The more we know, the less we agree[*]

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Very preliminary, please do not quote!

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Abstract

Stylized facts on the increased trading volume around public announcements in various financial markets give the impression that public information releases increase disagreement among traders. However, it is widespread opinion in the economic literature that under Bayesian updating and common priors, public information cannot cause divergence of opinion. We show that this is true only under specific circumstances. In this paper we will argue that in a standard rational expectation model – where some traders has to liquidate before others, there are new entrants and the information sets of early traders and new entrants are not too strongly connected –, public announcements increase disagreement among traders, which generates large trading volume. The result is closely related to the effect of higher-order expectations on trading activity.

Keywords: confirmatory bias, public announcements, trading volume, higher-order expectation, short-term traders.

JEL classification: D4, D8, G11, G12.

1 Introduction

It is a well established stylized fact in financial markets that public announcements are followed by intense trading, high trading volume and volatile prices. Recently, there is also evidence from high-frequency data (both FX and Government bond) that although at the time of the announcement there is an initial price adjustment, it is followed by a prolonged period with increased, more volatile and more informative trading (Evans and Lyons, 2003 Love and Payne,2003, Fleming and Remolona, 1999, Love 2004). Apparently, disagreement increases due to public announcements and the different opinions are channeled into prices by higher trading intensity.

The problem with this explanation is that economists tend to think that in a fully rational environment opinions cannot go farther apart after public information release. In behaviour economics,

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experimental observations of the contrary are labeled as confirmatory bias. For example, Rabin and Schrag (1999) put it as follows:

> "... a large and growing body of psychological research suggests that the way people process information often departs systematically from Bayesian updating. In this paper we formally model and explore the consequences of one particular departure from Bayesian rationality: confirmatory bias. [...] The most striking evidence for the confirmatory bias is a series of experiments demonstrating how providing the same ambiguous information to people who differ in their initial beliefs on some topic can move their beliefs farther apart." Rabin and Schrag(1999, pp 38,43.)

On the same ground, theorists in finance also argued that disagreement increasing public announcements cannot be explained by Bayesian models. They – just like Rabin and Schrag in behavioural economics – suggested to accept that agents interpret the same information differently i.e. agents look at the same information, but they see something different (Evans and Lyons, 2001, Kandel and Pearson, 1995, Harris and Raviv, 1993, Varian, 1989).

In this paper, we will argue that the statement which says that under Bayesian updating and common priors public information cannot cause divergence of opinion, is true only under specific circumstances. These circumstances may hold in experimental environments used by behavioural economists, but certainly do not hold in financial markets. Furthermore, we present a standard rational expectation model – where some traders has to liquidate before others, there are new entrants and early traders' and new entrants' private information sets are different enough – where public announcements increase disagreement, which generates large trading volume.

Let us present two examples which demonstrate the difference between environments with and without disagreement increasing public information releases. The first example is similar to the motivating examples of Rabin and Schrag (1999), where public information should not move beliefs farther apart. The second example illustrates our story well, where public information increase disagreement because it makes existing differences in information relevant, which were not relevant before.

Example 1 (GM food) *Alex and Betty do not agree on the usefulness of genetical manipulation in the food industry. They are rational, so their different opinions are based on different evidence they saw. If they are confronted with the same evidence, their opinion cannot diverge.*

Example 2 (Alex and Betty meet the famous Professor F.) *Alex and Betty, are two Ph.D. student in the department. They have the same interest, the same academic background and the same talent. Every Wednesday, there is a seminar with guest talkers. This is a Tuesday, and they do not know yet who will give the seminar next day. Until it is not announced, Alex and Betty will fairly agree on the expected quality of the talk. They both have seen several professors presenting (they do not necessarily have seen the same ones), they liked some of them and did not like others, so they will fairly agree. Let us suppose that in the evening, it is announced that the guest will be the famous Professor F. Alex gets very exited as he has never heard him and he knows how famous he is. However,* *Betty gets disappointed as she has seen him talking the previous year, and she remembers that she did not understand anything. They agreed before the announcement, but they do not agree after it.*

Note, that Betty had the information about Professor F.'s presentation style before the announcement, but it was not relevant as Professor F. was just one in the very numerous possible presenters. The public information that he will give the talk, made the private information relevant and caused the divergence in opinion. The main difference between the two examples is that in the first one, new public information is – conditionally – independent of the existing information i.e. the effect of the new piece of information does not depend on which private information the agent possessed. Note also, that there is a clear conceptual difference to the misinterpretation argument. In the second example, if Alex and Betty were sharing their private information, they would agree at the end: they cannot "agree to disagree" as it is a common prior environment. However, if they were interpreting the announcement differently – they had different priors, they believed in different likelihood functions in different models etc. – sharing their ideas would not help.

Similarly to the second example, in our model increased trading volume is not the result of different interpretation of public information, but of the fact that it makes the existing differences in agents' opinion relevant. This existing private information will be poured into the market during the intense trading period.

Our work is naturally related to the theoretical work on trading volume and public information releases. The literature can be divided into two groups. Rational models – models with Bayesian updating and common priors (Brown and Jenning, 1989, Kim and Verecchia, 1991, He and Wang, 1995) – do not deliver disagreement increasing announcements, consequently they have problems matching empirical observations. The basic structure of these models are nested in our set up, so we will be able to point out the difference which makes our model capable of explaining high trading volume around announcement. Non-Bayesian models in contrast (Varian, 1986, Harris and Raviv, 1993, Kandel and Pearson, 1995, Evans and Lyons, 2001) can produce disagreement and volume, but for the expense of assuming non-common priors (or alike). They argue that as rational models are inherently incapable of explaining the observed stylized facts, these assumptions are necessary. We will show that rational models can deliver similar findings, if the information structure is rich enough.

Our model also fits well to an other flow of applied theoretical papers which analyse the effect of higher-order expectations – expectations on expectations of others – on financial markets[1]. The leading metaphor in this literature is the famous beauty contest example of Keynes which compares speculative trading to those beauty contests where gifts are distributed among those who voted for the winner. Similarly to the metaphor with the contestants, the problem of speculative traders is to choose those assets which future traders will consider valuable – so the resale price will be high –, which do not necessarily coincide with those that they consider undervalued. The main observation of these papers is that higher-order expectations in asymmetric information environments may behave very differently

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[1] This literature originates from the application of results from the global games literature on currency crisis by Morris and Shin (1998), but recently it has been extended to non-global games environments. In particular, monetary economics seems to be a fruitful area in the higher-order expectations literature (see Woodford, 2001, Hellwig 2002, Adams, 2003, Amato and Shin 2003).

to first-order expectations i.e. the low of iterated expectations is violated in a systematic way. This fact in turn, can explain stylized facts of financial markets which were considered to be inconsistent with Bayesian models. The first paper in this literature is Allen, Morris and Shin (2003), which shows that assuming short-horizon traders in the standard dynamic asymmetric information model of Brown and Jennings (1989) implies that prices will be oversensitive to public information in the early periods, because higher-order expectations overreacts the public signal. Similarly to our model, Kondor (2004) allows for an information structure, where private information sets of early traders are less connected. As a consequence, expectations on the resale price (higher-order expectations) can move in the opposite to expectations on the fundamental value (first-order expectations) as traders' private information change. This effect results in severe inefficiency and mispricing in a set-up where traders trade off short term gains versus long-term gains. In the current work, our results also related to the weaker-than-usual connection between traders private information sets across time, but the driving force is different. It is the fact that the mean absolute deviation of higher-order expectations can increase after an announcement even when the mean absolute deviation of first-order expectations decrease. We will return to this issue at the end of section 2.

The structure of this paper is as follows. In the next section we discuss formally the difference between environments with and without disagreement increasing public announcements and their relation to trading volume and higher-order expectations in the simplest possible set up. In section 3, we present the full model and the results. In section 4 we confront our findings with existing empirical results and we present some preliminary new empirical evidence on an implication of our model. Finally we conclude.

2 The problem of disagreement increasing announcements

In this section, we highlight the three main points of our argument in an intuitive manner. In the first subsection, we show that public information releases can cause divergence in opinion, if the same (public) piece of information implies something different when it is coupled with different (private) pieces of existing information. In the second subsection, we will argue that disagreement in general will cause volume in asymmetric information models of asset pricing. In the final subsection, we show that in a dynamic model with short-term traders, disagreement is naturally related to the mean absolute deviation of higher-order expectations. Furthermore, if the connection between the information sets of early traders and late comers is not too strong, the mean absolute deviation – and the variance – of higher-order expectations increase due to a public announcement, which never happens with first-order expectations.

Hence, the three points together give the following line of arguments. Because of the described property of higher-order expectations (last subsection), a financial market is a natural environment where public information can increase disagreement (as in first subsection). This disagreement leads to high trading volume (second subsection).

2.1 Disagreement and public information

Suppose that there are two people who have different information but they agree. The next proposition shows that the fact that they are both confronted with the same new piece of information does not guarantee that they will necessarily continue to agree.

Lemma 1 *$P(F|P_1) = P(F|P_2)$ for all F,in a given partition implies $P(F|P_1 \cap C) = P(F|P_2 \cap C)$ for all F and C in that partition, if and only if $P(P_1 \cap C|F) = P(P_2 \cap C|F)$ for all F and C in that partition.*

Proof. It is a straightforward application of Bayes' Rule. ■

It holds, for example if P_1 and C and P_2 and C are pair-wise conditionally independent and $P(P_1) = P(P_2)$. This is the case at Bernoulli experiments, like in the GM food example. If the state space is binary (GM food is good or bad) and the private signals are binary as well, then $P(F|P_1) = P(F|P_2)$ holds only if the difference between the number of good and bad signals in Alex's and Betty's case are the same. Then if we observe the same signals (C), we will continue to agree. But this is not true in general. This is where Rabin and Schrag are imprecise.

We give a simple and intuitive example, where $P(F|P_1) = P(F|P_2)$ does not implies $P(F|P_1 \cap C) = P(F|P_2 \cap C)$.

Example 3 (In a Hungarian bar) *A foreigner goes to a bar in Budapest. She wants to order a good drink (event F). There are two drinks on the menu: "Nescafe & tejszín" and "Coca-Cola & tejszín". She figures out what Nescafe is (information P_1) and what Coca-Cola is (information P_2), but she has no idea what "tejszín" is. In this stage she may be indifferent between the two drinks i.e. $P(F|P_1) = P(F|P_2)$. However, if the waitress tells her that tejszín means cream (information C), then she most probably would prefer coffee with cream, i.e. $P(F|P_1 \cap C) > P(F|P_2 \cap C)$.*

Although this is a structurally different example than the one where Alex and Betty meet Professor F., the idea is the same. The public information puts the existing private information in different context. Hence, it is not the public, but the existing private information that makes the difference.

