Network Asymmetries and Access Pricing in Cellular Telecommunications*

Viktória Kocsis†

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Abstract

Network shares and retail prices are not symmetrical in the telecommunications market with multiple bottlenecks which give rise to new questions of access fee regulation. We consider a model with asymmetry in the supply side coming from different timing of entry, that is different costs among firms and brand loyalty. As a result firms have divergent preferences over the access fee. In case of linear and non-linear prices the access fee might still act as the instrument of collusion, but only if a side-payment is permitted which is generally welfare decreasing. Moreover, in contrast with the European regulatory framework, the access fee on the basis of termination cost might not necessarily be a socially preferable solution.

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1 Introduction

In the last decade the deregulation of telecommunications - formerly seen as a natural monopoly - plays an important role within policy and in economic literature. In this paper we focus on mobile telephony, particularly in the EU, which operated as a monopoly until the early 1990s and since then has functioned as an oligopoly[1]. In the beginning of the millennium a new framework for communications networks was introduced which was designed to harmonize European regulation in order to reduce entry barriers and facilitate effective competition to the benefit of consumers. Under the new regulatory regime each country was required to establish a national regulatory authority to monitor the competition in communications markets and to define the relevant market segments. Furthermore they were to decide whether an operator had significant market power (SMP) on a particular segment and to assess the appropriate regulatory obligations. On the basis of an EU communications directive on access and interconnection, the wholesale mobile voice call termination has been

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†Corvinus University of Budapest and Tinbergen Institute. Address: Tinbergen Institute, Roetersstraat 31, 1018 WB Amsterdam, The Netherlands. E-mail: kocsis@tinbergen.nl

[1] About the evolution of mobile telecommunications in Europe see (Gruber, 2001) and (Gruber and Verboven, 2001).

defined as a relevant market since each operator is a monopolistic provider of its bottleneck and charges an access fee for its usage[2]. In the last years the access fee of call termination was freely negotiated among the firms; however under the new regulatory framework the termination charge set by the SMP operators is controlled on the basis of long run incremental cost (LRIC). The LRIC is a forward-looking methodology which measures the additional cost an operator incurs to provide termination in its network, or in other words, the cost that the operator would avoid if it decided not to provide access.

On the telecommunications market several asymmetries can be observed. This requires an individual consideration of each firm with regard to regulation. In this paper we focus on supply side asymmetries. Liberalizing the telephony market provided opportunity for new entry and as a consequence caused asymmetries among firms entering the market early ("incumbent") and late ("entrant"). The reasons for this may be as follows: First, the entrant had the opportunity to introduce a more recent technology which implies a lower cost of servicing. On the other hand the incumbent who entered the market earlier may gain an extra advantage because of reliability or reputation for quality, realized in the form of extra utility for those consumers who wanted to subscribe for the incumbent's services (i.e. brand loyalty). This extra utility can be reckoned as a switching cost because if a consumer subscribed to the incumbent wants to switch network, he has to face additional costs like the administrative cost of switching or the reimbursement of discounts resulting from a longer contract or buying a new cellular device. In the presence of this type of cost a consumer is willing to change between networks only if the additional cost of switching is smaller than the extra surplus gained from the other firm's lower price[3].

Thus all these asymmetries, together with rapidly developing technologies call for a reconsideration of the role of the regulatory mechanism. The following questions arise: is the cost-based access fee tenable, or do other ways of asymmetric or symmetric access price regulation seem reasonable? What is the effect of access price regulation on firms' profit and on consumer surplus?

This paper is an extension of the common symmetric models of network interconnection and competition. The first articles published in this topic are (Armstrong, 1998) and (Laffont et al., 1998*a*), in which the authors analyze the problem of two-way access pricing in the setup of a symmetric cost structure, uniform pricing and reciprocal access fee. They claim that for small access charges and poor network substitution, there is a unique, symmetric and stable equilibrium, while for strong substitution, there exists no equilibrium. The main and principally cited results of their articles are that (i) under linear retail pricing the price is increasing in the access fee, and therefore the access fee might be an instrument of collusion, and (ii) under two-part tariffs the profit is independent of the access fee. If termination-based price discrimination is

[2]The most pertinent and normative European (tele)communications authority is Ofcom (formerly Oftel) in the UK. For more details in accordance with the EU harmonization in cellular market see (Ofcom, 2004).

[3]Even though, because of the learning process of consumers about the quality of the operators, the decrease of switching costs, and as a consequence of competition, the additional incentives to innovate and to reduce costs, the brand loyalty and the cost advantage might disappear over time and can result a more symmetric competition.

considered in the model, as (Laffont et al., 1998*b*) shows, then under linear prices the access fee can be the instrument of collusion only in case of weak substitution; however in case of two-part tariffs firms will agree on cost-based access fee thus causing the cease of price discrimination.

The profit-neutrality property is confirmed in a more general setup by (Dessein, 2004) and (Hahn, 2004), who assume consumer heterogeneity in terms of demand for calls and calling pattern.

In (Carter and Wright, 1999), (Carter and Wright, 2003) and (Peitz, 2005) the authors consider an asymmetric market in the presence of brand loyalty and leave the other segments (for instance costs) symmetric. In case of reciprocal access fees, Carter and Wright state that if the market shares are symmetrical, the firms are indifferent over the access fee. Otherwise the firm with brand loyalty prefers a cost-based access fee and if brand loyalty is sufficiently strong, the other firm has the same preference. Moreover the cost-based access fee is in most cases socially optimal. Emphasizing the use of a non-reciprocal access fee, Peitz claims that asymmetric access price regulation (i.e. allowing positive access mark-up for the entrant) can stimulate entry and increase consumer surplus at the same time.

In (de Bijl and Peitz, 2002) and (de Bijl and Peitz, 2004) the authors focus on the same type of asymmetry although they analyze the market in a dynamic setup and use simulation to derive policy implications. They obtain that independently of the type of access price regulation, the asymmetry among firms becomes less; the entrant's profit and the consumer surplus increase over time and finally a symmetric equilibrium emerges. In (de Bijl and Peitz, 2004) their model is extended with cost asymmetry and from the simulations similar results are derived to those the authors found before.

The other relevant paper related to cost asymmetry is (Armstrong, 2002) in which the author considers a market where each consumer makes one unit of calls to every other consumer. As a policy implication he derives that the firms have divergent preferences over the access fee. For a given assumption (that is strong brand loyalty) the total industry profit changes in the same direction as the bigger firm's larger profit, so that a side payment to the smaller firm might compensate for an access fee which is against the smaller firm's interest. The larger the retail price, the smaller the consumer surplus; however, because of the inelastic demand, the increment in the industry profit is equal to the decrease in the consumer surplus, thus resulting in welfare-neutrality of the equilibrium.

In the present paper we generalize the previous models: in the presence of brand loyalty, cost asymmetry and continuum telephony consumption we seek to find the equilibrium and derive policy implications. First we present Armstrong's (2002) model as a benchmark case. In the second part of the paper we extend his model to linear demand for telephony consumption and to uniform linear retail prices. We formalize a market with two firms competing for consumers with horizontally differentiated products. The firms are interconnected: they set two-way access fee for call termination and afterwards they maximize their profit charging retail prices simultaneously and non-cooperatively. Since we obtain quadric and asymmetric first derivatives, the main results arise from simulations. The equilibrium exists if the difference between access fees is sufficiently small and the networks are weak substitutes. Studying the effect of different access fees under weak brand loyalty, we obtain similar results to Armstrong; however the welfare-neutrality property of the equilibrium does not hold any more. For stronger brand loyalty the firms prefer the same access fee difference. This might result in a negative access mark-up for the entrant.

Finally we extend the model to the case of competition in two-part tariffs, where each firm sets a uniform per-minute charge and a fixed fee at the same time. We find that the profit-neutrality result of the symmetric models is not valid. The firms' preferences over the access fee are different, and therefore they might negotiate a termination charge which is more preferable for one of the firms, since this also increases industry profit. In case of small cost difference a positive access mark-up for the entrant favors both the entrant itself and the consumers. However for larger cost difference the rise in access fee lowers the consumer surplus.

The structure of the paper is as follows. In Section 2 we define all the terms and conditions which are used in the paper and then as a benchmark case we present Armstrong's model. In Section 3 we extend his model to competition with uniform linear tariffs under linear demand for calls, in Section 4 we derive the equilibrium for non-linear tariffs. In Section 5 we conclude. The proofs are shown in Appendix A and simulation results in Appendix B.

2 Benchmark case: unit consumption

First we present the stylized model of Armstrong[4] in which he assumes a market with unit telephony consumption and seeks to find arguments in the interest of implementing non-reciprocal access fee. This section defines the most important terms and assumptions used in this paper, and other versions presented will be expressed in terms of deviation from this setup.

2.1 Cost structure and access fee

Consider a market with two networks indexed by $i = 1, 2$ and denote 1 the incumbent and 2 the entrant[5]. Firms compete for consumers by means of a horizontally differentiated service on a segment $[0, 1]$, and we assume maximal product differentiation, that is the firms are located at the two ends of the segment ($x_1 = 0,\ x_2 = 1$). Each firm incurs three types of costs: (i) Connection independent cost. This can be for instance, the fee for a license, or the cost of building-up and improving facilities. These costs will be considered in the model as sunk cost. (ii) Connection dependent but traffic independent cost. This is the fixed cost f_i of serving a consumer, and without loss of generality we assume that the fixed cost is the same for both firms, i.e. $f_1 = f_2 = f$. (iii) Traffic dependent cost. This is a unit cost c_{iO} of originating and c_{iT} of terminating a call. For simplicity we assume that $c_{iO} = c_{iT} = c_i^0$, and that the

[4] See (Armstrong, 2002).

