The effects of the minimum wage in an economy with tax evasion

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The paper investigates the role of the minimum wage in an economy where tax evasion by employed labour is made possible by partial or total participation in the informal economy. A model of tax evasion where detection is imperfect even in case of auditing is developed and the effects of introducing a minimum wage are explored. The minimum wage plays its role by posing a lower bound on the amount of earnings that has to be declared to enjoy formal working status. One of the implications of the model is that a spike in the declared wage distribution will appear in correspondence with the minimum wage. It also implies a positive correlation between the size of the spike and the size of the informal economy and that tax revenues are increasing with the minimum wage when enforcement is not too effective.

1 Introduction

What are the fiscal implications of introducing or increasing the minimum wage? How can we explain the very high spike at the minimum wage level appearing in the wage distribution of some countries? This paper contributes to answering these questions by studying the effects of the interaction between tax evasion and minimum wage legislation.

I thank the Department of Economics at the Central European University, where this research was conducted, for its hospitality. Comments and suggestion by participants at the lunch seminars at CEU and IIES have been greatly helpful. Financial support by the Jan Wallanders och Tom Hedelius Stiftelse is gratefully acknowledged.
A simple model of the labour market is built where underreporting of earnings is made possible by imperfect detection of tax evasion. When choosing their reporting behavior agents trade off the costs of non-compliance, higher expected penalties, with its benefits, reduced payments of due taxes and social security contributions. The introduction of the minimum wage induces some workers to increase compliance, while pushing others out of the formal labour market into the black economy or into inactivity. The overall effect when enforcement is not too effective is to unambiguously increase fiscal revenues. The distribution of the fiscal burden is also altered. Moreover, an otherwise smooth distribution of declared earnings is transformed by the introduction of the minimum wage into a distribution presenting a spike at the minimum wage level. The intuition behind this result is that the minimum wage poses a constraint to reporting behavior, as agents have to choose whether to report nothing or to report at least the minimum wage. When faced with such a restriction agents may prefer to increase their reporting to the minimum wage level than to decrease it to zero.

The model also predicts a positive correlation between the size of the spike at the minimum wage level and the estimated size of the informal economy. Such a positive correlation is documented in the paper for some European countries, mainly in Central and Eastern Europe.

The minimum wage is the subject of a rich literature and policy debate. Large efforts have also been devoted to the theoretical and empirical study of tax evasion and the shadow economy. However, the aspects of the interaction between minimum wage legislation and tax evasion investigated in this paper have, to the best of my knowledge, never been analysed before.

The importance for policymakers of contrasting tax evasion and increasing budget revenues by establishing lower bounds to the amount of taxes and social security contributions a person in formal employment has to pay is being stressed by two recent reforms in Bulgaria and Croatia. In Bulgaria minimum wage is fixed by national level negotiations, but since the beginning of 2003 employers have been required to pay social security contributions on the basis of minimum social insurance thresholds, varying along economic activity and occupational groups (Hristoskov, 2004, and Tomev, 2004.) In Croatia, where the minimum wage is not prescribed by law, a minimum basis

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1 See Brown (1999) for a review.
for calculation of obligatory social security contributions has been established in 2003 (Crnković-Pozaić, 2004.) These reforms have introduced instruments specifically aimed at tackling the issue of underreporting, at least with regards to social security contributions. In countries where such a reform has not been conducted, it is the minimum wage that is playing the role.

The literature on tax evasion has been mainly focused on personal income tax and on the compliance decision by an individual filling the tax declaration form. However, due to the tax withholding and information reporting systems present in many countries, this is not an accurate description for the case of employed labour. Indeed, the rate of non-compliance for wages and salaries at the stage of filling the tax declaration form is often negligible. For instance, Klepper and Nagin (1989) report a mere 0.1% non-compliance for wages and salaries in the US, lower than for any other income category. The study of tax evasion by employed labour is however of particular interest as fiscal imposition on labour in the form of social security contributions (SSC) and personal income tax (PIT) represents the bulk of fiscal revenues in many countries. A contribution of the paper is to present a tractable model of tax evasion by employed labour. By using imperfect detection alone, with fines imposed on risk neutral firms and the probability of auditing given, the model is able to generate an internal solution to the tax evasion problem.