2.2 Disagreement and volume

In the simplest, static CARA-Normal model (e.g. Grossman and Stiglitz, 1980), positions are given by:

$$d^i = \frac{E(\theta|I^i) - p}{var(\theta|I^i)\, a} \tag{1}$$

where θ is the true value, I_i is the information set of trader i, a is the risk-aversion parameter and p is the equilibrium price. The market clearing condition is

$$\int d^i = u.$$

Hence,

$$p = \bar{E}\left(\theta | I^i\right) - var\left(\theta | I^i\right) au \tag{2}$$

which is close to the "average" opinion. So the numerator of (1) can be seen as the disagreement of the trader with the average opinion, while the denominator can be seen as the precision of her opinion. The larger the disagreement and the larger the precision, the larger the trader's position. In any model where difference in opinion decreases after a public announcement, both the numerator and the denominator goes up, so the net effect of the announcement on trade is small. But if the difference in opinion increases, the volume can be very large. This is what happens in our model.

2.3 Disagreement and higher-order expectations

To strengthen our intuition, let us consider a dynamic version of the simple CARA-Normal set-up discussed in the previous subsection. Let us suppose that in each period there are myopic traders, who care only for the next period price as in Allen et al (2003). It is easy to see that (1) changes to

$$d_t^i = \frac{E\left(p_{t+1} | I_t^i\right) - p_t}{var\left(p_{t+1} | I_t^i\right) a},$$

in all periods $t < T$, where T is the last period, I_t^i is the information set of trader i in period t, and the price will be

$$p_t = \bar{E}\left(p_{t+1} | I_t^i\right) - var\left(p_{t+1} | I_t^i\right) au_t$$

,where u_t is the noisy supply in period t. Hence, the disagreement over the next period price matters and not the disagreement over the fundamental value (except the last period). As the last period price is closely related to the average opinion on θ, and in all other periods t, the price is closely related to the average opinion over the price in the next period $t+1$, – by backward induction – disagreement over next period prices can be expressed as higher order disagreement on θ:

$$E\left(p_{t+1} | I_t^i\right) - p_t \approx E\left(p_{t+1} | I_t^i\right) - \bar{E}\left(p_{t+1} | I_t^i\right) \approx E^{T-t+1}\left(\theta | I_t^i\right) - \bar{E}^{T-t+1}\left(\theta | I_t^i\right) \tag{3}$$

,where $E^{T-t+1}\left(\theta | I_t^i\right)$ stands for the $(T-t+1)\,th-$order expectations of the average expectations and $\bar{E}^{T-t+1}\left(\theta | I_t^i\right)$ is the $(T-t+1)\,th-$order average expectations of the average expectation. For example, if there are two periods before the last one, $t = T-2$,

$$\begin{aligned} E^{T-t+1}\left(\theta | I_t^i\right) &= E^3\left(\theta | I_t^i\right) = E\left(\bar{E}\left(\bar{E}\left(\theta | I_T^j\right) | I_{T-1}^k\right) | I_t^i\right) \\ \bar{E}^{T-t+1}\left(\theta | I_t^i\right) &= \bar{E}^3\left(\theta | I_t^i\right) = \bar{E}\left(\bar{E}\left(\bar{E}\left(\theta | I_T^j\right) | I_{T-1}^k\right) | I_t^i\right). \end{aligned}$$

Higher-order expectations do not collapse to first order expectations – the law of iterated expectations is violated (see Allen at al, 2003) – because information sets of agents in different time-periods are not nested in an asymmetric information set-up.

Observe that the left hand side of (3) is the deviation of a higher-order expectation term from its mean. Therefore, if we are interested in environments, where public announcement increases the average disagreement, we should focus on environments, where the mean absolute deviation[2] $(mad\,(z) = E\left(|z-\bar{z}|\right))$ of higher-order expectations increase due to announcements:

$$mad\left(E^{T-t+1}\left(\theta|I_t^i\right)\right) > mad\left(E^{T-t+1}\left(\theta|I_t^{i\prime}\right)|y\right) \tag{4}$$

,where $I_t^{i\prime}$ is the same information set as I_t^i except that it does not contain the public announcement.

In what follows, we will argue that (4) can hold for higher-order expectations even if it does not hold for first-order expectations. In particular, we will show that in the simplest case of normally distributed random variables, with a single private signal and a single public signal in each agent's information set, (4) will never hold with first-order expectations, but will hold for all higher-order expectations, if private signals across periods are not too closely related.

We assume that the fundamental value, the private signals and the public signal are jointly normally distributed:

$$\left(\theta, \mathbf{x}_1, ..., \mathbf{x}_t, ...\mathbf{x}_T, y\right) \sim N^{1+n_1+n_2+1}\left(0, \Sigma\right),$$

, where $\mathbf{x}_t$ is the vector of private signals in period t and we will refer the covariance of variables z_1 and $z_2$in Σ as σ_{z_1,z_2}, and σ_z^2 will be the variance of z. All covariances are assumed to be positive. For the sake of simplicity, we will assume the symmetric structure , where the variance of every private signal is σ_x^2, the covariance between any private signal and the fundamental value or the public signal are $\sigma_{\theta,x}$ and $\sigma_{y,x}$ respectively. Furthermore, the covariance between two private signals in the same period is $\sigma_{x,x}$, while the covariance between private signals in different periods is $\sigma_{x,x'}$. The only assumption we make on the relative sizes of covariance and variance terms is that the private signal x_t^i and the public signal, y, are a positive signals[3] on θ for agent i in period t, i.e. a higher x_t^i or a higher y will rise the fundamental expectation of agent i:

$$\begin{aligned} b_\theta &= \frac{\partial E\left(\theta|x_t^i, y\right)}{\partial x_t^i} = \frac{\sigma_{\theta,.x}\sigma_y^2 - \sigma_{x,y}\sigma_{\theta,y}}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} > 0 \\ c_\theta &= \frac{\partial E\left(\theta|x_t^i, y\right)}{\partial y} = \frac{-\sigma_{x,\theta}\sigma_{x,y} + \sigma_{\theta,y}\sigma_x^2}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} > 0. \end{aligned}$$

It is easy to check that $b_\theta, c_\theta > 0$ implies that the private signal is a positive signal on θ in the no-announcement case as well – i.e. $b_\theta^n = \frac{\partial E\left(\theta|x_t^i\right)}{\partial x_t^i} = \frac{\sigma_{\theta,x}}{\sigma_x^2} > 0$ – and that $|b_\theta^n| > |b_\theta|$. As in the normal case – where conditional expectations are linear – mean absolute deviation will depend only on the coefficient of the private signal, it is straightforward to show that public announcements will never

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[2] All of our results would continue to hold, if we used variance instead of mean absolute deviation. We chose mean absolute deviation, because it is more connected to volume than variance.

[3] In a more general set-up, this assumption would correspond to the assumumption of positive affiliation of $\left(\theta, x_t^i, y\right)$ for all t and i. (Our assumption is weaker, but works for the normal case only.)

increase disagreement over the fundamental value i.e. $b_\theta, c_\theta > 0$ implies

$$mad\left(E\left(\theta|x_t^i\right)\right) > mad\left(E\left(\theta|x_t^i, y\right)\right).$$

However, the next proposition shows that public announcements will increase disagreement over all higher-order expectations, if private information sets across periods are not too strongly correlated. The intuition behind this result is that with higher order expectations agents' guess on the fundamental value does not matter; the critical point is what each agent knows about another agent's signal[4].

Proposition 1 *If x_t^i and y are positive signals i.e. $b_\theta, c_\theta > 0$ and*

$$\frac{b_\theta \rho_{x,y}^2}{\left(b_\theta + \rho_{\theta,x}\left(1 - \rho_{x,y}^2\right)\right)} > \rho_{x',x}, \tag{5}$$

where $\rho_{z_1,z_2} = \frac{\sigma_{z_1,z_2}}{\sigma_{z_1}\sigma_{z_2}}$, then

$$E\left(mad\left(E^k\left(\theta|x_i\right)\right)|y\right) < E(mad\left(E^k\left(\theta|x_i, y\right)\right)|y)$$

holds for all $k > 1$.

Proof. Let us introduce b_x, and b_x^n for the coefficients of private signals in traders' conditional expectations on other traders private signals in other periods:

$$\begin{aligned} b_x &= \frac{\sigma_{x,x'}\sigma_y^2 - \sigma_{x,y}^2}{\sigma_x^2\sigma_y^2 - \sigma_{x,y}^2} = \frac{\partial E\left(x_u^j|x_t^i, y\right)}{\partial x_t^i} \text{ for all } u \neq t, \\ b_x^n &= \frac{\sigma_{x,x'}}{\sigma_x^2} = \frac{\partial E\left(x_u^j|x_t^i\right)}{\partial x_t^i} \text{ for all } u \neq t. \end{aligned}$$

As we compare the mean absolute deviation of two linear expressions with x_t^i as the only stochastic variable, it is enough to show that

$$\left|\left(b_x^n\right)^{k-1} b_\theta^n\right| = \left|\frac{\partial E^k\left(\theta|x_i\right)}{\partial x_i}\right| < \left|\frac{\partial E^k\left(\theta|x_i, y\right)}{\partial x_i}\right| = \left|\left(b_x\right)^{k-1} b_\theta\right| \tag{6}$$

for all $k > 1$. As $b_\theta^n > b_\theta > 0$, $|b_x^n|\, b_\theta^n < |b_x|\, b_\theta$ implies $|b_x^n| < |b_x|$. Therefore, $|b_x^n|\, b_\theta^n < |b_x|\, b_\theta$ implies (6). The condition in the proposition comes directly – after substitution and straightforward manipulation – from $|b_x^n|\, b_\theta^n < |b_x|\, b_\theta$. ■

In this section we presented the intuitive arguments that (1) public announcements can increase disagreement (2) disagreement over higher-order expectations increase after public announcements if information sets are not very correlated (3) in financial models with short-term traders disagreement

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[4] In contrast, the results in Kondor (2004) are driven by the observation thatt $b_\theta, c_\theta > 0$ and $\rho_{x,x'} < \rho_{x,y}\rho_{x',y}$ imply $\frac{\partial E^2\left(\theta|x_t^i,y\right)}{\partial x_t^i} < 0$, eventhough $\frac{\partial E\left(\theta|x_t^i,y\right)}{\partial x_t^i} > 0$. Hence, in a two period model with new entrants in the second period, first period traders' expectation on the interim price will move opposite to their fundamental expectation with respect to their private signal, which result in inefficency and mispricing.

over higher-order expectations leads to high trading volume. In the next section we show that this intuition goes through in a rigorous, dynamic, asymmetric information model of asset pricing with Bayesian learning.