[5] We name firms as incumbent and entrant to make a distinction according to their reputation or brand loyalty, though we analyze a simultaneous move game.

two firms have different costs, in the sense that one firm has lower termination cost. Assume that firm 2, the entrant, is more efficient, i.e.

$$0 \leq c_2^0 < c_1^0.$$

Denote $C_i \equiv 2c_i^0 + f_i$ and $\Delta^c \equiv c_2^0 - c_1^0$.

The networks are interconnected which means that if a consumer subscribed to a firm originates a call, this call might be terminated in the rival firm's network. In this case the firm has to purchase access to ensure that its subscribers are able to call all consumers independently of service provider, and as a price after each unit call terminated in the rival's network the firm has to pay an access fee. In the model network i charges a per-unit access charge τ_i for terminating its rival's off-net call and pays τ_j per-unit fee to network j for terminating its off-net call in network j. We assume that the access fee is not a decision variable of the firm and can be determined by negotiation among the firms or by the government. Hereafter denote $\Delta^\tau \equiv \tau_2 - \tau_1$.

On the grounds of the above definitions the unit cost of a call depending on its termination can be determined. If a call is originated and terminated in the same network (i.e. on-net call), say in network i, the total unit cost is equal to $c_i \equiv 2c_i^0$. However if a call is originated in network i but terminated in network j (i.e. off-net call), network i incurs a unit cost of $c_i^0 + \tau_j$, and if a call of network j is terminated in network i, network i's unit cost is equal to $c_i^0 - \tau_i$.

2.2 Demand structure and consumer surplus

A consumer decides about connecting to a network, and then consumes one unit of the service independently of the price, or in other words, the demand for consumption is inelastic. The consumers are homogeneous in the sense that each consumer gains the same fixed surplus of consumption (v_0). On the other hand, the consumers are heterogeneous since they have different *a priori* preferences: a consumer values a service more when it is 'closer' to his preference. Let x denote the characteristic (location) of the consumer on the segment $[0, 1]$ and assume that the consumers are uniformly distributed on this segment. As the preference of a consumer is different from the characteristics of service supplied by the networks (i.e. different location), the consumer has to pay 'transportation cost' ($t > 0$) which cost measures the disutility of not being the consumer of the ideal firm. We assume that t is the same for all consumers and the total transportation cost is a linear function of the distance.

Therefore the total utility of a consumer is

$$v_0 - t\,|x - x_i| + t\beta_i - p_i,$$

where

1. v_0 is assumed to be high enough to provide full consumer participation in the market, that is
$$v_0 \geq \frac{p_1 + p_2 - t\beta}{2} + t$$

2. the second term indicates the total transportation cost since the consumer travels a distance of $|x - x_i|$, where x_i denotes the location of the firm which can be 0 or 1

3. β_i is the extra utility a consumer gets when subscribed to network i (i.e. brand loyalty or switching cost). Assume that $\beta \equiv \beta_1 - \beta_2 > 0$, that is the incumbent has stronger reputation on the market

4. p_i denotes the uniform retail price set by firm i. A price is uniform if a company does not differentiate prices depending on which network a call is terminated in.

Since the consumers are uniformly distributed on $[0, 1]$, in order to find the optimal proportion of consumers in each network, we aim at finding the consumer who is indifferent between being a subscriber of network 1 or that of network 2. A consumer located in α is indifferent to the networks if

$$v_0 - t\alpha + t\beta - p_1 = v_0 - t(1 - \alpha) - p_2.$$

Then α, the market share of firm 1, is equal to

$$\alpha(p_1, p_2) = \frac{1 + \beta}{2} + \sigma(p_2 - p_1),$$

where $\sigma = \frac{1}{2t}$ is the measure of substitution. The inverse ratio between σ and t means that the larger the transportation cost, i.e. the more 'painful' it is to be further from the ideal network, the weaker the substitution between the networks. Denote firm i's market share

$$\alpha_i \equiv \frac{1 \pm \beta}{2} + \sigma(p_j - p_i),$$

where the upper sign of β belongs to firm 1 and the lower to firm 2, and we know that $\alpha_1 + \alpha_2 = 1$. When the prices are equal, if $0 \leq \beta \leq 1$ the market shares are non-negative; if $\beta = 0$, then the market is symmetric; if $\beta > 0$, then the incumbent has larger market share, and in the extreme case of $\beta = 1$, the entrant has no chance to extract any subscriber from the incumbent.

In the model we use the assumption of *balanced calling pattern* which means that every consumer initiates the same number of calls he receives. This implies that in case of homogenous consumers, the fraction of calls originating in a network which terminate in the other network is proportional to the latter network's market share.

2.3 Price competition

Before defining the firms' optimization problem, we illustrate the firms' benefits and costs in the next figure.

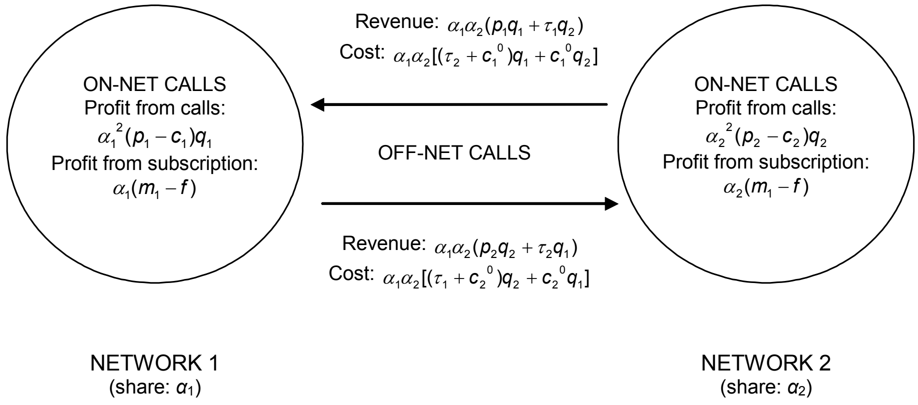

Figure 1. Costs and revenues in a market of interconnection and competition.

The figure indicates all the parameters and variables which are used in the most general model of this paper. In that situation firm i sets a two-part tariff such that its consumer has to pay a per-minute charge p_i and a fixed fee m_i. As a result of balanced calling pattern and consumer heterogeneity, firm i originates the α_i^2 share of on-net calls and the $\alpha_i\alpha_j$ share of off-net calls, and terminates the $\alpha_i\alpha_j$ share of off-net calls as well. In this section we present a specific model in which each consumer buys one unit of a call (that is $q_1 = q_2 = 1$) and pays only price p_i for it (that is $m_i = 0$).

We model the market as a noncooperative game in strategic form and seek to find a unique Nash-equilibrium[6]. There are two firms in the market ($i = 1, 2$) with the previously described characteristics competing for consumers by choosing a price profile $s = (p_1, p_2)$. As it can be tracked from Figure 1, firm i has the following profit function:

$$\pi_i(s) = \underbrace{\alpha_i\left(p_i - C_i\right)}_{\text{retail profit}} + \underbrace{\alpha_i\alpha_j(\tau_i - \tau_j)}_{\text{access profit}}, \tag{1}$$

where the first part measures the retail profit from telephony consumption and the second part comprises benefits and costs derived from providing access to its rival and using its rival's facilities. Firm i maximizes its profit function (1) by choosing a price p_i given its rival's price p_j and subject to

$$\begin{aligned} 0 &\leq p_i, \\ 0 &\leq \alpha_i \leq 1, \qquad i = 1, 2. \\ 0 &\leq \pi_i, \end{aligned}$$

Unlike the symmetric models of telecommunications, the asymmetry in the profit function suggests that the equilibrium will not be symmetric. From the first order conditions of the profit maximization one obtains the following Nash-equilibrium[7].

[6] We use the definition of Nash-equilibrium stated in (Vega-Redondo, 2003), p. 39.

[7] The proof is given in Appendix A.

Proposition 1 *If*

$$|\Delta^\tau| < \frac{1}{2\sigma},$$

there exists an equilibrium which is unique, asymmetric and can be shared or cornered.

If

$$|\beta + 4\sigma\Delta^c| < 3,$$

then the equilibrium is shared. Firm i sets price p_i^*, gets market share α_i^* and profit π_i^* which are the following:

$$p_i^* = \frac{1}{2\sigma} \pm \frac{\beta}{6\sigma} + \frac{2C_i + C_j}{3} - \frac{\Delta^\tau}{3}(\beta + 4\sigma\Delta^c), \tag{2}$$

$$\alpha_i^* = \frac{1}{2} \pm \frac{1}{6}(\beta + 4\sigma\Delta^c), \tag{3}$$

$$\pi_i^* = (\alpha_i^*)^2 \left[\frac{1}{\sigma} + \tau_i - \tau_j\right]. \tag{4}$$

For large cost difference and strong substitution (that is $\beta + 4\sigma\Delta^c < -3$) the entrant and for strong brand loyalty (that is $\beta + 4\sigma\Delta^c > 3$) the incumbent corners the market.

