prices in the period of the shock, a change in the fraction of backward looking firms, ω , acts like a change in the probability of fixed prices, ξ in

³⁰Impact reaction is also influenced by the degree of the shock's persistence, ρ_μ for $\forall \omega$. Higher ρ_μ implies a larger nominal reaction on impact. This is due to the effect of a constant fraction $(1 - \xi)(1 - \omega)$ of firms resetting their prices in a forward looking way in a given period: these firms would frontload higher expected future inflation rates in case of a higher ρ_μ . The reaction of all the other firms is independent of ρ_μ .

decreasing the contemporaneous nominal impact of the monetary shock

In subsequent periods, changes in the aggregate price index are determined by both forward looking and backward looking types of firms. The presence of backward looking firms introduces inflation inertia. This however does not necessarily imply slower price increases for longer periods like price rigidity would do. On the contrary, backward looking prices, \tilde{p}_t^b turn out to increase relatively fast to relatively high levels a couple of periods after the shock. Large values of ω therefore tend to imply a relatively high average of newly set prices and thereby a high aggregate price index in the given periods.

By the inflation rigidity induced by backward looking firms, the aggregate price index, \tilde{p}_t hence starts to increase slower after the shock but would then increase at a faster pace and to higher levels, the more so, the higher ω . The aggregate price index therefore reaches the flexible price level faster when the fraction of backward looking firms is larger.

With ω high enough, the aggregate price level may even overshoot the flexible price level in the medium run. The threshold value of ω at which overshooting in the aggregate price index sets in, depends on the underlying parameters of the model.

The way the *output* reaction to a monetary shock is modified by backward looking firms, follows directly from their influence on the aggregate price level. Note in particular that cyclical fluctuations around the steady state can also be observed in the output transition path when the fraction of backward looking firms is large enough.

4 Inflation Rigidity and Business Cycles

In the previous section we have described in which way backward looking behavior modifies the price, inflation and output responses to a monetary shock. It is however not straightforward to conclude what these modifications imply for the model's capacity to reproduce stylized empirical facts about monetary business cycles.

This section studies the influence of inflation rigidity on the volatility and the dynamics of inflation and output when shocks to the money supply growth process are the only source of fluctuations. For the study of persistence, we shall define an indicator which allows us to evaluate and to compare the persistence of complex dynamic processes.

4.1 Volatility

In order to measure the volatility of inflation and output, we have calculated those variables' unconditional standard deviation³¹ σ_x generated by an in-

³¹Note that the unconditional variance of a variable is the limit of its forecasting error variance an infinite number of periods ahead. To see why note that, with mon-

dependent white noise process of monetary shocks $\{\epsilon_{t+j}\}_{j=0}^{\infty}$ for different levels of ω under the baseline calibration ($\sigma_{\epsilon} = 1\%$, $\rho_{\mu} = 0.5$). The results are displayed in Figure 4.

As can be observed in the figure, *backward looking behavior increases output volatility compared to the purely forward looking setting. In contrast, the volatility of the inflation rate seems to be practically unaffected by inflation rigidity for any plausible level of backward lookingness.* Note that the purely forward looking calibration of our model would already account for a relatively large output volatility (1.31 percent)³². Setting ω to higher values may additionally imply a significant rise in σ_y .

How are these implications of inflation rigidity for the output volatility related to the influence of the standard Calvo-type price rigidity? To find out, we have compared the effect of backward looking behavior, ω , to the effect of the probability of fixed prices, ξ , on the standard deviation of output. As one might have expected, the degree of inflation rigidity as measured by ω , and the degree of price stickiness captured by ξ , act as substitute inputs in generating real volatility by monetary shocks in our model.

Figure 5 displays the iso-volatility curves of output in the $(\omega; \xi)$ space. An iso-volatility curve connects the points $(\omega; \xi) \mid \sigma_y = \text{constant}$. The negative slope of those curves shows that a decrease in the probability of fixed prices may be offset by an increasing fraction of firms behaving in a backward looking way. This result is not surprising as both the assumptions of fixed prices and of backward looking behavior introduce nominal rigidity into our model which then account for the real impact of a monetary shock. The concavity of the iso-volatility curves is a result of an increasing marginal effect of changes in ξ or in ω .

Finally, it should be noted, that the volatility of both inflation and output depend on the persistence of the monetary shock ρ_{μ} . The more persistent the shock, the higher the volatility of inflation and of the output gap in response to the shock. At the same time, the volatility of output is increasing in both ξ and ω for any degree of persistence of the monetary shock.

etary shocks being the only source of fluctuations, the forecasting error can be expressed as $FE_s(\tilde{x}) \equiv \tilde{x}_{t+s} - E_t(\tilde{x}_{t+s} | I_t) = c_0 \epsilon_{t+s} + c_1 \epsilon_{t+s-1} + \dots + c_{s-1} \epsilon_{t+1}$, where $I_t = \{\tilde{x}_{t-1}, \tilde{x}_{t-2}, \dots, \epsilon_t, \epsilon_{t-1}, \dots\}$. With ϵ_{t+j} being elements of an independent white noise process, the forecasting error variance is $FEV_s(\tilde{x}) = \sigma_{\epsilon}^2 \sum_{j=0}^{s-1} c_j^2$. When $s \rightarrow \infty$, $FEV_s(\tilde{x}) \rightarrow \sigma_x^2$. For a more detailed discussion see e.g. Hamilton (1994, Ch.11).

³²To compare, Stock and Watson (1999) estimate a value of 1.66 percent for the standard deviation of U.S. GDP in the postwar period. According to Gali (2003), other estimates in the literature report similar values. The important real effects of monetary shocks should not be overemphasized because of the simplicity of our model.

4.2 Dynamics

As noted in section 3, the similarity between fixed pricing firms and firms readjusting their prices in a backward looking way only holds in the period of the shock. This suggests that the presence of backward looking firms might introduce some qualitatively new features into the dynamics of the inflation and/or output response to a monetary shock. In the following subsection we present a persistence indicator by the means of which we can meaningfully compare the persistence of smooth and cyclically fluctuating series; after this, we shall discuss the effect of inflation rigidity on the persistence of impulse responses and on the dynamic link between inflation and output.