3 The model

3.1 The set-up

We modify a standard, dynamic, CARA-Normal, rational expectations model with asymmetric information (He and Wang, 1995, Brown and Jennings, 1989, etc.). As in any similar model since Grossman-Stiglitz (1980), preferences of our traders are given by $u_i(W_i) = -e^{-aW_i}$, where W_i is monetary wealth at the time of the exit, a is the absolute risk-aversion parameter and in each period traders submit demand curves to an auctioneer to buy up the random supply of assets: $u_t \tilde{} N\left(0, \frac{1}{\delta_t^2}\right)$. Traders base their portfolio decision on the private signal which they receive at the moment of their entry and all available public signals i.e. past and present prices and public announcements. They update their beliefs by Bayes' Rule. Prices, p_t, are determined by market clearing.

However, as a non-standard assumption, we will have two groups of traders – with continuum traders in each group – and 2+1 periods ($t = 0, 1, 2$). Traders in the first group trade among themselves in periods 0 and 1 and sell all of their remaining assets in period 2. Traders in the second group trade among each other in period 2 and liquidate for the uncertain value of θ at the end of the game. One may think of the model as a 24-hour day in the USD/GBP market, where the first group represents traders based in London, while the second group is based in New York. Period 0 and 1 are daylight periods in London, so Londoners trade among themselves twice, then they go to sleep, so they sell all their holdings to New Yorkers. They do not hold positions overnight[5]. Period 2 is daylight in New York, so New Yorkers trade among themselves and get θ in the evening. We assume that if there is a public announcement,y, then it will be released at the beginning of period 1. Hence, we will focus on the differences in trading patterns of Londoners with and without announcement.

The driving force of our model lies in the information structure. We assume that the fundamental value of the asset – the exchange rate – is given by

$$\theta = \theta_s + \theta_k + \theta_w$$

where $\theta_s, \theta_k, \theta_w$ are the US factor, the UK factor and the world factor respectively. We assume that the private signal that Londoners receive contains noisy information on the UK factor and the world factor, but does not contain information on the US factor: $x_i = \theta_k + \theta_w + \varepsilon_i$, while the private signals of New Yorkers contain information on the US factor and the world factor, but not on the UK factor: $z_j = \theta_s + \theta_w + \varepsilon_j$. Hence, the world factor simply represents the common element in the information set of agents in different groups, while the US factor and the UK factor represent group-specific information. The public signal contains information on fundamental value: $y = \theta + \eta$.

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[5] Although we use the interpretation of a 24-hour FX market only for expositional reasons, it happens to be a stylized fact among FX dealers that they do not hold positions overnight (see Lyons, 2001).

We assume that all factors and noise terms are iid and normally distributed:

$$\theta_k, \theta_s \sim N\left(0, \frac{1}{\kappa}\right), \theta_w \sim N\left(0, \frac{1}{\omega}\right), \varepsilon_i, \varepsilon_j \sim N\left(0, \frac{1}{\alpha}\right), \eta \sim N\left(0, \frac{1}{\beta}\right).$$

The following two tables helps to compare our structure to information in other rational expectations models. The first one presents the structure of other models, while the second one shows how our set up nests all the others.

model	private signal	public signal	liquidation value
Brown-Jennings, Kim-Verecchia	$\theta + \varepsilon_i$	$\theta + \eta$	θ
He and Wang	$\theta + \varepsilon_i$	$\theta + \eta$	$\theta + \xi$

	priv	*pub*	*fund*	$p_2(\cdot)$	model
$\kappa, \delta_2 \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w$	two period -one factor, B-J
$\kappa \to \infty$	$\theta_w + \varepsilon_i$	$\theta_w + \eta$	θ_w	$\approx \theta_w + u_2$	He and Wang (1995)
$\omega \to \infty$	$\theta_k + \varepsilon_i$	$\theta_s + \eta$	$\theta_k + \theta_s$	$\approx \theta_s + u_2$	independent information sets

Note, that all these models – together with the majority of asymmetric information models in finance [6] – use a one-factor framework. The problem is that the assumption that all signals are noisy versions of the fundamental, imposes a very rigid structure on the information sets. Namely, all covariances between any two of the random variables equals the variance of the fundamental value: $cov(x_i, x_j) = cov(x_i, y) = cov(x_i, \theta) = var(\theta)$. Our structure represents a partial relaxation of this assumption. We allow for weaker correlation between private signals across groups. We presented the structure with the help of the specific story about the FX traders just for exponential purposes. We believe that our model applies to any financial markets, where traders cannot be sure to be able to hold their positions until it is optimal and where those whom they will sell to, do not necessarily have a very similar information set to their own.

In the one-factor structure disagreement never increases due to the announcement. In particular, in the simplest Brown and Jennings model, there is no effect of public announcement at all: the increased precision and the decreased disagreement exactly cancels out. In the Kim-Verecchia case there is some volume due to different precision of signals of different traders, but the effect is small and always proportional to the price change. In He and Wang, there is an additional random factor in the liquidation value, ξ, which is not included in the union of traders' information set. This induce traders to follow a more complex dynamic strategy over time, which allows traders to bet in advance on the price effect of the public announcement. Hence, they will build up positions before the announcement and liquidate these positions when the announcement is released. This effect works only with expected announcements. Our model will deliver this bet-in-advance effect as well, but we will have an additional effect coming from the increased disagreement.

Our model nests these information structures. When $\kappa \to \infty$, the non-common factors, θ_s, θ_k lose their importance and we end up in a one-factor structure with θ_w only. When also $\delta_2 \to \infty$, second

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[6] Foster and Viswanathan (1996) is a notable exception. For a detailed review of the literature, see Brunnermeier (2001).

period price, p_2, will be fully revealing, so Londoners will behave as if they could liquidate for θ_w, which is the only relevant factor remaining. Hence, in the case of $\kappa, \delta_2 \to \infty$, effectively we have a two period model with one factor as in Brown and Jennings. When δ_2 is finite, the model resembles to He and Wang (1995) as the liquidation value for Londoners, p_2, will contain the random term u_2 as well. However, when only $\omega \to \infty$, we have a very different structure, where the private information sets of Londoners and New Yorkers get separated.

3.2 Analytical results

In the first part of this section, we show that an equilibrium of the present model exists. In the second part, we present results on the equilibrium volume.

3.2.1 Equilibrium and existence

We search for a linear equilibrium, so we assume that prices are given by the functions

$$\begin{aligned} p_2 &= c_2 y + b_2 (\theta_s + \theta_w) + f_2 q_1 + g_2 q_0 - e_2 u_2 \\ p_1 &= c_1 y + b_1 (\theta_k + \theta_w) + f_1 q_0 - e_1 u_1 \\ p_0 &= b_0 (\theta_k + \theta_w) - e_0 u_0 \end{aligned} \tag{7}$$

where $c_t, b_t, e_t, f_1, f_2, g_2$ are undetermined coefficients, while q_1, q_0 are specified below. Prices together with past prices and the public information are informationaly equivalent with the following price signals

$$\begin{aligned} q_2 &= \frac{1}{b_2}(p_2 - c_2 y - f_2 q_1 - g_2 q_0) = (\theta_s + \theta_w) - \frac{e_2}{b_2} u_2 \\ q_1 &= \frac{1}{b_1}(p_1 - c_1 y - f_1 q_0) = (\theta_k + \theta_w) - \frac{e_1}{b_1} u_1 \\ q_0 &= \frac{1}{b_0} p_0 = (\theta_k + \theta_w) - \frac{e_0}{b_0} u_0. \end{aligned} \tag{8}$$

Below, we will show that the equilibrium trading activity is determined only by the noisiness of these price signals, so we define τ_t^2 as the precision of q_t:

$$\frac{1}{\tau_t^2} = \frac{e_t^2}{b_t^2 \delta_t^2} \text{ and } \frac{\tau_t}{\delta_t} = \frac{b_t}{e_t}.$$

Let us also define the following coefficients of variables in traders' information sets in the different conditional expectations of New Yorkers and Londoners:

$$
\begin{aligned}
E\left(\theta | z_j, y, q_2, q_1, q_0\right) &= \bar{b} z_j + \bar{c} y + \bar{e} q_2 + \bar{f} q_1 + \bar{g} q_0 && (9)\\
E\left(\theta_s + \theta_w | x_i, y, q_1, q_0\right) &= b_s x_i + c_s y + e_s q_1 + f_s q_0 && (10)\\
E\left(\theta | x_i, q_0\right) &= E\left(\theta_k + \theta_w | x_i, q_0\right) = E\left(q_1 | x_i, q_0\right) = E\left(y | x_i, q_0\right) = b_y x_i + e_y q_0. && (11)
\end{aligned}
$$

From standard results(e.g. Brown and Jennings, 1989), we know that the demand functions of New Yorkers in period 2 and Londoners in period 1 will be

$$
\begin{aligned}
d_2^j &= \frac{E\left(\theta | z_j, y, p_2, q_1, q_0\right) - p_2}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = \frac{\bar{c} y + \bar{b} x_j + \bar{f} q_1 + \bar{e} q_2 + \bar{g} q_0 - p_2}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} && (12)\\
d_1^i &= \frac{E\left(p_2 | x_i, y, p_1, q_0\right) - p_1}{avar\left(p_2 | x_i, y, p_1, q_0\right)} = \frac{c_2 y + b_2\left(b_s x_i + c_s y + e_s q_1 + f_s q_0\right) + f_2 q_1 + g_2 q_0 - p_1}{avar\left(p_2 | x_i, y, p_1, q_0\right)} && (13)
\end{aligned}
$$

Finding the demand function in period 0 is a bit more subtle. Londoners maximize the following expected utility in period 0:

$$
E\left(-\exp\left(-a\left(p_1 - p_0\right) d_1 - \frac{E\left(p_2 | q_1, y, x_j, q_0\right) - p_1}{avar\left(p_2 | q_1, y, x_j, q_0\right)}\left(p_2 - p_1\right)\right) | x_i, q_0\right).
$$

The source of the difficulty is that there are two random variables in this expression, p_1 (or q_1) and y In the appendix, we show that as the outcome of this maximization – the demand function in period 0 – is

$$
d_0^i = \frac{\left(E\left(p_1 | x_i, q_0\right) - p_0\right)\left(\sigma_q s + 1\right)}{a\left(c_1^2 \sigma_y + c_1^2 s \sigma_y \sigma_q - c_1^2 s \sigma_{yq}^2 + \sigma_q b_1^2 + 2 c_1 \sigma_{yq} b_1\right)} + \frac{b_s E\left(d_1^i | x_i, q_0\right)\left(c_1 \sigma_{yq} + b_1 \sigma_q\right)}{a\left(c_1^2 \sigma_y + c_1^2 s \sigma_y \sigma_q - c_1^2 s \sigma_{yq}^2 + \sigma_q b_1^2 + 2 c_1 \sigma_{yq} b_1\right)} \tag{14}
$$

where

$$
\begin{pmatrix} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{pmatrix} = var\left(\begin{pmatrix} y \\ q_1 \end{pmatrix} | x_i, q_0\right).
$$

is the variance-covariance matrix of y and q_1 conditional on a London-trader's information set in period 0. Intuitively, the first part in expression (14) represents the short-term demand component, while the second part represents the hedging component for demand in period 1.