The proposition claims that the equilibrium exists and is unique if $|\Delta^\tau| < 1/2\sigma$; under this condition the second order conditions hold and therefore the profit functions are strictly quasi-concave. Moreover we have that $0 < r_i'(.) < 1$ and $r_i'(.)$ is constant which provides the single crossing property of the reaction curves $r_i(p_j)$. In accordance with this condition, the equilibrium exists if the substitution between the networks is weak and the difference between the access fees is small enough. The underlying intuitions are similar to the symmetric model of (Armstrong, 2002) and (Laffont et al., 1998*a*).

The incumbent always sets higher price because it has higher cost and faces stronger brand loyalty. However, since each firm has an advantage over the other, if its advantage is strong enough, it can choose a price so as to drive out the other firm from the market. If brand loyalty is very strong ($\beta >> 3$) or the cost difference is large and the substitution is strong ($4\sigma\,|\Delta^c| >> 3$), a monopoly outcome emerges: in the first case the incumbent corners the market, in the latter the entrant. These conditions are realized by extremely high parameter values which can only result in an infant market. In general, when brand loyalty is less strong ($\beta < 3$) or the cost difference is lower and the networks are weaker substitutes ($4\sigma\,|\Delta^c| < 3$), a shared-market equilibrium emerges. Henceforth we will eliminate the extreme parameter values and focus on the shared-market outcomes.

Before generalizing the results, two special cases will be considered. If the cost difference is equal to zero (that is firms are symmetric), in spite of its higher price the incumbent gets larger market share, since from (2) and (3)

$$\Delta^p \equiv p_2 - p_1 = -\frac{\beta}{3\sigma} < 0,$$

$$\Delta^\alpha \equiv \alpha_2 - \alpha_1 = -\frac{\beta}{3} < 0.$$

If the access fees are on the basis of cost, the incumbent has higher profit. However if the entrant is allowed to set an access fee larger than the termination cost to compensate for its disadvantage, it might occur that the difference in access profits exceeds the difference in retail profits; therefore the entrant gets higher profit.

In the special case of zero brand loyalty, the entrant sets a lower price and attracts more subscribers:

$$\Delta^p = \frac{2\Delta^c}{3} < 0,$$

$$\Delta^\alpha = -\frac{4\sigma\Delta^c}{3} > 0.$$

Following the same train of thought as before, in case of cost-based access fee the entrant gets higher profit; although if the incumbent may set an access fee higher than its cost, it can realize higher profit.

In general, when $\Delta^c > 0$ and $\beta > 0$, the entrant's price is lower since

$$\Delta^p = -\frac{\beta}{3\sigma} + \frac{2\Delta^c}{3} < 0,$$

however the market shares and the profits vary according to the intensity of asymmetry; from (3):

$$\Delta^\alpha = -\frac{\beta + 4\sigma\Delta^c}{3},$$

thus

$$\Delta^\alpha < 0 \Leftrightarrow \beta + 4\sigma\Delta^c > 0.$$

The condition $\beta + 4\sigma\Delta^c > 0$ holds if brand loyalty is strong and for a given substitution Δ^c is small or for a given Δ^c the services are weak substitutes. As a result, the incumbent obtains more subscribers. On the other hand, if β is small and for a given substitution the cost difference is large or the networks are strong substitutes, then $\beta + 4\sigma\Delta^c < 0$, causing a larger market share for the entrant. As for the firms' profit, the difference depends on the market shares and the access mark-up: the stronger the brand loyalty or the smaller the cost difference and the access mark-up, the more likely that the incumbent gets higher profit. Otherwise the entrant is better off.

2.4 How access fee affects the equilibrium?

To derive a relation between the access fee, firms' preferences and social preference, we analyze the effect of Δ^τ on the equilibrium prices and profits. As formula (4) shows, in an asymmetric market a firm with higher market share which charges a higher access fee gets higher profit. Since the market shares are independent of the access fee (see (3)), the larger Δ^τ, the better for firm 2 and the worse for firm 1 which implies that the firms have divergent preferences over the access fee. As for the whole industry, a change in the industry profit ($\pi \equiv \pi_1 + \pi_2$) is equal to

$$\frac{\partial \pi}{\partial \Delta^\tau} = \frac{\partial \pi}{\partial \tau_2} = \Delta^\alpha,$$

therefore if $\Delta^\alpha < 0$, a smaller Δ^τ, or alternatively, a larger Δ^τ is more favorable. If brand loyalty is strong, the incumbent has a higher market share and as a result industry profit is decreasing in Δ^τ. In a more symmetric market where brand loyalty is weaker and the cost difference dominates, the entrant has a higher market share. In this case the total industry profit varies in the same direction as the entrant's profit: the larger Δ^τ, the larger the industry profit. Therefore in the first case the incumbent might compensate the entrant with a side-payment in favor of setting a lower difference in access fees, otherwise the entrant has the opportunity to choose a higher access fee and compensate the incumbent.

The change in consumer surplus also depends on brand loyalty and cost difference, but in the opposite way to before. Since

$$\frac{\partial p_i}{\partial \Delta^\tau} = -\left(\beta + 4\sigma\Delta^c\right),$$

if $\beta + 4\sigma\Delta^c > 0$, then equilibrium prices are decreasing in Δ^τ which favors consumers. Otherwise the prices are increasing, thus lowering consumer surplus.

However, in case of unit consumption for a given market share the total welfare (which is equal to profits plus consumer surplus) is constant and since market shares do not depend on the access fee, welfare is also independent of the access fee. The following proposition summarizes the above results[8].

Proposition 2 *Suppose that the incumbent is subject to cost-based access fee. The effect of a larger Δ^τ is the following:*

(i) Firms have divergent preference over the access fee. If brand loyalty is strong (that is $\beta+4\sigma\Delta^c > 0$), the incumbent's profit varies in the same direction as the industry profit which is decreasing in Δ^τ, meaning a lower access fee for the entrant. In this case the incumbent might compensate the entrant for a lower access fee. If the cost difference is large (that is $\beta + 4\sigma\Delta^c < 0$), the industry profit varies with the entrant's profit, giving the opportunity to set a larger access fee and compensate the incumbent.

(ii) Consumer surplus varies in the opposite direction to industry profit.

(iii) Welfare remains unchanged.

3 Linear demand and linear tariffs

In this chapter we extend the previous model to a model with linear demand for telephony consumption and seek an answer to the following questions: what is the effect of cost asymmetry and brand loyalty on the equilibrium and its properties? What are the preferences of the firms and the social planner over the access fee?

3.1 Demand structure and consumer surplus

Since we relax the assumption of unit consumption and extend it to a linear demand function, a consumer faces a two-step problem: in the first step he decides on whether to connect to a network and in the second step he chooses the amount of telephony consumption. We leave the first group of characteristics of

[8] The proposition can be proved easily by the above argument.

consumers as it was set in section 2.2, that is the fixed surplus v_0, the network specific surplus β_i (and $\beta \equiv \beta_1 - \beta_2 > 0$) and different *a priori* preferences with respect to the consumer's location x and the 'traveling cost' t. Applying these assumptions, the total utility of a consumer subscribed to firm i is equal to

$$v_0 - t\,|x - x_i| + t\beta_i + u(q) - p_i q,$$

where the first three terms measure the utility of being connected (traffic independent surplus), and the last two terms measure the utility from telephony consumption (traffic dependent surplus): $u(q)$ is the utility from telephony consumption q assumed to be the same for all consumers and p_i denotes the uniform linear price of firm i.

We can solve the consumer's two-step problem by backward induction. In the second step, given the retail prices, a consumer decides about telephony consumption. His utility function is considered quadratic, i.e.[9]

$$u(q) = q - \frac{q^2}{2},$$

therefore his demand function for telephony consumption is linear:

$$q(p) = 1 - p.$$

As a consequence, the variable net surplus gained from consumption is as follows:

$$v(p) = \max_q \{u(q) - pq\} = \frac{(1-p)^2}{2}.$$

In the first step a consumer chooses a network. The optimal network share of firms can be determined by finding a consumer, located in α, who is indifferent between the networks, that is

$$v_0 - t\alpha + t\beta + v(p_1) = v_0 - t(1-\alpha) + v(p_2).$$

Then α, the network share of firm 1, is

$$\alpha(p_1, p_2) = \frac{1+\beta}{2} + \sigma\left[v(p_1) - v(p_2)\right],$$

where $\sigma = \frac{1}{2t}$ is the measure of substitution. Introduce the notation $v_i \equiv v(p_i)$, $q_i \equiv q(p_i)$ and

$$\alpha_i \equiv \frac{1 \pm \beta}{2} + \sigma\left(v_i - v_j\right),$$

where the upper sign of β belongs to firm 1 and the lower to firm 2.

