Wage Inequality in a Burdett-Mortensen World*

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Abstract

This paper examines the development of wage inequality in the context of a Burdett-Mortensen (1998) model that is extended to incorporate worker heterogeneity through skill requirements in the production process. In this environment, wage dispersion is a natural consequence of firms pursuing different wage strategies as well as a result of worker and firm production heterogeneity. Changes in the wage distribution are then explained by changes in the productivity of heterogeneous firms. The resulting change in theoretical steady state wage distributions as a result of changes in relative productivity is consistent with many of the observed changes in distribution of wages in the US in recent decades. In particular, an increase in the productivity of less efficient firms may reduce between-group inequality while at the same time increase within-group inequality as observed during the 1970s. On the other hand, an increase in productivity of more efficient firms will tend to increase both between- and within-group inequality as observed during the 1980s and 1990s.

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1 Introduction

There has been a well-documented increase in wage inequality in the US and the UK over the last three decades. Katz and Murphy (1992) for the US and Machin (1996) for the UK, identify this as being caused by an increase in both between- and within-group inequality, where groups are defined according to education and experience level. The increase in between-group inequality can be viewed as an increase in the returns to education. More educated workers have always tended to receive higher wages but the wage differentials between college and high school graduates expanded in the 1980s and early 1990s after a period of contraction during the 1970s. Attracting less attention, but of perhaps even more quantitative importance is the increase in within-group wage inequality. This increase in within-group inequality represents increased wage dispersion among workers with similar observable characteristics. Interestingly, the increase in within-group wage inequality begins in the early 1970s, predating the increase in between-group inequality, and continues throughout the 1980s and 1990s.

Strong empirical support has been provided for the hypothesis that skill-biased technical change has been the primary cause for the increase in between-group inequality. Proponents of this theory note that much of the increase in wage inequality coincided with the rapid diffusion of new technology and in particular, the introduction of computers into the workplace. Krueger (1993) finds a positive correlation between high-paying jobs and computer usage and provides indirect but supportive evidence that this correlation represents a causal relationship. Autor et al. (2003) identify particular tasks which theory suggests that computers are complementary with and tasks for which they are substitutable. Data indicate that skilled labour demand increased in tasks in which computers are complementary and unskilled labour demand decreased in tasks for which computers are substitutable, providing further evidence of the role of computers in changing labour demand.

It is clear how skill-biased technical change can bring about an increase in between-group inequality but it is only relatively recently that macroeconomists have devoted attention to explaining the causes of the observed increase in within-group inequality. Aghion (2002) and Violante (2002) argue that the nature of technological progress may be able to simultaneously explain the evolution of both between- and within-group wage inequality. These papers rely upon the slow diffusion of technology and vintage-specific skills that are only partially transferable across different vintages. In such an environment, increased technological growth leads to increased within-group wage inequality as a result of workers requiring luck to find employment at high-tech, high wage firms. In addition, increased technological growth in an environment with limited skill transferability leads to an increase in the dispersion of skills within the economy and hence wages within groups with similar observable characteristics.

This paper provides an alternative explanation for how technical change affects both between- and within-group inequality, by examining in a wage-setting environment the response of firms' wage policies to technological change. In such an environment, it is demonstrated that skill-biased technological change can increase both within- and between-group inequality, consistent with changes in the wage structure during the 1980s and 1990s. On the other hand, increases in productivity that favour low-skilled workers may simultaneously increase within-group wage inequality for at least some groups, while decreasing between-group wage inequality as experienced during the 1970s. Taking search frictions and wage policies of firms into account, can then provide rich wage dynamics that help explain some of the features of the evolution of the distribution of wages.

A natural setting to examine the interaction between productivity, wage-setting and wage inequality seems to be the framework developed by Burdett and Mortensen (1998), hereafter BM. They noted that when firms post wages, they face a tradeoff between attracting labour and dividing the surplus generated from employment. Some firms may follow a relatively aggressive wage strategy, offering higher wages to attract labour more quickly and retain labour for longer periods but receive a smaller share of the surplus. In contrast, other firms may pursue a relatively passive strategy of offering low wages. These firms receive a larger share of match surplus but attract and retain a smaller share of the labour force. In equilibrium, there is wage dispersion since firms with identical characteristics, resolve this wage-setting tradeoff by pursuing different wage strategies.

However, the original BM model was designed to explain wage dispersion among individuals with identical skills. This homogeneity in worker skills makes it inappropriate to address issues such as wage inequality, where between-group differences are so important. Hence this paper modifies the BM model to incorporate worker production heterogeneity through the assumption of skill requirements in production tasks. In particular, we assume that workers are required to have a certain level of skills to be able to work for certain firms. The greater the productivity of a firm, the greater the necessary skill level of the worker required. It is the introduction of worker heterogeneity in skill levels that allows discussion of within- and between-group inequality in a meaningful manner. Importantly, the model still remains tractable. With a continuum of worker and firm types, sufficient conditions for the existence of a monotone equilibrium, where more productive firms post higher wages, are derived. Furthermore, solutions to the distribution of wages received by workers, the distribution of wages offered by firms, and measures of between- and within-group inequality are provided for given parameter values. This framework also leads to some intuitive results. More skilled workers have higher average wages, lower unemployment rates and greater within-group inequality, as observed in the data. Furthermore, more productive firms offer higher wages and attract workers more rapidly, conditional on workers having the required skill level.

In this environment, the effects of changes in productivity are examined in detail. When faced with a productivity shock, firms that pursue different wage strategies will react in different ways. In particular, firms that rely upon receiving a large share of match surplus and only attracting a small share of labour have little incentive to increase their wage rate in response to increased productivity. Contrastingly, firms that pursue an aggressive, high-wage strategy of attracting many workers, need to increase their wage by a greater amount to retain a similar sized workforce. Hence a productivity increase that affects a set of firms will tend to increase the dispersion of wage offers within that set. This dispersion in wage offers leads naturally to an increase in wage inequality.

Within the model, skill-biased technical change is naturally viewed as an increase in productivity of the more productive firms. For the reasons described above, this results in an increase in dispersion of wage offers from more productive firms and leads to an increase in between-group inequality and increased within-group inequality for more skilled workers who are employed by this set of firms. These movements in inequality are broadly consistent with the observed movements during the 1980s. The impact of a productivity increase among less productive firms is also examined. This increases the average level and the dispersion of wages offered by firms with lower productivity. The impact on the observed wage distribution of workers is to reduce between-group inequality and increase within-group inequality, at least among less skilled workers. Hence, considering the interaction of productivity shocks with the response of wage-setting strategies provides rich dynamics that are able to capture some stylised features of the evolution of the wage structure.

The most similar paper to this one, in terms of emphasing the role of wage determination in explaining wage inequality is Shi (2002). Shi also examines the wage setting behaviour of firms in response to productivity changes and provides broadly similar results however there are some important differences. Most importantly, Shi generates within-group wage inequality only among low skilled workers while the model considered in this paper generates within-group inequality among all types of workers. This is crucial since the impact of productivity shocks has different implications for within-group inequality depending upon skill level. Thus our model is able to generate additional implications regarding the evolution of wage inequality that are absent from Shi's work. For example, this paper suggests that during periods of skill-biased technical change, within-group wage inequality should increase primarily among the skilled workforce since firms compete more for skilled labour. This is shown to be supported in the data in recent empirical work by Lemieux (2004) that illustrates using CPS data that within-group inequality increased more for the college graduates than for high school graduates over the last two decades.

Finally, we feel that this paper makes an important contribution in extending the BM model to an environment in which worker heterogeneity exists. To our knowledge, the only other paper that considers ex ante heterogeneity in such a framework is Postel-Vinay and Robin (2002) although the wage-setting environment they examine is quite different, with firms engaging in Bertrand competition when determining wages. This paper shows that by modeling worker heterogeneity as skill requirements in the production process that the model remains tractable and will hopefully provide an avenue for future empirical research.

The next section outlines a simple two-worker, two-firm theoretical model that features both between- and within-group wage inequality. The third section extends the model to a continuum of workers and firms. Following that, the wage dynamics associated with productivity changes are analysed in the fourth section and it is discussed how technological change that favours less productive firms may be consistent with rising within-group inequality and decreasing between-group inequality as observed in the 1970s and how technological change that favours more productive firms increases both between- and within-group inequality. The final section concludes.