We can show that in equilibrium, demand functions can be characterized completely by the equilibrium values of τ_t in the following manner:

$$
\begin{aligned}
d_2^j &= \frac{\tau_2}{\delta_2}\left(z_j - q_2\right) = \frac{\tau_2}{\delta_2}\varepsilon_j + u_2 && (15)\\
d_1^i &= \frac{\tau_1}{\delta_1}\left(x_i - q_1\right) = \frac{\tau_1}{\delta_1}\varepsilon_i + u_1 && (16)\\
d_0^i &= \frac{\tau_0}{\delta_0}\left(x_i - q_0\right) = \frac{\tau_0}{\delta_0}\varepsilon_i + u_0 && (17)
\end{aligned}
$$

The right hand sides of the three equations show that in each period, total positions consist of two parts. There is a risk-sharing part, u_t, which is purchased by each agent regardless of her information, and there is a speculative part, $\frac{\tau_t}{\delta t}\varepsilon_i$, which depends on the difference between the agent's signal and the true value of the factor, ε_i or ε_j, and the fraction $\frac{\tau_t}{\delta t}$. It is apparent that $\frac{\tau_t}{\delta t}$ determines how intensively the trader uses her private information to bet against the others, so we will label this fraction as trading intensity in period t. Here, we only present the steps which lead to (15). Expressions (16) and (17) are obtained very similarly.

From (12), the market clearing condition is

$$D_2 = \frac{\bar{c}y + \bar{b}\left(\theta_s + \theta_w\right) + \bar{f}q_1 + \bar{e}q_2 + \bar{g}q_0 - p_2}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = u_2.$$

Using (8) and rearranging gives

$$\bar{c}y + \bar{b}\left(\theta_s + \theta_w\right) + \bar{f}q_1 + \bar{e}q_2 + \bar{g}q_0 - avar\left(\theta | z_j, y, p_2, q_1, q_0\right)u_2 = b_2q_2 + c_2y + f_2q_1 + g_2q_0.$$

As the two sides has to be equal in equilibrium for any realizations of u_1, u_2 and η,

$$c_2 = \bar{c} \tag{18}$$

$$f_2 = \bar{f} \tag{19}$$

$$g_2 = \bar{g} \tag{20}$$

must hold. This implies

$$\bar{b}\frac{e_2}{b_2}\left(\theta_s + \theta_w\right) - avar\left(\theta | z_j, y, p_2, q_1, q_0\right)\frac{e_2}{b_2}u_2 = \left(b_2 - \bar{e}\right)\frac{e_2}{b_2}q_2.$$

As $q_2 = \theta_s + \theta_w - \frac{e_2}{b_2}u_2$, this gives us

$$\frac{\bar{b}}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = \frac{b_2}{e_2} = \frac{\left(b_2 - \bar{e}\right)}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)},$$

consequently,

$$b_2 = \bar{b} + \bar{e}. \tag{21}$$

Note, that expressions (18)-(20) and (21) determine the equilibrium value of the coefficients in the price function,(7), in terms of coefficients of the conditional expectation of θ.

If we substitute out p_2, c_2, g_2, f_2 from the left hand side of (12), we get

$$E\left(\theta | z_j, y, p_2, q_1, q_0\right) - p_2 = b_2\left(z_j - q_2\right),$$

which – together with the definition of τ_2 and q_2 – implies

$$d_2^i = \frac{\bar{b}\left(z_j - q_2\right)}{avar\left(\theta | z_j, y, p_2, q_1, q_0\right)} = \frac{\tau_2}{\delta_2}(z_j - q_2) = \frac{\tau_2}{\delta_2}\varepsilon_j + u_2. \tag{22}$$

Very similar steps applied to (13) and (14) gives us

$$\begin{aligned} d_1^i &= \frac{1}{ab_2}\frac{b_s}{var_i\left(\theta_s + \theta_k\right) + \frac{1}{\tau_2^2}}\left(x_i - q_1\right) = \frac{\tau_1}{\delta_1}\left(x_i - q_1\right) = \frac{\tau_1}{\delta_1}\varepsilon_i + u_1 && (23)\\ d_0^i &= \frac{\left(\sigma_q s + 1\right)\left(\left(c_1 + b_1\right)b_y + s\left(1 - b_y\right)\frac{c_1\sigma_{yq} + b_1\sigma_q}{\sigma_q s + 1}\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}\left(x_i - q_0\right) = \frac{\tau_0}{\delta_0}\left(x_i - q_0\right) = \frac{\tau_0}{\delta_0}\varepsilon_i + u_0 && (24)\end{aligned}$$

From this procedure – similarly to expressions (18)-(20) and (21) – we also gain expressions for c_1 and b_1 in terms of coefficients of the conditional expectations (9)-(11)[7]. All of these, together with the expectational coefficients are given in the appendix. The last step is to find the equilibrium trading intensities,$\frac{\tau_t}{\delta_t}$, which give the equilibrium demand functions. For this, we simply plug in the expressions for $b_s, \bar{b}$,b_2 and the conditional variances into (22) and (23) and equate the coefficients of $(z_j - q_2), (x_i - q_1)$ and $(x_i - q_0)$ in the two sides of the equations (22)-(24). This gives a system of three equations with the three unknowns of τ_1, τ_2, τ_0 :

$$\begin{aligned} \tau_2 &= f^2\left(\tau_2, \tau_1, \tau_0\right) = && (25)\\ &= \delta_2\frac{1}{a}\alpha\frac{\tau_0^2\omega + \tau_1^2\omega + \kappa\omega + \tau_0^2\kappa + \tau_1^2\kappa + \kappa^2}{\tau_0^2\kappa + \tau_0^2\tau_2^2 + \tau_0^2\omega + \alpha\tau_0^2 + \alpha\tau_1^2 + \alpha\omega + \kappa\alpha + \kappa^2 + 2\kappa\omega + \kappa\tau_2^2 + \omega\tau_2^2 + \tau_1^2\tau_2^2 + \tau_1^2\omega + \tau_1^2\kappa}\\ \tau_1 &= f^1\left(\tau_2, \tau_1, \tau_0\right) = && (26)\\ &= \delta_1\tau_2^2\alpha\frac{\kappa^2 - \omega\beta}{a\left(\kappa\tau_0^2\tau_2^2 + \kappa\omega\tau_2^2 + \tau_2^2\kappa^2 + \omega\tau_1^2\tau_2^2 + 2\alpha\omega\tau_2^2 + \kappa\tau_1^2\tau_2^2 + 2\kappa\alpha\tau_2^2 + \tau_0^2\omega\tau_2^2 + \alpha\kappa^2 + \kappa\omega\alpha\right)}\\ \tau_0 &= f^0\left(\tau_2, \tau_1, \tau_0\right) = \frac{\delta_0\left(\sigma_q s + 1\right)\left(c_1 + b_1\right)b_y + s\left(1 - b_y\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}. && (27)\end{aligned}$$

Note, that all the building-blocks of $f^0\left(\tau_2, \tau_1, \tau_0\right)$ – which are $\sigma_q, \sigma_y, \sigma_{yq}, b_y, c_1, b_1$ and s – can be expressed as functions of τ_2, τ_1, τ_0 only (see appendix). Furthermore, it is easy to see that the corresponding equilibrium intensities when there is no announcement will be given by the same equations by setting $\beta = 0$. When it could cause misunderstanding, we will distinguish between τ_2, τ_1, τ_0 of the announcement and the no-announcement cases by the subscript n, for no-announcement.

Hence, the equilibrium exists if and only if this system of equations has a fix point. The following proposition states that this will be the case for any parameter values.

Theorem 1 (Existence) *From (25)-(27) any equilibrium is a fixed-point of a system:*

$$\tau_2 = f^2\left(\tau_2, \tau_1, \tau_0\right), \qquad \tau_1 = f^1\left(\tau_2, \tau_1, \tau_0\right), \qquad \tau_0 = f^0\left(\tau_2, \tau_1, \tau_0\right)$$

---

[7] Actually, we also obtain similar expressions for the other coefficients in the price functions – e_2, f_1, e_1, b_0, e_0 – but as they are not relevant for our purposes, we omit them to save space.

There is always at least one equilibrium of this system both for the announcement and the no-announcement cases.

Proof. The proof is in the Appendix. ■

3.2.2 Announcement and volume

As the focus of this paper is the effect of announcement on trading volume, we will be interested in the change of volume in period 1 due to the announcement. From equations (13) and (14), the amount of trading of trader i in period 1 will be given by

$$v_1^i = d_1^i - d_0^i = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i - u_0 + u_1.$$

Just as total positions, the total amount of trade of individual i consist of two parts. There is an information-independent risk-sharing part, $u_1 - u_0$, and there is a speculative part $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i$, which is determined by the difference of trading intensities in the two periods and the private information of the trader. If we aggregate across traders, we get the following expression for total volume in period 1:

$$\begin{aligned} V_1 = \frac{1}{2}\int \left|d_1^i - d_0^i\right| di = \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i > u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right) d\varepsilon_i - \\ - \frac{1}{2}\int_{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i < u_0 - u_1} \left(\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0\right)\phi\left(\alpha\varepsilon_i\right) d\varepsilon_i = \\ = \left|\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\right| \frac{1}{\sqrt{\alpha}}\phi\left(T\right) + sgn\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\left(u_1 - u_0\right)\frac{1}{2}\left(1 - 2\Phi\left(T\right)\right) \end{aligned}$$

with $T = \alpha\frac{u_0 - u_1}{\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)}$,where we used the result that if $\zeta \sim N\left(\mu, \sigma^2\right)$, then

$$\int_{\zeta > L} \zeta\frac{\phi\left(\frac{\zeta - \mu}{\sigma}\right)}{\Phi\left(\alpha\right)} d\zeta = E\left(\zeta | \zeta > L\right) = \mu + \sigma\lambda\left(\alpha\right)$$

with $\lambda\left(\alpha\right) = \frac{\phi(\alpha)}{1 - \Phi(\alpha)}$ and $\alpha = \frac{L - \mu}{\sigma}$.