For a given $s = (p_1, p_2)$ strategy profile the total consumer surplus can be defined in the following way:

$$CS(s) = \alpha_1 v_1 + \alpha_2 v_2 - D\left(\alpha_1\right),$$

where $D(\alpha)$ measures the average disutility originating from the difference between the *a priori* preferences of consumers and the characteristics of services

[9] We use quadratic utility function because it fulfils the general assumptions and at once makes the analysis easier since a linear demand function can be derived from the first order condition of utility maximization. We might also set up a more general model using the utility function $u(q) = aq - b\frac{q^2}{2}$, however this generality does not modify the main results.

offered by the networks. Or in other words, the function $D(\cdot)$ is the average traveling cost between the location of consumers and firms which is for any α equal to

$$D(\alpha) = \frac{1}{2\sigma}\left[\alpha\frac{\alpha}{2} + (1-\alpha)\frac{1-\alpha}{2}\right] = \frac{1}{2\sigma}\left[\frac{\alpha^2 + (1-\alpha)^2}{2}\right].$$

3.2 Price competition

The basic characteristics of firms remain the same as were presented in section 2.1. We look for the Nash-equilibrium of a static game in which firms ($i = 1, 2$) compete for consumers by setting a price profile $s = (p_1, p_2)$. As can be seen from figure 1 and recalling that $m_i = 0$, firm i's profit is equal to

$$\pi_i(s) = \alpha_i\left[(p_i - c_i)\, q_i - f\right] + \alpha_i\alpha_j\left[(\tau_i - c_i^0)q_j - (\tau_j - c_i^0)q_i\right]. \tag{5}$$

In the Nash-equilibrium, firm i maximizes its profit function (5) according to its price p_i given the rival's price p_j, subject to

$$\begin{aligned} 0 &\leq p_i \leq 1, \\ 0 &\leq \alpha_i \leq 1, \qquad i = 1, 2. \\ 0 &\leq \pi_i, \end{aligned}$$

Since the access profit depends on $\alpha_i\alpha_j q_i$ which is a fifth-degree function of p_i, the first derivatives of the profit function are quadric, and therefore it is not possible to find a closed form solution for the reaction functions and for the equilibrium prices. Moreover, the second derivatives are cubic which makes the specification of equilibrium conditions very complicated. To avoid this calculation problem, we ran simulations[10], and from the results we derive conclusions about the existence of the equilibrium and about policy implications.

3.3 Comparison and policy implications

We obtain that for very large access fee difference and very strong substitution, there exists no equilibrium, which is a robust result of (Armstrong, 2002) and (Laffont et al., 1998*a*).

Similarly to the unit consumption case, a firm with enormous advantage over the other firm might corner the market. Denote $\Delta^v \equiv v_2 - v_1$. Monopoly equilibrium can emerge, if one of the following conditions holds:

$$v_1 - \frac{1-\beta}{2\sigma} \geq v_2 \Leftrightarrow \Delta^v \leq -\frac{1-\beta}{2\sigma}$$

or

$$v_1 + \frac{\beta}{2\sigma} \geq v_2 - \frac{1}{2\sigma} \Leftrightarrow \Delta^v \geq \frac{1+\beta}{2\sigma}$$

In the first case the incumbent drives out the other firm from the market, in the second the entrant. Because of the same reason as in the benchmark case,

[10] For simulations we used Mathematica 5.0.

the entrant always charges lower price; therefore its consumers realize higher variable net surplus. Since $\Delta^v > 0$, the first condition fulfils if $\beta >> 1$, that is strong brand loyalty, and the second condition holds if for a given β, $|\Delta^c|$ is sufficiently large. The cornering conditions require extreme parameter values; therefore we will solely focus on the more general cases, namely the shared-market equilibria.

We can draw up other characteristics of the equilibrium. Equilibrium prices are decreasing in substitution between the networks, that is the stronger the substitution, the smaller the prices, which means that the substitution effect stimulates competition, thus lowering firms' profit and causing smaller network share for the incumbent (the effect of brand loyalty becomes weaker and the effect of the cost difference becomes stronger). Moreover, from a welfare point of view, it is preferable to have more fierce competition in the market (see Figure 2 and 3 in Appendix B.1).

The further brand loyalty and the cost difference are from zero, the further is the equilibrium from the symmetric case. Since the two effects of different timing of entry are opposite, we need to separate them so as to compare the outcomes. As the data shows, the entrant's profit is generally higher than the incumbent's profit. For a given degree of brand loyalty, the smaller the cost difference, the larger the incumbent's profit and the smaller the entrant's profit, though the resultant is ambiguous just as is the change in consumer surplus and welfare. However for stronger brand loyalty, the equilibrium price difference becomes larger, and therefore the incumbent's market share and profit decreases, while the entrant's profit increases. The consumer surplus varies opposite to the industry profit. In a more asymmetric market the welfare increases with the industry profit; however in a more symmetric market the effect of Δ^τ on consumers surplus is stronger than on the industry profit, thereby causing higher welfare (see Figure 4 and 5 in Appendix B.1).

The most important and most controversial point in the analysis is the determination of the access fee. As we discussed in the introduction, the relevant questions are whether to impose cost-based access fee regulation, or allow freely negotiated access fee and for the less efficient firm to set an access fee containing a positive mark-up. As before, the main results for policy makers are derived from simulations (see Figure 6-11 in Appendix B.2).

Let us use the difference in access fees (Δ^τ) for the policy implications, and assume that the incumbent is subject to a cost-based access fee. *For weak brand loyalty* the equilibrium prices are increasing in Δ^τ, and the larger the Δ^τ, the lower the net surplus a consumer might receive, thus lowering consumer surplus (see Figure 6 and 7).

The intuition behind the increasing equilibrium prices is the following. If Δ^τ increases (that is larger τ_2), for a given market share the incumbent has to face larger per-consumer access deficit (or smaller access profit); therefore the total access profit can be reduced by decreasing the product of market shares ($\alpha_1\alpha_2$). Since $\alpha_1\alpha_2$ is smaller if the difference in market shares is larger, the incumbent is interested in moving away from the symmetric case which can be achieved by a higher retail price. The entrant has the same preference over its own price change. The entrant wants to lower the net outflow of calls to raise the per-consumer access profit, which can be obtained by charging a higher retail price. As a consequence the entrant gets higher per-consumer retail profit, and since the incumbent is interested in lowering its market share, the entrant obtains higher total retail profit.

Despite the increasing prices, if Δ^τ is larger, the entrant with the lower cost gets higher and increasing equilibrium profit, while the incumbent gets lower and decreasing profit. This means that the firms have divergent preference over the access fee (see Figure 8-10). The industry profit changes in the same direction as the entrant's profit, thus obtaining similar result to Armstrong's: the entrant can offer a side-payment for the less efficient firm to compensate for a larger Δ^τ.

Up to now we analyzed the case of zero brand loyalty. *The stronger the brand loyalty*, the larger the incumbent's retail price and the smaller the entrant's price compared to the case of zero brand loyalty; moreover the prices become monotone decreasing function of Δ^τ. Therefore the overall effect of increasing Δ^τ on consumer surplus is positive. Because of the decreasing prices, not only the incumbent's profit but also the entrant's profit is decreasing, resulting in a lower industry profit. In this case firms have similar preference over the access fee, that is a smaller Δ^τ, which might result a negative access mark-up for the entrant.

Welfare changes in the same direction as consumer surplus, that is under low brand loyalty a larger Δ^τ, otherwise a smaller Δ^τ is more favorable (see Figure 11).

4 Linear demand and two-part tariffs

This section analyzes network interconnection and competition in two-part tariffs. In the symmetric models of (Armstrong, 2002) and (Laffont et al., 1998*a*), they get a profit-neutrality result, i.e. the firms' profit is independent of access fee and equal to the Hotelling-profit. Since the firms are indifferent over the access fee, the zero access mark-up (i.e. cost-based access fee) which is the socially optimal solution might be carried through. In this section we look for the equilibrium in the presence of network asymmetry, and answer the question of whether the profit-neutrality property still holds, or if not, what the socially preferable access charge would be.

4.1 Demand structure and consumer surplus

The consumers' utility and demand functions are the same as were set in Section 3. However in the present model each network offers a two-part tariff, therefore a consumer of network i making q amount of call pays

$$T_i(q) = m_i + p_i q,$$

where m_i is the fixed fee, e.g. the monthly charge of usage, and p_i is the per-minute charge. As a consequence, the net surplus a consumer obtains subscribing to network i is equal to

$$w_i \equiv w(p_i, m_i) = v(p_i) - m_i.$$

The network shares can be determined by finding the consumer, located in α, who is indifferent between the two networks. This indifference means that

$$w(p_1, m_1) - t\alpha + t\beta = w(p_2, m_2) - t(1 - \alpha).$$

From this expression, firm 1's network share is

$$\hat{\alpha}(\hat{s}) = \frac{1+\beta}{2} + \sigma \left[w(p_1, m_1) - w(p_2, m_2) \right],$$

where $\sigma = \frac{1}{2t}$ is the measure of substitution and $\hat{s} = (p_1, m_1, p_2, m_2)$ is a possible price profile of the firms. Henceforth denote $v_i \equiv v(p_i)$, $q_i \equiv q(p_i)$ and

$$\alpha_i \equiv \frac{1 \pm \beta}{2} + \sigma(w_i - w_j),$$

where the upper sign of β belongs to firm 1 and the lower to firm 2.

For a given $\hat{s}$ strategy profile the consumer surplus is the following:

$$CS(s) = \alpha_1 w_1 + \alpha_2 w_2 - D(\alpha_1),$$

where $D(\alpha)$ measures the average disutility originating from the difference between the *a priori* preferences and the characteristics of services.