2 The Basic Model

This section extends the standard BM model of wage dispersion to incorporate two-sided heterogeneity to allow analysis of wage inequality. Begin by considering the case where there are only two types of firms and two types of workers. A firm type is denoted by j where $j \in \{H, L\}$ and the objective of both types of firms is to maximise the discounted present value of future profits. To produce output a firm must be matched with a worker in which case p_j is the value of output produced per unit of time. Assume without loss of generality that H firms are more productive so $p_H > p_L$. There is a continuum of risk neutral workers with measure normalized to size one, who maximise the present value of discounted future consumption. Heterogeneity among the workforce is modelled by having two different types of workers, where a worker's type is denoted by $i \in \{S, U\}$, where S denotes skilled workers while U denotes unskilled workers. Assume that the mass of workers who are of type U is equal to α while the mass of S workers is $1 - \alpha$. Worker heterogeneity is assumed to take the form of skill requirements in the production process, with different types of workers capable of matching with different types of firms. In particular, S workers are able to match with both H and L firms, while U workers are assumed to only be able to match with L

firms.

Time is continuous and matching between workers and firms is a time consuming and stochastic process. Workers are obviously able to match with firms when unemployed but are also allowed to search on-the-job. It is assumed that the opportunity for a worker to match with a type L firm arrives with a Poisson probability of λ_L and with a Poisson probability of λ_H for H firms. The rate at which possible matches arrive is taken as exogenous and reflects the relative number of firms in the economy and their effectiveness in matching with workers. For simplicity, the matching rate of workers is independent of their employment status. In addition, once a match is formed, it is exogenously destroyed with a Poisson probability of δ.

As is standard in search models, once a match is formed there is positive surplus to be divided between a worker and a firm. This division of surplus is determined via a wage determination mechanism. A common assumption is to impose a Nash Bargaining Solution so that wages divides the match surplus according to some exogenous parameter that depends upon the relative bargaining strength. The BM model moves away from this paradigm by considering a wage-posting environment. In particular, firms announce the wage they are offering, w, prior to meeting a worker. The distribution from which type L and H firms draw their wage offers from will be denoted $F_L(w)$ and $F_H(w)$, respectively. In equilibrium, the wage that a firm offers will maximise profits conditional upon the wage offers of other firms and the behaviour of workers.

When a worker meets a firm, the worker is faced with the decision of accepting the job offer at the announced wage rate of w, or rejecting the offer and continuing in his current employment state. Assuming there is no disutility associated with work and because the rate at which workers match with firms is independent of employment status, it is straightforward to verify that the optimal strategy for employed workers is to accept employment from firms that post wages above their current wage rate and reject wage offers below that level. For unemployed workers, the optimal strategy is to accept any wage offer that exceeds their reservation flow utility.[1]

Given the exogenous matching rates and that workers follow optimal strategies, it becomes possible to solve for the steady state unemployment and the distribution of wages in the economy conditional on the wage offer curves, $F_L(w)$ and $F_H(w)$. The level of unemployment of worker type i will be denoted u_i and the fraction of employed type i workers, that earn a wage less than w will be denoted by $G_i(w)$. Denote the corresponding steady state values, u_i^* and $G_i^*(w)$. With exogenous matching

[1] This result is shown formally by Burdett and Mortensen (1998). Further, it is straightforward to generalise to the case in which there is a constant disutility of work of b across all types of employment. In this case, unemployed workers will accept employment if the wage offered exceeds b and employed workers will continue to accept offers from firms that offer the higher wages.

rates λ_L and λ_H and job destruction rate, δ, the following relationships describe the evolution of the unemployment for different skill types,

$$\begin{aligned} \dot{u_U} &= \delta(\alpha - u_U) - \lambda_L u_U \\ \dot{u_S} &= \delta(1-\alpha-u_S) - (\lambda_L + \lambda_H)u_S. \end{aligned}$$

The first term of each equation represents the flow of employed workers into unemployment as a result of exogenous match destruction. The second term represents the flow of unemployed workers into the employed workforce. In a steady state, $\dot{u_i} = 0$, which implies,

$$u_U^* = \frac{\alpha\delta}{\delta + \lambda_L} \tag{1}$$

$$u_S^* = \frac{(1-\alpha)\delta}{\delta + \lambda_L + \lambda_H}. \tag{2}$$

It is possible to derive steady state distribution of wages observed in the economy for employed workers of type i, $G_i^*(w)$, in a similar fashion. In particular, conditional upon the distribution of wage offers that firms are making, the evolution of the distributions of wages satisfies the following,

$$\begin{aligned} \dot{G_U}(w) &= \lambda_L F_L(w) u_U - (\delta + \lambda_L(1 - F_L(w)))(\alpha - u_U)G_U(w) \\ \dot{G_S}(w) &= (\lambda_L F_L(w) + \lambda_H F_H(w))u_S \\ &\quad -(\delta + \lambda_L(1-F_L(w)) + \lambda_H(1-F_H(w)))((1-\alpha) - u_H)G_H(w). \end{aligned}$$

The first term in the each of the above equations reflects the inflow of workers from unemployment into the workforce at wages below w. The second term represents the outflow of workers from employment at wages below w. Some of this outflow occurs due to exogenous job destruction but a component of it occurs due to workers transiting to firms offering higher wages. In a steady state equilibrium, $\dot{G_i}(w) = 0$, which implies,

$$G_U^*(w) = \frac{\lambda_L F_L(w) u_U^*}{(\delta + \lambda_L(1-F_L(w)))(\alpha - u_L^*)} \tag{3}$$

$$G_S^*(w) = \frac{(\lambda_L F_L(w) + \lambda_H F_H(w))u_S^*}{(\delta + \lambda_L(1-F_L(w)) + \lambda_H(1-F_H(w)))((1-\alpha) - u_S^*)}. \tag{4}$$

Having derived the steady state distribution of workers conditional upon the distribution of wages offered, it is also possible to describe the profits earned by firms in this environment. The expected flow profit of a firm with a vacancy that offers a wage of w depends upon the rate at which a firm is able to hire workers. In equilibrium, workers follow an optimal strategy of accepting employment at firms that offer wages above their current wage. Hence, firms offering a wage of w are able to attract workers from the stock of the unemployed labour force and from workers who are employed but currently receive a wage less than w. As a result, the probability of a worker of type i accepting a wage offer once contacted is defined as $h_i(w)$ and can be expressed as the following,

$$\begin{aligned} h_U(w) &= u_U + (\alpha - u_U)G_U(w) \\ h_S(w) &= u_S + (1 - \alpha - u_S)G_S(w). \end{aligned}$$

Here it becomes explicit that offering higher wages will attract workers more rapidly since $G_i(w)$ is an increasing function of w. There is of course a tradeoff involved, firms that offer higher wages will receive less of the surplus from any matches that are formed. The steady state flow Bellman equations can be used to solve for the value of a firm with productivity p_j that employs a type i worker paid a wage of w. This value is denoted $J_i(p_j, w)$ and is given below,

$$\begin{aligned} J_U(p_j, w) &= \frac{p_j - w}{r + \delta + \lambda_L(1 - F_L(w))} \\ J_S(p_j, w) &= \frac{p_j - w}{r + \delta + \lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w))}. \end{aligned}$$

The expected value to a firm of employing a type j worker is the flow profit, $p_j - w$, discounted by the interest rate and the rate at which workers are expected to leave the firm. This rate depends upon the exogenous rate of job destruction, δ, as well as the rate at which workers receive outside wage offers of greater value, $\lambda_L(1 - F_L(w))$ for unskilled workers and $\lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w))$ for skilled workers.

Using the above equations, it is possible to calculate the expected equilibrium flow profit of a vacancy for a firm of type j offering a wage of w. Denote this value as $\pi_j^*(w)$, then the following equations hold,

$$\pi_H^*(w) = h_S(w)J_S(p_H, w)$$
$$\pi_L^*(w) = h_S(w)J_S(p_L, w) + h_U(w)J_U(p_L, w).$$

The expected flow profit of a vacancy for a high productivity firm is the probability of hiring a skilled worker multiplied by the value of hiring a skilled worker. Low productivity firms on the other hand are able to employ both skilled and unskilled workers so the flow profit of a vacancy depends upon the rate at which both types are hired as well as the value of hiring each type of worker. It is possible to use the above relationships to solve for expected profit as a function of the exogenous parameters and the endogenous wage offer curves, $F_L(w)$ and $F_H(w)$. Assuming for simplicity that the interest rate approaches zero, some straightforward algebra leads to the following,

$$\pi_H^*(p_H, w) = \frac{\alpha\delta(p_H - w)}{[\delta + \lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w))]^2} \tag{5}$$
$$\pi_L^*(p_L, w) = \frac{\alpha\delta(p_L - w)}{[\delta + \lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w))]^2} + \frac{(1-\alpha)\delta(p_L - w)}{[\delta + \lambda_L(1 - F_L(w))]^2}. \tag{6}$$

The first equation describes the steady state profits of high productivity firms from hiring skilled workers by offering a wage of w and is analogous to the profit functions derived by BM. The second equation describes the steady state profits of low productivity firms. The first term captures the expected profits from hiring a skilled worker while the second term represents the expected profits from hiring an unskilled worker.