Hence, the aggregate volume depends only on the realization of $u_1 - u_0$, the precision of the private signals α, and the distance between trading intensities, $\left|\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\right|$. As the first one is unrelated to information or announcements, we will focus on the speculative volume, which we define as the volume when there is no risk-sharing trade:

$$V_1^S = V_1|_{u_0 = u_1} = \left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right| \frac{1}{\sqrt{\alpha 2\pi}}.$$

It must be clear now that – for results on the effect of announcement on speculative volume – we only have to compare the change in trading intensities in the announcement case, $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$, and the no-announcement case, $\left|\frac{\tau_1^n}{\delta_1} - \frac{\tau_0^n}{\delta_0}\right|$. The main result of this paper that the outcome of this comparison will depend heavily and systematically on the information structure i.e. on the relative importance of the common factor, θ_w, and the individual factors θ_s, θ_k. The following proposition shows that as the individual factors are getting less important and the common factor becomes more important, volume disappears. This result is in line with our earlier observation that in a rational model with one factor trading volume around announcements is small, because the effects of increasing precision of opinions and decreasing disagreement cancel out.

Proposition 2 *As* $\delta_2, \kappa \to \infty$

$$\frac{\tau_2}{\delta_2} = \frac{\tau_1}{\delta_1} = \frac{\tau_0}{\delta_0} = \frac{\tau_2^n}{\delta_2} = \frac{\tau_1^n}{\delta_1} = \frac{\tau_0^n}{\delta_0} = \frac{\alpha}{a}$$

, *hence* $V_1 = V_1^n = 0$ *in this limit.*

Proof. For $\frac{\tau_2}{\delta_2}, \frac{\tau_1}{\delta_1}, \frac{\tau_2^n}{\delta_2}$ and $\frac{\tau_1^n}{\delta_1}$, the result comes from the simple observation that the ordered limit

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f^2(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{a}$$

and after substitution of $\tau_2 = \frac{\alpha}{a}$

$$\lim_{\delta_2 \to \infty} \lim_{\kappa \to \infty} f^1(\tau_2, \tau_1, \tau_0) = \frac{\alpha}{a}.$$

The result for $\frac{\tau_0}{\delta_0}$ and $\frac{\tau_0^n}{\delta_0}$ can be obtained in a similar, but much more tedious way, if we take the limit of all the building-blocks of $f^0(\tau_2, \tau_1, \tau_0)$ and plug them in. ■

We confront this result with the next proposition, where we show that if one measures the effect of announcement by the proportion of volume in the announcement case to volume in the no-announcement case, this proportion will be arbitrary large as the common factor, θ_w, loses its importance. The same is true for the proportion of speculative positions in both periods.

Proposition 3 *If* ω *is large enough* $D_1^s = \frac{1}{2} \int \left| \frac{\tau_1}{\delta_0} \varepsilon_i \right| di > \frac{1}{2} \int \left| \frac{\tau_1^n}{\delta_0} \varepsilon_i \right| di = D_1^n$ *and* $V_1 > V_1^n$ *and as* $\omega \to \infty$,$\frac{D_1}{D_1^n} \to \infty$,$\frac{D_0}{D_0^n} \to \infty$. *Furthermore,* $\frac{V_1}{V_1^n} \to \infty$ *(for almost all parameters).*

Proof. The proof is in the appendix. ■

The intuition of this result goes as follows. Let us suppose that ω is large, so the common factor, θ_w is unimportant i.e. only the individual factors, θ_s and θ_k matter. Traders can bet only on those variables which are not part of the public information set. From the Londoners point of view in period 1 (the second period when they trade), the only variable in p_2 which is not part of the public information set – apart from the noise, u_2 – is θ_s. But first period traders have no information on θ_s, only on θ_k. Hence, they will agree that they do not know anything (i.e. there guess will be the a priori mean, 0), and there will be agreement and no speculative trade. But if Londoners do not trade on their private information, their private information cannot be channeled into prices, so p_1 will be pure noise. So if we go one period back, in period 0, Londoners should bet on p_1 and p_2, but they do not have any information neither on p_1, as it is pure noise , nor on p_2, because they do not know anything

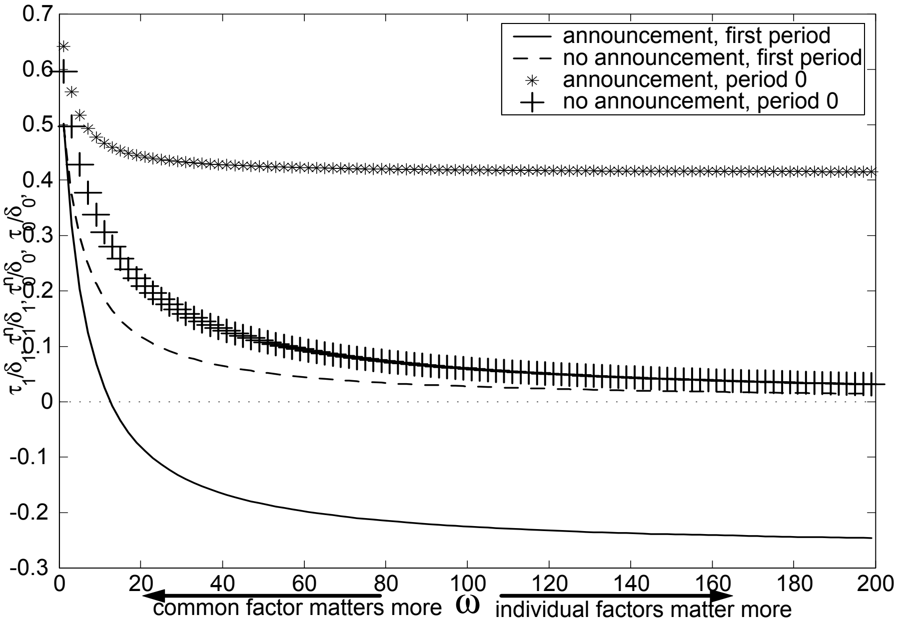

Figure 1: Trading intensities in periods 0 and 1 in the cases of announcement and no-announcement (the coefficient of ε_i in the expression d^i_t) as the information set of traders across groups becomes more separated i.e. ω increases. Parameter values are $\delta_2 = \kappa = 5$, $\alpha = \beta = \delta_1 = \delta_0 = 1$.

about θ_s. Hence, there will be no speculative trade in period 0 either. It means that the speculative volume, the difference between individual speculative positions in period 0 and period 1, will also be zero. However, with public announcement the situation changes. The public announcement is $y \approx \theta_s + \theta_k + \eta$, if θ_w is unimportant. So Londoners will have some information on the sum of θ_s and θ_k. But together with their private information on θ_k, it gives them some information on θ_s. What is more, as they have different guesses on θ_k due to their different private signals, their guesses on θ_s will also be different. Therefore, public announcement increases disagreement. With the opposite logic as in the no-announcement case, there will be trade in all periods and there will be volume.

3.3 Numerical Results

We calculated numerically the fix point of the system (25)-(27) with several parameter combinations. A typical graph of trading intensities $\frac{\tau_t}{\delta_t}$ and $\frac{\tau^n_t}{\delta_t}$ is shown in Figure 1. The middle two lines are $\frac{\tau^n_0}{\delta_0}$ and $\frac{\tau^n_1}{\delta_1}$ (the trading intensity in the no-announcement case in period 1 and 0). Both goes to 0 as $\omega \to \infty$ as it is stated in the proof of Proposition 3 and in line with our intuitive story when θ_w is unimportant. With announcement, in period 0 traders bet intensively on the size of y based on their private information (line with asterisk), and in period 1 they bet intensively on p_2 (solid line). The larger the distance between the two lines, the larger the trading volume around announcements. When ω is small, we are close to the standard information case (one factor model). It is apparent that at this extreme, all lines are almost equal, so trading volume is small. If κ and δ_2 would be large enough, all lines would coincide as it is stated in Proposition 2. It is spectacular that as ω grows – private information sets become separated across groups – the lines corresponding to the announcement cases fan out. This shows the potential of our story in explaining the jump in trading volume around announcements.

4 Confrontation with empirical results

4.1 Established stylized facts

Recent evidence from high-frequency data-sets (Evans and Lyons, 2001,2003, Love and Payne, 2003, Love 2004, Fleming and Remolona, 1999) show a prolonged intense trading period after announcements when

1. there is a simultaneous increase in buying orders and selling orders[8]
2. order flows, the difference between buying and selling orders, are more volatile
3. order flows are more informative i.e. they influence price formation more.

Our rational expectations model is of a reduced form. All traders submit whole demand schedules and orders are executed on the market clearing price. Hence, in our model there is no order flow. In reality, a market maker neither observes whole demand schedules nor observes all of them at the same time. She has to map them through time. She quotes a price which is good for any amount, some traders make transactions on this price, and the market maker updates her quote depending on the received orders. Order flow is like the aggregate trades of a small group of traders executed at a close-to-equilibrium price. We believe that the nearest we can get in our model to the behaviour of order flow, is to consider the behaviour of individual orders at equilibrium price, because the aggregation of a small number of trades must have similar characteristics to the parts of this aggregation. We will argue that if one is willing to accept individual trades as a proxy for order flow, then our model is consistent with the three observed facts above.

1. It is easy to see that both buys and sells increase due to announcement in our model as individual orders are given by

$$d_1^i - d_0^i = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)\varepsilon_i + u_1 - u_0 = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)x_i - \left(\frac{\tau_1}{\delta_1}q_1 - \frac{\tau_0}{\delta_0}q_0\right)$$

   and we showed that $\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)$ can be much larger when there is announcement, than when there is none.