4.2 Price competition

Firm i's benefits and costs can be read from figure 1, and the profit is as follows

$$\pi_i(\hat{s}) = \hat{\alpha}_i(\hat{s}) \left[(p_i - c_i) q_i + m_i - f \right] + \hat{\alpha}_i(\hat{s}) \hat{\alpha}_j(\hat{s}) \left[\left(\tau_i - c_i^0\right) q_j - \left(\tau_j - c_i^0\right) q_i \right].$$

To simplify the profit maximization, we rewrite the profit function of firm i using a modified strategy profile $s = (p_1, w_1, p_2, w_2)$:

$$\pi_i(s) = \alpha_i \left[(p_i - c_i) q_i + v_i - w_i - f \right] + \alpha_i \alpha_j \left[(\tau_i - c_i^0) q_j - (\tau_j - c_i^0) q_i \right], \quad (6)$$

Hereafter denote $A_i \equiv (\tau_i - c_i^0) q_j - (\tau_j - c_i^0) q_i$ firm i's per-consumer access profit.

We intend to find the Nash-equilibrium, in which each firm maximizes its profit functions (6) according to its price and the offered net surplus, subject to

$$\begin{aligned} 0 &\leq p_i, \leq 1, \\ 0 &\leq w_i, \qquad\qquad i = 1, 2. \\ 0 &\leq \alpha_i \leq 1, \\ 0 &\leq \pi_i, \end{aligned}$$

The first order conditions of profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \alpha_i \left[-p_i + c_i + \alpha_j (\tau_j - c_i^0) \right] = 0$$

and

$$\frac{\partial \pi_i}{\partial w_i} = \sigma[(p_i - c_i) q_i + v_i - f - w_i - \alpha_i/\sigma - 2\sigma(w_i - w_j) A_i] = 0.$$

The first order condition according to w_i is quadric, therefore the equilibrium cannot be given in the regular closed form. However, the first order condition according to p_i is linear for non-negative market shares; therefore the following proposition can be claimed about the equilibrium and its properties[11].

Proposition 3 *If for* $i = 1, 2$

$$|A_i| < \frac{1}{\sigma},$$

there is an equilibrium which is unique and characterized as

$$p_i = c_i + \alpha_j \left(\tau_j - c_i^0\right), \tag{7}$$

$$m_i = f + \frac{\alpha_i}{\sigma} - \alpha_j \left(\tau_j - c_i^0\right) q_i + \left(\alpha_i - \alpha_j\right) A_i, \tag{8}$$

$$\begin{aligned} \alpha_i \;=\; & \frac{1}{2} \pm \frac{\beta}{6} + \frac{\sigma}{3} [v_i - v_j + \alpha_j \left(\tau_j - c_j^0\right) q_i - \alpha_i \left(\tau_i - c_i^0\right) q_j \\ & + \left(c_j^0 - c_i^0\right) \left(\alpha_1 q_1 + \alpha_2 q_2\right)], \end{aligned}$$

$$\pi_i = \alpha_i^2 \left[\frac{1}{\sigma} + A_i\right].$$

We get first best retail prices (7) since they are equal to the perceived marginal cost of a unit call, that is the marginal cost of an on-net call ($\alpha_i 2 c_i^0$) plus the marginal cost of an off-net call ($\alpha_j(\tau_j + c_i^0)$). The fixed fee (8) covers the fixed cost and is modified (reduced or raised) with the access profit.

4.3 Comparison and policy implications

We start comparing the results in case of cost-based access fee and then we extend the analysis to a particular situation when the entrant is allowed to charge a higher access fee than its termination cost ($\tau_2 - c_2^0 > 0$).

First we redefine the equilibrium to the case of a cost-based access fee, substituting $\tau_i = c_i^0$, that is

$$p_i = c_i + \alpha_j \left(c_j^0 - c_i^0\right), \tag{9}$$

$$m_i = f + \alpha_i \left[\frac{1}{\sigma} - \left(c_j^0 - c_i^0\right) q_i\right],$$

$$\alpha_i = \frac{1}{2} \pm \frac{\beta}{6} + \frac{\sigma}{3} [v_i - v_j + \left(c_j^0 - c_i^0\right) \left(\alpha_1 q_1 + \alpha_2 q_2\right)],$$

$$\pi_i = \alpha_i^2 \left[\frac{1}{\sigma} - \left(c_j^0 - c_i^0\right) q_i\right].$$

For the comparison denote $\Delta^p \equiv p_2 - p_1, \Delta^q \equiv q_2 - q_1, \Delta^v \equiv v_2 - v_1, \Delta^m \equiv m_2 - m_1, \Delta^\alpha \equiv \alpha_2 - \alpha_1$ and $\Delta^\pi \equiv \pi_2 - \pi_1$. The difference between the equilibrium per-minute prices is equal to the cost difference ($\Delta^p = \Delta^c$), i.e. the incumbent with the larger cost charges a larger per-minute price. As a consequence, the consumers of the incumbent initiate less call ($\Delta^q = -\Delta^c$) thus

[11] The proof is given in Appendix A.

obtaining lower net surplus. When the entrant's access fee increases, firm 1's perceived marginal cost increases, therefore it sets a more higher per-minute price ($\Delta^p = \Delta^c - \alpha_2(\tau_2 - c_2^0) < 0$).

Since firms charge their perceived marginal cost as a retail price, they get zero retail profit; therefore the profit arises from the connection dependent profit and from the access profit. The difference in equilibrium profits is closely related to the difference in fixed fees, which is in case of a cost-based access fee is equal to

$$\Delta^m = \frac{\Delta^\alpha}{\sigma} + \Delta^c \left(\alpha_1 q_1 + \alpha_2 q_2\right). \tag{10}$$

This expression can be either negative or positive, and since the second part of (10) is always negative, the sign of (10) depends on which firm has larger market share.

As for the market shares, the difference varies in response to the strength of brand loyalty and the cost difference in the following way:

$$\Delta^\alpha = \frac{-\beta - 4\sigma\Delta^c \left(1 - \alpha_1 c_1 - \alpha_2 c_2\right)}{3 + \sigma \left(\Delta^c\right)^2}. \tag{11}$$

The denominator of (11) is always positive, and therefore the entrant has higher market share only if the numerator is also positive. Let us start from zero cost difference. In case of symmetric costs only brand loyalty has an effect on market shares, and in spite of its higher per-minute price the incumbent obtains higher market share ($\Delta^\alpha = -\beta/3 < 0$). If $\Delta^\alpha < 0$ and $\Delta^c = 0$, then $\Delta^m < 0$, meaning that the incumbent sets a higher fixed fee. The behavior of firm 1 is straightforward: it faces a positive per-consumer access profit; therefore it can increase its total access profit by raising the product of market shares ($\alpha_1\alpha_2$). This product is maximal if firms are symmetric, that is $\alpha_1\alpha_2 = 1/4$. Firm 1 can enlarge this product by reducing its (larger) market share by means of a higher access fee.

In case of zero brand loyalty, for reasonably low termination costs ($\alpha_1 c_1^0 + \alpha_2 c_2^0 < 1$), the numerator of (11) is positive, causing higher market share for the entrant ($\Delta^\alpha = -4\sigma\Delta^c \left(1 - \alpha_1 c_1 - \alpha_2 c_2\right) / (3 + \sigma \left(\Delta^c\right)^2) > 0$). The intuition is similar to the linear pricing case. Since the entrant faces a negative per-consumer access profit, it is interested in lowering the total access profit and raising the total retail profit, that is moving further from the symmetric case through a higher market share. A higher market share can be obtained by simply setting a lower price or by charging a lower fixed fee at the same time. Therefore if $\beta = 0$, it is ambiguous which firm sets a higher fixed fee: Δ^m depends on how cost difference and brand loyalty relate to each other. For a given brand loyalty, the larger the cost difference, the higher the entrant's market share and the more likely that it charges higher fixed fee.

The same argument can be applied to the difference between π_2 and π_1 which is equal to

$$\Delta^\pi = \frac{\Delta^\alpha}{\sigma} + \Delta^c \left(\alpha_1^2 q_1 + \alpha_2^2 q_2\right).$$

If the entrant can slightly deviate from the cost-based access fee, a small access mark-up does not affect the equilibrium market shares[12], therefore the above results remain unchanged.

[12] The proof is similar to lemma 1's proof in (Peitz, 2005), p. 356.

In a neighborhood of cost-based access fee $\partial\alpha_i/\partial\tau_2 = 0$ holds. Applying this shows that the effect of the increment in τ_2 on equilibrium profits can be measured by the following derivatives:

$$\left.\frac{\partial\pi_1}{\partial\tau_2}\right|_{\tau_2=c_2^0} = \alpha_1^2\left(\alpha_2\Delta^c - q_1\right) < 0,$$

$$\left.\frac{\partial\pi_2}{\partial\tau_2}\right|_{\tau_2=c_2^0} = \alpha_2^2 q_1 > 0.$$

On the grounds of these expressions we state that profit-neutrality of the symmetric equilibrium no longer holds: the incumbent's profit decreases and the entrant's profit increases in response to an increase in τ_2. If the termination costs are the same, then each firm's profit depends only on its market share. Therefore the industry profit changes in the same direction as the larger firm's, which is in this case the incumbent, profit. The larger the cost difference, the larger the entrant's market share, thus the more likely that the industry profit varies the same direction as the entrant's profit. As a consequence we obtain that in case of zero cost difference, the access mark-up favors the entrant, and for larger cost difference it favors the industry as well.