It now becomes possible to define a suitable steady state equilibrium concept.

Definition 1. *A steady state equilibrium describes the distribution of workers across different states, the wage offer distributions offered by different firms and the profits made by different firms,* $(u_U^*, u_S^*, G_U^*(w), G_S^*(w), F_L^*(w), F_H^*(w), \pi_L^*(w), \pi_H^*(w))$, *that satisfies the following conditions:*

- *the steady state allocation of workers across states satisfies equation (1) through (4),*
- *firms are maximizing profits, so that the profits associated to firm of type i from offering a wage on the support of F_i are at least as high as profits from any other w.*

The above definition ensures that the distribution of workers across different wage rates and between employment and unemployment is unchanging when workers are transiting optimally between different states. The second condition of equilibrium simply ensures that firms are setting wages in an optimal fashion to maximise profits.

It is straightforward to see how this model could be extended to a situation with any finite number of worker and firm types although we do not elaborate upon this point, our first result relates to the existence of equilibrium in such a general setting and is stated below.

Proposition 1. *In any game with a finite type space there exists a steady state equilibrium.*

To prove this proposition we first assume that there are only finitely many possible wage levels that may be offered. Then this game becomes a finite game, which has a (stationary) equilibrium. After taking a limit of such finite games where the wage space becomes dense in the real numbers the limit of those equilibria is shown to be an equilibrium of the original game without any restriction on the wage space. The details are in the Appendix.

The basic BM model considers a similar environment, with the exception that workers and firms are homogenous and that there are no skill requirements. One of the key insights of BM is that the distribution of wages offered by firms is continuous. A similar argument applies in this framework. In particular, the wage offer distribution of the whole economy, which is the weighted wage offer distribution of H and L firms does not display any mass points or gaps in the distribution. If a discrete mass of firms offered a wage w', one of these firm could deviate and offer a marginally higher wage rate $w' + \epsilon$, increasing the supply of labour by a discrete amount while only having a minimal effect on the profit provided by a unit of employed labour and hence would increase total profits. Similarly, there are no gaps in the distribution of wages offered by firms in this economy, otherwise firms offering wages immediately above this gap, could reduce wages and increase per worker profit without altering the ability to attract or retain labour. Hence the distribution of wage offers is continuous.

The BM model with firm heterogeneity but without skill requirements features a unique equilibrium in which the wages of firms with higher productivity exceed wages offered by low productivity firms. For example, in the simplest environment with only two firm types, the low productivity firms only offer wages below some cutoff $\hat{w}$ and high wage firms only offer wages above $\hat{w}$. We will define an equilibrium of this type, in which the wage offers of different types of firms do not overlap as a separating equilibrium. These separating equilibrium exist in a world without skill requirements because the marginal cost of increasing the wage is identical across different types of firms the marginal benefit is strictly greater for more productive firms. Hence, more productive firms will be willing to offer higher wages to attract larger labour forces.

However, in a world with skill requirements this intuition regarding marginal cost and marginal benefit no longer applies. Firms now offer wages to compete for different workforces, as some firms are able to employ both skilled and unskilled labour while others only hire skilled labour. As a result, both the marginal benefit and the marginal cost of increasing wages will depend upon firm type. The following result indicates that the possibility of the existence of a separating equilibrium depends upon the parameter values.

Proposition 2. *If the difference in p_H and p_L are close enough, then H firms will have an incentive to deviate from a separating equilibrium and offer wages less than w^*.*

The proof of this result is contained in the appendix but essentially, the intuition is that firms are competing for labour services via wages. If the productivity difference between firms is large enough, the more productive firms will be willing to offer higher wages to attract skilled labour away from the less productive firms. However, if the productivity difference is small, some less productive firms may be willing to offer higher wages than some H firms as they compete against each other for the services of both types of labour. In the following section, sufficient conditions are provided under which an equilibrium where more productive firms offer higher wages will exist when there are a continuum of worker and firm types.

3 Continuum of Firm and Worker Types

The section extends the model to deal with a continuum of worker and firm types. To do so, define $v \in [0,1]$ be the type of a worker and $V(v)$ the corresponding atomless, strictly increasing continuous distribution function. Similarly, let $p \in [0,1]$ denote the type of firm and $P(p)$ the corresponding atomless, strictly increasing, continuous distribution function. Also, assume that the density functions exist, are positive and are bounded. Extending the concept of skill requirements used in the previous section, it will be assumed that a type p firm can employ a worker with type $v \geq p$ to produce output $R(p)$, where R is a strictly increasing, continously differentiable and $R(0) = 0$. The unit of time will be normalised so that the arrival rate of job offers, λ is equal to one and δ remains the rate of job destruction.

This section will focus upon determining necessary and sufficient conditions for the existence monotone equilibrium, in which $w(p)$ is strictly increasing in p. Furthermore, the equilibrium, if it exists is characterised. Let

$$H_v(w) = Pr(wage \leq w | type = v)$$

be the distribution function of the wage (in a stationary equilibrium) of a type v worker. Since we consider equilibrium in which $w(p)$ is strictly increasing, and the following notation can be introduced:

$$T_v(p) = Pr(wage \leq w(p) | type = v).$$

This yields the probability that a worker has a wage less than $w(p)$. Analogous to the BM case, the law of motion is such that

$$\dot{T_v(p)} = (1 - T_v(p))\delta - T_v(p)(P(v) - P(p))$$

when $v \geq p$ and $\dot{T_v(p)} = 0$ otherwise. The first term defines the flow of workers into unemployment from workers receiving a wage greater than w, while the second term describes the flow of workers from wages below $w(p)$ to wages above. In equilibrium, this implies

$$T_v(p) = \frac{\delta}{\delta + P(v) - P(p)}$$

if $v > p$ and $T_v(p) = 1$ if $v < p$ since a worker will never obtain an offer above $w(v)$ which is less than $w(p)$.

Note, that the probability of being unemployed for a type v worker, $u(v)$ is such that

$$u(v) = H_v(0) = T_v(0).$$

Note that this implies the total level of unemployment in the economy is given by the following,

$$U = \int_0^1 \frac{\delta\, V'(v)}{\delta + P(v)}\, dv.$$

The above analysis describes the distribution of workers in the economy across different firms and employment states. We now focus upon the wage-setting decision of a firm. In particular, the objective function of a firm with type p who bids $w(\hat{p})$ can be written as follows:

$$\pi(p,\hat{p}) = (R(p) - w(\hat{p}))M(p,\hat{p}),$$

where,

$$M(p,\hat{p}) = \begin{cases} \int_p^1 \frac{V'(v)\delta}{(\delta+P(v)-P(\hat{p}))^2}\,dv & \text{for } p > \hat{p} \\ \int_{\hat{p}}^1 \frac{V'(v)\delta}{(\delta+P(v)+P(\hat{p}))^2}\,dv + \int_p^{\hat{p}} \frac{V'(v)\delta}{\delta^2}\,dv & \text{for } p < \hat{p}. \end{cases}$$

is the expected time, possibly zero, that the firm employs a worker if it bids $w(\hat{p})$ and has type p. The first order condition, that should hold as an equality at $p = \hat{p}$ implies that in any equilbrium,

$$(R(p) - w(p))M^{(2)}(p,p) - w'(p)M(p,p) = 0$$

or equivalently,

$$w'(p) = (R(p) - w(p))\frac{M^{(2)}(p,p)}{M(p,p)} \tag{7}$$

where $M^{(2)}(p,\hat{p})$ is the derivative of M with respect to the second variable. Since $R(0) = 0$, it is also known that $w(0) = 0$. If a monotone equilibrium exists, profit maximisation implies $w(p)$ must satisfy the differential equation implied by the first order condition in equation (7) as well as the boundary condition of $w(0) = 0$. Defining $\tau(p) = \frac{M^{(2)}(p,p)}{M(p,p)}$, the solution to this problem is given by the following,

$$w(p) = \frac{\int_0^p R(x)\tau(x)Exp\left(\int_0^x \tau(u)du\right)dx}{Exp\left(\int_0^p \tau(u)du\right)}. \tag{8}$$

This function provides the solution for the wages posted by firms as a function of productivity when a monotone equilibrium exists. The sufficient second-order conditions that ensure existence are given below,

$$\pi^{(2)}(p,\hat{p}) < 0 \quad \text{if } p < \hat{p}$$
$$\pi^{(2)}(p,\hat{p}) > 0 \quad \text{if } p > \hat{p}.$$

The first condition implies that a firm of type p that deviates to a higher wage, $w(\hat{p})$, has an incentive to reduce the wage offer towards $w(p)$. The second condition implies that a firm of type p that deviates to a lower wage has an incentive to increase the offered wage. To facilitate the analysis, it is assumed that $V(x) = P(x)$, that is the distributions of worker skill levels, coincides with the distribution of firm skill requirements. Then, one may assume that $V(x) = P(x) = x$, without loss of generality.[2] The appendix proves the following theorem.