4.2.1 Persistence Indicator

Measuring the persistence of inflation and output is not a straightforward exercise. Equations (19) and (20) describe complex dynamic processes which may imply cyclical fluctuations. Under these circumstances, the usual measure of a shock's half-life³³ loses its relevance. We therefore need to define a new persistence indicator that can capture the intertemporal distribution of a shock's impact on a variable, while being independent of the sign of the variable's deviations from its steady state in response to the shock.

To discuss persistence, we shall focus on the case where uncertainty is restricted to a single period t . That is, all information about the economy's state is known up to period $t - 1$. An unexpected temporary shock to the money supply growth rate, with volatility σ_ϵ and an autoregression coefficient $\rho_\mu \in [0, 1[$, may then hit the economy in period t , after which the money supply evolves in a perfectly foreseen manner according to its exogenous law of motion.

The difference between a variable's realization s periods after the shock, \tilde{x}_{t+s} and its expected value based on information up to the shock, $E_{t-1}(\tilde{x}_{t+s}|I_{t-1})$ can then be expressed as:

$$\tilde{x}_{t+s} - E_{t-1}(\tilde{x}_{t+s}|I_{t-1}) = c_s \epsilon_t, \quad \forall s,$$

with $I_{t-1} = \{\tilde{y}_{t-2}, \tilde{y}_{t-3}, \dots, \epsilon_{t-1}, \epsilon_{t-2}, \dots\}$. The coefficient c_s is a function of ρ_μ and of the model's structural parameters.

The conditional variance of variable \tilde{x} s periods ahead, generated by the stochastic shock in period t , would then be equal to:

$$V_{t-1}(\tilde{x}_{t+s}) = E_{t-1} \left[(\tilde{x}_{t+s} - E_{t-1}(\tilde{x}_{t+s}|I_{t-1}))^2 \right] = c_s^2 \sigma_\epsilon^2, \quad \forall s.$$

Let $ISV_{t-1}(\tilde{x}_{t+s})$ denote the intertemporal sum of variances, i.e. the across-time sum of a variable's conditional variances between the periods t

³³For a description of the half-life of a shock as a measure of persistence see Chari, Kehoe, McGrattan (2000) or Woodford (2003, Ch.3).

and $t + s$, caused by the stochastic shock of period t :

$$ISV_{t-1}(\tilde{x}_{t+s}) \equiv \sum_{j=0}^s V_{t-1}(\tilde{x}_{t+j}) = \sigma_\epsilon^2 \sum_{j=0}^s c_j^2.$$

Let us define a variable's intertemporal total volatility, ITV_x as the across-time sum of conditional variances caused by the shock over indefinite time:

$$ITV_x \equiv \sum_{j=0}^{\infty} V_{t-1}(\tilde{x}_{t+j}) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} c_j^2.$$

Considering non-explosive solution paths only, the intertemporal total volatility is finite³⁴. Intertemporal total volatility is hence the finite limit to which the across-time sum of variances converges when the number of observed periods s increases, i.e. when $s \rightarrow \infty$, $ISV_{t-1}(\tilde{x}_{t+s}) \rightarrow ITV_x$.

Having said this, we shall define the indicator of a variable's persistence as:

$$\Psi_s(\tilde{x}) \equiv \frac{ITV_x - ISV_{t-1}(\tilde{x}_{t+s})}{ITV_x},$$

where $0 \leq \Psi_s(\tilde{x}) \leq 1$, by construction.

On the basis of the above described considerations, the persistence indicator $\Psi_s(\tilde{x})$ expresses the *percentage of the variable's intertemporal total volatility generated by a shock in period t , which is to take place later than s periods after the shock*. Is the variable's response not persistent at all, this fraction would be equal to 0 in the period of the shock. That is, the total volatility of the variable in response to a monetary shock would take place in the period of the shock, and no more variability would follow. The *more persistent* the effect of the shock on the variable, the greater a fraction of the variable's intertemporal total volatility takes place in periods further away from the shock. This implies a *higher value* for $\Psi_s(\tilde{x})$ in any given period $t + s$. Should the backward looking firms' behavior increase the persistence of a variable's response, this would hence be indicated by $\Psi_s(\tilde{x} | \omega_1) \geq \Psi_s(\tilde{x} | \omega_2)$ when $\omega_1 > \omega_2$ for $\forall s$.

Note, that the persistence of a variable's response is different from the 'rigidity' of this variable. Hence, the inflation response to a monetary shock would be persistent even in the NKPC setting, i.e. when inflation is said to be fully flexible. While the rigidity of a variable may influence its response's persistence, it is not the only factor contributing to persistence.

³⁴Note that in the case when all fluctuations come from a single source, the unconditional variance σ_x^2 generated by an independent white noise process of consecutive shocks will be equal to the intertemporal total volatility ITV_x caused by a single stochastic shock in period t over indefinite time.

4.2.2 Persistence

Calculating the value of $\Psi_s(\tilde{\pi})$ and $\Psi_s(\tilde{y})$ under the baseline calibration we find that *increasing the fraction of backward looking firms, ω , increases the persistence of the inflation rate; however, the effect of an increase of ω on the persistence of the output response is ambiguous.*

Figure 6 displays $\Psi_4(\tilde{\pi})$ and $\Psi_4(\tilde{y})$, i.e. the percentage of total inflation respectively output volatility that takes place more than one year after the shock, as a function of ω . Table 1 lists values of $\Psi_s(\tilde{\pi})$ and $\Psi_s(\tilde{y})$ as function of ω for different periods s .

The first panel of the figure shows that $\Psi_4(\tilde{\pi})$ increases in ω . This finding is confirmed and generalized to different periods by values listed in Table 1. The values indicate a clear rise of persistence for any period when ω increases. This finding is not surprising: it confirms the intuitive expectation that the increasing rigidity of the inflation rate, as implied by the increasing fraction of backward looking firms, ω , increases the persistence of the inflation response.

The graph of $\Psi_4(\tilde{y})$ displays a somewhat different pattern. *The persistence of the output gap seems to decrease first with an increasing fraction of backward looking firms, and to increase only when the fraction of backward looking firms is large enough.* The greater real impact of a monetary shock which can be observed with a larger fraction of backward looking firms, does hence not necessarily go together with an increased persistence of the real response.