The literature on minimum wage has been particularly concerned about its impact on the distribution of earnings. A spike at the minimum wage level has been observed in several instances (see for instance DiNardo et al., 1996, Rama, 2001, Neumark et al. 2004, Dickens and Manning, 2004). Such a spike has been defined as a "puzzle" for several standard types of labour market models (Brown, 1999) and as an "anomalous finding from the standpoint of the standard model of the low wage labor market" (Card and Krueger, 1995, p. 152.) Proposed rationalizations include reductions in non-wage compensation or increases in required effort to offset a binding minimum wage, flatter earnings profiles, adjustments in the amounts of hours worked. The model presented here proposes an alternative rationale for the observed spike in the framework of a competitive model. The positive correlation between the size of the spike at the minimum wage level and the estimated size of the informal economy in the data presented below suggests that, at

\[ In EU15 as a whole labour taxes contributed in 2002 around 50% of total tax receipts (Eurostat, 2004), while in several Central and Eastern European countries social security contributions alone represent more than a third of total tax revenues (UNECE, 2004.) \]
least in some countries, the mechanism analyzed in this paper contributes to shape the observed distribution of earnings.

The following section provide some evidence about the relevance of underreporting of earnings and the relationship between the size of the spike at minimum wage level and the informal economy. In section 3 some of the related previous literature is discussed. The model is introduced in the fourth section. First it is solved in absence of a minimum wage, then the various effects of introducing the minimum wage are explored. In section 5 the quantitative impact of the model is investigated through a simple numerical exercise [to be done]. Section 6 presents some extention to the basic model. The last section concludes.

2 The relevance of underreporting

Undeclared work is a serious issue in many countries. Reliable data on its extension are difficult to come by, but raw estimates indicate that the phenomenon is relevant, particularly in transition and developing countries, but also in some OECD countries (see Schneider and Enste, 2003, pp 43-53.)

In a report for the European Commission about “undeclared work in an enlarged union” (Renooy et al., 2004) the authors stress how the practice of paying “envelope wages” above the officially declared minimum “exists in practically all of the Central and Eastern European countries” and in particular in the less developed. For instance, “in Latvia, underreporting (not non-reporting!) of income dominates all other forms of undeclared work”, while “in Bulgaria and Romania, it is also a well-known practice used in all sectors of the economy.”

According to the Lithuanian statistical office (Statistics Lithuania, 2003) “economic entities in 2002 did not declare at least 23 per cent of wages and salaries formally and informally paid to employees” while “the number of non-registered employees is considered to reach 104 thousand” (by comparison total employment was 1.4 millions in 2002).

An OECD study of the Baltic countries (OECD, 2003) also reports as a common practice the payment of supplements above the officially declared wage and estimates that in Latvia and Lithuania 20% of private-sector employees earn more than what is officially reported\(^4\). An employers’ survey

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\(^4\)The Latvian Central Statistical Office publishes data on earnings under the heading "Gross wage of employed excluding all kinds of irregular payments by kind of activity"
puts the percentage of wage on which social insurance contributions are actually paid at 86% in Lithuania and at 91% in Latvia. In Bulgaria as of March 2003, 22.6% of people working under an employment contract were estimated to receive actual wages higher than those declared for tax and social security purposes (Tomev, 2004.)

The phenomenon is not limited to CEE economies. An OECD study finds that, among OECD members, beside Hungary, also Mexico and South Korea had "actual social security contribution receipts [...] about 30% short of what could be expected on the basis of scheduled contribution rates and ceilings, compared with total wages and salaries in national accounts which include estimates of undeclared incomes", while Italy, Poland, Spain and Turkey had an estimated shortfall above 20% (OECD, 2004.)