Proposition 3. *When types are normalised such that $V(x) = P(x) = x$, a sufficient condition for the existence of a unique monotone equilibrium is that $R(p)$ is a (weakly) convex function.*

It can also be verified that a monotone equilibrium may not exist. This result should not be too surprising, since in the two-type model, when firms with higher types do not have a large enough productivity advantage, a separating equilibrium fails to exist. In the above proposition, the convexity of $R(p)$ ensures that the productivity differences between firms are large enough.

Under the conditions described in the theorem, it becomes possible to characterise the distribution of wages in a monotone equilibrium. Begin by defining the following, $w(p) = w$ and let $p = w^{-1}(w)$, be the corresponding inverse function. Then,

$$H(w) = Pr(wage \leq w)$$

for an arbitrary worker. It then follows that integrating across worker types that,

[2]To see this start with a model with general (but equal) distribution functions. Then the distribution function of the maximum production a worker is capable of is

$$\Pr(R(v) \leq y) = V(R^{-1}(y)).$$

Then consider a model where the distributions are uniform and the production function is

$$\widetilde{R}(p) = R(V^{-1}(p)).$$

Then

$$\Pr(\widetilde{R}(v) \leq y) = \widetilde{R}^{-1}(y) = V(R^{-1}(y)),$$

which implies that the two specifications describe the same model just rescaling variables.

$$H(w) = \int_0^1 V'(v) H_v(w)\, dv$$

and the distribution of wages for employed workers is given by the following,

$$\widetilde{H}(w) = \frac{H(w) - U}{1 - U},$$

where recall that U defines the level of unemployment. Define the corresponding density functions, $H'(w)$ and $\widetilde{H}'(w)$. Returning to the special case in which $V(x) = P(x) = x$, the appendix derives the following results regarding the density functions,

$$H'(w) = \frac{1 - w^{-1}(w)}{w'(p)(\delta + 1 - w^{-1}(w))} \tag{9}$$

and

$$\widetilde{H}'(w) = \frac{H'(w)}{1 - U}.$$

For the case of $R(p) = p$ it is possible to show that the cumulative distribution function $H(w)$ must be concave. This contrasts with the convex wage distribution derived by Burdett and Mortensen (1998) for their case of homogenous productivity. The within-group wage distribution, that is the distribution for a particular type of worker may still display convexity, but aggregation "smoothes things out".

Using the above information, the appendix details how the expected wage of a type v worker, conditional upon employment may be described as the following,

$$\widetilde{E}_v(w) = \int_0^v \frac{\delta(\delta + v) w(p)}{(\delta + v - p)^2 v}\, dp.$$

Similarly, the variance of wages of a type v worker conditional upon employment is given by,

$$\widetilde{Var}_v(w) = \int_0^v \frac{\delta(\delta + v) w^2(p)}{(\delta + v - p)^2 v}\, dp - \left(\int_0^v \frac{\delta(\delta + v) w(p)}{(\delta + v - p)^2 v}\, dp \right)^2$$

It is possible to perform a variance decomposition exercise to analyse how much of the variance in the wage distribution is caused by variation in v and how much is due to frictions in search:

$$\widetilde{Var}(w) = E[\widetilde{Var}_v(w)] + Var[\widetilde{E}_v(w)].$$

The first component is the variance caused by within-group dispersion, while the second is caused by the change in the underlying productivity of different workers. In the simple case where $R(x) = x$ and $\delta = 0.05$, the first component is 0.011 while the second component is 0.065, indicating that search frictions only explain a modest share of total wage variability.

The empirical literature, discussed within Mortensen (2003) demonstrates that the observed probability density function of wages approximates a log-normal distribution. This contrasts with the BM model, in that the case with homogenous productivity generates a convex CDF for wages with a corresponding increasing pdf. When extended to heterogeneous firm productivity without skill requirements, there exist underlying distributions of firm productivity that can broadly explain the observed wage distribution with an internal mode, but rely upon the distribution of firm productivity being skewed with a long right tail.

A number of probability density functions associated with different revenue functions are summarised in Figure 1. Here, it is apparent that the model with an underlying uniform probability density function for worker and firm types generates a decreasing pdf or equivalently a concave CDF. This seems reasonable at the upper end of the wage distribution but comes at the expense of generating other problems. In particular, the pdf of wages, now features a large proportion of workers earning low wage. Although it is possible to generate an interior mode this only occurs for relatively high values of δ which imply a high level of unemployment within the economy. This implies that perhaps incorporating skill requirements in the production process among firms with high productivity may be valuable but perhaps less relevant for explaining the distribution of wages among low productivity firms.

4 Impact of Productivity Changes

The previous section derives an equilibrium wage distribution in the presence of a continuum of heterogeneous firms and workers. Wage dispersion arises as a natural consequence of a combination of firms pursuing different wage strategies and heterogeneity between firms and workers. This section investigates the impact of changes in firm productivity upon the wage distribution and relates these results to historical movements in wage inequality. Two scenarios are examined. Firstly, the impact of skill-biased technological change is examined and the impact upon wage and inequality is evaluated. Secondly, an increase in productivity that favours firms that are less productive is also examined. For the following examples, attention is restricted to the case in which $P(x) = V(x) = x$ and $\delta = 0.05$. This generates a rather high unemployment rate of 15 per cent. Although the probability of job destruction is small relative to the arrival rate of matches, some workers are ineligible for most jobs in the economy and have an exit rate from unemployment of close to zero. This raises the question as to whether unemployment within the model should be interpreted as encompassing both unemployed workers and the population not participating in the labour force.

4.1 Technical change favouring productive firms

It is natural to think of skill-biased technical change as an increase in the function $R(p)$ for more productive firms. Essentially, this is equivalent to R becoming more convex. This section analyses the impact upon the distribution of wages of moving from a case where $R(p) = p$, to the following revenue function,

$$R(p) = \begin{cases} p & \text{if } p < 1/2, \\ \frac{4}{3}p - \frac{1}{6} & \text{if } p \geq 1/2, \end{cases}$$

which increases the slope of the revenue function for more productive firms ($p > 1/2$) but has no impact upon less productive firms. This productivity change does not have an impact upon behaviour of firms with $p < 1/2$ or consequently, workers with $v < 1/2$. The economic outcomes of this set of agents remains unchanged, with the wages offered by these firms, their ability to attract workers and the distribution of wages received by workers unaffected.

There are however important changes in behaviour of firms with $p > 1/2$. This is captured in Figure 2, which displays the wage offers of firms with different productivities and their response to the increase in $R(p)$. The increase in revenue for these firms makes them more aggressive in attracting workers and leads to higher posted wages. This increase in wage offers that follows a productivity increase is somewhat futile, in the sense that no firm manages to attract a greater labour force on average. Despite this, all firms with $p > 1/2$ must increase their wage or risk losing their labour force to competitors.

This increase in wage offers, flows through to effect other aspects of the economy. In particular,

Figure 3 displays how the increase in productivity has an impact upon expected wages of different types of workers conditional on being employed. Since firms with ($p < 1/2$) do not change their wage offers, the equilibrium expected wages of workers with $v < 1/2$ remain unchanged. However, workers with $v > 1/2$ now receive higher wages from the more productive firms and this leads to an increase in the expected wages that these workers receive in equilibrium. This is clearly an increase in the degree of between-group inequality, with more skilled workers now being paid relatively higher wages than previously.

Similarly, the increase in wage offers flows through to increase the variance of wages that workers receive. Figure 4 displays the variance of wages of a type v worker, conditional on employment under the alternative equilibrium. Since low productivity firms offer unchanged wages, there is no impact upon the variance of wages of workers of type $v < 1/2$. However, technical change that increases productivity of more productive firms leads to an increase in the variance of wages of more skilled workers. This is consistent with an increase in within-group inequality and suggestive that periods of technical change favouring productive firms should lead to increased within-group inequality among more skilled workers and have relatively small impact upon unskilled labour.

