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Who Becomes A Stockholder? Expectations, Subjective Uncertainty, and Asset Allocation

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Abstract. We develop a model of portfolio selection with subjective uncertainty and learning in order to explain why some people hold stocks while others don't. We model heterogeneity in information directly, which is an alternative to the existing explanations that emphasized heterogeneity in transaction costs of investment. Our approach leads to a model of learning with new implications such as zero optimal risky assets, or ex post correlation of income or wealth and optimal portfolio composition. It also points to two factors in probabilistic thinking that should have a major impact on stock ownership. These are the level and the precision of stock market expectations. We use subjective proxy measures of expectations from the Health and Retirement Study (HRS) over stock returns and economic growth. We also use an index of precision over many subjective probabilities from the 1992-2002 waves of HRS, a measure introduced by Lillard and Willis (2001). Our results indicate that stock ownership and the probability of becoming a stockholder are strongly positively correlated with the indices of the level and precision of expectations. They also significantly reduce the partial correlation of stock ownership and education, race and wealth, which indicates that the latter are probably at least as much proxies for expectations and probabilistic thinking as measures of direct transaction costs.

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When looking at portfolio choice, much of the theoretical finance literature has focused on how people should behave given their preferences. Considerably less is known about how people behave in reality. As part of the optimal choice, normative finance theory prescribes strategies for using the available information the best possible way. In contrast, a positive approach should ask what information people actually have when they make portfolio choices, how they use that information, and how they acquire it. In this paper we aim at contributing to the latter approach. In particular, we try to understand how heterogeneity in the perception of uncertain outcomes translates into heterogeneity in portfolio choices by individual households, and how informational heterogeneity arises in the first place.

In the past, the dominance of defined benefit public and private pension plans meant that the financial security of retirees depended chiefly on decisions made by firms and governments. Knowing more about individual investment behavior is becoming increasingly important because of the growing importance of defined contribution pension plans in which individuals have discretion about contribution rates and portfolio composition. In addition, judgments about the advantages and disadvantages of proposals to encourage the development of individual accounts as part of the Social Security system often rest on views about people's willingness to bear risk and about how well informed they are about risks of stocks and alternative forms of wealth. This paper does not attempt to address these policy issues directly. Rather, our goal is to use individual-level longitudinal data from the Health and Retirement Study to establish the extent of heterogeneity in beliefs about returns in the stock market and to trace through the possible implications of these heterogeneous beliefs for actual investment behavior.

Our paper is related to recent developments in behavioral finance. Andrei Shleifer (2000) has shown that financial markets may not be efficient if there are agents with "erroneous" beliefs about the distribution of market returns. These agents he calls noise traders. Here we try to understand how these beliefs affect behavior and how they arise. Contrary to the behavioral approach, however, we stay in an expected utility framework. Conditional on their beliefs everyone behaves optimally, and beliefs are formed by learning in an optimal way. The model we develop is quite simple but it leads to interesting implications such as the idea that the optimal portfolio structure should depend on wealth even when preferences are isoelastic and asset returns are i.i.d. Throughout the paper we focus on the ownership margin. That is, we ask why some people hold certain assets while others do not. We show that, among other things, imprecise beliefs can lead to zero risky assets in the portfolio. Following Shliefer's terminology, these agents we may call "noise non-traders".

Another problem our paper helps to understand is the "stockholding puzzle." The usual explanation for why some people hold stocks while others don't is heterogeneity in transaction costs of investment (see, e.g., Halliosos and Bertaut, 1995; Bertaut, 1998; or Vissing-Jørgensen, 2001). Usually, these costs are defined in a broad way. Among other things, people's imperfect knowledge about stock returns has been viewed as part of the costs of investment. In this paper we take a different approach. While acknowledging that some kind of fixed transaction costs are likely to affect investment decisions, we model heterogeneity in information in a more direct way.

There are two main advantages of this approach. First, we can use survey measures of people's perceptions about the distribution of stock market returns to calibrate our model, and use the model to derive the implied optimal portfolios. The data are not available yet, but we expect a lower and more heterogeneous optimal share of risky assets than implied by the traditional approach (which would imply the same optimal portfolio shares for everybody).

Secondly, we can introduce learning into the portfolio selection problem. We show that the perspective of future learning can lead to zero risky assets in the portfolio in a setup where no learning would always imply a positive share. We have also started to model heterogeneity in learning, from exogenous factors (such as memory) and endogenous reasons (higher expected labor earnings makes it more worthwhile to acquire information). The extension helps to connect cognitive capacities to observed portfolio allocation. It may also help in understanding the ex post correlation of labor income (uncorrelated with stock returns) and portfolio allocation: people with higher expected earnings may find it worth acquiring more information and have a better understanding of the stock returns process, which may increase the share of stocks in their portfolio.

In the second part of the paper we examine the determinants of stock ownership using longitudinal data from the Health and Retirement Study (HRS). The major empirical novelty of our paper is that we relate the theoretical parameters to data on subjective measures of preferences and expectations from HRS. These include survey measures of cognitive capacity, risk aversion, and measures of two aspects of probabilistic beliefs: the level and the dispersion of expectations.

We use two measures for the level of people's expectations. One is the subjective expectation of positive stock returns, measured in 2002. The other is the expectation of economic growth (the negative of the probability of a major depression) which is available for 1992, 1994, and 1996.

While the relevant measures of expectations are connected to the stock market, we have no such measure for the precision of beliefs. Instead, we use a general index of precision over many aspects of probabilistic thinking, based on a large battery of subjective expectations questions of HRS. Lillard and Willis (2001) have shown that one can interpret the propensity to give nonfocal answers to all subjective probability questions (measured by the fraction answers other than 0, 50, or 100 percent) as a proxy for the individuals' general precision when facing uncertainty.

The paper is organized the following way. The first part motivates the problem by showing that the fraction of households that own stocks is less than one even among those with financial assets (the stockholding puzzle). We also show that stock ownership is correlated with education, wealth, and demographic variables, correlations that are hard to reconcile with the standard theory. The second part presents the theoretical model of portfolio choice. The third part discusses the various measures we use in explaining household choices. The fourth section presents the empirical model of stock ownership and the results. The last part concludes.

1. Motivation: stockholding puzzles

Standard portfolio choice models (see section 2) imply that everyone should own some level of stocks. Yet microeconomic data show that many households don't have any. The absence of stocks from the portfolio also comes up in conventional financial counseling and advice for households engaged in long term saving for retirement (see Campbell and Visceira, 2002, for an excellent survey of the literature). In this section, we illustrate some of the empirical patterns of stock holding using ten years of longitudinal data from households of Health and Retirement Study (HRS) participants born in 1941-51 who were age 51-61 at the first wave in 1992. (The data are described in more detail below in Section 3.)

Table 1 shows that in the HRS cohort, less than 40 percent live in households that own stocks (or mutual funds) directly. By direct ownership we mean ownership outside retirement accounts. The number grew somewhat between 1992 and 2000 but not very much. The fraction of people who live in households that own stocks in a broader sense is significantly larger, around 60 percent. The broader definition also includes IRA ownership.[1] We don't know much about the portfolio composition of those accounts, and for the time being we treat them as if they all contained stock market investments. This introduces an upward bias in our estimates of overall stock ownership.

Table 2 shows the results of simple linear probability models where the dependent variable is one if the person lives in a household that owns stock directly or indirectly through retirement accounts. In order to summarize information we pooled all six waves of observations. Since individuals in the same household and observations of the same households from different are obviously not independent, we allowed for arbitrary clustering within households. The right-hand side variables include education, gender, marital status, race, age, year of birth, and wealth.

The results are familiar from earlier research. Education, race, and wealth are very important determinants of stock ownership. The results for direct and broader stock ownership are very similar, the latter being stronger in general. After controlling for wealth and demographics, people with less than high-school education are 13 percent less likely than those with a high-school degree to live in stockholder households (direct ownership); people with a college degree are 12 percent more likely. Couples are 7 percent more likely to own stocks, and

[1] For the years we for which we have the appropriate data (1994 through 1998), the number is about the same if we include those that have defined constribution pension accounts. We don't present those results.

blacks and Hispanics are 10-12 percent less likely. The wealth effects are strong and mildly concave.[2]

Table 3 shows evidence of considerable mobility in stock ownership. Between 1992 and 1994, for example, 11 percent of households who did not own stocks in 1992 became owners by 1994 while 7 percent of former stockholders became non-owners. These patterns show considerable stability over time.

2. Portfolio Selection With Subjective Uncertainty and Bayesian Learning

In this section we present a model of portfolio selection under subjective beliefs, uncertainty, and learning. We would like to relate stock ownership to expectations about asset returns and the precision of those expectations. Our goal is to focus on the intuition behind the models and to introduce a simple analytical framework to facilitate our empirical investigations. Therefore, we keep things as simple as possible. The model is an application of the well-known continuous time portfolio choice model of Merton (1969), augmented with subjective beliefs and Bayesian updating as derived by Gennotte (1986) and Brennan (1998). We also consider some extremely simplified versions of the model to understand the main intuition. We allow for heterogeneity and look at the results from the angle of stock ownership, an application not considered in the previous literature.

2.1 Portfolio selection with known parameters

[2] Households with no positive financial wealth are 25 percent less likely to be stockholders. Since stocks are financial assets this is many times a mechanical relationship (although not always: for a few households debts counterbalance the value of stocks). This variable is included mainly to partial out the other effects, that is to show that the main relationships hold among those with positive financial wealth.

Consider an individual who saves for retirement. For simplicity, assume that at time 0 she has wealth W_0 to invest and she wants to maximize the expected utility of W_T, her wealth when she retires at some predetermined time T. Assume that she has a conventional constant relative risk aversion (CRRA) utility function with γ being the parameter of relative risk aversion. She can choose between investing in the risk-free asset with known instantaneous rate of return r and one risky asset. The instantaneous rate of return of the risky asset, denoted by dS/S, is assumed to follow a Brownian motion with constant mean μ and variance σ^2. The investment decision consists of choosing an optimal fraction of wealth invested into the risky asset for each time t between 0 and T, which we denote by α_t.

The equation of motion for the instantaneous return to the risky asset is given by

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz\ , \tag{1}$$

where dz is the increment to a standard Wiener process. This is a continuous time generalization of a random walk with drift, where the instantaneous drift is μ and the variance is σ^2. Throughout the analysis we assume that the investor knows the random walk nature of the process and that its parameters are constant. For now we also assume that she also knows the parameters themselves, an assumption we will relax later.

With fraction α_t of wealth W_t invested into the risky asset at each time t, wealth also follows a geometric Brownian motion given by

$$\frac{dW}{W} = \left(r + \alpha_t\left(\mu - r\right)\right)dt + \alpha_t\,\sigma\, dz\ , \tag{2}$$

where r is the known instantaneous rate of return on the risk-free asset.

Subject to this budget constraint, the investor's problem is

$$\max_{\alpha_t} E_t \frac{W_T^{1-\gamma}}{1-\gamma} \quad . \tag{3}$$

Assuming that $\gamma > 1$,[3] the standard solution to this problem (Merton, 1969) is a constant fraction of wealth invested into the risky asset

$$\alpha^* = \frac{\mu - r}{\gamma\sigma^2} . \tag{4}$$

The optimal share invested into stocks is increasing in its mean return, decreasing in the return of the risk-free asset, and decreasing in the variance and the degree of risk aversion.