As (Peitz, 2005) claims in case of symmetric costs, the consumer surplus increases as the entrant is allowed to use an access mark-up. This result holds true if the cost difference remains small: an increase in τ_2 is followed by a larger consumer surplus, though the consumer surplus is lower than that of zero cost difference. Moreover for large cost differences the consumers are worse off.

The previous results are summarized in the following proposition[13]:

Proposition 4 *In case of symmetric costs, in a neighborhood around cost-based access prices an increase in the entrant's access price gives rise to higher profit for the entrant and higher consumer surplus. With increasing cost differences, the industry profit might also increase though it leads to a smaller consumer surplus.*

5 Conclusions

In the present paper we analyzed network interconnection and competition under asymmetric cost structure and brand loyalty. First, following the article of (Armstrong, 2002), we presented a unit consumption model as a benchmark case, and showed the results of welfare-neutrality and divergent firms' preferences over Δ^τ. When we extended the model to linear demand for telephony consumption and linear prices, the property of divergent preferences remained valid only if brand loyalty is weak: the early mover (the incumbent) prefers a negative access markup for the late mover (the entrant), while the entrant prefers a positive one. Since the industry profit changes with the entrant's profit, the access fee might act as the instrument of collusion, but only if a side-payment is permitted. Welfare changes with the consumer surplus, and in the opposite direction to the entrant's profit and the industry profit. Therefore

[13] The proof is given in Appendix A.

the socially desirable access fee is characterized by a negative mark-up for the entrant. When brand loyalty is strong, the firms prefer smaller access fee difference, resulting in a negative access mark-up for the entrant; although this access fee is welfare reducing.

Extending the model to competition in two-part tariffs, neither the profit-neutrality nor the welfare-neutrality properties of the equilibrium remain valid. The firms' preferences related to Δ^τ are different, though the change in the industry profit is ambiguous. If the cost difference is small, the industry profit varies in line with the incumbent's decreasing profit, causing a negative access mark-up for the entrant; although consumers prefer a positive access mark-up. However as a result of increasing cost differences and a higher access fee for the entrant, the industry obtains higher profit, while consumers are worse off.

As the results show, the access fee regulated on the basis of termination cost might not necessarily provide the socially optimal solution. In case of an infant market, when the incumbent derives advantage from earlier presence in the market, an access mark-up for the entrant might increase the consumer surplus and the welfare. However in a more symmetric market when the importance of the incumbent's reputation is eliminated, a cost-based access fee or even a negative mark-up for the entrant might provide a socially favorable outcome.

6 Appendices

A Proofs

Proof. Proposition 1. *Existence and uniqueness of the equilibrium.*

The first order condition for firm i is

$$\frac{\partial \pi_i}{\partial p_i} = -\sigma(p_i - C_i) + \left(\frac{1 \pm \beta}{2} + \sigma(p_j - p_i)\right) + \sigma\left[2\sigma(p_j - p_i) \pm \beta\right](\tau_i - \tau_j) = 0,$$

where the upper sign of β applies to firm 1, and the lower to firm 2. From this first order condition firm i's reaction curve is

$$r_i(p_j) = \frac{(1 + 2\sigma(\tau_i - \tau_j))\, p_j + C_i + (1 \pm \beta)/2\sigma - \beta\Delta^\tau}{2\,(1 + \sigma(\tau_i - \tau_j))}.$$

Since

$$\frac{\partial r_i}{\partial p_j} = \frac{1 + 2\sigma(\tau_i - \tau_j)}{2\,(1 + \sigma(\tau_i - \tau_j))},$$

each reaction curve is a monotonic function of the rival firm's price (that is firms are strategic complements) if and only if

$$\tau_j - \tau_i < \frac{1}{2\sigma}. \tag{12}$$

Moreover the slope of the reaction function is constant and $0 < r_i'(.) < 1$ holds. Therefore the reaction curves intersect only once in the relevant price range and a unique equilibrium exists. The second derivative is the following

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2\sigma\,(1 + \sigma(\tau_i - \tau_j)) < 0,$$

which is negative if (12) holds. In this case the profit function is strictly quasi-concave, so therefore it has a unique maximum.

From the reaction curves the equilibrium prices, market shares and maximal profits can be derived as follows:

$$\begin{aligned} p_i^* &= \frac{1}{2\sigma} \pm \frac{\beta}{6\sigma} + \frac{2C_i + C_j}{3} - \frac{\Delta^\tau}{3}\left(\beta + 4\sigma\Delta^c\right) \\ \alpha_i^* &= \frac{1}{2} \pm \frac{1}{6}\left(\beta + 4\sigma\Delta^c\right) \\ \pi_i^* &= (\alpha_i^*)^2\left[\frac{1}{\sigma} + \tau_i - \tau_j\right]. \end{aligned} \tag{13}$$

If (12) holds, the equilibrium prices and profits are positive.

Shared- or cornered-market outcome. (I) Suppose that firm 1 corners the market and sets a low price such that (i) at this price the closest consumer to firm 2 also wants to buy from firm 1, that is

$$v_0 + \frac{\beta}{2\sigma} - \frac{1}{2\sigma} - p_1 \geq v_0 - p_2, \tag{14}$$

and (ii) this price is a Nash-equilibrium price, since none of the firms wants to unilaterally deviate from it. Substituting (13) into (14), we get the condition for a cornered-market, that is

$$\beta + 4\sigma\Delta^c \geq 3.$$

Since $\Delta^c < 0$ and $\sigma > 0$, this condition holds if $\beta >> 3$. In this case firm 1 gets positive profit, and firm 2 gets zero since it does not have any subscribers. If firm 2 attempts to mimic firm 1 with the same price, it will achieve a positive market share, but receives negative per-consumer profit. Therefore it is not profitable to deviate from the equilibrium.

(II) Suppose now that only firm 2 attains subscribers. It can be possible for an equilibrium price p_2 which satisfies the following condition:

$$v_0 + \frac{\beta}{2\sigma} - p_1 \leq v_0 - \frac{1}{2\sigma} - p_2.$$

This condition in the equilibrium holds if and only if

$$\beta + 4\sigma\Delta^c \leq -3,$$

that is $4\sigma\left|\Delta^c\right| >> 3$. We can now follow the same argument as in point (I).

(III) If

$$\left|\beta + 4\sigma\Delta^c\right| < 3,$$

then the equilibrium market shares are positive, i.e. the shared market outcome emerges. ■

Proof. Proposition 3. Suppose that profit function (6) is twice continuously differentiable. The first order conditions of profit maximization are

$$\frac{\partial \pi_i}{\partial p_i} = \alpha_i\left[-p_i + c_i + \alpha_j(\tau_j - c_i^0)\right] = 0$$

and

$$\frac{\partial \pi_i}{\partial w_i} = \sigma \left[(p_i - c_i) q_i + v_i - f - w_i - \frac{\alpha_i}{\sigma} - 2\sigma (w_i - w_j) A_i \right] = 0,$$

where $A_i = \left(\tau_i - c_i^0\right) q_j - \left(\tau_j - c_i^0\right) q_i$ is firm i's per-consumer access profit. From these conditions we get the equilibrium values stated in the proposition:

$$p_i = c_i + \alpha_j \left(\tau_j - c_i^0\right),$$

$$m_i = f + \frac{\alpha_i}{\sigma} - \alpha_j \left(\tau_j - c_i^0\right) q_i + (\alpha_i - \alpha_j) A_i,$$

$$\begin{aligned} \alpha_i \quad = \quad & \frac{1}{2} \pm \frac{\beta}{6} + \frac{\sigma}{3} [v_i - v_j + \alpha_j \left(\tau_j - c_i^0\right) q_i - \alpha_i \left(\tau_i - c_j^0\right) q_j \\ & + \left(c_j^0 - c_i^0\right) (\alpha_i q_i + \alpha_j q_j)], \end{aligned}$$

$$\pi_i = \alpha_i^2 \left[\frac{1}{\sigma} + A_i \right].$$

For the existence of the equilibrium, the following second order conditions (SOCs) should hold:

(i)

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -\alpha_i < 0$$

which holds if $\alpha_i > 0$ and $\partial^2 \pi_i / \partial p_i^2 = 0$ if firm j corners the market.

(ii)

$$\frac{\partial^2 \pi_i}{\partial w_i^2} = -2\sigma \left(1 + \sigma A_i\right) < 0 \Leftrightarrow |A_i| < \frac{1}{\sigma}$$

This expression states that the SOC according to w_i holds if and only if for a given σ the per-consumer access profit is small enough or if σ is small so that there is weak substitution between the networks.

(iii) Since

$$\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} = \frac{\partial^2 \pi_i}{\partial w_i \partial p_i} = -\sigma \alpha_i \left(\tau_j - c_i^0\right),$$

therefore

$$\frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_i}{\partial w_i^2} > \left(\frac{\partial^2 \pi_i}{\partial p_i \partial w_i} \right)^2 \Leftrightarrow \alpha_i \left(\tau_j - c_i^0\right)^2 < 2 \left(\frac{1}{\sigma} + A_i \right).$$

■

Proof. Proposition 4. The structure of the proof originates in (Peitz, 2005) who applied the methodology of supermodular games and monotone comparative statics of the equilibrium.