4.2 Technical change favouring less productive firms

While skill-biased technical change is naturally thought of as an increase in the convexity of R, technical change favouring low-productivity firms is naturally thought of as a decrease in convexity. Hence, this considers the case of an economy moving from a revenue function of

$$R(p) = \begin{cases} \frac{2}{3}p & \text{if } p < 1/2, \\ \frac{4}{3}p - \frac{1}{3} & \text{if } p \geq 1/2, \end{cases}$$

to a revenue function in which $R(p) = p$. In this situation, the distribution $R(p)$ becomes less convex. Unlike the previous case all firms adjust their wage offers to some degree. The equilibrium wage-setting schedules are displayed in Figure 5. Again, the increase in productivity makes firms more aggressive in posting wages. Despite this increase in wages, the ability of an individual firm to attract labour remains unchanged. Note even though there is no productivity change for firms of type $p = 1$, they must increase their wages, since other firms are competing more aggressively.

The impact upon the expected wage received by workers conditional upon employment is displayed in Figure 6. The increase in the expected wage of workers is a direct result of the increase in wage offers. If workers are divided into two groups; those with $p < 1/2$ and those with $p > 1/2$, it can be shown that there is a decrease in between-group inequality with the relative ratio of high skilled to low skilled wages getting smaller.

Finally, Figure 7 displays the impact of the change in wage offers upon the variance of wages of workers according to type, conditional on employment. Here we get the interesting result that within-group inequality increase for the workers in the middle and lower-end of the skill distribution, but decreases for workers at the top-end of the wage distribution. This decrease in within-group inequality at the top-end of the distribution follows since search frictions imply that skilled workers will be distributed across firms of many different types. An increase in the wages offered by low productivity firms relative to high productivity firms will increase the wages of skilled workers in the less productive firms, bringing their wages closer to the mean value and hence reducing within-group inequality. On the other hand, the within-group inequality of less skilled workers will tend to increase. Since less skilled workers are unable to earn higher wages, an increase in dispersion of wages offered by low productivity firms will naturally result in an increase in within-group inequality.

These results illustrate that taking into account the wage-setting strategies of firms can produce rich dynamics that may help explain developments in wage inequality. In particular, skill-biased technical change tends to increase simultaneously both between- and within-group wage inequality. In contrast, an increase in productivity of less productive firm tends to reduce between-group inequality but can have differential effects upon within-group inequality. In particular, within-group inequality of more skilled workers may decrease while within-group inequality of less skilled workers may increase.

5 Conclusion

This paper demonstrates that incorporating a reasonable mechanism for wage determination in a labour market with frictions, helps provide a plausible explanation of changes in the wage distribution in the US over the last three decades. This is shown by extending the BM model to incorporate production heterogeneity among workers, enabling discussion of between-group wage inequality. This environment allows firms to adjust their wages optimally to productivity shocks and creates rich wage dynamics and discussion of between- and within-group inequality.

The effect of two types of productivity shocks are examined and it is found that an increase in the productivity of less efficient firms may reduce between-group wage inequality while simultaneously increasing within-group wage inequality. This is consistent with the dynamics of the wage structure of the US in the 1970s. On the other hand, increases in the productivity of more efficient firms increases both between- and within-group wage inequality which corresponds to the development of wages in the US in the 1980s and 1990s.

From a theoretical perspective, this paper takes some important steps in characterising the nature of equilibria in a BM model with worker production heterogeneity introduced through skill requirements in the production process. In the case of a continuum of worker and firm types, the sufficient conditions for a monotone equilibrium to exist were derived and the wage distribution was characterised in this case.

There are some final points to note. In particular, the model produces testable implications that are distinct from the implications produced by Shi (2002), Aghion (2002) or Violante (2002). Periods in which between-group inequality decreased should be associated with larger increases in within-group inequality among groups with lower average wages. Hence, the period of the 1970s, in which between-group inequality decreased should be associated with larger increases in within-group inequality for high school rather than college educated individuals. Conversely, periods in which between-group inequality increases should be associated with larger increases in within-group inequality for skilled labour than for unskilled labour.

6 Appendix

6.1 Proof of Proposition 1

The proof below shows the existence of a stationary equilibrium for our model with finite types. The formal proof is given for the case of two types but it should be clear that the same argument goes through for any (finite) number of types. We do not show that a separating equilibrium exists in general, a result which is in fact not true. Indeed, if p_H and p_L are close to each other then we provide an example where a stationary equilibium does not exist. Then our proof implies that a non-separating equilibrium must exist in that example.

The proof is presented in separate steps:

Step 1: First, discretize the wage space, i.e. require that firms choose from a finite set of nonnegative wage offers $B = \{b_1, b_2, ..., b_n\}$. For concreteness, assume that this admissible wage space is the same for all firms and if a firm offers exactly the same wage that the worker currently has then the worker is going to switch jobs. To respect participation constraints of the firms an offer of 0 is allowed, i.e. $0 \in B$.

Step 2: Fix the behavior of the other firms of the industry; they use distribution functions F_L (low type firms) and F_H (high type firms) over set $B = \{b_1, b_2, ..., b_n\}$. Let S_L and S_H be the set of admissible distributions on the finite set B. Formally,

$$S_L = S_H = \{(x_1, x_2, ..., x_n) \in R^n_+ : x_1 + ... + x_n = 1\}.$$

Step 3: Assuming that all the other firms employ these strategies one can calculate the probability of a worker accepting wage w and the subsequent separation frequencies (per unit time) as well. This implies that the profit function of a firm is pinned down by F_L and F_H according to equations (5) and (6):

$$\pi_L(w) = \frac{(1-\alpha)\delta(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w)))^2} + \frac{\alpha\delta(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)))^2}$$

and

$$\pi_H(w) = \frac{(1-\alpha)\delta(p_H - w)}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H(1 - F_H(w)))^2}.$$

Step 4: Then one can define the set of optimal wage offers of a firm with type L as follows:

$$O_L(F_L, F_H) = \{w \in B : w \in \arg\max \pi_L(w)\}.$$

This set is non-empty because the firm maximizes over a finite set B. Then let us define the best reply correspondence of a firm as the set of distributions that put positive weight only on optimal wage offers in B:

$$BR_L(F_L, F_H) = \{\widetilde{F}_L \in S_L : \widetilde{F}_L(w) = 0 \ if \ w \notin O_L(F_L, F_H)\}.$$

Similar concepts are defined for firms with high type.

Step 4: Define then the correspondence, $BR = (BR_L, BR_H) : (S_L, S_H) \rightrightarrows (S_L, S_H)$.

Step 5: We will apply Kakutani's fixed point theorem. For this we need to show that the correspondence BR is nonempty, convex valued for all (F_L, F_H) and has a closed graph (i.e. upper-hemicontinuous).

Step 6: Since O_L is nonempty thus BR is nonempty as well. By construction BR is convex valued for each (F_L, F_H) since a convex combination of optimal mixed strategies is also an optimal mixed strategy. The closed graph property of BR follows from the continuity assumptions of the model.

Step 7: Then BR has a fixed point (F_L^*, F_H^*), which is (part of) a stationary equilibrium of the model with the finite wage space w, since all agents are best replying and the stationarity properties hold by construction. Now, let us extend these distributions (F_L^*, F_H^*) such that the extension (E_L^*, E_H^*) takes the same values as (F_L^*, F_H^*) on wages that are in B and at other wage levels (E_L^*, E_H^*) has a zero derivative, i.e. it is a step-function.

Step 8: Take a sequence of those games with finite wage spaces such that the limit of this sequence $(B^1, B^2, ...)$ is dense in the real numbers, i.e. essentially all possible wage levels are permitted. Take the (extended) equilibrium distribution functions $((E_L^{1*}, E_H^{1*}), (E_L^{2*}, E_H^{2*}), ...)$ along this sequence. Since these functions are all (weakly) increasing in w, so an appropriately chosen subsequence has a limit (E_L^{**}, E_H^{**}) by Helly's selection Theorem.[3]

Step 9: Finally, one needs to show that (E_L^{**}, E_H^{**}) is a (stationary) equilibrium of the original game where the wage-offer space is not restricted. One can show that distribution functions (E_L^{**}, E_H^{**}) are strictly increasing and continuous. Then they are differentiable almost everywhere and we can define the support of these distributions as wage levels where the density function is strictly positive. Take any wage level w_L in the support of E_L^{**} and we will show that w_L is a best reply if the other firms use strategies (E_L^{**}, E_H^{**}), which concludes the proof. (A similar argument can be

[3]Kolmogorov and Fomin (1970) discusses Helly's selection Theorem on p. 373. This Theorem guarantees that a sequence of non-decreasing, uniformly bounded functions on an interval has a subsequence that converges to a nondecreasing function.

used for firms with high types.)