The simple and elegant result in (4) comes at the cost of being at odds with a number of empirical regularities. First of all, no matter how risk averse the investor is, α^* is always positive if the expected return is higher than the rate on the risk-free asset ($\mu > r$). Yet many people hold no stocks at all, as we could see it in the previous section. Moreover, given empirical evidence that stock market returns are high, the amount in stocks should be higher than observed if people had "sensible" risk preferences – this is the "equity premium puzzle" (Mehra and Prescott, 1985). In addition, many people choose the optimal portfolio at time zero and never change it, implying that no one becomes a stockholder at a later date or sells off an initial holding. And lastly, the "Tobin separation theorem" (Tobin, 1958) suggests that the composition of the optimal portfolio should be independent of the optimal level of wealth, while in the real world it is strongly correlated with wealth.

2.2 Portfolio Selection with Subjective Uncertainty

[3] If the coefficient of relative risk aversion is smaller than one and $\mu > r$, it is optimal to hold the entire portfolio in the risky asset. The reason is that if $\gamma < 1$ the concavity of the utility function is not sufficiently strong to offset the convex relationship between terminal wealth and the rate of return caused by compounding.

As noted in the Introduction, heterogeneity in transaction costs has been used to explain heterogeneity in stock ownership. Researchers usually interpret those costs in a very broad sense. In particular, they include the costs of acquiring and processing information. We take a different approach here, using a model by Brennan (1998) which augments the Merton model by considering individuals who are uncertain about the parameters of the process governing stock returns and who learn through Bayesian updating. By modeling heterogeneity in beliefs of the returns process directly, we show that Brennan's model can explain ownership differences without differences in transaction costs. More generally, our goal is to introduce a simple analytical framework to facilitate our empirical investigations. In particular, we would like to relate stock ownership to expectations about asset returns and the precision of those expectations.

Suppose that the investor does not know all the parameters of the returns process (μ and σ^2). To keep things simple, assume that she has a one-point belief about σ^2 but a distribution over μ. In particular, at time zero, she thinks that μ is drawn from a Normal distribution with mean m_0 and variance v_0. v_0 represents subjective uncertainty of those expectations. The inverse of v_0 is often called the precision of the beliefs. We will use the concept of precision in this sense, and will also refer to v_0 as the degree of prior imprecision.

With time she observes the realized returns, regardless of having invested into stocks or not. Based on the observed series, she continuously updates her belief about the distribution of μ.[4] The updated μ (conditional on the realization) is Normal, with parameters m_t and v_t, the former being a diffusion process itself while the latter is a deterministic function of time:

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[4] We assume that she does not update her beliefs about σ^2. If the price path induced by the Brownian motion is continuous, it is possible to estimate σ^2 over any short interval of data, perhaps justifying treating σ^2 as a known parameter.

$$dm = \frac{v_t}{\sigma^2}\left(\frac{dS}{S} - m_t\, dt\right) \tag{5}$$

$$dv = -\frac{v_t}{\sigma^2}\, dt\ , \tag{6}$$

where (6) can be solved to get

$$v_t = v_0 \exp\left(-\frac{t}{\sigma^2}\right). \tag{7}$$

These results were first derived by Lipster and Shiryayev (1978) and were used by Gennotte (1986) and Brennan (1998) for the portfolio selection problem.

Since the perceived parameters of the stochastic process of stock returns (and therefore wealth) change over time, the optimal fraction invested in risky assets will also vary with time. However, the investor's problem can be separated into first updating the parameters and then making the choice based on the posterior. As Brennan (1998) shows, the solution to this new problem is quite complicated. On the other hand, it can be represented by a closed form function ψ of γ and m. The optimal rule, then, is given by

$$\alpha_t^* = \frac{(m_t - r)}{\gamma\sigma^2} + \frac{v_t \psi(m_t, \gamma)}{\gamma\sigma^2}. \tag{8}$$

Note that the first term on the right hand side of (8) is the conventional expression for the optimal portfolio share from Merton's model where m_t is the expected rate of return on stocks, given current information. The second term in (8) represents what Brennan calls an "intertemporal hedging demand" for the risky asset that arises from subjective uncertainty about the “true” rate of return.[5] Brennan shows that sign of the hedging demand depends on the degree of risk aversion. Specifically, ψ is positive for $0 < \gamma < 1$ (mild risk aversion), zero for $\gamma =$ 1 (logarithmic utility), and negative for $\gamma > 1$ (strong risk aversion).

2.3 Learning and the ownership margin

Although Brennan does not emphasize this result, it is possible that parameter uncertainty and potential learning may be of sufficient importance that strongly risk averse persons with $\gamma >$ 1 may choose to hold no stocks, even if the expected return on the risky assets (m_t) exceeds the risk-free return. If sufficiently strong parameter uncertainty (large v_t) is accompanied by sufficiently strong risk aversion (leading to negative ψ), the expression in (8) can become nonpositive. Ruling out short sales so that α_t^* is non-negative, strong risk aversion and sufficient parameter uncertainty may therefore lead to a zero fraction invested into the risky asset.

Since this implication is central to the concerns of this paper, it is worth developing the intuition underlying it. To do so, we consider an extremely simple model in which an individual with CRRA utility seeks to maximize the utility of retirement wealth, which is consumed at the end of the final period. The return on the risky asset for person *i* in period *t*, R_{it}, can be either $\mu + \delta_{it}$ or $\mu - \delta_{it}$, with equal probability. Assume that the decision maker lives for two periods and maximizes her wealth at the end of period two (W_2) by choosing the share of risky assets in

[5] Note that hedging demand goes to zero as *t* goes to infinity since, from (7), uncertainty about the rate of return, v_t, disappears asymptotically as data on the history of stock prices increases, assuming that the underlying fundamentals are constant.

periods 1 and 2. Crucially, assume that she can reoptimize the portfolio after period 1. Thus, let her value function be

$$J\left(W_0\right)=\max_{s_1,s_2} Eu\left(W_2\right)$$

subject to the constraints

$$\begin{aligned}
&u\left(W_{i2}\right)=\frac{W_{i2}^{\;1-\gamma}}{1-\gamma},\\
&W_0=given,\\
&W_t=W_{t-1}\left(1-\alpha_t\right)\left(1+r\right)+W_{t-1}\alpha_t\left(1+R_{it}\right),\quad t=1,2,\\
&\Pr\left(R_{it}=\mu+\delta_{it}\right)=0.5,\quad \Pr\left(R_{it}=\mu-\delta_{it}\right)=0.5.
\end{aligned}$$

We consider two cases: one in which δ_{it} is *i.i.d.* so that observing R_{i1} in period 1 does not tell anything about the future. In this case, there is no role for learning. In the second case, $\delta_{i1}=\delta_{i2}=\delta_i$ so that observing R_{i1} leaves no uncertainty for period 2. This is the extreme case of perfect learning. The question we ask is how the possibility of learning affects the optimal allocation in period 1 before learning takes place. The problem is solved by backward induction.

Optimal allocation in period 1 takes into account the best choice in period 2. Figure 1 shows an example for the optimal share of the risky asset in period 1 by possible values of δ in the two cases where we assume r=0.05, μ=0.10 and γ=2. A larger δ corresponds to larger uncertainty, therefore the share of the risky asset is nonincreasing in δ. The pictures for larger values of γ are similar, with steeper curves which start to decline at lower values of δ. The two curves cross at some low optimal share. The possibility of learning makes it worth holding more risky assets in period 1 above that point, and less below it. For large enough δ, the share of risky assets reaches zero if learning is expected in period 2, and it stays positive no matter how large the risk if learning is not possible.

Why does the potential for learning lead to the possibility that it is optimal not to own any stock even if transaction costs are zero? To answer this question, it is useful first to review why a positive share will always be optimal if utility is CRRA, the expected return is greater than the risk free rate, and there is no learning. The reason is that, in the neighborhood of $\alpha = 0$, the investor has eliminated all but an arbitrarily small amount of risk from her portfolio. Since risk aversion is a second-order phenomenon, the investor should place at least some small amount of the risky asset in her optimal portfolio if its expected rate of return exceeds the risk free rate.

The situation is different if learning can occur and the investor can alter her portfolio in light of new information. Under these two assumptions the investor will not have eliminated risk from terminal wealth even if she chooses a totally "risk-free" portfolio in the initial period (i.e., chooses $\alpha_1 = 0$). The reason is that if sufficiently good news about stock returns occurs in the future, such an investor knows that she will then buy some stock because she will have increased her estimate of the expected rate of return, and risk has been reduced while, if bad news occurs, she will choose to remain fully invested in the safe asset. Thus, from the perspective of period 1 the investor's terminal wealth is stochastic even if she currently holds only safe assets.

With parameter uncertainty and future learning, it is not optimal to eliminate all risk because of the option value of acting on new information. Hence, sufficiently risk averse persons people will be at a corner with $\alpha_1^* = 0$. Note that this is true whether the source of the new information is changes in stock market prices, as in Brennan's model, or any other information that affects the individual's subjective beliefs about long term stock market returns such as news of the latest accounting scandal, the next big thing on the Internet or the anticipation of hot stock tips from Uncle Harry who is scheduled to visit next week.

2.4 Heterogeneous learning

We can develop a model with heterogeneous (and possibly endogenous) learning in the following way. Instead of remaining completely ignorant or becoming fully informed about R_2 by period 2, the investor knows with higher precision what it will be, by observing a number of signals.

To make things simpler, we assume that the return on the risky asset is a continuous random variable. The investor has some subjective belief about its mean, m_{it}.

$$R_{it} \sim N\left(m_{it}, v_{it}\right),\ t = 1, 2.$$

People start with some prior distribution, characterized by m_{i1} and v_{i1}. m_{i1} we call the investor's *optimism* about the return and v_{i1} her degree of *imprecision*. We assume that there are time-invariant "true" parameters out there, that R_t~N(μ,σ^2). The investor has a prior over period 1 parameters: we don't specify why people differ in those beliefs. For simplicity, we assume that conditional on m, R follows a normal distribution with known variance σ^2 (the true variance). It is uncertainty about m that leads to a higher overall variance than σ^2.

The investor then tries to estimate period 2 parameters the best way she can, using a number of signals to update her period 1 beliefs. She observes an i.i.d. sample of n_i signals, (s_{i1}, s_{i2}, ..., $s_{in(i)}$) . We assume that all signals are informative so that $E(s_{ij}) = \mu$, and they are i.i.d.

draws from the same normal distribution, s_{ij}~$N(\mu,\sigma_s^2)$. This is the simple Normal-Normal setup in Bayesian learning. The subjective distribution of period 2 returns is normal with mean m_{i2} and imprecision v_{i2}. The optimal updating rules are the following:

$$
\begin{aligned}
m_{i2} &= \frac{n_i\bar{s}_i + \left(v_{i1}/\sigma_s^2\right)m_{i1}}{n_i + v_{i1}/\sigma_s^2}, \quad \textit{and} \\
v_{i2} &= v_{i1}\frac{\left(v_{i1}/\sigma_s^2\right)}{\left(v_{i1}/\sigma_s^2\right)+n_i}
\end{aligned}
$$

where $\bar{s}_i = \sum_{j=1}^{n_i} s_{ij}/n_i$ is the sample mean of the observed signals. Besides differences in initial priors (m_{i1}, v_{i1}), individual heterogeneity in the beliefs of period 2 returns is a result of sheer luck (the value of the actual signals) and the number of those signals. The updated mean is a weighted average of its prior mean and average of the observed signals. The individual does not know in advance how she will update the mean. She knows, however, that the more signals she observes the more precise her parameter beliefs become: v_{i2} does not depend on the actual signals but it's deterministically decreasing in n_i .