Also, there appears to be a link between the size of the spike at the minimum wage level and the estimated size of the informal economy. The figure below contains data for the 12 countries for which:

1. minimum wage is fixed in terms of a monthly rate;
2. Eurostat reports the proportion of full-time employees with earnings on the monthly minimum wage;
3. Schneider (2005) estimates the size of the shadow economy;

Ten of the countries included are transition countries (all the Central and Eastern European countries that joined the EU in 2004 plus Romania and Bulgaria), the other two are Portugal and the Netherlands.

In Appendix B the raw data are reported and a short description of the method used to estimate the size of the informal economy is given (see Schneider, 2005, for details). Notice that the data collected by Eurostat are obtained from administrative sources.

A strong correlation clearly appears between the size of the spike at the minimum wage level and the estimated size of the informal economy. The correlation remains unaltered when controlling for a measure of how "biting" the minimum is in the wage distribution\(^5\).

\(^5\)The proportion of full-time employees with earnings on the minimum wage is regressed on the estimated size of informal economy (provided in Schneider, 2005) and on the minimum monthly wage as a proportion of average monthly earnings in industry and services.
3 Previous literature

The implications for tax evasion of the tax withholding system applied to employees have been studied in a series of papers by Yaniv (1988, 1992, 1995, 1996, 1998, 1999.) Tax evasion stems from separate decisions by the employer and the employee (Yaniv, 1988) or, more in line with the approach proposed here, from a collaboration among the two (Yaniv, 1992.) In the latter article evasion arises when a risk-neutral employer agrees to reduce the tax liability of a risk-averse employee by underreporting her wage, in exchange for paying a gross wage below the market rate. In case of detection, a fine is imposed on the employer and the employee has to pay due taxes. Under efficient bargaining and non-decreasing relative risk-aversion by the employee, the contract curve is shown to have a slope such that a higher declared wage corresponds to a higher paid wage. Moreover, with logarithmic utility, the tax withholding system is shown to entail greater evasion than a self-declaration system with the same enforcement parameters. Advanced tax payments are (provided by Eurostat). The latter is statistically insignificant, while the former remains significant.
also analysed using prospect theory (Yaniv, 1999). However, the papers by Yaniv are mainly concerned with analyzing the effects of the tax withholding system, while the effect of the minimum wage for evasion behaviour has not been considered.

Fugazza and Jacques (2003) study the effect of the minimum wage in a labour market where tax evasion takes the form of participation in the informal sector. They develop a matching model where workers can search for a job and firms can post vacancies either in the formal or informal sector. Workers differ in their subjective cost of operating in the informal sector and the total number of jobs is given. They determine the tightness of the two labour markets and their relative size in terms of active workforce and then study the effect of variations in labour market and fiscal institutions. The model have two different interior equilibria, depending on whether the instantaneous net return of a filled job is higher in the regular or irregular sector. In that framework the introduction of a minimum wage is interpreted as an increase in the exogenously given wage in the formal sector and is found to have an ambiguous effect on the relative size of the two labour markets in both equilibria. On one hand, formal employment is more attractive for workers. On the other hand, to induce firms to post vacancies in the formal sector the relative labour market tightness in the formal vis-à-vis the informal sector has to decrease, thus making search in the formal sector less attractive for workers. The effect on the government budget depends on which equilibrium the economy is in. While Fugazza and Jacques address some of the same issues dealt with in the present paper, the framework of analysis is rather different. In their paper, workers can either completely comply with regulation or be outside of the legal labour market, and wages in the two sectors are exogenously given.