To show this result we first use the fact that if w_L is in the support of E_L^{**} then

$$\text{for all } \epsilon > 0,\ \exists \underline{i} \text{ s.t. } i \geq \underline{i} \Rightarrow \exists w_L^i \in (w_L - \varepsilon, w_L + \varepsilon) \text{ s.t. } w_L^i \ \in Supp(E_L^{i*}). \tag{10}$$

Since in every game in the sequence firms employ best reply strategies it holds that if w_L^i is in the support of E_L^{i*} then it maximizes the payoff of firm L if the other firms use strategies (E_L^{i*}, E_H^{i*}). Formally,

$$\pi_L(w_L^i \mid (E_L^{i*}, E_H^{i*})) \geq \pi_L(w \mid (E_L^{i*}, E_H^{i*}))\ \forall w \in B^i. \tag{11}$$

Let us take a limit of such optimal wage offers w_L^i that also converge to w_L.[4] Then using continuity of the profit function implies

$$\lim_{i\to\infty} \pi_L(w_L^i \mid (E_L^{i*}, E_H^{i*})) = \pi_L(w_L \mid (E_L^{**}, E_H^{**})).$$

Also, using (11) and the fact that the wage space becomes dense as i goes to infinity we have that

$$\lim_{i\to\infty} \pi_L(w_L^i \mid (E_L^{i*}, E_H^{i*})) \geq \lim_{i\to\infty} \pi_L(w \mid (E_L^{i*}, E_H^{i*})) = \pi_L(w \mid (E_L^{**}, E_H^{**}))\ \forall w \in \mathbb{R}.$$

Putting together the last two results yields

$$\pi_L(w_L \mid (E_L^{**}, E_H^{**})) \geq \pi_L(w \mid (E_L^{**}, E_H^{**}))\ \forall w \in \mathbb{R},$$

which means that w_L is indeed a best reply in the game with unrestricted wage spaces.

6.2 Proof of Proposition 2

To see this, note in the region below w^* when there is a separating equilibrium, $F_H = 0$ and $F_L(w)$ must be such that the following is set to zero to satisfy the first order conditions,

$$\begin{aligned} \frac{\partial \pi_L}{\partial w} &= \alpha \left(\frac{2\lambda_L F_L'(w)(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H)^3} - \frac{1}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H)^2} \right) \\ &+ (1-\alpha) \left(\frac{2\lambda_L F_L'(w)(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)))^3} - \frac{1}{(\delta + \lambda_L(1 - F_L(w)))^2} \right) = 0 \end{aligned}$$

It is straightforward to show that the following must hold,

[4] Such sequence exists due to (10).

$$\frac{2\lambda_L F_L'(w)(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)))^3} - \frac{1}{(\delta + \lambda_L(1 - F_L(w)))^2} > \frac{2\lambda_L F_L'(w)(p_L - w)}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H)^3} - \frac{1}{(\delta + \lambda_L(1 - F_L(w)) + \lambda_H)^2} \quad (12)$$

but since there weighted average is equal to zero, it follows that the marginal profit that L firms receive from employing U workers must be higher than what they receive from hiring S workers. Part of the intuition being that U workers are less likely to leave and they are more likely to be attracted by a small increase in w than S workers who have greater outside options.

But this implies that for small enough p_H, that the marginal profit for H firms will be negative over the region from the reservation wage to w^*. This implies that by lowering wages, H firms will be able to expect an increase in profits and hence for small price differences H firms will not be able to sustain a separating equilibrium.

6.3 Proof of Proposition 3

The proposed monotone equilibrium is characterised by a function $w(p)$, that solves the differential equation (7) with boundary condition $w(0) = 0$. This section shows that when $V(x) = P(x) = x$ and $R(p)$ is convex, that a monotone equilibrium exists.

All we need to do is to check that global second order conditions are satisfied for this candiate equilibrium. Let $p' < p$ without loss of generality. For checking the second order conditions it is sufficient to prove that

$$\pi^{(2)}(p', p) < 0$$

and

$$\pi^{(2)}(p, p') > 0.$$

For example the second condition means that a high type that has always a marginal incentive to increase its bid from $w(p')$, which is sufficient to rule out that there is an incentive to deviate downward. The first condition works in the other direction.

Let us check the first condition. Then

$$\frac{\pi^{(2)}(p',p)}{M(p',p)} = (R(p') - w(p))\frac{M^{(2)}(p',p)}{M(p',p)} - w'(p) =$$

$$(R(p') - R(p))\frac{M^{(2)}(p',p)}{M(p',p)} + w'(p)(\frac{M^{(2)}(p',p)}{M(p',p)}\frac{M(p,p)}{M^{(2)}(p,p)} - 1).$$

Let

$$\beta = \frac{M^{(2)}(p',p)}{M(p',p)}\frac{M(p,p)}{M^{(2)}(p,p)} - 1.$$

Then

$$M(p',p) = \int_{p'}^{p} \frac{\delta}{\delta^2} dv + \int_p^1 \frac{V'(v)\delta}{(\delta + P(v) - P(p))^2} dv > M(p,p)$$

$$M^{(2)}(p',p) = \int_p^1 \frac{2\delta P'(p)V'(v)}{(\delta + P(v) - P(p))^3} dv + \int_{p'}^{p} 0 dv = M^{(2)}(p,p).$$

Then

$$\beta < 0$$

follows and since $p' < p$ it follows that

$$\pi^{(2)}(p',p) < 0$$

as needed.

Now, proceed to the second condition, where

$$\frac{\pi^{(2)}(p,p')}{M(p,p')} = \frac{(R(p) - w(p'))}{M(p,p')} M^{(2)}(p,p') - w'(p') =$$

$$(R(p) - R(p'))\frac{M^{(2)}(p,p')}{M(p,p')} + w'(p')(\frac{M^{(2)}(p,p')}{M(p,p')}\frac{M(p',p')}{M^{(2)}(p',p')} - 1).$$

At this point we use the fact that $V(x) = P(x) = x$ for all x. Then after calculating the integrals we obtain:

$$\frac{M^{(2)}(p,p')}{M(p,p')} = \frac{\int_p^1 \frac{2\delta P'(v)P'(p')}{(\delta + P(v) - P(p'))^3} dv}{\int_p^1 \frac{\delta P'(v)}{(\delta + P(v) - P(p'))^2} dv} = \frac{(2\delta + 1 + p - 2p')}{(\delta + 1 - p')(\delta + p - p')}.$$

Also,

$$\frac{M(p',p')}{M^{(2)}(p',p')} = \frac{\delta(\delta + 1 - p')}{(2\delta + 1 - p')}.$$

Condition (7) implies that for all p

$$w'(p) = (R(p) - w)\frac{(2\delta + 1 - p)}{(\delta + 1 - p)\delta}.$$

Thus

$$\widetilde{\beta} = \frac{M^{(2)}(p,p')}{M(p,p')}\frac{M(p',p')}{M^{(2)}(p',p')} - 1 =$$

$$-(p-p')\frac{\delta + 1 - p'}{(2\delta + 1 - p')(\delta + p - p')},$$

and

$$\frac{\pi^{(2)}(p,p')}{M(p,p')} = (R(p) - R(p'))\frac{M^{(2)}(p,p')}{M(p,p')} + w'(p')(\frac{M^{(2)}(p,p')}{M(p,p')}\frac{M(p',p')}{M^{(2)}(p',p')} - 1) =$$

$$(R(p) - R(p'))\frac{2\delta + 1 + p - 2p'}{(\delta + 1 - p')(\delta + p - p')} - w'(p')(p-p')\frac{\delta + 1 - p'}{(2\delta + 1 - p')(\delta + p - p')} \geq$$

$$\frac{1}{\delta + p - p'}[(R(p) - R(p'))\frac{2\delta + 1 - p'}{(\delta + 1 - p')} -$$

$$-(p-p')\frac{\delta + 1 - p'}{(2\delta + 1 - p')}w'(p')] =$$

$$\frac{1}{\delta + p - p'}[(R(p) - R(p'))\frac{2\delta + 1 - p'}{\delta + 1 - p'} - (p-p')\frac{(R(p') - w(p')}{\delta}],$$

where we used equation (7) repeatedly along with $p > p'$. Thus for our purposes it is sufficient to show then that for all $p' < p$

$$\frac{(R(p) - R(p'))}{(p-p')} - \frac{\delta + 1 - p'}{2\delta + 1 - p'}\frac{(R(p') - w(p')}{\delta} \geq 0.$$

We concentrate on the case when firms with higher type have sufficient incentives to bid more than firms with lower types, i.e. a monotone equilibrium exists. From the analysis of the two-type case one suspects that this holds when the productivity of higher type firms are much higher than that of the lower types. In a continuous type space model this condition can be captured by assuming that the marginal productivity from increasing p is increasing, i.e. that R is a convex function. Under that assumption

$$\frac{(R(p) - R(p'))}{(p-p')} \geq R'(p')$$

and thus it is sufficient for us to show that for all p'

$$R'(p') \geq \frac{\delta + 1 - p'}{2\delta + 1 - p'}\frac{(R(p') - w(p')}{\delta}$$

To prove that this condition indeed holds first note that equation (7) implies that

$$R(x) - w(x) \leq R'(x)\delta$$

for all x.[5] Then it follows that

$$R'(p') > \frac{\delta+1-p'}{2\delta+1-p'}R'(p') \geq \frac{\delta+1-p'}{2\delta+1-p'}\frac{(R(p')-w(p'))}{\delta},$$

which shows that our condition is satisfied for any δ.