The effects of cognitive capacities such as memory can affect the number of signals: $n_i = n(x_i)$, where the x_i are the personal characteristics. We can also think of n_i as a choice variable (possibly still affected by cognitive capacities): people can buy a sample of signals.[6] They do so in order to form a better view about the parameters of the return process, but the information is costly: a sample of size n_i costs $c(n_i)$. We can model the role of the exogenous factors x_i as factors affecting the cost of a sample: then, $c = c(n_i,x_i)$. This way we introduced another decision

[6] An alternative would be to have n fixed and let σ_s be heterogenous. Our intuition is that all qualitative results would be the same.

into our problem: the decision maker has to choose an optimal sample size n_i^*, by contrasting its costs to its expected benefits. In this case, the full model is the following.

$$J\left(W_{i0}, x_i\right) = \max_{n_i, s_{i1}, s_{i2}} Eu\left(W_{i2}\right)$$

$$u\left(W_{i2}\right) = \frac{W_{i2}^{\;1-\gamma_i}}{1-\gamma_i},$$

$$\begin{aligned}
&W_{i0} = given, \\
&W_{i1} = W_{i0}\left(1-\alpha_{i1}\right)\left(1+r\right) + W_{i0}\alpha_{i1}\left(1+R_{i1}\right) - c\left(n_i, x_i\right), \\
&W_{i2} = W_{i1}\left(1-\alpha_{i2}\right)\left(1+r\right) + W_{i1}\alpha_{i2}\left(1+R_{i2}\right), \\
&\alpha_{it} \in \left[0,1\right], \quad t = 1,2
\end{aligned}$$

and beliefs about the risky return (R) and learning about its period 2 value are described above.

We are still working on the solution of the problem, but we have a strong intuition about its comparative statics. Initial expectations (m_{i1}) should increase the likelihood of investing into stocks in both periods. Initial precision of beliefs ($1/v_{i1}$, the inverse of perceived variance) should have the same effect. The number of signals should initially have a positive effect on period 1 share of risky assets but that should reverse at high levels of initial uncertainty (just like on Figure 1.) Very strong learning and very high initial uncertainty should lead to zero optimal shares in period 1. Personal characteristics (x_i), such as memory, which lower the cost of information increase the number of signals one gets and should work through that channel. Unusually high signals should increase the period 2 share, while low signals should decrease it. The story of the 1990's could be modeled as a period during which signals were higher than m_{i1}, while the 2000-2002 bear market is the reverse.

One more subtle and probably more important implication of the endogenous learning model is that W_{i0} starts to matter, because wealthier people can afford more learning. We can therefore directly get a correlation between ex ante wealth and asset allocation, which results in a similar correlation between ex post wealth and asset allocation (the statistic we measure in our regressions).

3. Measuring Parameter Heterogeneity

The empirical novelty of this paper lies in the survey measures we use to proxy for the heterogeneous parameters that determine stock ownership. In this section we describe those measures in detail. We consider four types of measures: those that try to capture heterogeneity in prior expectations (m_{i1}); those that we think are related to prior precision (inverse of v_{i1}); risk preferences (γ_i); and general cognitive abilities, including education, that affect learning (x_i).

We define various measures for these variables from the expectations questions of the Health and Retirement Study (HRS). The HRS is a large household panel with detailed information on cognition, expectations, and asset ownership. In this paper, we use data on the initial HRS cohort who were born in 1931-41, who were first surveyed in 1992, and who have been resurveyed every two years. The latest available data is from the very preliminary release for 2002. See Juster and Suzman (1995) and Willis (1999) for more detailed descriptions of the HRS studies.

First, we describe our measures for expectations, next we turn to the measure of the precision of beliefs. The third subsection describes the measure of risk aversion, and the fourth subsection defines the cognitive measures we use. The last subsection looks at how all of these measures are related.[7]

3.1 Expectations

We use two measures to proxy prior stock market expectations. One is a direct, the other one is an indirect measure.

HRS 2002 contains two sets of questions about the respondents' expectations of future performance of the U.S. stock market. The questions are described in detail in the Appendix. In the core survey, two questions were asked of everybody. First, "By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?" This question is followed by a similar question asking the probability that the value of the mutual fund would rise by 10 percent or more by this time next year. The goal of the second question was to obtain an indication of the degree of spread in the respondent's subjective beliefs about stock market returns.[8] It does not appear that the second question contains additional information. In our analysis, we use the reported probability that the mutual fund will gain in value as a proxy for the respondent's subjective beliefs about the expected return in the stock market.[9] Clearly, this probability depends not only the expected value but also other moments of the subjective

[7] Note that in most cases we do not try to relate the magnitude of the measures to the theoretical variables. One reason is that, until HRS-2002, we do not have explicit measures for expectations about stock market returns, and the measures we use are therefore not related directly to the parameters. Instead, they serve as proxies for them with arbitrary units of measurement. The only quantitative comparisons we make are to show which measures are the most powerful in predicting behavior.

[8] In a separate experimental module (Module 12), more detailed questions of this type were asked from a random sample of respondents. See the Appendix for additional details.

[9] The order of the two questions was reversed in a random half of the sample. Since order effects do not appear to be significant, we simply use the probability of gain question as our measure of expected stock market returns. See the Appendix for additional details.

distribution, but the moments could not be elicited separately in the survey (again, see the Appendix).

The direct measure of stock market returns is available only in 2002. In earlier waves, HRS asked people about their expectations about economic growth. On the HRS age eligible subsample, the following question was administered in the 1992-1996 surveys[10]: "What do you think are the chances that the U.S. economy will experience a major depression sometime during the next 10 years or so?" For ease of interpretation, we use the probability of the complementary event: that is, one minus the subjective probability of a depression. For each period we examine, we use the beginning of period answer to this question as a proxy for prior expectation. For years after 1996, we use 1996 values when explaining changes from 1998 on. Table 5 shows the summary statistics of the variables (the subjective probability of no depression), scaled to be between 0 and 1. On average, expectations increased significantly between 1992 and 1994 but did not change much between 1994 and 1996. Through the years, expectations were positively correlated: those who were more optimistic than average in 1992 were likely to stay so.

In a companion paper, Kézdi and Willis (2003), we look at the effect of optimism in general. We consider two measures: optimism in weather forecast (forecast errors in predicting whether the next day will be sunny) and a common component in the level of all expectations. We find that both of them, and especially the common expectations component are strong and robust predictors of stock ownership and many other outcomes of economic decisions. We do not analyze them here because interpreting their effect is very complicated in itself (besides pure optimism, they may proxy cognition, human capital, health and wealth as well).

3.2 Subjective uncertainty

[10] After 1996 the question was asked only for those who were not interviewed before.

As the Appendix shows in detail, we cannot use the 2002 stock return questions to uncover the dispersion of beliefs about the returns. The two questions in the main survey do not appear to work well in identifying the dispersion of beliefs. Experimental Module 12 attempts to elicit a more detailed picture of an individual's subjective beliefs. However, it provides data only for a relatively small subsample and the responses are quite noisy.

Our strategy for measuring prior subjective uncertainty is based in part on the assumption that uncertainty about future stock returns is correlated with uncertainty about other future events. We treat this general uncertainty component as a fixed individual trait that stays constant over time (at least for the ten years of the survey). One way to formalize this is to assume that prior subjective uncertainty (v_{i1}) can be decomposed into two parts: one that is specific to the event (σ_{ui}^2) and one that is common across all events for any individual ($\sigma_{\delta i}^2$). This second component may be thought of as some kind of a cognitive trait. In that interpretation, some people understand uncertainty better than others. In our case, we assume that v_i, individual i's subjective parameter variance about stock returns, is the sum of an individual and domain-specific variance term and a variance term that is fixed for each individual through all domains:

$$v_{i1} = \sigma^2{}_{ui} + \sigma^2{}_{\delta i}\,. \tag{9}$$

Our two measures are proxies for the common variance term $\sigma^2{}_{\delta i}$.

The measure we use is the fraction of nonfocal probability answers, where focal answers are 0, 50-50, or 100 percent. Lillard and Willis (2001) show that if people form a subjective distribution about the probabilities and report the mode of this distribution when asked in a survey, the fraction of focal answers directly measures $\sigma^2_{\delta_i}$. The intutiion can be explained as follows. An individual's beliefs about the probabliity of a given event are given by a subjective prior distribution whose dispersion indicates the degree of imprecision or ambiguity about the person's beliefs about the "true value" of the probability. As the degree of uncertainty increases, the prior spreads out and the shape of the prior distribution changes in a systematic way. If uncertainty is small, the prior distribution tends to have a single mode that does not diverge much from its mean. As uncertainty increases, the distribution begins to spread out until eventually it tends to become J-shaped, with a mode at zero or one, depending on whether the mean of the prior is greater or less than one-half. (If the mean is exactly one-half, the distribution changes in shape from uni-modal to uniform as uncertainty increases.) As uncertainty increases still further, the prior distribution tends to become bi-modal, with modes at zero and one.

The modal choice hypothesis advanced by Lillard and Willis (2001) is that, at least in a survey situation with limited time think about an answer, individuals asked to give a probability will answer with the value they consider to be the most likely. That is, they are hypothesized to respond by giving the mode of their prior distribution. In particular, if they have significant uncertainty about the true value, they will tend to give answers of zero or one hundred percent for probabilities they believe to be small or large and answers of fifty percent for mid-range probablities.[11]

On theoretical grounds, Lillard and Willis (2001) argue that individuals who are more uncertain (i.e., have less precise beliefs about probabilities) tend to display more risk aversion. This is easy to see in the context of uncertainty about the returns from stock. As was discussed above in Section 2.1 (see especially footnote 3), if the coefficient of relative risk aversion is

[11] Lillard and Willis (2001) present evidence confirming the internal validity of this interpretation of focal responses to the probability questions.

greater than one, an individual's expected utility is a concave function of the rate of return. Such a person will tend prefer an asset with a certain rate of return to an uncertain one.

There is a fairly strong empiricial tendency for respondents who give focal answers to a given probability question to give focal answers on other questions in unrelated domains. This tendency provides some justification for treating precision as a fixed trait, measured by one minust the proportion of all probability answers to which focal answers are given. Even if we do not rely on the "modal choice" hypothesis of survey response, this measure is intuitively appealing in that answers of zero, fifty or a hundred percent probably reflect a very crude understanding of probabilities. Table 5 describes the fraction of exact (nonfocal) probability answers for our sample. A higher value of this index reflects higher degree of precision over all the subjective probability questions in HRS.

3.3 Risk aversion

HRS measures risk aversion based on responses to hypothetical gambles over lifetime earnings. Barsky, Juster, Kimball, and Shapiro (1997) show that those measures contain reasonable information about risk preferences. HRS 1992 asks the questions from the whole sample. In 1994, 1998, and 2000 the risk preference questions were administered for subsamples of 6-8% of the HRS age eligible; there was no such question in 1996.[12] Table 6 describes the measures in four risk aversion category for the age eligible, in the four waves of HRS where the question was asked. The distribution is stable over time, with around 60 percent of people showing very strong risk aversion. They would turn down a gamble that would double their

[12] The risk aversion measures were administered in HRS-2002 to all respondents less than age 65. Two alternative versions not involving a job scenario were given to 10 percent samples of persons age 65 or more. We have not attempted to use these measures in this paper.

lifetime earnings or could result in a 20% decrease, both with a fifty percent chance. The distribution is even across the three other categories.