The second panel of Table 1 confirms that the output response is least persistent for values of ω between³⁵ 0.4 and 0.5. Note especially, that the degree of output persistence, $\Psi_s(\tilde{y})$, is strictly lower with $\omega = 0.3$, i.e. the value estimated by GG (1999) and GGL (2001), than in the purely forward looking case $\omega = 0$. To generate a degree of output persistence $\Psi_s(\tilde{y})$ similar to the degree implied by the purely forward looking setting, the fraction of backward looking producers needs to be set as high as $\omega = 0.6$ or 0.7 , i.e. approximately the values implied by the Christiano et al. (2001) specification.

The pattern of $\Psi_s(\tilde{y})$ as a function of ω reflects the dynamic impact of backward lookingness described in section 3: the increasing fraction of backward looking firms tends to shorten the time necessary for the output to reach its steady state level after the shock; this then has a decreasing effect on the output persistence, $\Psi_s(\tilde{y})$ for lower levels of ω . However, when the fraction of backward looking firms is large enough, cyclical fluctuations in the output path would set in. This then induces larger deviations of the output from its steady state in later periods, and thereby increases the

³⁵The value of ω implying minimum persistence of the output gap may vary with the monetary shock's persistence ρ_μ . With $\rho_\mu = 0$, the minimum value of $\Psi_s(\tilde{y})$ can be found for $\omega \in [0.3, 0.5]$ while $\rho_\mu = 0.9$ $\min \Psi_s(\tilde{y})$ for $\omega \in [0.5, 0.7]$.

persistence of those deviations as measured by $\Psi_s(\tilde{y})$.

While the level of $\Psi_s(\tilde{y})$ depends on the persistence of the monetary shock ρ_μ , it is important to note that the above described pattern of $\Psi_s(\tilde{y})$ as a function of ω seems to be robust across different degrees of money shock persistence: the output persistence decreases first with an increasing fraction of backward looking firms reaches a minimum and increases again for higher values of ω independently of the level of ρ_μ .

The effect of backward looking behavior on the real persistence is in contrast to the effect of a change in the probability of fixed prices ξ : while an increase in ω has an ambiguous effect on $\Psi_s(\tilde{y})$, an increase in ξ would clearly raise the persistence of the output response³⁶. In this sense, increasing ω cannot act as a substitute to ξ in generating persistent business cycles.

It should be noted, that the Calvo model has been criticized for its shortcoming in generating empirically observed degrees of business cycle persistence when assuming plausible levels of price rigidity³⁷. The fact that the assumption of backward looking firms does not increase the persistence of the real response to a monetary shock with plausible degrees of backward looking behavior suggests that models featuring inflation rigidity cannot come up for this failure of the NKPC.

4.2.3 Dynamic Link between Output and Inflation

Another empirical shortcoming of the Calvo model is that it fails to match the observed dynamic link between output and inflation.

As reported in Christiano et al. (1999) and Christiano et al. (2001) for instance, the response of the output gap to a monetary policy shock precedes that of the inflation rate by several quarters.

Empirical cross-correlations of output and inflation also indicate that the output gap leads the inflation rate over the cycle. GG (1999) present cross-correlations of the current output gap with leads and lags of inflation computed on postwar U.S. time series. They report that the current output gap is positively correlated with future inflation while it co-moves negatively with lagged inflation. The highest positive correlation can be found between the current output gap and inflation four quarters ahead. The lowest negative correlation is reported between current output gap and inflation lagged by six quarters.

As pointed out by Fuhrer, Moore (1995) and by GG (1999) however, the Calvo model implies that inflation should lead output, i.e. the opposite of what can be found in the data.

³⁶For a discussion of the influence of increasing price rigidity on the output persistence see also Jeanne (1998).

³⁷See e.g. Chari et al. (2000).

Figure 7 displays the cross-correlations of the current output gap with lags and leads of inflation, $Corr(\tilde{y}_t, \tilde{\pi}_{t+j})$, generated by an independent white noise process of consecutive monetary shocks in our model. Theoretical cross-correlations have been evaluated under the baseline calibration for three different values of ω .

While the empirical values reported by GG (1999) reflect the combined effects of shocks of any origin during their sample period for the US economy, in our model, business cycles are driven by monetary shocks only. It is not surprising that the cross-correlation pattern generated by our model for any level of ω does not come close to the observed values.

Nevertheless, two features indicated in Figure 7 seem to be worth noting.

First, higher levels of ω shift the cross-correlations to the right, in the sense of implying higher positive correlations of the current output gap with future inflation and lower values with past inflation. This is a confirmation of the finding described in the previous section: backward looking behavior increases the persistence of inflation while it has an ambiguous effect on the persistence of the output response. This then delays the effect of a monetary shock on inflation compared to its effect on output. Inflation rigidity thereby contributes to explaining the asymmetries in the time profile of the output and inflation impulse responses; hence, inflation rigidity also contributes to reproducing relatively high observed correlations between current output and leads of inflation.

A second and more surprising effect of inflation rigidity on the cross-correlations between output and inflation is that it may contribute to explaining negative cross-correlation values even in the case when business cycles are driven by monetary shocks only (see panel 3). In fact, monetary shocks drive the output gap and the inflation rate in the same direction: i.e. an expansionary (restrictive) monetary shock implies a rise (decline) in both output and inflation. This should then imply positive correlations of current output with both leads and lags of inflation. The reason that inflation rigidity can still lead to negative cross-correlation values lies in the fact that relatively high levels of ω induce cyclical fluctuations in the transition paths of inflation and output; the initial expansionary (restrictive) effect of a positive monetary shock would then turn into temporary contraction (expansion) a couple of periods after the shock.

5 Conclusion

In this paper we have compared the hybrid Phillips curve, as specified by Gali, Gertler (1999), to the standard New Keynesian Phillips curve from a theoretical point of view.

We have studied in a simple closed economy model the way inflation rigidity, incorporated into the hybrid Phillips curve by the assumption of backward looking behavior, modifies the model economy's reaction to a monetary shock. We have then discussed the effects of those modifications on the model's capacity to reproduce stylized facts about monetary business cycles.