6.4 Derivation of distributional results

Deriving results regarding the wage distribution:

$$\begin{aligned} H(w) = \int_0^1 V'(v)H_v(w)\,dv &= \int_0^{w^{-1}(w)} V'(v)\,dv + \int_{w^{-1}(w)}^1 V'(v)T_v(w^{-1}(w))\,dv \\ &= V(w^{-1}(w)) + \int_{w^{-1}(w)}^1 V'(v)\frac{\delta}{\delta+P(v)-P(w^{-1}(w))}\,dv \end{aligned}$$

To simplify matters, restrict attention to the case where $P(x) = V(x) = x$. Then,

$$H'(w) = w^{-1'}(w)\int_{w^{-1}(w)}^1 V'(v)T_v'(p)\,dv$$

$$\begin{aligned} H'(w) &= w^{-1\prime}(w)\int_{w^{-1}(w)}^1 V'(v)T_v'(p)dv \\ &= \delta w^{-1\prime}(w)\int_{w^{-1}(w)}^1 \frac{1}{(\delta+v-p)^2}dv \\ &= \frac{\delta[\frac{-1}{\delta+v-p}]_p^1}{w'(p)} = \frac{\delta(\frac{1}{\delta}-\frac{1}{\delta+1-p})}{w'(p)}. \end{aligned} \quad (13)$$

Evaluating the above, we find the following,

[5] To see this observe that

$$R(0) - w(0) = 0$$

and whenever

$$R(x) - w(x) = \delta R'(x)$$

holds it follows that

$$w'(x) = R'(x)\frac{2\delta+1-x}{\delta+1-x} > R'(x).$$

Then at that point $R - w$ is decreasing, while the right hand side, $\delta R'$ is increasing because R is convex.

$$H'(w) = \frac{\delta(\frac{1}{\delta} - \frac{1}{\delta+1-p})}{w'(p)} \tag{14}$$

Generally, the empirical evidence focuses upon the wage distribution of workers who are employed. Hence, let

$$\widetilde{H}(w) = Pr(wage \leq w | employed).$$

and

$$U = Pr(unemployed) = \int_0^1 \frac{\delta}{\delta + v}\, dv = \delta\left(ln(\delta + 1) - ln(\delta)\right)$$

Then,

$$H(w) = U + (1 - U)\widetilde{H}(w)$$

or

$$\widetilde{H}(w) = \frac{H(w) - U}{1 - U}$$

and thus

$$\widetilde{H}'(w) = \frac{H'(w)}{1 - U}.$$

We also may evaluate the distribution of a particular type of worker

$$H_v(w) = \frac{\delta}{\delta + v - w^{-1}(w)}.$$

The density becomes then

$$H_v'(w) = \frac{\delta w^{-1\prime}(w)}{(\delta + v - w^{-1}(w))^2} =$$

$$= \frac{\delta}{(\delta + v - w^{-1}(w))^2} / w'(w^{-1}(w)),$$

which is infinite at $w = 0$, so it must be decreasing for small w's. For higher wages (a computation with Mathematica) the density is increasing again. Again, the wage density conditional on employment is just a constant times $H'(w)$:

$$\widetilde{H}_v'(w) = \frac{H_v'(w)}{1 - U(v)}$$

The expected wage a type obtains in equilibrium can be calculated as follows:

$$\widetilde{E}_v(w) = \int_0^{w(v)} w\widetilde{H}'_v(w)dw = \int_0^{w(v)} w\frac{\frac{\delta}{(\delta+v-w^{-1}(w))^2}/w'(w^{-1}(w))}{1-U(v)}dw =$$

$$= \int_0^{v} w(p)\frac{\frac{\delta}{(\delta+v-p)^2}/w'(p)}{\frac{v}{\delta+v}}w'(p)dp = \int_0^{v} \frac{\delta(\delta+v)w(p)}{(\delta+v-p)^2 v}dp.$$

Similarly, the variance is

$$\widetilde{Var}_v(w) = \int_0^{v} \frac{\delta(\delta+v)w^2(p)}{(\delta+v-p)^2 v}dp - (\int_0^{v} \frac{\delta(\delta+v)w(p)}{(\delta+v-p)^2 v}dp)^2.$$

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6.5 Figures

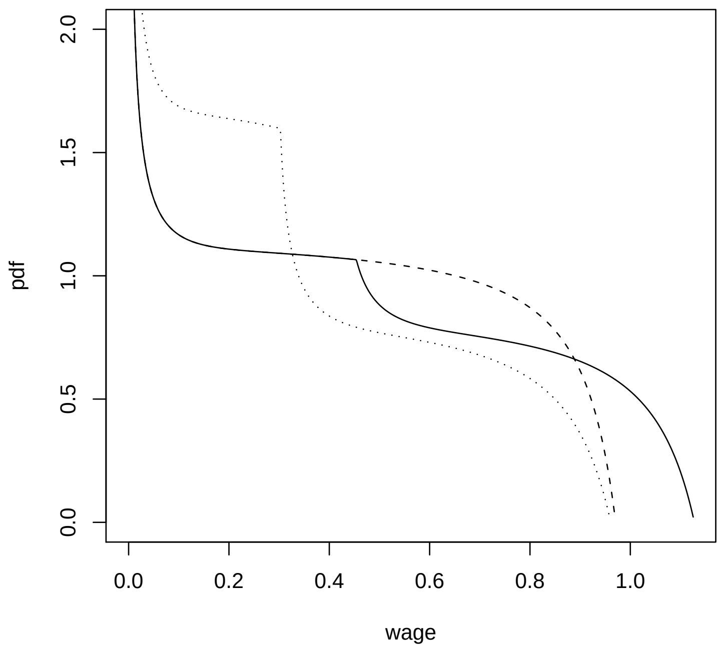

Figure 1: Comparison of alternative pdfs. Solid line is pdf of wages when $R(p) = p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{6}$ for $p > 1/2$. Dashed line is case of linear technology where $R(p) = p$, and dotted line is pdf when $R(p) = \frac{2}{3}p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{3}$ for $p > 1/2$. The distribution of types is $P(x) = V(x) = x$.

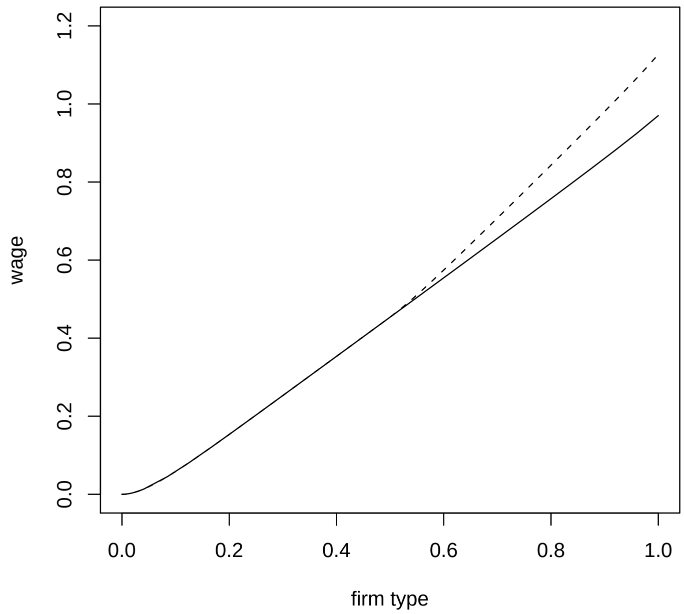

Figure 2: Wage offered by a firm of type p. Solid line represents the $w(p)$ when $R(p) = p$. Dashed line represents $w(p)$ when $R(p) = p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{6}$ for $p > 1/2$.