---

[8] In the FX market, all dealers act as market makers, so all submit bid and ask prices simultaneously to the brokerage system, which are good for any amount. Then any of the dealers can initiate transactions on the best submited quotes. A transaction is called a buying (selling) order, if the initiator of the transaction buys (sells) the comodity currency. Hence, the number of buys and the number of sells in any FX dataset can move independently of each other. (See Lyons, 2001, for detailed discussion on the microstructure of FX markets.)

2. Similarly, the volatility of individual orders depend positively on $\left|\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right|$ as well, since

$$var\left(d_1^i - d_0^i\right) = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)^2 \frac{1}{\alpha} + \frac{1}{\delta_1^2} + \frac{1}{\delta_2^2}$$

.

3. Note, that in the mapping process of demand curves described above, the information content of individual orders depends on how strongly they are correlated with private information. It is so, because the more heavily a trader uses her information on her speculative betting, the easier the market maker can deduct the private information of the trader. Since for a given price, q_1,

$$cov\left(d_1^i - d_0^i, x_i\right) = \left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right).$$

The informativeness of prices depends again only on$\left(\frac{\tau_1}{\delta_1} - \frac{\tau_0}{\delta_0}\right)$.

Evans and Lyons (2001) argue that the finding of increased informative trade due to announcements is inconsistent with a common-priors model:

> "...when (1) information is publicly observed and (2) all market participants agree on the mapping from that information to price, then price adjustment occurs independently of order flow. Our finding that the adjustment of price to announcements depends on order flow suggests a violation of the second condition that all participants agree on the mapping."

Although in our model (1) and (2) is true, there is a lot of price adjustment through trade. However, it is not the public information, which is built in this way, but the existing private information becomes relevant and flows into the market.

4.2 Other testable implications

We find three other implications of our model which may be testable.

1. Our model predicts large increase in volume even if the announcement is unexpected (unlike in He and Wang). Therefore it would be intriguing to investigate the differences in market reaction to expected and unexpected announcements.

2. If ω is large, $cov\left(p_2, p_1 | y, q_0\right)$ – the conditional covariance of "equilibrium" prices of London morning and NY afternoon – should be small in no-announcement times, and much larger in announcement times.

3. Our model shows that announcements can increase trading activity independently of their content, because they help to channel private information into prices. Putting it differently, the

model implies that announcements have an event-effect apart from a size-effect.[9]

The first two implications are for future research, but we present here preliminary results on the third one. We use the same data-set as Love and Payne (2003) and Love (2004) used. The data comes from the Reuters D2000-2 electronic brokerage system, which is one of the two dominant systems for inter-dealer trades on the FX spot market. It covers the period between 2nd December 1999 and 24th July 2002 of the markets of the three major currency pairs: EUR/GBP, USD/GBP and EUR/USD. It is transaction level data, which is aggregated on the minute frequency. Although it does not contain the size of the transactions, previous analysis showed that transactions are approximately of the same size, so the number of transactions in each minute – number of buys plus number of sells – is a good proxy of the volume. We filter the data by dropping out the low-activity periods[10]. The second part of the data-set contains US, UK and Euro-area scheduled macro-announcements along with the time-stamp of release and the market expectation of the announcements provided by Standard and Poor[11]. From this announcement data, a surprise-variable is constructed by subtracting the expectation of the news and dividing it by the standard deviation of the news group. For example, the size of the surprise from the inflation announcement on a given date, is the announced inflation, minus the expected inflation at that date, divided by the standard deviation of announced inflation in the given geographical area in the whole data-set. We intend to asses how excess trade due to announcements is related to the size of the surprise and the event of the announcement. We expect that there will be an event effect as well. Because there is an obvious intra-day time pattern of volume in the data[12] , we measure excess trade as the difference between volume in a given minute and average volume at the same minute on no-announcement days[13].

As our focus is on the activity at, and after the announcement, we construct 1,5,10,15 and 20-minute windows starting from the minute of the announcement and aggregate excess volume within the windows. We pool the data for all currency pairs and all announcements, which gives us 244 data points. Then we regress this aggregate excess volume series on the size of the surprise and on dummy variables which identifies the effect of announcements originating form different geographical areas on different currency pairs (e.g. the dummy variable $D^{uk}_{USD/GBP}$, takes the value 1 in data points corresponding to announcements of UK macro-news and excess volume on the USD/GBP market). We estimate a simple OLS regression with White-heteroscedasticity consistent standard errors. Different

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[9] Kandel and Pearson (1995) observes a related result on equity markets around earning announcements. They find that there is trading volume increase around announcements, even if prices do not change. They claim that in rational models there is no volume without price movement, and this claim motivates their non-Bayesian model. It is easy to see that in our Bayesian model, speculative volume is unrelated to price changes as well.

[10] We drop weekends, public holidays, periods which correspond to overnight periods both in London and New York, and periods where there are no trading activity through consequent 20 minutes which is considered to be a sign of a system brake-down.

[11] For further details on the whole dataset, see Love and Payne (2003).

[12] Intra-day trading activity shows an M-shaped pattern. There is a peak when most UK-traders enter around 9 GMT and another one when US-traders enter around 14 GMT. (See Love and Payne, 2003)

[13] As a robustness check we repeated the excercise with using the 32-point Fourier transformation of the avarage volume instead of the simple avarage volume. This method was introduced to the literature by Gallant (1981) and used with an increased popularity, because of its favourable properties. The results with this method are very similar to the reported ones.

windows give very similar results[14], so we report only the results from the 5-minute window and the 20-minute window exercise.

X/Y	ex. volume (20-minute)	ex volume (5-minute)	ex volume (20-minute)
constant	11.01 (2.5)**	–	–
size	17.97 (3.5)***	5.25 (2.87)***	15.88 (3.14)***
$D^{uk}_{USD/GBP}$	–	16.43 (3.80)***	19.67 (2.19)**
$D^{us}_{USD/EUR}$	–	19.78 (5.54)***	35.14 (3.66)***
$D^{us}_{USD/GBP}$	–	5.72 (2.27)**	6.54 (0.97)
$D^{uk}_{GBP/EUR}$	–	8.80 (2.87)***	12.21 (1.67)*
$D^{eu}_{USD/EUR}$	–	1.50 (0.61)	-1.79 (-0.28)
$D^{eu}_{GBP/EUR}$	–	1.83 (0.94)	-2.72 (-0.52)
$\bar{R}^2$	0.04	0.12	0.10
N of observations	244	244	244

The results show that although the size of the surprise matters as well, there seems to be an independent effect of the event of announcements too. It is clear for UK announcements on both the USD/GBP and the EUR/GBP market and for US announcements on the EUR/USD market. The independent effect of US announcements on the USD/GBP market is weaker, but significant with the 5-minute window. Euro-area announcements definitely do not have a significant effect on trading activity. This is consistent with the results of Love (2004) and Love and Payne (2003), who report that Euro-area news had much less effect on the market in this period, than UK and US news. Although this is a preliminary exercise, we consider these results supportive for the hypothesis that announcement events has an independent positive effect on trading volume apart from their informational role.

5 Conclusion

We showed that common knowledge public announcements can move opinions further apart, when agents are rational and the model is common knowledge. The main intuition behind our model is that the implication of public information can be very different if it is coupled with different private information sets. We showed that this fact can cause large, volatile and informative trading volume around public announcements. We had the following critical assumptions. We assumed that there were some early traders (Londoners) who had to resale the asset to new entrants (New Yorkers). We allowed for weaker correlation between private information sets across two groups, and it was necessary that public announcement was not independent of any of the information sets. In the second half of the paper we argued that implications are largely consistent with empirical observations, but future research is needed.

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[14] The larger the window, the less significant the coefficents, but the only coefficient which loses its significance is $D^{us}_{USD/GBP}$.

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Appendix

A.1 Demand in period 0

In period zero traders maximize the expected utility

$$
\begin{aligned}
&E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{avar\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)|x_i,q_0\right)= \\
&=E\left(E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{E\left(p_2|q_1,y,x_i,q_0\right)-p_1}{avar\left(p_2|q_1,y,x_i,q_0\right)}\left(p_2-p_1\right)\right)q_0,y,x_i,q_1\right)|x_i,q_0\right)= \\
&=E\left(-\exp\left(-a\left(p_1-p_0\right)d_0^i-\frac{\left(E\left(p_2|q_1,y,x_i,q_0\right)-p_1\right)^2}{2var\left(p_2|q_1,y,x_j,q_0\right)}\right)|x_i,q_0\right)= \\
&=E\left(-\exp\left(-a\left(b_1q_1+c_1y+f_1q_0-p_0\right)d_0^i-\frac{1}{2}s\left(x_i-q_1\right)^2\right)|x_i,q_0\right)
\end{aligned}
$$

where

$$
s=\frac{b_s^2}{\left(var\left(\theta_s+\theta_w|q_1,y,x_j,q_0\right)+\frac{1}{\tau_2^2}\right)}=b_2a\frac{\tau_1}{\delta_1}b_s.
$$

If we write the expression in the inner bracket into matrix form and we use the standard result for the expectation of exponentials with quadratic forms[15], we get

---

[15] If c is constant scalar, L is a $nx1$ constant vector, N is an nxn constant matrix and M is an $nx1$ stochastic matrix and I is an information set, then

$$
\begin{aligned}
&E\left(-\exp\left(c+L'M-M'NM'\right)|I\right)= \\
&\qquad -|W|^{-1/2}\left|2N+W^{-1}\right|^{-1/2}\exp\left(c+L'Q-Q'NQ+\frac{1}{2}\left(L'-2Q'N\right)\right)\left(2N+W^{-1}\right)^{-1}\left(L-2NQ\right)\right)
\end{aligned}
$$

where $Q=E\left(M|I\right)$ and $W=var\left(M|I\right)$ (see Brunnermeier, 2001, page 110).