The principal findings of our analysis are that inflation rigidity increases the real volatility generated by a monetary shock. The assumptions of inflation rigidity and of the standard Calvo-type price rigidity have similar implications for the model's reaction in the period of the shock. The main differences between the two assumptions lie in their implications for the dynamics of the inflation and output responses.

As opposed to staggered price setting, the assumption of backward looking rule-of-thumb behavior does not clearly raise the persistence of the output response. Indeed, implausibly large fractions of backward looking producers need to be assumed in order to observe an increase in output persistence compared to the purely forward looking setting. On the other hand, inflation rigidity delays the effect of a monetary shock on inflation compared to its effect on output. Moreover, by the cyclical fluctuations induced by higher degrees of inflation rigidity, negative dynamic cross-correlation values between inflation and output can be generated even in the case when shocks to the monetary policy are the only source of fluctuations.

While the hybrid Phillips curve can thus contribute to explaining the observed dynamic link between inflation and output, the actual persistence of those variables appears to be predominantly governed by factors different from price or inflation rigidity. One plausible explanation of persistence might be frictions in the economy, which go beyond price and inflation rigidity.

Jeanne (1998) e.g. makes a case for real rigidities. In his paper he shows that a low degree of real wage rigidity can account for a significant degree of output persistence. Christiano et al. (2001) have presented a model assuming Calvo type staggered price and nominal wage setting with backward looking behavior as well as frictions on the real side of the economy as consumption habit persistence, capital adjustment costs and variable capital utilization. The authors compare the effect of different nominal and real rigidity assumptions on their model's performance in generating output persistence and find that their assumption of nominal wage rigidities has played a more important role than their specification of price and inflation rigidities in replicating persistence in business cycles; in addition, they emphasize

the importance of real rigidities, too. Gali, Gertler (1999) also stress the importance of labor market rigidities in explaining the short run dynamics of the output gap and the inflation rate. Future research should be devoted to further investigate this set of hypotheses.

In our paper we have analyzed the effects of a stochastic shock to exogenous monetary policy only. In future research, we intend to study the effects of real shocks with endogenous monetary policy. This would also allow us to investigate the way inflation rigidity influences the transmission of systematic monetary policy.

Finally, we are also interested in extending our analysis to an open economy framework.

6 Appendix

In this Appendix we describe the solution of the log-linearized model for output. The solution for inflation can be found in a similar way.

The solution for output can be found from equations (17) and (18):

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t(\tilde{\pi}_{t+1}) + \lambda \tilde{y}_t, \quad (\text{A.1})$$

and

$$\tilde{\pi}_t = \tilde{y}_{t-1} - \tilde{y}_t + \tilde{\mu}_t, \quad (\text{A.2})$$

knowing the exogenous process governing the money growth $\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t$.

Substituting out for $\tilde{\pi}_t$, $\tilde{\pi}_{t-1}$ and $E_t(\tilde{\pi}_{t+1})$ using equation (A.2) gives a third order stochastic difference equation linking output and money growth:

$$\gamma_f E_t \tilde{y}_{t+1} - (1 + \lambda + \gamma_f) \tilde{y}_t + (1 + \gamma_b) \tilde{y}_{t-1} - \gamma_b \tilde{y}_{t-2} = \gamma_f E_t \tilde{\mu}_{t+1} - \tilde{\mu}_t + \gamma_b \tilde{\mu}_{t-1}. \quad (\text{A.3})$$

This equation reduces to the solution presented in Jeanne (1998) when $\omega = \gamma_b = 0$.

The conjectured solution is

$$\tilde{y}_t = \nu_1 \tilde{y}_{t-1} + \nu_2 \tilde{y}_{t-2} + \nu_{\mu 1} \tilde{\mu}_t + \nu_{\mu 2} \tilde{\mu}_{t-1}. \quad (\text{A.4})$$

Making use of this conjecture to rewrite the difference equation (A.3) and equating the coefficients yields:

$$\begin{aligned} \nu_1 &= \frac{1 + \gamma_b + \gamma_f \nu_2}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_2 &= -\frac{\gamma_b}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_{\mu 1} &= \frac{1 - \gamma_f(1 - \nu_{\mu 1})\rho_\mu + \gamma_f \nu_{\mu 2}}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_{\mu 2} &= -\frac{\gamma_b}{1 + \gamma_f(1 - \nu_1) + \lambda}. \end{aligned} \quad (\text{A.5})$$

Rearranging the expressions for ν_1 and ν_2 implies the following third degree polynomial for ν_1 :

$$\gamma_f^2 \nu_1^3 - 2(1 + \gamma_f + \lambda) \gamma_f \nu_1^2 + [(1 + \gamma_f + \lambda)^2 + \gamma_f(1 + \gamma_b)] \nu_1 - (1 + \gamma_f + \lambda)(1 + \gamma_b) + \gamma_f \gamma_b = 0. \quad (\text{A.6})$$

Under any plausible calibration this polynomial has a unique stable root³⁸, which will then be chosen as the coefficient ν_1 . The remaining parameters can be calculated recursively.

³⁸For a comprehensive discussion of the stability conditions see Hamilton (1994 Ch.1)

The coefficients ν_2 and $\nu_{\mu 2}$ are found to be equal and can be expressed as

$$\nu_2 = \nu_{\mu 2} = \frac{-\gamma_b}{1 + \lambda + \gamma_f - \gamma_f \nu_1}.$$

Finally, the coefficient of impact $\nu_{\mu 1}$ is given by:

$$\nu_{\mu 1} = \frac{1 - \gamma_f \rho_\mu + \gamma_f \nu_{\mu 2}}{1 + \lambda + \gamma_f - \gamma_f \nu_1 - \gamma_f \rho_\mu}.$$

Note that in the purely forward looking case, the coefficients ν_2 and $\nu_{\mu 2} = 0$, and the solution of the output gap thus reduces to

$$\tilde{y}_t = \nu_1 \tilde{y}_{t-1} + \nu_{\mu 1} \tilde{\mu}_t.$$

This result corresponds to Jeanne's (1998) solution in the case of perfect competition in the labor market.