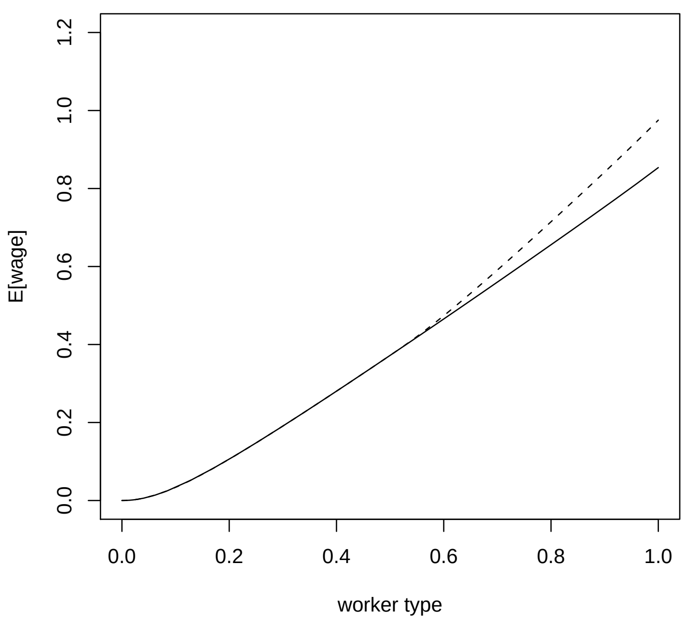

Figure 3: Expected wage of a worker of type v. Solid line represents the case$R(p) = p$. Dashed line represents case when $R(p) = p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{6}$ for $p > 1/2$.

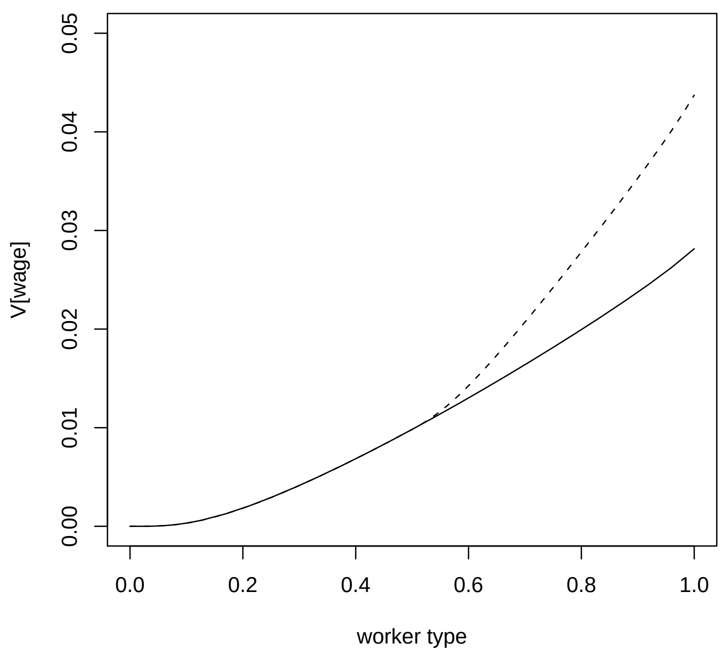

Figure 4: Variance of wage of worker type, v. Solid line represents the case $R(p) = p$. Dashed line represents case when $R(p) = p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{6}$ for $p > 1/2$.

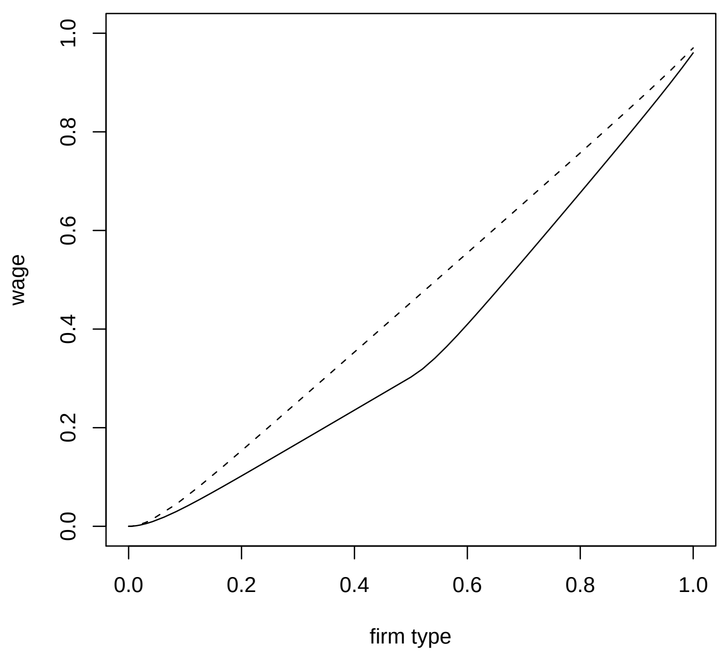

Figure 5: Wage offered by a firm of type p. Solid line represents the $w(p)$ when $R(p) = \frac{2}{3}p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{3}$ for $p > 1/2$. Dashed line represents $w(p)$ when $R(p) = p$.

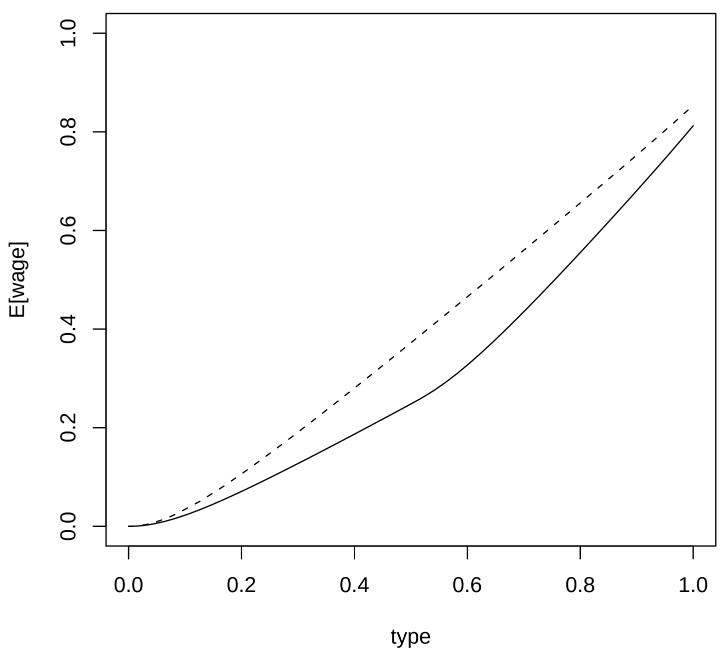

Figure 6: Expected wage of a worker of type v. Solid line represents the case $R(p) = \frac{2}{3}p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{3}$ for $p > 1/2$. Dashed line represents case when $R(p) = p$.

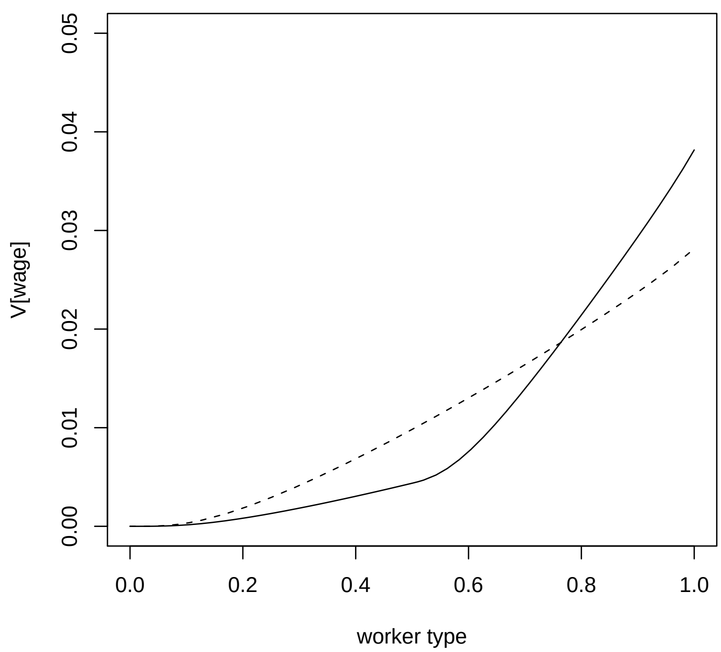

Figure 7: Variance of wage of worker type, v. Solid line represents the case $R(p) = \frac{2}{3}p$ for $p < 1/2$ and $\frac{4}{3}p - \frac{1}{3}$ for $p > 1/2$. Dashed line represents case when $R(p) = p$.