$$E\left(-\exp\left(\begin{array}{c} -\frac{1}{2}sx_i^2 - a\left(f_1q_0 - p_0\right)d_0^i + \begin{pmatrix} -ac_1d_0^i & \left(-ab_1d_0^i + sx_i\right)\end{pmatrix}\begin{pmatrix} y \\ q_1 \end{pmatrix} \\ -\begin{pmatrix} y & q_1 \end{pmatrix}\left(\frac{1}{2}s\right)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y \\ q_1 \end{pmatrix} \end{array}\right)|x_i, q_0\right) =$$

$$= -\exp\left(\begin{array}{c} -\frac{1}{2}sx_i^2 - a\left(f_1q_0 - p_0\right)d_0^i + \begin{pmatrix} -ac_1d_0^i & \left(-ab_1d_0^i + sx_i\right)\end{pmatrix}\begin{pmatrix} \mu_y \\ \mu_q \end{pmatrix} + \\ \frac{1}{2}\left(\begin{pmatrix} -ac_1d_0^i & \left(-ab_1d_0^i + sx_i\right)\end{pmatrix} - 2\begin{pmatrix} \mu_y & \mu_q \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}s \end{pmatrix}\right) \\ \left(\begin{pmatrix} 0 & 0 \\ 0 & s \end{pmatrix} + \begin{pmatrix} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{pmatrix}^{-1}\right)^{-1}\left(\begin{pmatrix} -ac_1d_0^i \\ -ab_1d_0^i + sx_i \end{pmatrix} - 2\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}s \end{pmatrix}\begin{pmatrix} \mu_y \\ \mu_q \end{pmatrix}\right) \\ -\begin{pmatrix} \mu_y & \mu_q \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}s \end{pmatrix}\begin{pmatrix} \mu_y \\ \mu_q \end{pmatrix} \end{array}\right)$$

where

$$\begin{aligned} \mu_q &= E\left(q_1|x_i, q_0\right) \\ \mu_y &= E\left(y|x_i, q_0\right). \end{aligned}$$

and

$$\begin{pmatrix} \sigma_y & \sigma_{yq} \\ \sigma_{yq} & \sigma_q \end{pmatrix} = var\left(\begin{pmatrix} y \\ q_1 \end{pmatrix}|x_i, q_0\right).$$

Maximizing the term in the bracket with respect to d_0^i gives the demand function

$$d_0^i = \frac{\left(c_1\mu_y + \mu_q b_1 + q_0 f_1 - p_0\right)\left(\sigma_q s + 1\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)} + \frac{s\left(x_i - \mu_q\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}$$

, which is identical with the one in the text. The second order condition of the maximization is

$$\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0.$$

A.2 Expectations, variances and coefficients in the price-functions

We give here the conditional expectations and variances obtained by standard results on normal variables (see e.g. Brunnermeier, 2001, p12). We also give the equilibrium expressions for coefficients in the price function. The method to obtain the latter is described in the text.

$$
\begin{aligned}
\sigma_y &= \frac{2\kappa\omega\beta+\kappa^2\beta+\beta\alpha\omega+\beta\alpha\kappa+\beta\tau_0^2\omega+\beta\tau_0^2\kappa+\kappa\alpha\omega+\kappa^2\alpha+\kappa\tau_0^2\omega+\kappa^2\tau_0^2+\kappa^2\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\kappa\beta} \\
\sigma_q &= \frac{\tau_1^2\omega+\tau_1^2\kappa+\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega}{\left(\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega\right)\tau_1^2} \\
\sigma_{yq} &= \frac{\omega+\kappa}{\alpha\omega+\kappa\alpha+\tau_0^2\omega+\kappa\tau_0^2+\kappa\omega} \\
b_y &= (\omega+\kappa)\frac{\alpha}{\alpha\omega+\alpha\kappa+\tau_0^2\omega+\tau_0^2\kappa+\kappa\omega} \\
c_1 &= (b_2 c_s+\bar{c}) \\
b_1 &= \left(b_2\left(b_s+e_s\right)+\bar{f}\right)
\end{aligned}
$$

where
$b_2 = \frac{\left(\alpha\omega\tau_0^2+\alpha\omega\tau_1^2+\kappa\omega\alpha+\kappa\alpha\tau_0^2+\kappa\alpha\tau_1^2+\alpha\kappa^2+\tau_0^2\omega\tau_2^2+\tau_1^2\omega\tau_2^2+\kappa\omega\tau_2^2+\kappa\tau_0^2\tau_2^2+\kappa\tau_1^2\tau_2^2+\tau_2^2\kappa^2\right)}{B}$
with

$$
B=\left(\begin{array}{c}
\alpha\omega\beta+\kappa^2\beta+\tau_1^2\kappa^2+\tau_2^2\kappa^2+\alpha\kappa^2+\kappa^2\omega+\tau_0^2\kappa^2+\alpha\kappa\beta+\kappa\omega\tau_1^2+\tau_1^2\omega\tau_2^2+ \\
\kappa\omega\tau_2^2+2\kappa\tau_0^2\tau_2^2+\tau_0^2\omega\tau_2^2+2\kappa\tau_1^2\tau_2^2+\kappa\omega\tau_0^2+\kappa\omega\alpha+ \\
+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\tau_2^2\beta\kappa+\tau_1^2\tau_2^2\beta+\tau_0^2\tau_2^2\beta+\tau_2^2\omega\beta+2\kappa\omega\beta \\
+2\kappa\alpha\tau_0^2+2\kappa\alpha\tau_1^2+\alpha\omega\tau_1^2+\alpha\tau_1^2\beta+\alpha\omega\tau_0^2+\alpha\tau_0^2\beta
\end{array}\right)
$$

and
$\bar{f} = \frac{1}{B}\tau_1^2\left(\alpha+\tau_2^2+\kappa\right)(\omega+\kappa)$
$\bar{c} = \frac{\beta\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$
$c_s = \frac{\beta\left(\kappa+\tau_1^2+\alpha+\tau_0^2\right)(\omega+\kappa)}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$
$e_s = \frac{\tau_1^2\kappa^2-\tau_1^2\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$
$b_s = \frac{\alpha\kappa^2-\alpha\omega\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$
$var\left(\theta_s+\theta_w|x_i,q_1,q_0,y\right)=$
$= \frac{\tau_1^2\omega+2\tau_1^2\kappa+\omega\beta+\kappa^2+\kappa\omega+\alpha\omega+2\alpha\kappa+\tau_1^2\beta+\alpha\beta+2\tau_0^2\kappa+\tau_0^2\omega+\beta\kappa+\tau_0^2\beta}{\kappa^2\omega+2\kappa\omega\beta+\kappa\omega\tau_0^2+\tau_0^2\kappa^2+\kappa^2\beta+\alpha\kappa^2+\kappa\omega\alpha+\kappa\omega\tau_1^2+\tau_1^2\kappa^2+\tau_0^2\kappa\beta+\tau_1^2\omega\beta+\tau_1^2\kappa\beta+\tau_0^2\omega\beta+\alpha\kappa\beta+\alpha\omega\beta}$
$var\left(\theta|z_j,q_2,q_1,q_0,y\right) = \frac{\left(\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa\right)}{B}$

A.3 Proof of existence

The equilibrium is given by the fixed point of the system

$$\begin{aligned}
\tau_2 &= f^2(\tau_2,\tau_1,\tau_0) = \\
&= \delta_2 \frac{1}{a}\alpha \frac{\tau_0^2\omega+\tau_1^2\omega+\kappa\omega+\tau_0^2\kappa+\tau_1^2\kappa+\kappa^2}{\tau_0^2\kappa+\tau_0^2\tau_2^2+\tau_0^2\omega+\alpha\tau_0^2+\alpha\tau_1^2+\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2+\tau_1^2\tau_2^2+\tau_1^2\omega+\tau_1^2\kappa} \\
\tau_1 &= f^1(\tau_2,\tau_1,\tau_0) = \\
&= \delta_1\tau_2^2\alpha \frac{\kappa^2-\omega\beta}{a\left(\kappa\tau_0^2\tau_2^2+\kappa\omega\tau_2^2+\tau_2^2\kappa^2+\omega\tau_1^2\tau_2^2+2\alpha\omega\tau_2^2+\kappa\tau_1^2\tau_2^2+2\kappa\alpha\tau_2^2+\tau_0^2\omega\tau_2^2+\alpha\kappa^2+\kappa\omega\alpha\right)} \\
\tau_0 &= f^0(\tau_2,\tau_1,\tau_0) = \frac{\delta_0(\sigma_q s+1)(c_1+b_1)b_y+s(1-b_y)(c_1\sigma_{yq}+b_1\sigma_q)}{a\left(c_1^2\sigma_y+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2+2c_1\sigma_{yq}b_1\right)}
\end{aligned}$$

We show the existence in three steps.

Lemma 2 *Let us fix* $\tau_0=\bar\tau_0$ *at any arbitrary level. The system* $\tau_2=f^2(\tau_2,\tau_1,\bar\tau_0)$, $\tau_1=f^1(\tau_2,\tau_1,\bar\tau_0)$ *will have at least one fix point,* (τ_1^*,τ_2^*). *Additionally,* $\tau_2^{\min}\leq\tau_2^*<\delta_2\frac{1}{a}\alpha$ *where* $\tau_2^{\min}$ *is the single root of* $\delta_2\frac{1}{a}\alpha\frac{\kappa\omega+\kappa^2}{\alpha\omega+\kappa\alpha+\kappa^2+2\kappa\omega+\kappa\tau_2^2+\omega\tau_2^2}=\tau_2$ *and* $\frac{\left(\kappa^2-\omega\beta\right)}{a2\kappa}<(>)\tau_1^*\leq(\geq)0$ *if and only if* $\kappa^2<(\geq)\omega\beta$. *Furthermore, let* $\tau_1^*(\tau_0)$ *and* $\tau_2^*(\tau_0)$ *are given as the fixed points corresponding to* $\bar\tau_0=\tau_0$ *with the smallest absolute value. Then these functions will be continuous.*

Proof. Notice first, that $\tau_1=f^1(\tau_2,\tau_1,\bar\tau_0)$ determines a third degree polinom in τ_1, which is monotone increasing so it gives a single root for every τ_0 and τ_2. Similarly,$\tau_2=f^2(\tau_2,\tau_1,\bar\tau_0)$ also determines a monotone increasing third degree polynom in τ_2 which gives a single unique root for every τ_0 and τ_1. It is also apparent that a marginal change in τ_0 or τ_2 in the first polynom or a marginal change in τ_0 or τ_1 in the second polynom will cause only a marginal change in the roots. This gives the continuity of $\tau_1^*(\tau_0)$ and $\tau_2^*(\tau_0)$.