The coefficients of autoregression of the *inflation rate* turn out to be equal to those of the output gap, i.e. $\nu_1 = \kappa_1$ and $\nu_2 = \kappa_2$. The contemporaneous impact of a monetary shock on the inflation rate is $\kappa_{\mu 1} = 1 - \nu_{\mu 1}$. This is a consequence of the CIA specification. The coefficient of lagged money growth in the solution of the inflation rate is $\kappa_{\mu 2} = \frac{-\gamma_f \lambda \rho_\mu}{(1 + \lambda - \gamma_f \kappa_1)(1 + \lambda + \gamma_f - \gamma_f \kappa_1 - \gamma_f \rho_\mu)}$.

It follows from these results, that $\kappa_2 = 0$, when all firms are forward looking. In contrast, $\kappa_{\mu 2}$ needs not necessarily be zero when $\omega = 0$. Instead, this coefficient is zero, when the monetary shock is not persistent³⁹. The solution of the inflation rate in the purely forward looking case is hence

$$\tilde{\pi}_t = \kappa_1 \tilde{\pi}_{t-1} + \kappa_{\mu 1} \tilde{\mu}_t + \kappa_{\mu 2} \tilde{\mu}_{t-1}.$$

³⁹To be precise, this coefficient can also be zero when all firms are backward looking, which would imply that the inflation rate does not react to the output gap, i.e. $\lambda = 0$. This case has, however, been ruled out.

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INFLATION AND OUTPUT PERSISTENCE

PSI _s (pi)		omega									
s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0	0.64	0.70	0.76	0.81	0.86	0.90	0.93	0.96	0.98	1.00	
2	0.21	0.23	0.26	0.30	0.36	0.45	0.55	0.68	0.81	0.93	
4	0.06	0.06	0.07	0.08	0.09	0.13	0.19	0.31	0.51	0.79	
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.09	0.39	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.14	
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.09	

PSI _s (y)		omega									
s	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0	0.82	0.80	0.79	0.78	0.78	0.79	0.81	0.84	0.88	0.94	
2	0.35	0.32	0.29	0.26	0.24	0.24	0.26	0.33	0.45	0.66	
4	0.11	0.09	0.08	0.06	0.04	0.03	0.04	0.08	0.18	0.42	
8	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.04	0.11	0.25	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.24	
16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.16	

Table 1: Persistence of inflation and output measured by $\Psi_s(x|\omega) \equiv \frac{ITV_x - ISV_{t-1}(\tilde{x}_{t+s})}{ITV_x}$ for different periods s , and different levels of ω . Increasing persistence is indicated by $\Psi_s(x | \omega_1) \geq \Psi_s(x | \omega_2)$ when $\omega_1 > \omega_2$ for $\forall s$.

IMPULSE RESPONSES

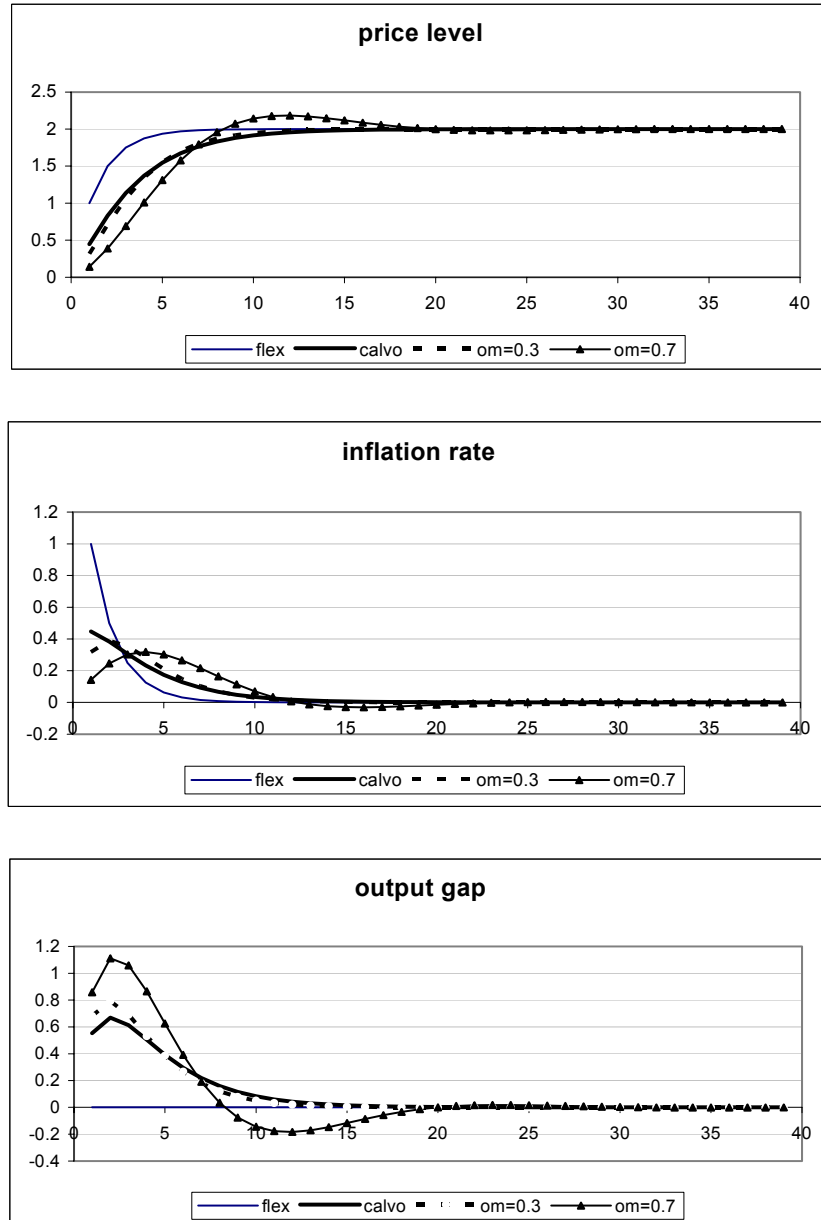


Figure 1: Dynamic impulse response of aggregate price level, inflation, output gap in percentage deviation from the steady state. Horizontal axis: periods in quarters. Baseline calibration. Monetary shock: $\epsilon_1 = 1\%$, $\rho_\mu = 0.5$. (flex: $\xi = 0, \omega = 0$, calvo: $\xi = 0.75, \omega = 0$, om=0.3: $\xi = 0.75, \omega = 0.3$, om=0.7: $\xi = 0.75, \omega = 0.7$).