Based on the 1992 and 1994 surveys, Barsky et al. (1997) also document that the risk preference measure is quite noisy. Although we do not present the corresponding figures, comparing the 1994, 1996, and 1998 measures also supports the presence of substantial measurement error. Despite the noise, Barsky et al. show that the measure predicts risky behavior such as smoking, heavy drinking, or not having health or life insurance, after controlling for demographics. They also show that the measure predicts stock ownership.

3.4 Cognition

One factor that probably affects learning is cognitive abilities. For measuring learning abilities, we use education and the various cognitive measures of HRS. In the context of our current setup, these measures can be a thought of as proxies for prior parameter uncertainty: those with weaker memory or analytical and numerical abilities might have learned less so they have more diffuse priors at the beginning of our time period.

We use all cognitive test scores in HRS: items from the WAIS IQ test contained in HRS 1992 (they ask respondents to define the relationship of two different things like an orange and a banana, or praise and punishment); immediate and delayed word recall tests from all waves; counting back by seven from 100; and screening questions for dementia 1996, 1998, and 2000. The latter include questions about the date (day, month, and year), the day of the week, the President and Vice President of the U.S., and naming two things after hearing their definition (scissors and cactus). We extracted one common factor in each of the four groups (IQ, word recall, counting back by sevens, dementia screen) for all survey waves together, and we also created a factor from all cognitive questions. Table 7 shows the correlation of the different factors with each other. All factors are positively correlated, and the overall factor shows a strong correlation with the individual factors. Table 8 shows the correlation of the overall cognition factor with each of the items in each wave. The correlation is always positive and statistically significant. The results suggest that the overall cognition factor captures both memory properties (word recall) and numerical abilities (counting back by sevens). Its relationship to IQ type questions and the dementia control variables is weaker, though. Therefore, we shall use the domain-specific factors separately in our analysis.

4. Empirical models

We estimate three models: the probability of being a stockholder in the individual surveys (1992, 1994, 1996, and 2002), the probability of becoming a stockholder between two surveys conditional on not being one at the beginning of the period, and the probability of selling all stocks between two observations (conditional of having stocks in the first point of observation). We present results on a both a narrow and a broader definition of stock ownership (direct ownership of stocks and mutual funds and including IRA ownership).

Our aim is to see whether cognition, risk aversion, expectation, and precision of beliefs predict stock ownership and whether they replace, or at least reduce the predictive power of education, wealth and demographics. The theoretical model outlined in section 2.4 highlights the direct effect of expectations and the dispersion of the subjective distribution, partly through the precision of the beliefs. It also predicts a role for cognition, and also wealth and other characteristics like education through learning costs. Therefore, the reduced form effects of the latter (like in Table 2) contain the results of the learning process besides any possible transaction costs to investing. If we have perfect measures of the subjective probabilities, including those in a regression would perfectly separate the learning effects from the direct transaction costs. On the other hand, survey measures of subjective beliefs are far from perfect. Therefore the coefficients on these measures will be biased downward through measurement error, as will the reduction of the correlation between wealth and the personal characteristics.[13]

4.1 The empirical models

We estimate linear probability models for easier interpretation. Probit and logit counterparts give essentially the same results for average marginal effects. We estimate the following regressions:

$$s_{it} = \alpha_0 + \alpha_m' m_{it} + \alpha_x' x_{it} + u_{it}, \tag{10}$$

$$\Delta^+ s_{it} = \beta_0 + \beta_m' m_{it} + \beta_x' x_{it} + v_{it}, \tag{11}$$

$$\Delta^- s_{it} = \gamma_0 + \gamma_m' m_{it} + \gamma_x' x_{it} + w_{it}, \tag{12}$$

where s_{it} denotes stock ownership (0 or 1), $\Delta^+ s_{it} = s_{it+1} - s_{it}$ (0 or 1) is and indicator for becoming a stockholder conditional on no stocks at the beginning of the time period, and $\Delta^- s_{it} = s_{it+1} - s_{it}$ (0

[13] The Lillard and Willis (2001) "modal choice" hypothesis about survey responses to probability questions implies that measures of subjective probabilities will tend to be noisy.

or -1) is an indicator for selling all stocks conditional on having stocks at the beginning of the time period.

m_{it} is the vector of subjective measures: the measures of expectations (economic growth in 1992-1996 and stock market expectations in 2002); the of precision of beliefs (index of precision taken over all probability variables); and cognition. The measures of risk aversion have no predictive power in any of the models so they were left out from all models. x_i are demographic variables (age, coupleness, gender if single, education, race), together with the initial (t) level of total wealth. All variables refer to the year of the interview (t_0 where changes are modeled), except for the time-invariant demographic variables.

We estimate (10)-(12) for each time period and also for a pooled sample of the 1992-1996 observations for the effect of expectations over economic growth. In accordance with the descriptive analysis in Section 1, the ownership is assigned to each member of a couple. The standard errors are estimated by allowing for clustering at the household level, both across individuals, and for the pooled sample, across time. The subjective measures are signed in such a way that we predict all to have a positive correlation. We use standardized values of wealth for easier interpretation.

4.2 **Results**

Table 9 presents the main results for equations (10)-(12). The first two and last four columns are estimated on the pooled sample of the three survey waves for which expectations over economic growth are available (1992, 1994, and 1996), while the third and fourth columns use the 2002 sample that contains stock market expectations. In each of the four blocks, the first column estimates the probability of direct stock ownership and the second column the probability of stock and IRA ownership. Columns 1-4 look at ownership (equation 10), columns 5-6 at becoming a stockholder if not being one at t_0 (11) and columns 7-8 look at becoming a non-holder if having stocks at t_0 (12).

The results are similar for the three different equations, with a clear ranking in predictive power. Ownership shows the strongest correlations for the subjective measures, selling out all stocks the weakest, and becoming a stockholder is somewhere in-between. This is consistent with the hypothesis that stock ownership is measured with noise, and therefore first differences are noisier than levels. Selling out might be the noisiest because the upward trends in the 1990's resulted in a higher signal-to-noise ratio for buying than selling.

All subjective measures and cognitive variables have the predicted sign except for the imprecisely estimated zero correlation between expectations over economic growth and becoming a non-holder. People with higher expectation about stock market returns and economic growth, more precise probabilistic beliefs in general, and higher cognitive capacity are significantly more likely to be stockholders, more likely to become stockholders if they are non-holders, and are somewhat less likely to become non-holders. Expectations about the stock market seem to have a larger effect than expectations over economic growth in general (although we cannot look at their effect jointly), a fact that supports that the former is a better measure of true stock market expectations.

Entering the subjective measures increases the predictive power of the models. More importantly, they decrease the effect of race, education, and wealth. This supports our claim that what previous research identified as fixed transaction costs of investment do contain informational elements. Even our very crude measures explain a significant part of their covariance with stock ownership. This, of course, does not rule out the possibility that other types of transaction cost are important.

Tables 10-15 replicate the results of Table 9 for the separate survey years. They also show changes in the stockholder status between 2000 and 2002 regressed on 2002 stock market expectations (t_1), which we intend as a proxy for 2000 expectations (t_0). In each table, the one before last column shows the effect of stock market expectations without partialing out the effect of the other covariates.

The tables show that the main results are stable over time (except for the insignificant correlation between 1996 direct stock ownership and 1996 expectations about economic growth). The subjective measures are significant determinants of stock ownership and buying stocks if not being a stockholder before. They substantially decrease the effect of race, wealth, and education. The results also show that the stock market expectations have a very large and robust total correlation with ownership (and also with becoming a stockholder). When the other covariates entered these correlations decrease substantially but remain significant.

5. Discussion and conclusions

The theoretical model of portfolio choice with subjective uncertainty and learning offers implications about what kind of heterogeneity should matter in determining who does and who does not become a stockholder. Our results support the role of prior expectations and the precision of beliefs as important determinants of who becomes a stockholder, the major implication of the theory of portfolio selection with learning. Stock market expectations matter, although they are measured with a lot of noise. Expectations about economic growth also are a good proxy of stock market expectations. The index of precision (fraction of nonfocal answers to all expectations questions) also predicts who becomes a stockholder. This result provides further support for Lillard and Willis (2001) who argue that this index is a useful measure of how well people think in probability terms. Cognitive capacities also have a significant effect in predicting stock ownership and transitions between ownership statuses.

Expectations, precision of beliefs and cognition are not only significant determinants of who becomes a stockholder but they also explain a large part of reduced-form covariation between stock ownership and education, race, and wealth. This is very much in line with the theoretical approach we take: subjective probabilities and cognitive capacities that enter in forming and using those probabilities are an important (if not the most important) determinants of investment choices. Modeling and measuring them directly offers a promising alternative to treating them as parts of "transaction costs" of investment.

The empirical results are all the more interesting because the measures we use to proxy the subjective probability distribution of stock returns are very crude. Improving these measures could probably offer a clearer picture of how people use probabilities to make decisions and perhaps also how they form those beliefs. An important next step in this research program is to examine in depth the implications of heterogeneity of beliefs and differential ability in probabilistic thinking for policies designed to improve the welfare of American workers looking forward to retirement.

References

Barsky, Robert B., F. Thomas Juster, Miles S. Kimball, and Matthew D. Shapiro (1997), "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics*, 537-579.

Bartaut, C. 1998. "Stockholding Behavior of U.S. Households: Evidence from the 1983-1989 Survey of Consumer Finances." *Review of Economics and Statistics*, 80(2) 263-275.

Bassett, W. F., and R. L. Lumsdaine (1999), "Outlook, Outcomes and Optimism" unpublished manuscript, Brown University.

Bassett, W. F., and R. L. Lumsdaine (2001), "Probability Limits," *Journal of Human Resources*, 36(2), 327-363.

Brennan, M. J. (1998), "The Role of Learning in Dynamic Portfolio Decisions," *European Finance Review* 1: 295-306.

Campbell, John Y. and Viceira, Luis M. (2002), *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press.

Genotte, G. (1986), "Optimal Portfolio Choice under Incomplete Information," *Journal of Finance*, 61, 733-749.

Haliassos, M. and C. Bertaut. (1995) "Why Do So Few Hold Stocks?" *Economic Journal*, 1110-1129.

Juster, F. T. and R. Suzman (1995) "An Overview of the Health and Retirement Study," *The Journal of Human Resources,* 30, S7-S56.

Kézdi, Gábor and Willis, Robert J., (2003) "Optimism and Stock Market Investment: Cognitive Bias, Personality Trait, Fortunate Past or What?" (in progress).

Lipster, R.S. and A. N. Shirayayev (1978), *Statistics of Random Processes*, Springer-Verlag, New York.

Lillard, Lee A. and Willis, Robert J. (2001), "Cognition and Wealth: The Importance of Probabilistic Thinking." Presented at the Third Annual Joint Conference for the Retirement Research Consortium "Making Hard Choices About Retirement," May 17-18, 2001, Washington, DC.

Mehra, Rajnish and Edward C. Prescott, (1985), "The Equity Premium: A Puzzle*," Journal of Monetary Economics* 15:145-161.

Merton, R. C. (1969), "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, 51(3), 247-257.

Poterba, J. M. (2001), "The Rise of the 'Equity Culture': U.S. Stockownership Patterns, 1989-1998." *Mimeo.*

Shliefer, Andrei (2000) *Inefficient Markets: An Introduction to Behavioral Finance*. Oxford University Press.

Tobin, James (1958). "Liquidity Behavior as Behavior Toward Risk" *Review of Economic Studies*, No. 67, 65-86.