Assume that firm 1 is subject to cost-based access fee, and firm 2 can freely set its access charge. Substituting the equilibrium prices (9) into the modified profit function (6) we obtain a pseudo-profit function:

$$\hat{\pi}_i(w_1, w_2, p_j) = \alpha_i \left(v \left[c_i + \alpha_j \left(\tau_j - c_i^0\right) \right] - w_i - f + \alpha_j \left(\tau_i - c_i^0\right) q_j \right)$$

from which the first derivative according to w_i is

$$\begin{aligned}\frac{\partial \hat{\pi}_i}{\partial w_i} &= \sigma\left(v\left[c_i+\alpha_j\left(\tau_j-c_i^0\right)\right]-w_i-f+\alpha_j\left(\tau_i-c_i^0\right)q_j\right)\\ &\quad +\alpha_i\left(\sigma\left(\tau_j-c_i^0\right)q\left[c_i+\alpha_j\left(\tau_j-c_i^0\right)\right]-1-\sigma\left(\tau_i-c_i^0\right)q_j\right).\end{aligned}$$

If the second derivative according to w_j is positive, the slope of the pseudo-reaction curve is positive (i.e. strategic complements), and if the second derivative according to τ_2 is also positive, the reaction curve moves outward in response to an increase in τ_2, which means that the net equilibrium surplus increases with τ_2.

First consider the first derivative of firm 1's pseudo-profit:

$$\begin{aligned}\frac{\partial \hat{\pi}_1}{\partial w_1} &= \sigma\left(v\left[c_1+\alpha_2\left(\tau_2-c_1^0\right)\right]-w_1-f\right)\\ &\quad +\alpha_1\left(\sigma\left(\tau_2-c_1^0\right)q\left[c_1+\alpha_2\left(\tau_2-c_1^0\right)\right]-1\right).\end{aligned}$$

The second derivative according to w_2 is equal to

$$\frac{\partial^2 \hat{\pi}_1}{\partial w_1 \partial w_2}=\sigma\left(1-\sigma\left(\tau_2-c_1^0\right)q\left[c_1+\alpha_2\left(\tau_2-c_1^0\right)\right]\right).$$

If the access mark-up is small so that $c_2^0<\tau_2<c_1^0$, then the cross-derivative is positive, which means that firm 1's best response function is upward sloping. This is also the case if $\tau_2<c_2^0<c_1^0$ (there is a negative mark-up). A sufficient condition for strategic complements is that $\tau_2<c_1^0$.

As for the other second derivative,

$$\frac{\partial^2 \hat{\pi}_1}{\partial w_1 \partial \tau_2}=-\sigma\left(\Delta^\alpha q\left[c_1+\alpha_2\left(\tau_2-c_1^0\right)\right]+\alpha_1\alpha_2\left(\tau_2-c_1^0\right)\right) \tag{15}$$

is positive if $\tau_2<c_1^0$ and Δ^α is negative: that is, the incumbent has a larger market share. This can occur in case of small cost difference and strong brand loyalty:

$$\beta+4\sigma\Delta^c\left(1-\alpha_1c_1-\alpha_2c_2\right)>0.$$

In this case firm 1's reaction curve moves outward, therefore its consumers benefit from a larger τ_2.

Consider now firm 2's first derivative function:

$$\begin{aligned}\frac{\partial \hat{\pi}_2}{\partial w_2} &= \sigma\left(v\left[c_2-\alpha_1\Delta^c\right]-w_2-f+\alpha_1\left(\tau_2-c_2^0\right)q_1\right)\\ &\quad -\alpha_2\left(\sigma\Delta^c q\left[c_2-\alpha_1\Delta^c\right]+1+\sigma\left(\tau_2-c_2^0\right)q_1\right).\end{aligned}$$

The cross-derivative according to w_1 is equal to

$$\frac{\partial^2 \hat{\pi}_2}{\partial w_2 \partial w_1}=\sigma\left(\sigma\Delta^c\left(q_1+q\left[c_2-\alpha_1\Delta^c\right]-\alpha_2\right)+2\sigma\left(\tau_2-c_2^0\right)q_1+1\right),$$

which is positive - in the neighborhood of cost-based access fee - if the cost difference is small:

$$\left|\Delta^c\right|<\frac{2\left(\tau_2-c_2^0\right)q_1+\frac{1}{\sigma}}{q_1+q\left[c_2-\alpha_1\Delta^c\right]-\alpha_2}.$$

The second derivative according to τ_2 is equal to

$$\frac{\partial^2 \hat{\pi}_2}{\partial w_2 \partial \tau_2} = -\sigma \Delta^\alpha q_1, \tag{16}$$

which is positive if the cost difference is small and brand loyalty is strong. The larger the cost difference, the more likely the reaction curve moves inward.

As can be seen from (15) and (16)

$$\left| \frac{\partial^2 \hat{\pi}_1}{\partial w_1 \partial \tau_2} \right| < \left| \frac{\partial^2 \hat{\pi}_2}{\partial w_2 \partial \tau_2} \right|,$$

it is more likely that firm 1's reaction curve moves inward. The overall effect of τ_2 on the consumer surplus depends on the size of Δ^α. If $\Delta^\alpha > 0$, both reaction curves move inward thus lowering the total consumer surplus. If $\Delta^\alpha < 0$, but $|\Delta^\alpha| < \alpha_1\alpha_2(\tau_2 - c_1^0)/q_1$, only firm 1's reaction curve moves inward, and the effect of τ_2 on w_2 is always positive, though its effect on w_1 is ambiguous: w_1 can be lower or higher. However if $\Delta^\alpha < 0$ and $|\Delta^\alpha| > \alpha_1\alpha_2(\tau_2 - c_1^0)/q_1$, both reaction curves move outward yielding higher net surplus for every consumer. ■

B Simulation results

In this section we show some results from simulations: first we illustrate a comparison for different parameter values, then for different access fees.

B.1 Comparison according to parameter values

In the following figures we present equilibrium profits (individual and industry, π_i and $\pi \equiv \pi_1 + \pi_2$ respectively), consumer surplus (CS) and welfare ($W \equiv \pi + CS$) for different values of the substitution parameter (σ), and for decreasing difference in termination costs (Δ^c) by different brand loyalty (β). For simplicity we assume that the firms are subject to an access fee on the basis of termination cost.

The effect of increasing strength of substitution:

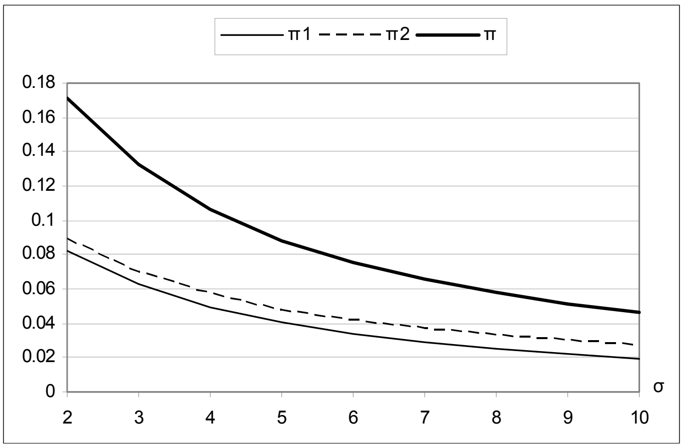

Figure 2. Maximal individual and industry profit, increasing σ

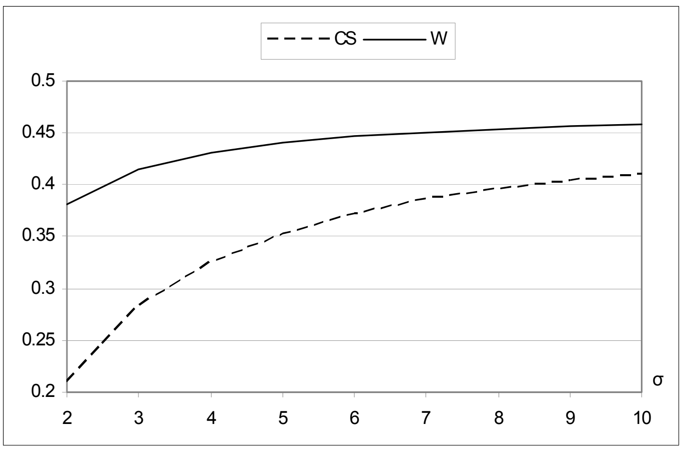

Figure 3. Consumer surplus and social welfare, increasing σ

The effect of decreasing cost difference for different levels of brand loyalty (grey curves show cases of $\beta = 0$, and black curves of $\beta = .5$):