IMPACT: $\nu_{\mu 1}$, $\kappa_{\mu 1}$

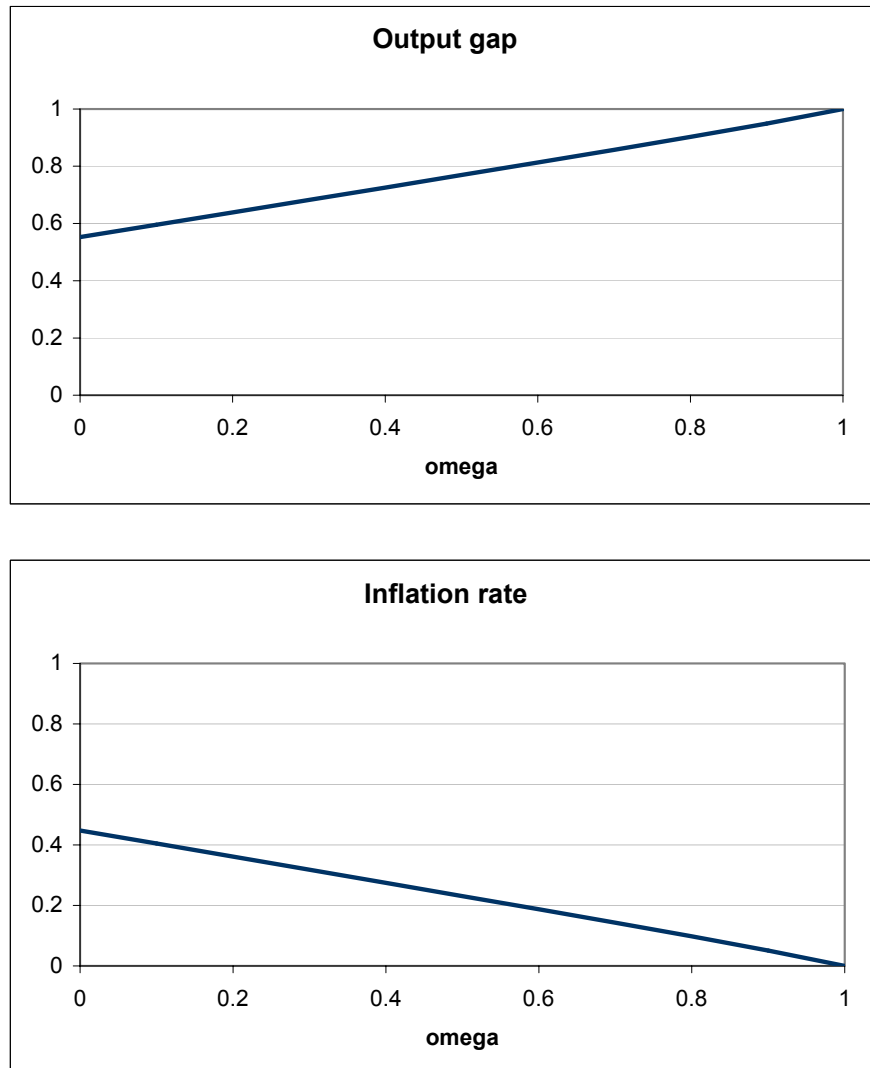


Figure 2: Contemporaneous impact of monetary shock ($\epsilon_t = 1\%$) on output ($\nu_{\mu 1}$) and on inflation ($\kappa_{\mu 1}$) as function of backward looking firms ratio, ω under baseline calibration.