Vissing-Jørgensen, Annette. 2000. "Non-Financial Income and Participation Cost Structures." Department of Economics, University of Chicago.

Willis, Robert J. (1999) "Theory Confronts Data: How the HRS Is Shaped by Economics of Aging and How the Economics of Aging Will Be Shaped by HRS," *Labour Economics*, 6(2), 119-145.

Tables

Table 1. Fraction of people living in stockholder households (percent).
HRS age eligibles (born 1931-41).

	All		**Those with positive net financial wealth**	
Year	Direct stock ownership	Direct stock ownership and IRA's	Direct stock ownership	Direct stock ownership and IRA's
1992	35	58	44	70
1994	39	60	48	71
1996	38	60	48	70
1998	38	61	47	71
2000	38	61	47	71
2002	37	59	47	70

Notes. Net financial wealth: Stocks, bonds, bank accounts, cd-s, net of all debts except mortgage.
Sample selection: HRS age eligibles, those who were interviewed in each survey from 1992 to 2002. The ownership data refer to the whole family (couple).
HRS 2002: preliminary data.
Observations are weighted by household weights (2000 weights in 2002).

Table 2. Stock ownership, education, demographics, and wealth
Pooled sample of HRS age eligibles (born 1931-41) who were interviewed in all waves.

	Direct stock ownership	Direct stock ownership and IRA's
Less than high school	-0.133	-0.218
	(12.4)**	(15.9)**
College degree	0.125	0.117
	(9.6)**	(10.1)**
Female	0.009	0.006
	(1.2)	(0.7)
Lives in couple	0.070	0.125
	(6.3)**	(10.4)**
Age	0.000	0.002
	(0.6)	(2.5)*
Year of birth	0.000	-0.002
	(0.1)	(1.2)
Black	-0.117	-0.205
	(11.2)**	(14.5)**
Hispanic	-0.104	-0.182
	(7.7)**	(10.2)**
Nonpositive net financial wealth (0-1)	-0.251	-0.280
	(31.2)**	(29.7)**
Standardized net worth	0.147	0.109
	(10.6)**	(9.3)**
Standardized worth squared	-0.0033	-0.0025
	(6.1)**	(5.7)**
Constant	-0.122	4.475
	(0.0)	(1.4)
Observations	35,034	34,876
Clusters (household)	4,711	4,711
R-squared	0.18	0.24

Robust t-statistics in parentheses
* significant at 5% level; ** significant at 1% level

Table 3. Changes in stock ownership outside retirement accounts

HRS age eligibles, those interviewed in all waves; household weights

	1994		
1992	no	yes	total
no	54	11	65
yes	7	28	35
total	61	39	100

	1996		
1994	no	yes	total
no	54	7	61
yes	7	32	39
total	61	39	100

	1998		
1996	no	yes	total
no	54	8	62
yes	8	30	38
total	62	38	100

	2000		
1998	no	yes	total
no	54	8	62
yes	8	30	38
total	62	38	100

	2002		
2000	no	yes	total
no	54	8	62
yes	9	29	38
total	63	37	100

Table 4. Probability of no economic depression (one minus the probability of an economic depression in the near future). Summary statistics. HRS cohort. Weighted by t0 person weight.

	mean	std	obs	min	max
1992	0.45	0.26	9086	0	1
1994	0.62	0.28	7878	0	1
1996	0.61	0.28	7427	0	1
average	0.54	0.22	9086	0	1

Correlation

	1992	1994	1996	average
1992	1.00			
1994	0.34	1.00		
1996	0.32	0.43	1.00	

Table 5. Fraction of exact answers. Summary statistics. HRS age eligibles, participants in survey between 1992 and 2002

Variable	# obs	Mean	Std. D	Min	Max
Fraction exact, 1992	11,879	0.51	0.25	0.00	1.00
Fraction exact, 1994	10,635	0.41	0.22	0.00	1.00
Fraction exact, 1996	10,099	0.39	0.23	0.00	1.00
Fraction exact, 1998	9,548	0.38	0.23	0.00	1.00
Fraction exact, 2000	8,950	0.41	0.22	0.00	1.00
Fraction exact, avg.	7,460	0.42	0.17	0.00	0.94

Correlations
(obs=7460)

	1992	1994	1996	1998	2000	avg
Fraction exact, 1992	1.00					
Fraction exact, 1994	0.34	1.00				
Fraction exact, 1996	0.33	0.49	1.00			
Fraction exact, 1998	0.29	0.43	0.46	1.00		
Fraction exact, 2000	0.27	0.41	0.44	0.45	1.00	
Fraction exact, avg.	0.59	0.75	0.78	0.73	0.72	1.00

Table 6. Distribution of respondents in four risk preference categories (percent)
HRS age eligibles, person weights.

Risk preference categories	**1992**	**1994**	**1998**	**2000**
I. very strong risk aversion ($\gamma > 4$)	64.7	63.2	58.2	64.1
II. strong risk aversion ($4 > \gamma > 2$)	12.0	12.9	16.2	14.4
III. weak risk aversion ($2 > \gamma > 1$)	10.6	13.2	9.6	8.5
IV. very weak risk aversion ($1 > \gamma > 0$)	12.7	10.7	16.0	13.0
All	100.0	100.0	100.0	100.0
Number of observations	9,089	591	628	760

Table 7. Cognition: correlation between different factors

	Memory	IQ	Countig back by 7	Dementia control
Memory	1.00			
IQ	0.36	1.00		
Countig back by 7	0.58	0.38	1.00	
Dementia control	0.36	0.33	0.47	1.00

Table 8. Cognition: correlation between the overall factor and the different items HRS cohort. p-values in parentheses.

Question	1992	1994	1996	1998	2000	IQ-type questions	1992
Immediate word recall	0.60 (0.00)	0.64 (0.00)	0.71 (0.00)	0.74 (0.00)	0.72 (0.00)	Orange & banana	0.38 (0.00)
Delayed word recall	0.57 (0.00)	0.61 (0.00)	0.69 (0.00)	0.73 (0.00)	0.73 (0.00)	Table & chair	0.30 (0.00)
Counting back by sevens			0.65 (0.00)	0.67 (0.00)	0.68 (0.00)	Eye & ear	0.44 (0.00)
Date month			0.21 (0.00)	0.20 (0.00)	0.24 (0.00)	Egg & seed	0.33 (0.00)
Date day			0.22 (0.00)	0.24 (0.00)	0.24 (0.00)	Air & water	0.27 (0.00)
Date year			0.23 (0.00)	0.26 (0.00)	0.20 (0.00)	Fly & tree	0.30 (0.00)
Day of week			0.15 (0.00)	0.14 (0.00)	0.15 (0.00)	Praise & punishment	0.25 (0.00)
Scissors			0.11 (0.00)	0.10 (0.00)	0.12 (0.00)		
Cactus			0.43 (0.00)	0.39 (0.00)	0.42 (0.00)		
President			0.30 (0.00)	0.29 (0.00)	0.29 (0.00)		
Vice president			0.40 (0.00)	0.44 (0.00)	0.46 (0.00)		
Counting back from 20			0.25 (0.00)	0.27 (0.00)	0.24 (0.00)		
Counting back from 86			0.37 (0.00)	0.36 (0.00)	0.36 (0.00)		

Table 9. Probability of stock ownership *(1 if stockholder, 0 otherwise)*, **becoming a stocholder** *(1 if became a stockholder, 0 otherwise)*, **and becoming a non-holder** *(-1 if sold out , 0 otherwise)*. **Linear probability models. HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)**

	Stockholder, 1992-1996		Stockholder, 2002		Becoming a stock-holder, 1992-1998		Becoming a non-holder, 1992-1998	
	narrow	broad	narrow	broad	narrow	broad	narrow	broad
Less than high-school	-0.072	-0.158	-0.100	-0.182	-0.044	-0.062	-0.037	-0.040
	(5.21)**	(9.68)**	(4.97)**	(8.05)**	(5.36)**	(5.58)**	(1.39)	(2.69)**
College	0.078	0.082	0.071	0.044	0.043	0.064	0.028	0.008
	(5.11)**	(6.08)**	(3.50)**	(2.62)**	(3.17)**	(3.15)**	(2.08)*	(1.06)
Female	-0.006	-0.001	0.015	0.003	-0.015	-0.032	-0.012	-0.007
	(0.67)	(0.15)	(1.08)	(0.22)	(2.22)*	(3.36)**	(1.20)	(1.16)
Couple	0.055	0.105	0.046	0.094	0.031	0.044	-0.006	0.021
	(4.13)**	(7.66)**	(2.56)*	(5.40)**	(3.37)**	(3.82)**	(0.37)	(2.23)*
Age	0.004	0.003	-0.002	0.001	-0.011	-0.008	-0.005	-0.000
	(1.92)	(1.49)	(0.64)	(0.44)	(4.17)**	(2.54)*	(1.35)	(0.16)
Year of birth	0.003	-0.003	-0.003	-0.003	-0.009	-0.003	-0.007	-0.003
	(0.92)	(1.30)	(0.95)	(0.96)	(3.18)**	(0.83)	(1.75)	(1.54)
Black	-0.078	-0.153	-0.131	-0.248	-0.032	-0.039	-0.167	-0.142
	(5.93)**	(9.04)**	(6.17)**	(10.13)**	(3.69)**	(3.58)**	(4.18)**	(5.44)**
Hispanic	-0.082	-0.149	-0.166	-0.250	-0.046	-0.035	-0.075	-0.095
	(4.89)**	(6.94)**	(6.45)**	(7.85)**	(4.93)**	(2.70)**	(1.26)	(2.81)**
Negative fin. wealth	-0.256	-0.286	-0.158	-0.126	-0.045	-0.073	-0.192	-0.111
	(24.71)**	(21.89)**	(8.60)**	(6.71)**	(5.15)**	(6.52)**	(4.65)**	(6.50)**
Net wealth (std)	0.232	0.182	0.246	0.217	0.144	0.097	0.050	0.034
	(10.18)**	(8.45)**	(10.48)**	(11.38)**	(4.60)**	(2.46)*	(3.50)**	(4.32)**
Net wealth squared	-0.019	-0.016	-0.020	-0.019	-0.013	-0.015	-0.004	-0.003
	(4.02)**	(3.54)**	(5.93)**	(5.75)**	(1.40)	(1.29)	(2.23)*	(2.77)**
Cognition	0.049	0.041	0.023	0.028	0.020	0.010	0.029	0.026
	(7.05)**	(5.43)**	(1.99)*	(2.56)*	(4.32)**	(1.70)	(2.80)**	(4.74)**
Econ.growth	0.060	0.083			0.033	0.034	-0.000	-0.003
	(3.78)**	(5.33)**			(2.64)**	(2.27)*	(0.01)	(0.26)
Precision	0.090	0.114	0.030	0.083	0.026	0.042	0.050	0.020
	(4.41)**	(5.82)**	(0.91)	(2.73)**	(1.55)	(2.00)*	(1.95)	(1.46)
Stock mkt. up			0.079	0.128				
			(2.95)**	(5.25)**				
Constant	-4.890	7.025	6.720	6.338	18.643	6.508	14.540	6.461
	(0.89)	(1.34)	(1.00)	(1.02)	(3.23)**	(0.90)	(1.72)	(1.48)
Observations	16243	16208	4391	4371	10417	6958	5657	9018
R-squared	0.23	0.29	0.21	0.26	0.07	0.07	0.03	0.05

Robust t-statistics in parentheses
* significant at 5% level; ** significant at 1% level