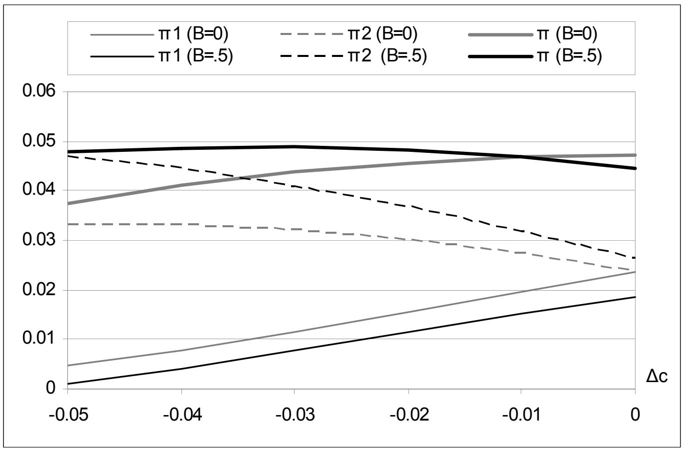

Figure 4. Maximal individual and industry profit, decreasing $|\Delta^c|$

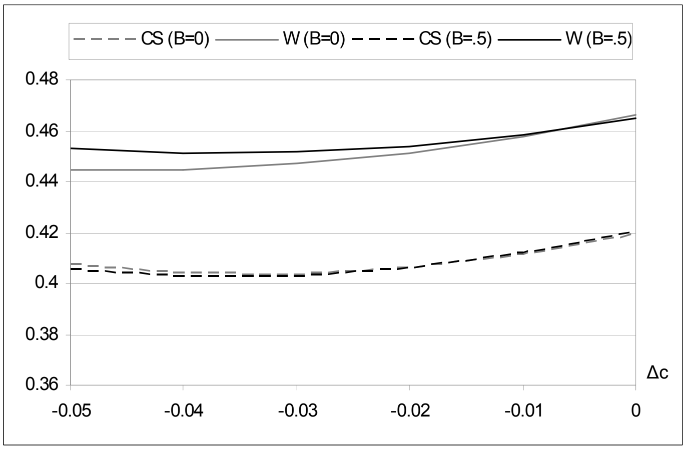

Figure 5. Consumer surplus and social welfare, decreasing $|\Delta^c|$

B.2 Comparison according to access fee

To compare the effect of different access fees, we fixed the following parameter values

$$c_1^0 = 0.02, c_2^0 = 0.01, \sigma = 10, f = 0, \tau_1 = 0.02.$$

In this case, the more efficient firm sets a cost-based access fee, and only firm 2's access fee is changing. If $\Delta^\tau = 0$, then the access fees are reciprocal, and if $\Delta^\tau = -0,01$, both firms set a cost-based access fee. In the following figures equilibrium prices (p_i^*), consumer surplus (CS), profits (individual and industry, π_i and $\pi \equiv \pi_1 + \pi_2$ respectively) and social welfare (W) are shown by different values of brand loyalty (β). In the figures we indicate Δ^τ on the horizontal axis and the above mentioned equilibrium values on the vertical axis.

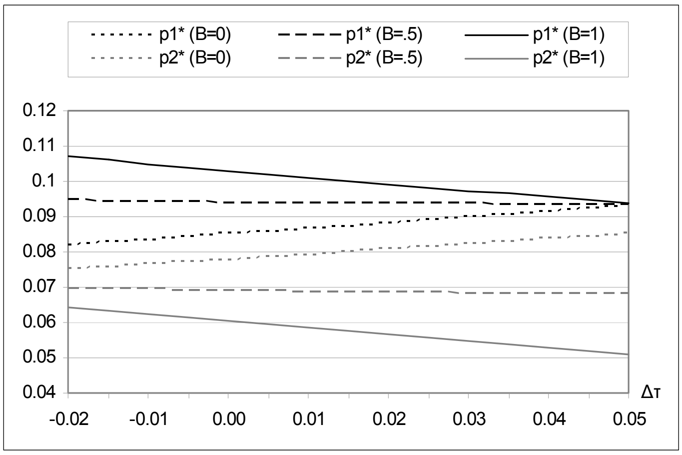

Figure 6. Equilibrium prices, increasing Δ^{τ}

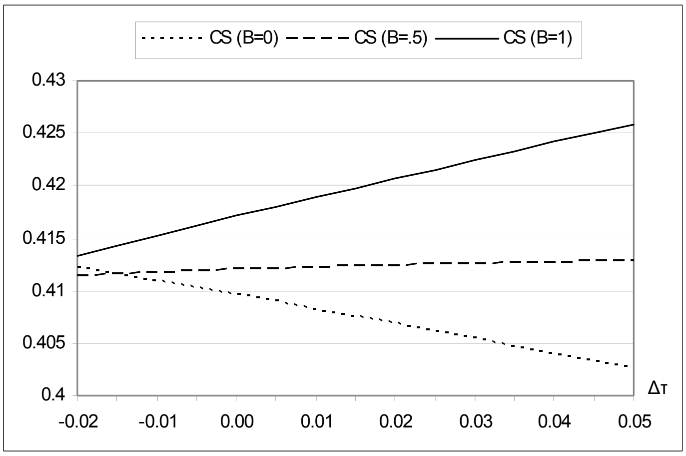

Figure 7. Consumer surplus, increasing Δ^{τ}

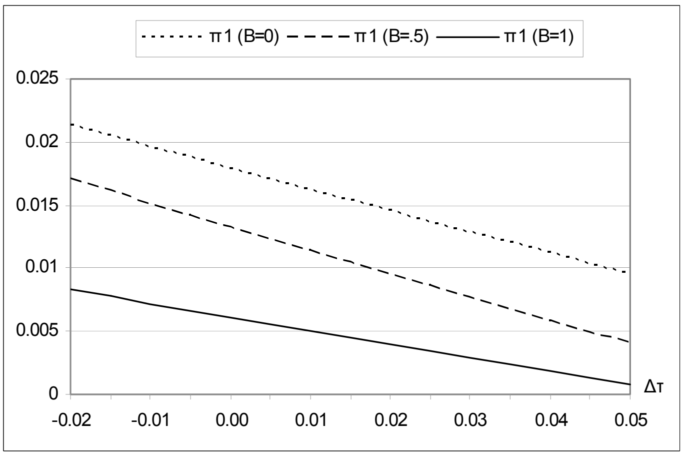

Figure 8. Maximal profit of firm 1, increasing Δ^{τ}

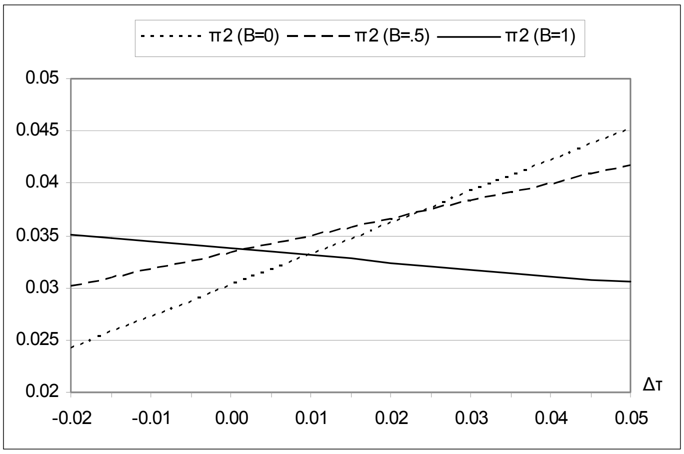

Figure 9. Maximal profit of firm 2, increasing Δ^{τ}

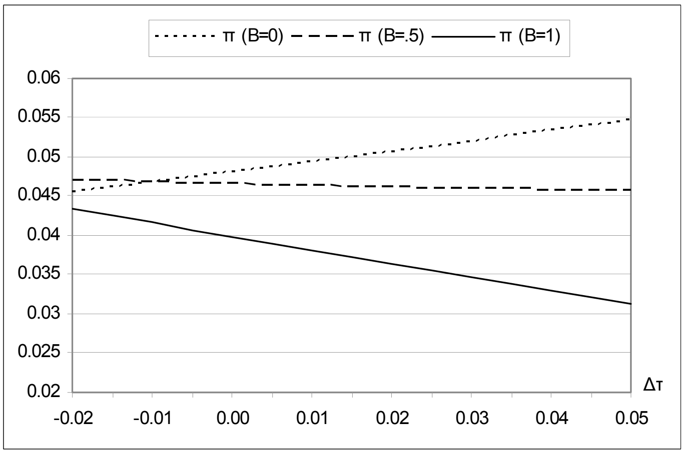

Figure 10. Maximal industry profit, increasing Δ^{τ}

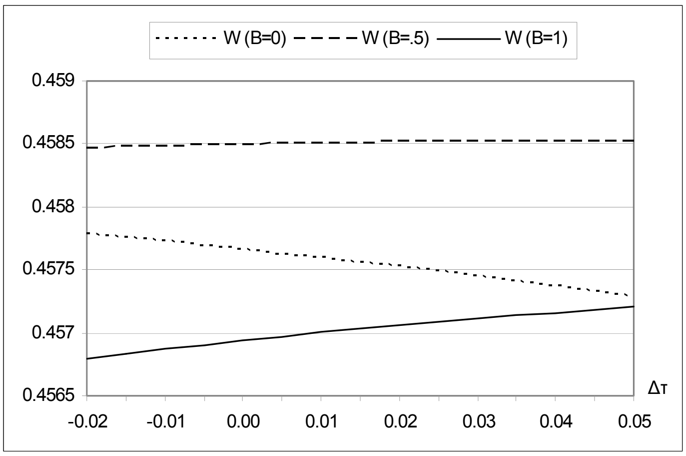

Figure 11. Social welfare, increasing Δ^{τ}

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