IMPULSE RESPONSE: PRICE SETTING

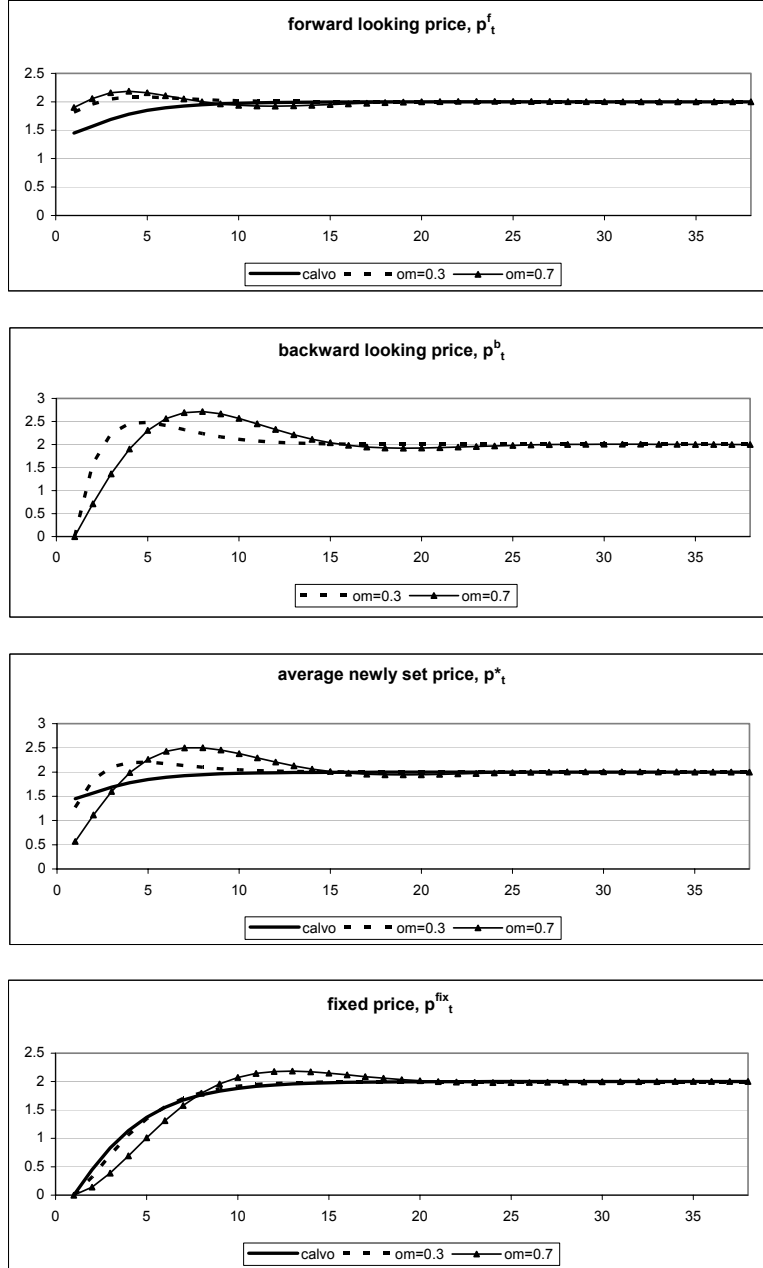


Figure 3: Dynamic impulse response of price indexes in percentage deviation from the steady state. Monetary shock: $\epsilon_1 = 1\%$, $\rho_\mu = 0.5$. calvo: $\xi = 0.75$, $\omega = 0$, om=0.3: $\xi = 0.75$, $\omega = 0.3$, om=0.7: $\xi = 0.75$, $\omega = 0.7$. (no \tilde{p}_t^b in Calvo model). The average of newly set prices is $\tilde{p}_t^* = \omega \tilde{p}_t^b + (1 - \omega) \tilde{p}_t^f$. The aggregate price level (see Fig. 1) is the weighted average $\tilde{p}_t = \xi \tilde{p}_t^{fix} + (1 - \xi) \tilde{p}_t^*$.

VOLATILITY: STANDARD DEVIATION

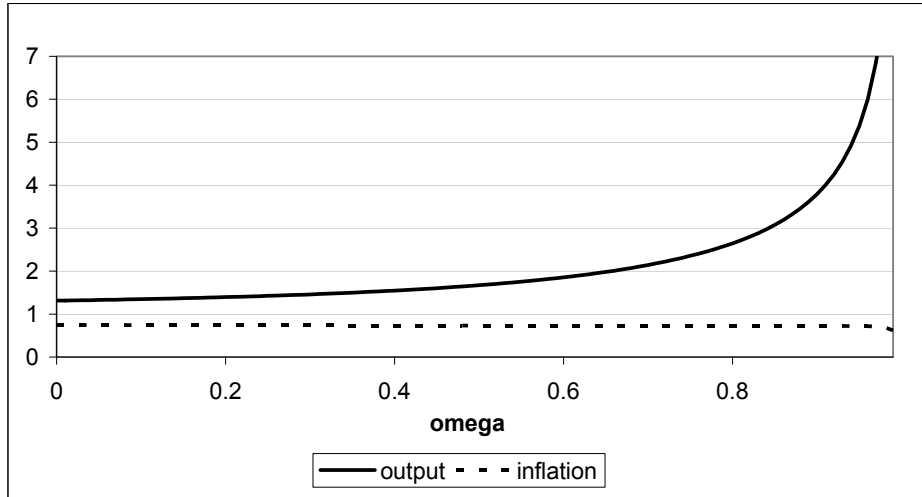


Figure 4: Standard deviation of inflation and output in pct points, as function of backward looking firms ratio ω , generated by monetary shocks ($\sigma_\epsilon = 1\%$, $\rho_\mu = 0.5$).

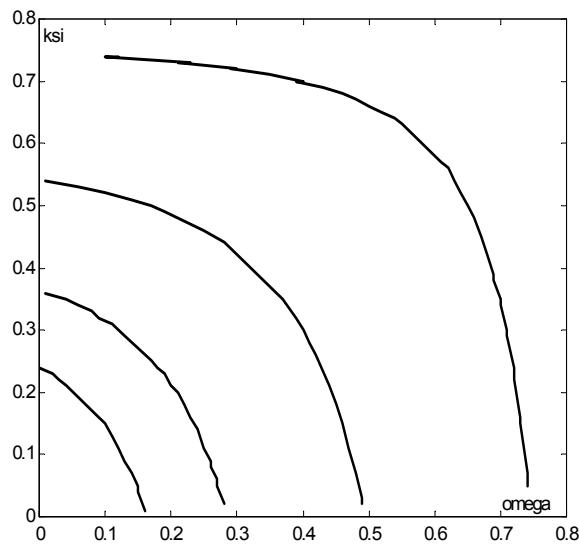


Figure 5: Iso-volatility of output in the $(\omega; \xi)$ space. Joining point couples $(\omega; \xi) \mid \sigma_y = \text{constant}$.