Table 10. Probability of *direct stock ownership*. Linear probability models *(1 if stockholder, 0 otherwise)*. HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)

	1992	1994	1996	2002	**2002**
Less than	-0.070	-0.073	-0.070		-0.100
high-school	(4.43)**	(4.18)**	(4.06)**		(4.97)**
College	0.098	0.055	0.074		0.071
	(5.35)**	(2.94)**	(3.98)**		(3.50)**
Female	-0.006	-0.001	-0.011		0.015
	(0.54)	(0.12)	(1.03)		(1.08)
Couple	0.033	0.052	0.070		0.046
	(2.11)*	(3.14)**	(4.32)**		(2.56)*
Age	-0.007	-0.034	0.003		-0.002
	(0.51)	(2.58)**	(0.22)		(0.64)
Year of birth	-0.007	-0.036	0.000		-0.003
	(0.51)	(2.69)**	(0.01)		(0.95)
Black	-0.052	-0.085	-0.087		-0.131
	(3.12)**	(4.74)**	(4.99)**		(6.17)**
Hispanic	-0.054	-0.115	-0.067		-0.166
	(2.81)**	(5.57)**	(2.99)**		(6.45)**
Negative fin.	-0.223	-0.269	-0.266		-0.158
wealth	(15.45)**	(17.48)**	(17.54)**		(8.60)**
Net wealth	0.343	0.225	0.225		0.246
(std)	(12.95)**	(8.75)**	(8.87)**		(10.48)**
Net wealth	-0.049	-0.015	-0.018		-0.020
squared	(7.85)**	(3.79)**	(4.80)**		(5.93)**
Cognition	0.042	0.047	0.056		0.023
	(5.26)**	(5.27)**	(6.24)**		(1.99)*
Econ.growth	0.096	0.062	0.010		
	(4.01)**	(2.60)**	(0.47)		
Precision	0.068	0.112	0.104		0.030
	(2.58)**	(3.49)**	(3.30)**		(0.91)
Stock mkt. up				0.281	0.079
				(10.20)**	(2.95)**
Constant	13.637	71.276	-0.122	0.284	6.720
	(0.52)	(2.70)**	(0.00)	(18.79)**	(1.00)
Observations	5603	5342	5298	4422	4391
R-squared	0.24	0.23	0.24	0.03	0.21

Table 11. Probability of *direct and idirect stock ownership* (incl. IRA). Linear probability models *(1 if stockholder, 0 otherwise)*. HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)

	1992	1994	1996	2002	**2002**
Less than	-0.155	-0.159	-0.158		-0.182
high-school	(8.33)**	(7.92)**	(8.08)**		(8.05)**
College	0.099	0.079	0.063		0.044
	(6.23)**	(4.85)**	(3.87)**		(2.62)**
Female	-0.003	-0.008	0.007		0.003
	(0.31)	(0.70)	(0.64)		(0.22)
Couple	0.079	0.113	0.115		0.094
	(4.85)**	(6.85)**	(7.02)**		(5.40)**
Age	-0.001	-0.010	-0.002		0.001
	(0.06)	(0.77)	(0.12)		(0.44)
Year of birth	-0.007	-0.017	-0.007		-0.003
	(0.52)	(1.35)	(0.57)		(0.96)
Black	-0.130	-0.158	-0.164		-0.248
	(6.66)**	(7.37)**	(7.75)**		(10.13)**
Hispanic	-0.132	-0.167	-0.143		-0.250
	(5.31)**	(6.31)**	(5.21)**		(7.85)**
Negative fin.	-0.288	-0.279	-0.278		-0.126
wealth	(15.47)**	(14.42)**	(14.73)**		(6.71)**
Net wealth	0.284	0.167	0.186		0.217
(std)	(11.87)**	(7.74)**	(8.32)**		(11.38)**
Net wealth	-0.045	-0.013	-0.016		-0.019
squared	(7.67)**	(3.18)**	(4.47)**		(5.75)**
Cognition	0.037	0.041	0.040		0.028
	(4.45)**	(4.49)**	(4.34)**		(2.56)*
Econ.growth	0.090	0.091	0.060		
	(3.80)**	(3.86)**	(2.75)**		
Precision	0.072	0.114	0.167		0.083
	(2.79)**	(3.79)**	(5.63)**		(2.73)**
Stock mkt. up				0.344	0.128
				(13.48)**	(5.25)**
Constant	13.669	33.651	14.625	0.486	6.338
	(0.53)	(1.35)	(0.58)	(31.36)**	(1.02)
Observations	5603	5338	5267	4402	4371
R-squared	0.30	0.28	0.29	0.04	0.26

Table 12. Probability of buying stocks if not a stockholder (*direct stock ownership)*. Linear probability models *(1 if bought stocks, 0 otherwise)*. **HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)**

	1992-4	1994-6	1996-8	2000-2	**2000-2**
Less than	-0.047	-0.037	-0.049		-0.048
high-school	(3.15)**	(2.88)**	(3.96)**		(2.66)**
College	0.016	0.042	0.072		0.051
	(0.71)	(1.92)	(3.21)**		(1.97)*
Female	-0.006	-0.013	-0.024		0.014
	(0.58)	(1.34)	(2.10)*		(0.94)
Couple	0.034	0.041	0.014		0.046
	(2.14)*	(3.02)**	(0.91)		(2.65)**
Age	0.001	0.001	-0.006		-0.017
	(0.06)	(0.06)	(0.51)		(0.97)
Year of birth	0.000	0.002	-0.002		-0.012
	(0.03)	(0.12)	(0.12)		(0.65)
Black	-0.047	-0.014	-0.032		-0.069
	(3.14)**	(0.95)	(2.28)*		(4.09)**
Hispanic	-0.083	-0.018	-0.037		-0.088
	(5.54)**	(1.19)	(2.41)*		(5.02)**
Negative fin.	-0.043	-0.052	-0.036		-0.058
wealth	(2.63)**	(3.90)**	(2.55)*		(3.25)**
Net wealth	0.197	0.135	0.129		0.102
(std)	(4.47)**	(3.17)**	(3.02)**		(3.34)**
Net wealth	-0.044	-0.006	-0.004		-0.003
squared	(4.43)**	(0.34)	(0.48)		(2.71)**
Cognition	0.022	0.023	0.013		-0.000
	(2.95)**	(3.02)**	(1.73)		(0.04)
Econ.growth	0.072	0.027	0.028		
	(2.92)**	(1.29)	(1.44)		
Precision	0.023	0.002	0.028		-0.004
	(0.86)	(0.07)	(0.98)		(0.11)
Stock mkt. up				0.146	0.092
				(5.05)**	(3.19)**
Constant	-0.546	-2.889	3.429	0.094	23.555
	(0.02)	(0.12)	(0.14)	(7.21)**	(0.67)
Observations	3770	3340	3307	2632	2557
R-squared	0.06	0.07	0.08	0.01	0.07

Table 13. Probability of buying stocks if not a stockholder (*direct and indirect stock ownership throuh IRAs)*. Linear probability models *(1 if bought stocks, 0 otherwise)*. **HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)**

	1992-4	1994-6	1996-8	2000-2	**2000-2**
Less than	-0.066	-0.068	-0.053		-0.029
high-school	(3.42)**	(3.70)**	(2.98)**		(1.23)
College	0.082	0.043	0.065		0.086
	(2.52)*	(1.40)	(1.90)		(2.25)*
Female	-0.029	-0.003	-0.059		-0.034
	(1.93)	(0.22)	(3.65)**		(1.76)
Couple	0.061	0.038	0.024		0.047
	(3.19)**	(2.14)*	(1.26)		(2.14)*
Age	0.013	-0.006	-0.019		0.003
	(0.74)	(0.39)	(1.13)		(0.13)
Year of birth	0.013	0.003	-0.012		0.016
	(0.73)	(0.16)	(0.71)		(0.67)
Black	-0.042	-0.028	-0.041		-0.056
	(2.28)*	(1.48)	(2.47)*		(2.63)**
Hispanic	-0.028	-0.035	-0.041		-0.083
	(1.19)	(1.71)	(1.94)		(3.16)**
Negative fin.	-0.066	-0.086	-0.060		-0.076
wealth	(3.27)**	(5.01)**	(3.32)**		(3.39)**
Net wealth	0.166	0.057	0.166		0.203
(std)	(2.57)*	(1.64)	(2.23)*		(3.23)**
Net wealth	-0.046	-0.007	-0.027		-0.013
squared	(3.21)**	(0.76)	(1.14)		(2.14)*
Cognition	0.012	-0.001	0.016		0.010
	(1.34)	(0.14)	(1.63)		(0.71)
Econ.growth	0.056	0.042	0.027		
	(1.97)*	(1.62)	(1.09)		
Precision	0.034	0.030	0.046		0.069
	(1.07)	(0.76)	(1.13)		(1.41)
Stock mkt. up				0.120	0.048
				(3.09)**	(1.27)
Constant	-25.319	-4.641	24.481	0.118	-30.969
	(0.73)	(0.14)	(0.73)	(6.95)**	(0.65)
Observations	2561	2225	2172	1618	1568
R-squared	0.07	0.06	0.08	0.01	0.11

Table 14. Probability of not selling off all stocks if a stockholder (*direct stock ownership)*. Linear probability models *(-1 if sold out all stocks, 0 otherwise)*. HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)

	1992-4	1994-6	1996-8	2000-2	**2000-2**
Less than	-0.055	0.013	-0.074		0.001
high-school	(1.21)	(0.33)	(1.69)		(0.01)
College	0.008	0.047	0.024		0.045
	(0.36)	(2.30)*	(0.98)		(1.80)
Female	-0.008	-0.032	0.005		-0.020
	(0.47)	(2.12)*	(0.31)		(1.09)
Couple	-0.005	-0.005	-0.012		-0.025
	(0.19)	(0.19)	(0.41)		(0.87)
Age	-0.026	0.033	-0.038		-0.021
	(1.27)	(1.84)	(2.00)*		(0.89)
Year of birth	-0.029	0.028	-0.036		-0.023
	(1.41)	(1.51)	(1.88)		(0.93)
Black	-0.238	-0.192	-0.062		-0.216
	(3.44)**	(3.07)**	(0.91)		(3.00)**
Hispanic	-0.047	-0.055	-0.102		-0.166
	(0.49)	(0.59)	(1.05)		(1.56)
Negative fin.	-0.226	-0.123	-0.218		-0.137
wealth	(3.13)**	(1.94)	(3.27)**		(1.56)
Net wealth	0.050	0.048	0.069		0.042
(std)	(1.60)	(3.12)**	(3.32)**		(4.95)**
Net wealth	-0.009	-0.003	-0.006		-0.001
squared	(1.67)	(2.35)*	(2.63)**		(4.13)**
Cognition	0.018	0.064	0.003		0.012
	(1.08)	(4.13)**	(0.16)		(0.63)
Econ.growth	-0.011	-0.033	0.019		
	(0.26)	(0.93)	(0.53)		
Precision	0.082	-0.019	0.104		-0.066
	(1.89)	(0.45)	(2.25)*		(1.33)
Stock mkt. up				-0.025	-0.063
				(0.61)	(1.52)
Constant	57.872	-55.872	71.973	-0.218	45.069
	(1.40)	(1.52)	(1.88)	(8.92)**	(0.93)
Observations	1789	1949	1919	1734	1694
R-squared	0.04	0.05	0.04	0.00	0.04