For the existence, note that from the root of the polynom $\tau_1=f^1(\tau_2,\tau_1,\bar\tau_0)$, $\tau_1^2\left(\tau_2^2\right)$ is a well defined continuous function. Therefore, the equilibrium is given by the fixed point of $\tau_2=\frac{1}{a}\alpha\delta_2 g^2(\tau_2)=f^2\left(\tau_2,\tau_1^2\left(\tau_2^2\right),\bar\tau_0\right)$ where $g(\cdot)$ continuously maps τ_2 to the unite interval. As $\lim_{\tau_2\to 0}\frac{1}{a}\alpha\delta_2 g(\tau_2)=\delta_2\frac{1}{a}\alpha\frac{\kappa\omega+\kappa^2}{\tau_1^2(0)\kappa+\kappa^2+\tau_1^2(0)\omega+\tau_0^2\kappa+\alpha\kappa+\tau_0^2\omega+\alpha\omega+2\kappa\omega}>0$, where $\tau_1^2(0)$ is finite and $0<\frac{1}{a}\alpha\delta_2 g(\tau_2)\leq\delta_2\frac{1}{a}\alpha$, there has to be a fixed point. The rest of the lemma comes from simple observation. ■

Lemma 3 *The second order condition of the maximization problem in period 0 always holds, so the denominator of* $f^0(\tau_1,\tau_2,\tau_0)$

$$a\left(c_1^2\sigma_y+c_1^2 s\sigma_y\sigma_q-c_1^2 s\sigma_{yq}^2+\sigma_q b_1^2+2c_1\sigma_{yq}b_1\right)>0$$

Proof. Note that the matrix

$$Q=\left(\begin{pmatrix}0&0\\0&s\end{pmatrix}+\begin{pmatrix}\sigma_y&\sigma_{yq}\\\sigma_{yq}&\sigma_q\end{pmatrix}^{-1}\right)^{-1}$$

is positive definite as $s > 0$. Consequently

$$0 < xQx^T$$

for all x.The lemma comes from the choice of

$$x = \left(\left(\begin{array}{cc} -ac_1d_1 & (-ab_1d_1 + sx_i) \end{array} \right) - 2 \left(\begin{array}{cc} \mu_y & \mu_q \end{array} \right) \left(\begin{array}{cc} 0 & 0 \\ 0 & \frac{1}{2}s \end{array} \right) \right)$$

as then

$$0 < xQx^T = \left(\begin{array}{c} \frac{a^2\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)}{\sigma_q s+1} d_1^2 - \\ -\frac{2ac_1\sigma_{yq}sx_i - 2\sigma_q ab_1\mu_q s - 2ac_1\sigma_{yq}\mu_q s + 2\sigma_q ab_1 sx_i}{\sigma_q s+1} d_1 \\ + \frac{s^2\sigma_q\left(x_i - \mu_q\right)^2}{\sigma_q s+1} \end{array} \right),$$

which is possible for all d_1 only if

$$a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right) > 0.$$

■

Proposition 4 *There is always at least one equilibrium.*

Proof. From Lemma 1, we have to show that the expression $\tau_0 = g^0(\tau_0) = f^0(\tau_2^*(\tau_0), \tau_1^*(\tau_0), \tau_0)$ has at least one fix point. We proceed in 4 steps.

1. Note, that $g^0(\tau_0) = g^0(-\tau_0)$ for all τ_0. It is so, because τ_0 enters as τ_0^2 to all building-blocks of $g^0(\tau_0)$.
2. We show that $\lim_{\tau_0\to\infty} g^0(\tau_0) = 0$. Let us check the building-blocks separately. As $\tau_0 \to 0$, τ_2^* goes to a constant, τ_1^* goes to 0 by the order of $\frac{1}{\tau_0^2}$, hence c_1, σ_y, s goes to constants, σ_{yq} and b_y goes to 0 by the order of $\frac{1}{\tau_0^2}$, σ_q goes to infinity by the order of τ_0^4 and b_1 goes to 0 by the order of $\frac{1}{\tau_0^2}$. So the nominator of $g^0(\tau_0)$, $(\sigma_q s + 1)(c_1 + b_1)b_y + s(1 - b_y)(c_1\sigma_{yq} + b_1\sigma_q)$, goes to infinity by the order of τ_0^2 from the speed of convergence of the term $\sigma_q s(c_1 b_y + b_1)$. The denominator,$a\left(c_1^2\sigma_y + 2c_1\sigma_{yq}b_1 + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2\right)$ also goes to infinity but by the order of τ_0^4 given by the term of $c_1^2 s\sigma_y\sigma_q$. Hence, $\lim_{\tau_0\to\infty} g^0(\tau_0) = 0$.
3. The function $g^0(\tau_0)$ is continuous. It comes by the positivity of the denominator, which holds because of Lemma 2, and the continuity of all building-blocks.
4. Hence, $g^0(\tau_0)$ will cross the 45° line necessarily, because it is symmetric, continuous and goes to 0 as τ_0 increases without bound. Therefore, there will be a fixed point with $\tau_0^* \geq (<)\, 0$ if $g^0(0) \geq (<)\, 0$.

■

A.4 Proof of Proposition 3

Proof. The second half of the statement implies the first half as both the aggregate holdings and the volume are continuous functions of ω. For the second half of the statement, it is sufficient to show that $\lim_{\omega\to\infty}\left|\frac{\tau_0^n}{\delta_0}-\frac{\tau_1^n}{\delta_1}\right| = \lim_{\omega\to\infty}\left|\frac{\tau_0^n}{\delta_0}\right| = \lim_{\omega\to\infty}\left|\frac{\tau_1^n}{\delta_1}\right| \to 0$ while $\left|\frac{\tau_0}{\delta_0}-\frac{\tau_1}{\delta_1}\right| \to C_1, \left|\frac{\tau_1}{\delta_1}\right| \to C_2$ and $\left|\frac{\tau_0}{\delta_0}\right| \to C_3$, where C_1, C_2 and C_3 are non zero constants. In the no-announcement case, the equilibrium is characterized by the following equations:

$$\begin{aligned}
\tau_2^n &= f^2\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \\
&= \frac{\delta_2 \frac{1}{a}\alpha\left(\tau_0^{n2}\omega + \tau_1^{n2}\omega + \kappa\omega + \tau_0^{n2}\kappa + \tau_1^2\kappa + \kappa^2\right)}{\tau_0^{n2}\kappa + \tau_0^{n2}\tau_2^2 + \tau_0^{n2}\omega + \alpha\tau_0^{n2} + \alpha\tau_1^{n2} + \alpha\omega + \kappa\alpha + \kappa^2 + 2\kappa\omega + \kappa\tau_2^{n2} + \omega\tau_2^{n2} + \tau_1^{n2}\tau_2^{n2} + \tau_1^{n2}\omega + \tau_1^{n2}\kappa} \\
\tau_1^n &= f^1\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \\
&= \delta_1\tau_2^2\alpha\frac{\kappa^2}{a\left(\kappa\tau_0^{n2}\tau_2^2 + \kappa\omega\tau_2^2 + \tau_2^2\kappa^2 + \omega\tau_1^{n2}\tau_2^2 + 2\alpha\omega\tau_2^2 + \kappa\tau_1^{n2}\tau_2^2 + 2\kappa\alpha\tau_2^2 + \tau_0^{n2}\omega\tau_2^{n2} + \alpha\kappa^2 + \kappa\omega\alpha\right)} \\
\tau_0^n &= f^0\left(\tau_2^n, \tau_1^n, \tau_0^n\right) = \delta_0\frac{b_y^n + \sigma_q^n s^n}{b_1^n a\sigma_q^n}.
\end{aligned}$$

It is apparent, that for any τ_1^n, τ_0^n, $\lim_{\omega\to\infty}\tau_2^n$ is a finite, positive constant, and for any finite, positive τ_2^n and any τ_0^n $\lim_{\omega\to\infty}\tau_1^n = 0$. Consequently, $\lim_{\omega\to 0}\sigma_q^n = \lim_{\tau_1\to 0}\sigma_q^n = \infty$. Hence,

$$\lim_{\omega\to\infty}\delta_0\frac{b_y^n + \sigma_q^n s^n}{b_1^n a\sigma_q^n} = \lim_{\omega\to\infty}\frac{s^n}{b_1^n} = \lim_{\omega\to\infty}\frac{\frac{b_s^n}{\left(\sigma_s^n + \frac{1}{\tau_2^2}\right)}}{b_2^n\frac{\left(b_s^n + e_s^n\right)}{b_s^n} + \frac{\bar{f}}{b_s}} = 0$$

, which holds because $\lim_{\omega\to\infty} b_s^n = 0$, but $\lim_{\omega\to\infty} b_2^n$, $\lim_{\omega\to\infty}\sigma_s^n$ and $\lim_{\omega\to\infty}\frac{(b_s+e_s)}{b_s} = \lim_{\omega\to\infty}\frac{\tau_1^2\kappa^2+\alpha\kappa^2}{\alpha\kappa^2} = 1$ are non zero constants.

In the announcement case, our equilibrium determining equations go to the following ones as $\omega \to \infty :$

$$\begin{aligned}
\tau_2 &= \delta_2\frac{1}{a}\alpha\frac{\left(\kappa + \tau_0^2 + \tau_1^2\right)}{\left(\tau_0^2 + 2\kappa + \tau_2^2 + \tau_1^2 + \alpha\right)} \\
\tau_1 &= -\delta_1\tau_2^2\alpha\frac{\beta}{a\left(\tau_1^2\tau_2^2 + \kappa\tau_2^2 + \tau_0^2\tau_2^2 + 2\alpha\tau_2^2 + \kappa\alpha\right)} \\
\tau_0 &= \frac{\delta_0\left(\sigma_q s + 1\right)\left(c_1 + b_1\right)b_y + s\left(1 - b_y\right)\left(c_1\sigma_{yq} + b_1\sigma_q\right)}{a\left(c_1^2\sigma_y + c_1^2 s\sigma_y\sigma_q - c_1^2 s\sigma_{yq}^2 + \sigma_q b_1^2 + 2c_1\sigma_{yq}b_1\right)},
\end{aligned}$$

where the building-blocks of the last equations are all of the corresponding limiting functions. By the observation of expressions for the building-blocks, it is apparent that all of them are going to finite, non-zero constants as $\omega \to \infty$. Hence, just by the same reasoning as in the existence theorem, there must be at least one equilibrium where all τ_2, τ_1, τ_0 will be finite and non-zero. If $\lim_{\omega\to\infty}\frac{\tau_1}{\delta_1}$ and $\lim_{\omega\to\infty}\frac{\tau_0}{\delta_1}$ happened to be equal with certain combinations of the parameters, small perturbation on the parameters (for example perturbing δ_0) would unambiguously make $\lim_{\omega\to\infty}\left|\frac{\tau_0}{\delta_0}-\frac{\tau_1}{\delta_1}\right| > 0$. ■