DYNAMICS 1: PERSISTENCE

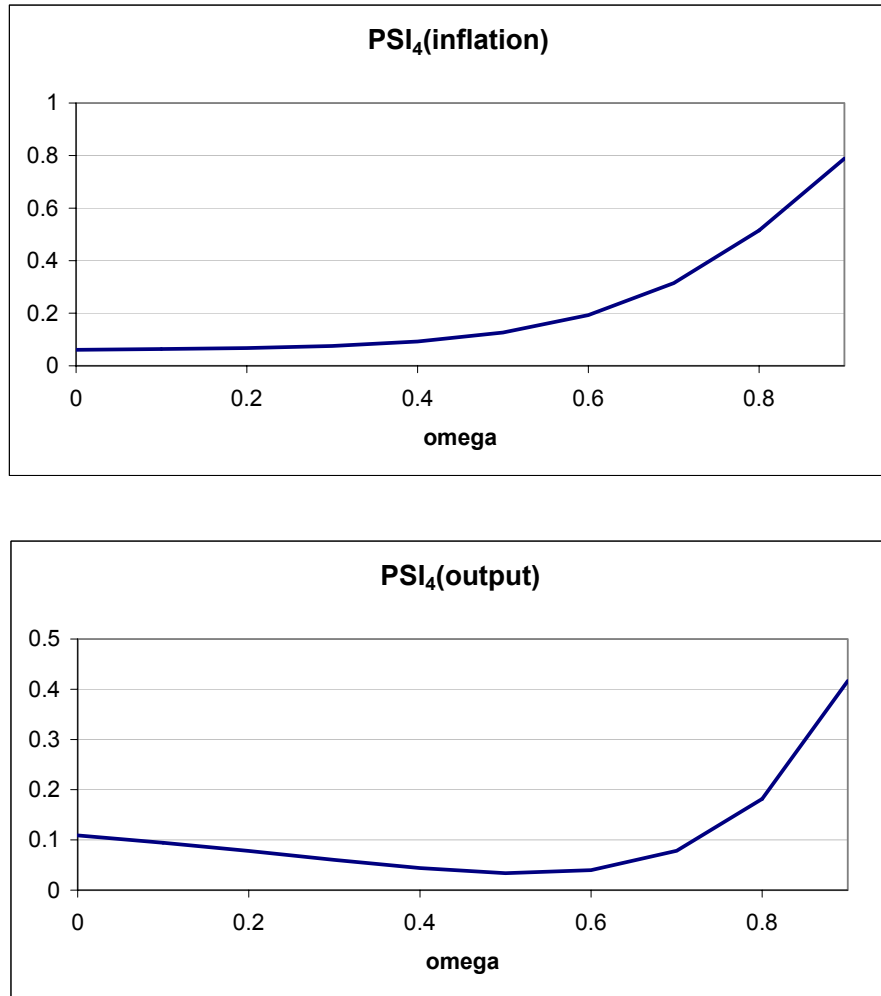


Figure 6: Persistence of output and inflation response to a stochastic monetary shock in t , measured by $\Psi_s(x) \equiv \frac{ITV_x - ISV_{t-1}(\tilde{x}_{t+s})}{ITV_x}$, $s = 4$, as function of backward looking firms ratio ω . Increasing persistence is indicated by $\Psi_s(x | \omega_1) \geq \Psi_s(x | \omega_2)$ when $\omega_1 > \omega_2$.

DYNAMICS 2: CROSS-CORRELATIONS $\text{corr}(y_t, \pi_{t+j})$

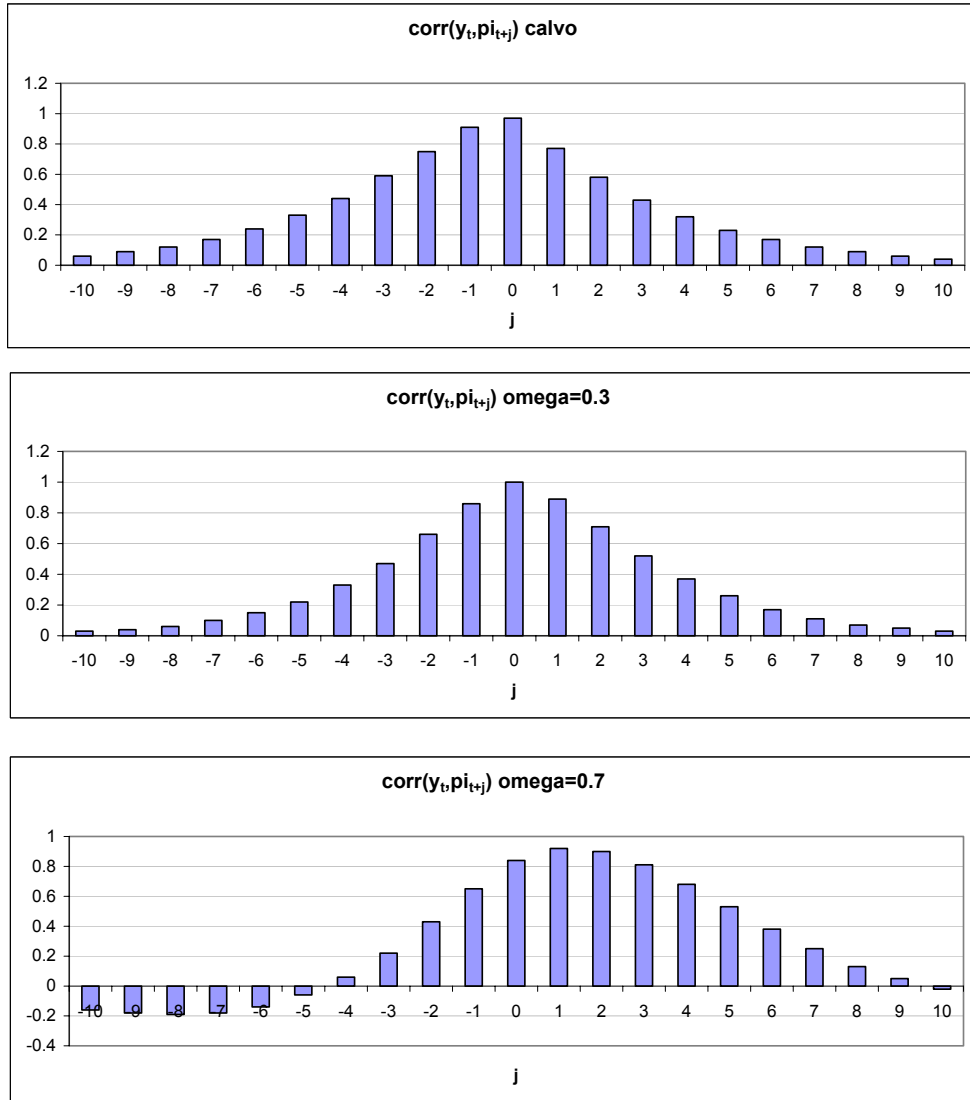


Figure 7: Cross-correlations between \tilde{y}_t and $\tilde{\pi}_{t+j}$ generated by monetary shocks ($\sigma_\epsilon = 1\%$, $\rho_\mu = 0.5$) for different fractions ω of backward looking firms. (calvo: $\xi = 0.75$, $\omega = 0$, omega=0.3: $\xi = 0.75$, $\omega = 0.3$, omega=0.7: $\xi = 0.75$, $\omega = 0.7$). Horizontal axis: number of leads/lags of inflation $\tilde{\pi}_{t+j}$.