Table 15. Probability of not selling off all stocks if a stockholder (*direct and indirect stock ownership throuh IRAs)*. Linear probability models *(-1 if sold out all stocks, 0 otherwise)*. **HRS age eligibles, participants of each survey between 1992 and 2002. (t values in parentheses)**

	1992-4	1994-6	1996-8	2000-2	**2000-2**
Less than	-0.050	-0.006	-0.059		-0.049
high-school	(2.01)*	(0.25)	(2.38)*		(1.65)
College	0.001	0.017	0.004		0.015
	(0.08)	(1.38)	(0.31)		(1.08)
Female	-0.006	-0.003	-0.010		0.005
	(0.71)	(0.28)	(1.09)		(0.46)
Couple	0.020	0.008	0.033		0.015
	(1.25)	(0.51)	(2.06)*		(0.89)
Age	0.007	0.009	-0.015		0.013
	(0.65)	(0.86)	(1.40)		(0.99)
Year of birth	0.003	0.004	-0.015		0.010
	(0.26)	(0.37)	(1.39)		(0.80)
Black	-0.151	-0.136	-0.131		-0.227
	(4.12)**	(3.39)**	(3.27)**		(4.65)**
Hispanic	-0.157	-0.060	-0.082		-0.205
	(2.69)**	(1.33)	(1.44)		(3.08)**
Negative fin.	-0.066	-0.168	-0.099		-0.090
wealth	(2.59)**	(5.16)**	(3.65)**		(2.58)**
Net wealth	0.029	0.043	0.044		0.028
(std)	(1.60)	(5.36)**	(4.43)**		(5.57)**
Net wealth	-0.006	-0.003	-0.004		-0.001
squared	(2.11)*	(3.28)**	(3.49)**		(4.51)**
Cognition	0.020	0.035	0.023		0.005
	(2.23)*	(3.65)**	(2.31)*		(0.46)
Econ.growth	0.014	-0.023	0.009		
	(0.57)	(1.06)	(0.43)		
Precision	0.011	0.034	0.018		0.030
	(0.49)	(1.38)	(0.77)		(1.04)
Stock mkt. up				0.086	0.063
				(3.91)**	(2.73)**
Constant	-5.706	-8.680	29.106	-0.150	-21.183
	(0.27)	(0.39)	(1.39)	(10.40)**	(0.81)
Observations	2993	3028	2997	2713	2649
R-squared	0.04	0.07	0.05	0.01	0.06

Figures

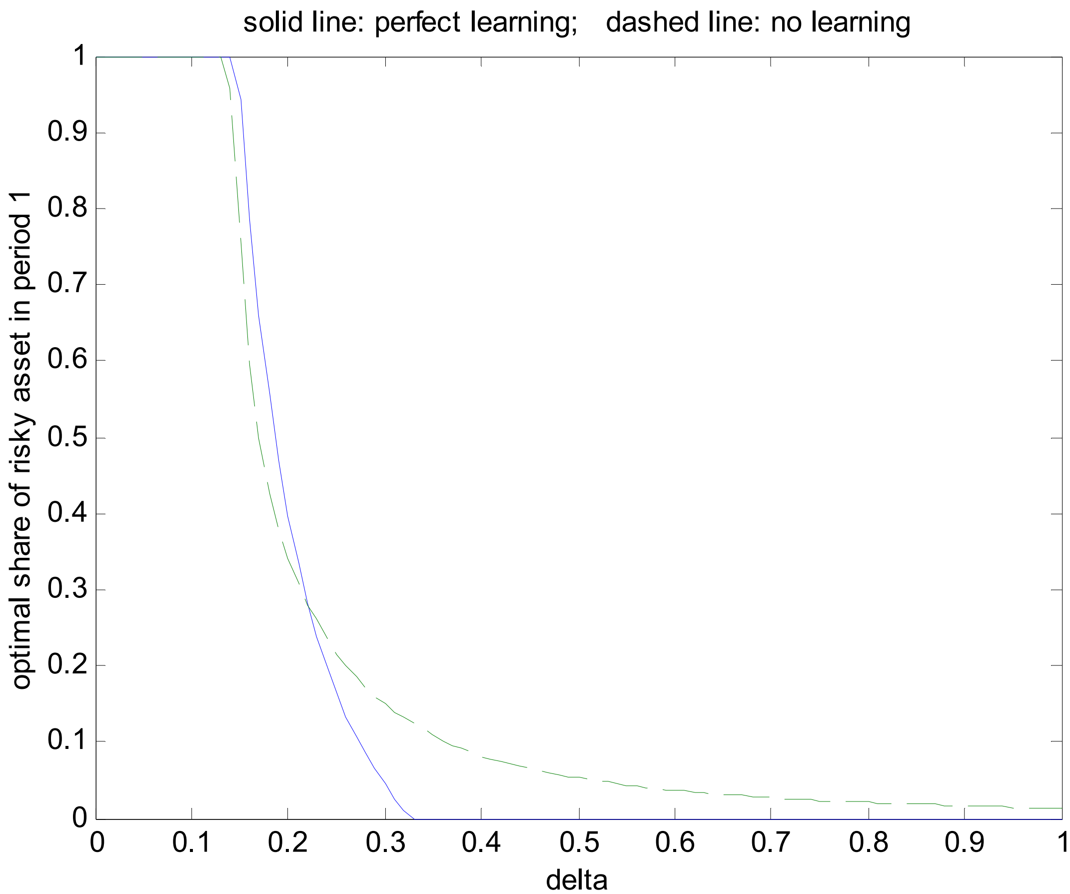


Figure 1.
Optimal share of risky assset in period 1 in the simple 2-period learning model with two possible returns. No learning and perfect learning by period 2

APPENDIX
THE HRS 2002 STOCK MARKET EXPECTATIONS QUESTIONS

HRS age eligibles, interviewed in each wave from 1992 to 2002

The stock market return questions

HRS 2002 contained two sets of questions about the respondents' expectations of future performance of the U.S. stock market. In the core survey, two questions were asked from everybody. One asked what the respondent thought the probability is that the market will go up at all, and another one about the probability that it will go up by at least 10 per cent. In a separate experimental module, more detailed questions of this type were asked from a random sample of respondents.
In this Appendix, we look at these questions. We focus on the sample analyzed in the main text: those HRS age eligible people who were interviewed in each wave from 1992 through 2002. The questions were administered on the whole 2002 sample. All results are qualitatively the same for the whole sample.

In the core survey, the two questions were asked in different orders following a random assignment of who gets them in what order. The questions themselves were phrased the following way.

The first sequence:
subsequent variable name: mktup1

We are interested in how well you think the economy will do in the next year.
By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?

subsequent variable name: mktup1_10

By next year at this time, what is the chance they will have grown by 10 percent or more?

The alternative sequence:
subsequent variable name: mktup2_10

We are interested in how well you think the economy will do in the next year.
By next year at this time, what is the percent chance that the value of mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will have grown by 10 percent or more?

subsequent variable name: mktup2

What is the chance they will be worth more a year from now than they are today?

The possible answers were integers on a 0-100 scale. In the first sequence, people who gave zero probability to the first question (mktup1) weren't asked the 10% question. In the second sequence, people who gave a 100% chance to the 10% question weren't asked about the other question (mktup2).

The experimental module contained the following questions

We are interested in how people think about investments in the stock market. I have some questions about how much someone might make or lose from an investment in the stock market. Imagine that you have a rich relative who unexpectedly leaves you $10,000. You are thinking of putting the money into a mutual fund invested in blue chip stocks like those in the Dow Jones Industrial Average.

We want to know what you think the chances are of how much you might gain or lose on that investment if you were to make it. Your answers can range from zero to one hundred, where zero means there is absolutely no chance, and one hundred means that it is absolutely certain. (For example, when weather forecasters report the chance of rain, a number like 20 percent means "not much chance", a number between 45 and 55 percent means "a pretty even chance", and a number like 80 percent means "a very good chance.")

Suppose you left the $10,000 in the mutual fund for one year, and didn't take out any dividends or interest, and then after one year you cashed in the mutual fund and took everything out. Assume that there are no commissions or fees for buying or selling this fund.

subsequent variable name: mktup3

What is the percent chance that you would have more than $10,000 when you cashed it in?

subsequent variable name: mktup3_10

What is the percent chance your mutual fund would have gone up by more than 10%; that is, you would have more than $11,000?

subsequent variable name: mktup3_20

What is the percent chance your mutual fund would have gone up by more than 20%; that is, you would have more than $12,000?

subsequent variable name: mktup3_30

What is the percent chance your mutual fund would have gone up by more than 30%; that is, you would have more than $13,000?

subsequent variable name: mktdown3

Now please think about the chances that you would have lost money; that is when you cashed in the mutual fund and took everything out you would have less than $10,000. What is the percent chance you would have less than $10,000?

subsequent variable name: mktdown3_10

What is the percent chance you would have lost more than 10%; that is, you would have less than $9,000?

subsequent variable name: mktdown3_20

What is the percent chance you would have lost more than 20%; that is, you would have less than $8,000?

subsequent variable name: mktdown3_30

What is the percent chance you would have lost more than 30%; that is, you would have less than $7,000?

subsequent variable name: invest

Instead of putting your money in the mutual fund, you could put your money in a guaranteed investment that will be worth $10,500 one year from now.

Would you put your money in the mutual fund or put it in the guaranteed investment?

subsequent variable name: follows

How closely do you follow the stock market: very closely, somewhat, or not at all?

The possible answers to the mktup3 and mktdown3 questions were again integers on a 0-100 scale. When a question was followed by an even higher or lower return, people with an answer of at most 5 % were not asked the next one. For example, if someone said there was 2% chance for the market to yield a 20% increase, the 30% increase question was not asked from the respondent.

The relevant parts of the questionnaire with the original variable names, the possible answers, and the skip logic are attached to this document.

The purpose of the questions

The purpose of the stock market questions was to elicit as much about people's subjective distribution as possible. Economic theory suggests that people's subjective beliefs about the whole distribution of the uncertain market returns matter when they make the decision whether and how much to invest. Even in the simplest investment models it is not only the expected return that matters but also its standard deviation. By looking at the series of answers people gave to the experimental module, one could in principle reconstruct a fairly detailed probability distribution of the uncertain returns. The two questions of the core survey are not very informative about the spread of people's beliefs. This is because a higher 10% growth answer with the same positive growth answer may correspond to a higher mean as well as a higher variance.

Descriptive analysis – Core survey questions

Summary statistics of the probability answers (DK and RF are coded as missing)

Variable	Obs	Mean	Std. Dev.	Min	Max
mktup1	1986	46.03978	27.50875	0	100

```
   mktup1_10 |      1781    42.51937   27.10511          0        100
      mktup2 |      2534    45.77822   29.68831          0        100
   mktup2_10 |      2665    37.27129   27.47276          0        100
```

Just by looking at the means and standard deviations, the sequence of the core questions seems to matter only in the more than 10% answers. There is a little difference in the average and standard deviation of the market will go up answers (mktup1 and mktup2), and there is a larger difference in the mean of the 10% or more increase answers (mktup1_10 and mktup2_10). Recall however that the two sequences differed in their skip logic. In the first case, the 10% question wasn't asked if the up at all answer was 0, while in the second case, the up at all question wasn't asked if the 10% answer was 100. The difference between the corresponding means in the two sequences are smaller if we condition on the common part of the skip logic [Pr(up)>0, Pr(up 10%)<1], but the standard deviations of the mktup1 and mktup2 answers become even larger.

```
. sum mktup1 mktup1_10 if mktup1>0 & mktup1_10<100

    Variable |     Obs        Mean   Std. Dev.       Min        Max
-------------+-----------------------------------------------------
      mktup1 |      1697    48.84502   24.23865          1        100
   mktup1_10 |      1697    39.67413   24.48125          0         99

. sum mktup2 mktup2_10 if mktup2>0 & mktup2_10<100

    Variable |     Obs        Mean   Std. Dev.       Min        Max
-------------+-----------------------------------------------------
      mktup2 |      2360    49.15339   27.93628          1        100
   mktup2_10 |      2401    37.27572    24.5821          0         99
```

We can conclude that the order of the questions matters somewhat for the distribution of the answers. Asking the up at all question first yields a somewhat smaller mean and standard deviation for the up at all question itself and a somewhat larger mean for the up by 10% question. The differences are, however, not very large.

Nonmonotonic answers

There are many non-monotonic answers. By the laws of probability, the probability of an event that is a subset of another event should no greater than the probability of the larger set. So the probability that the market will go up by at least 10% cannot be greater than the probability that the market will go up at all.

There are even more people who gave the same answer to the up and up by at least 10% questions. Although this does not violate the laws of probability, it is very hard to reconcile with well-behaved subjective beliefs unless both answers are 0% or 100%. These extreme cases are consistent with a 100% chance of a loss or a 100% chance of a more than 10% gain, respectively. In all other cases, however, the respondent would give a positive chance to a loss and a more than 10% gain but a zero chance to returns in between, which is strange to say the least.

The distribution of the core survey answers with respect to these problems is the following.

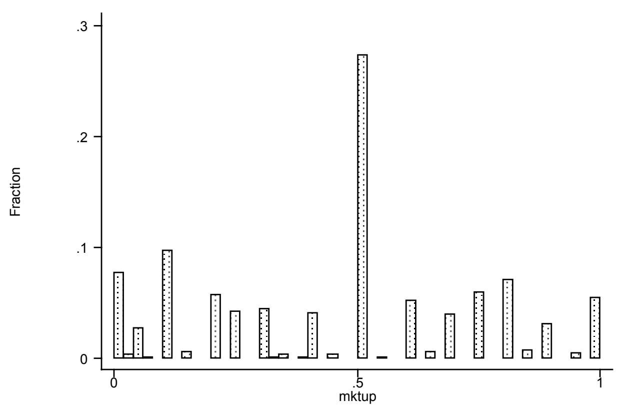
	1st sequence		2nd sequence	
	# cases	% distribution	# cases	% distribution
Pr(up) > Pr(up by 10%)	768	43	1109	44
Pr(up) = Pr(up by 10%) = 0	-	-	130	5
Pr(up) = Pr(up by 10%) = 1	35	2	-	-
Pr(up) = Pr(up by 10%) $\in$ (0,1)	651	37	947	37
Pr(up) < Pr(up by 10%)	327	18	348	14
Sum	4,909	100	6,431	100

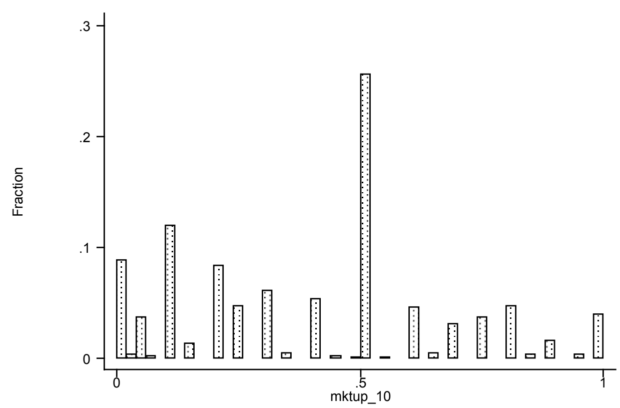
The fraction of strictly monotonic answers is only about 43-44 percent in both sequences. Two-thirds of the rest is equal answer to the two questions (37 percent), and the remaining are mostly nonmonotonic.

Heaping

The two sequences produced very similar answers for the purposes of most of the investigation. Therefore we put the two together and from now on, we analyze one mktup and one mktup10 variable by putting the two sequences together.

There is a lot of heaping, especially at 50%. See for example the distribution of the core survey mktup1 and mktup1_10 answers:





These distributions raise the possibility that some of the observed equal answers to the mktup and mktup10 questions may arise from heaping at 50%. This is indeed the case: of the 1598 equal but not 0 or 1 answers in the selected sample, 41 percent gave a “fifty-fifty” answer to both question. This is a large fraction but it also suggests that heaping does not determine equal answers completely.

Descriptive analysis – Module questions

Here, again, we focus only on the selected sample of HRS age eligibles who participated in each survey from 1992 to 2002.

```
Variable |     Obs        Mean   Std. Dev.       Min        Max
```

```
-------------+--------------------------------------------------------
      mktup3 |       278    46.15108    31.52087          0        100
   mktup3_10 |       268    31.97388    27.38249          0        100
   mktup3_20 |       254    19.25591    22.12312          0        100
   mktup3_30 |       228    13.00877    20.47024          0        100
    mktdown3 |       278    39.01439    30.93956          0        100
 mktdown3_10 |       261    31.03448    29.43798          0        100
 mktdown3_20 |       234    24.61111    27.76833          0        100
 mktdown3_30 |       211    21.90521    28.26508          0        100
```

278 respondents gave a valid answer to the "up at all" question in the module. In 45 cases (16%), the chance of a gain (mktup3) and the chance of a loss (mktdown3) add up to more than 100%. Here, two, there are many people who give a nonmonotonic or equal answer to some consecutive probability questions (e.g. the chance of a 20% gain is less than or equal to the chance of a 30% gain). Of the 278 answers, only 52 are "OK" and where probabilities don't add up to more than 100%. Another 51 people give an equal (and not 0 or 100%) answer somewhere in the sequence, which does not violate the laws of probability but are not strictly monotonic.

```
Nonmonotonic |
   and equal |
   answer to |
  the Module |   Gain + loss >100%
   questions |         0          1 |     Total
-------------+----------------------+----------
          OK |        52          6 |        58
 equal (0,1) |        51         12 |        63
nonmonotonic |       130         27 |       157
-------------+----------------------+----------
       Total |       233         45 |       278
```

Comparing the core and module questions

Since the core questions (mktup and mktup_10) were asked in the module once again, we can gain information about people's consistency by comparing them. The two measures are very weakly correlated, and the correlation does not depend on whether the answers to the core questions were "OK" (mktup>mktup_10). See the following table.

Correlation of module answer to core answer

Core survey monotonicity	Market will go up	Market will go up by 10%
All answers	.41	.29
Monotonic answers	.45	.27
Strictly monotonic answers	.49	.30

The mean and spread of subjective beliefs

According to economic theory, not only the expected value but the spread of the beliefs also should matter for investment decisions. People with beliefs with the same expected value but larger standard deviation should invest less into the risky asset (stocks) if they are risk-averse.

Unfortunately, the core survey questions do not allow for testing such a hypothesis: people with the same subjective probability of markets going up but a larger probability of a 10% increase have not only a more disperse distribution but also a higher mean. The two have opposing effects on the investment decision. We have estimated all 2002 regressions in the main text with mktup_10 entered along with mktup. Not surprisingly, mktup_10 doesn't have much (in most cases any) power after mktup is taken into account.

On the other hand, the experimental module asked a more complete set of questions to elicit the probability distribution. I created moments of the distribution the following way. To keep things as simple as possible, I assumed a discrete distribution: I assigned the probability of an interval to its midpoint. For the two half-open intervals (loss ≥ 30% or gain ≥ 30%) I assumed a boundary: 100% loss and 100% gain, respectively. All results are qualitatively the same for other endpoints. Formally, with midpoint k having a probability P_k, Mean = $\Sigma_k \, kP_k$, and Variance = $\Sigma_k \, k^2P_k$ – Mean^2. Summary statistics of the moments:

```
    Variable |     Obs        Mean   Std. Dev.       Min        Max
-------------+-----------------------------------------------------
all in the module with nonmissing answers
        mean |     186   -.0112634   .1535722        -.4         .4
    variance |     186    .0472362   .0444181          0     .27325
-------------+-----------------------------------------------------
if weakly monotonic and probabilities don't add up to more than 1
        mean |     151    .0034437    .152931        -.4         .4
    variance |     151    .0405376   .0374228          0      .1675
-------------+-----------------------------------------------------
if strictly monotonic and probabilities don't add up to more than 1
        mean |     100      .01527   .1646315        -.4         .4
    variance |     100    .0405148   .0422938          0      .1675
```

The mean and standard deviation calculated from the subjective distributions has no significant relationship with actual stock ownership (direct and direct plus IRA). On the other hand, they predict investment intentions in the module (the variable "invest", see above). The following tables show regression results for the investment intentions questions, which 1 if the respondent would invest in stocks and 0 otherwise (linear probability models). Standard errors are clustered at the household level.

all in the module with nonmissing answers

```
Regression with robust standard errors                 Number of obs =     180
                                                       F(  2,   179) =    7.36
                                                       Prob > F      =  0.0008
                                                       R-squared     =  0.0597
Number of clusters (HHID) = 180                        Root MSE      =  .42653
------------------------------------------------------------------------------
             |               Robust
      invest |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mean |    .685013   .1794421     3.82   0.000     .3309189    1.039107
         std |   -.099075   .2994678    -0.33   0.741    -.6900165    .4918664
       _cons |    .282028   .0656585     4.30   0.000     .1524637    .4115923
------------------------------------------------------------------------------
```

if weakly monotonic and probabilities don't add up to more than 1

```
Regression with robust standard errors                 Number of obs =     145
                                                       F(  2,   144) =    6.49
```

```
                                                       Prob > F      =  0.0020
                                                       R-squared     =  0.0820
Number of clusters (HHID) = 145                        Root MSE      =  .41431
------------------------------------------------------------------------------
             |               Robust
      invest |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mean |   .7322687   .2055612     3.56   0.000     .3259616    1.138576
         std |  -.4370968   .3268579    -1.34   0.183    -1.083156    .2089624
       _cons |   .3144286   .0702976     4.47   0.000     .1754802     .453377
------------------------------------------------------------------------------
```

if strictly monotonic and probabilities don't add up to more than 1

```
Regression with robust standard errors                 Number of obs =      98
                                                       F(  2,    97) =    3.44
                                                       Prob > F      =  0.0362
                                                       R-squared     =  0.0606
Number of clusters (HHID) = 98                         Root MSE      =   .4398
------------------------------------------------------------------------------
             |               Robust
      invest |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        mean |   .5783548   .2275821     2.54   0.013     .1266674    1.030042
         std |  -.4304051   .3399742    -1.27   0.209     -1.10516    .2443497
       _cons |   .3379567   .0759978     4.45   0.000     .1871221    .4887913
------------------------------------------------------------------------------
```

Errors in the stock return probability questions and stock ownership

We tried to see if nonmonotonic answers to the core survey stock returns questions could be used as proxies for how well people can think in terms of probabilities. One might think that people who deal worse with probabilities are less likely to invest into risky assets such as stocks. In the next four regressions, stock ownership is regressed on the two dummies that correspond to equal (mktup=mktup_10) and nonmonotonic (mktup<mktup_10). In the first set only the dummies are entered, while the second set controls for expectations (mktup), too. If we don't control for anything else, errors are negatively associated with stock ownership. When we control for expectations, however, this association becomes a lot weaker and it is seldom statistically significant. They are never significant if we enter standard demographics (those results are not shown). We don't present the results here. They are available upon request.