The view that a worker can be involved in both informal and formal activities is held by Cowell (1985). He studies a model where time can be allocated among leisure, work in the formal sector, and work in the informal sector. The effects of fiscal and enforcement parameters on the dimension of the informal sector is investigated. The paper contrasts a self-declaration tax system, where the exogenously given wage is identical in the two sectors of activity, to a Pay As You Earn (PAYE) system, where wages may instead differ. The potential relevance of a discontinuity arising when a worker passes from doing some legal work to being completely in the black economy is stressed. As in the previous paper, wages in the two sectors are exogenously given. Moreover, the implication for the wage distribution of the interaction
of tax evasion and minimum wage legislation is explored in none of the above mentioned papers and, to the best of my knowledge, has not been previously addressed in a formal model.

4 The model

The size of the working population is exogenously given and normalized to 1. Every individual has an exogenously given productivity $y_i$, distributed in the population according to pdf $f(y)$ and cdf $F(y)$ on the support $[y, \bar{y}]$, where $\bar{y} \geq 0$. We assume that the labour market is competitive, each firms employs one worker, there is no capital, and the production function is linear in the labour input. Moreover, there is free entry of firms, firms can observe workers’ productivity, and workers can move from one firm to another at no cost.

Gross wages are taxed at the proportional rate $t \in [0, 1]$. Firms have an obligation to withhold taxes and social security contributions and transfer them to the authorities. Firms are risk-neutral.

Free entry of firms implies that in equilibrium expected profit are zero and this in turn implies, in the full compliance case, that a worker with productivity $y_i$ would receive a gross wage $y_i$, from which firm would deduct taxes $ty_i$ leaving the worker a net wage $(1 - t)y_i$. Workers’ utility is strictly increasing in his net income.

In this economy, however, it is possible to evade taxes and social security contributions by not reporting to the authorities part or all of the workers’ earnings. A firm employing a worker with productivity $y_i$ has therefore to decide how much of the worker’s product to declare to tax authorities, $x_i$, and how much to conceal, $z_i = y_i - x_i$. If $x_i = y_i$, then the firm is fully compliant with regulations. If $x_i = 0$, then the full product is hidden from the authorities and the firm-worker pair operates completely in the black economy. If $x_i \in (0, y_i)$, then there is underreporting.

Tax authorities may inspect firms to find out whether they comply with fiscal regulation. We assume that there is an exogenously given probability of an audit being performed $\gamma \in [0, 1]$.

However, the fact that an audit is performed does not imply that the authority discovers for sure the true tax liability, but it may find evidence to impute an income $\hat{y}_i \in [0, y_i]$, where $y_i$ is the true product. For instance, Feinstein (1991) estimates that IRS examiners managed on average to detect
only half of the tax evasion in the forms they audited, while Erard (1997) rejects the null hypothesis of perfect detection in his empirical investigation of a model where detection can be either complete or null. We assume that the detection technology is such that the probability of finding an imputable product \( \hat{y}_i \) is uniform over the interval \([0, y_i]\), i.e. \( \hat{y}_i \sim U_{[0,y_i]} \).

Given a declaration of \( x_i \) and collected evidence of a true tax liability of \( y_i \) the tax authority imposes to the firm, in case \( \hat{y}_i > x_i \), the payment of due taxes \( t \) plus an additional fine proportional to evaded taxes \( \theta t > 0 \) on the amount of evaded product that it can prove \( \hat{y}_i - x_i \). We define \( \theta = (1+\bar{\theta}) > 1 \).

Given a true product \( y_i \) and a reported one \( x_i \in [0, y_i] \) the expected fine in case of auditing, \( f_i \), is:

\[
f_i = t\theta \int_{x_i}^{y_i} (\hat{y}_i - x_i) f(\hat{y}_i) d\hat{y}_i \quad \text{where} \quad f(\hat{y}_i) = \frac{1}{y_i}
\]

then:

\[
f_i = t\theta \frac{1}{y_i} \int_{x_i}^{y_i} (\hat{y}_i - x_i) d\hat{y}_i = t\theta \frac{1}{y_i} \frac{1}{2} (y_i - x_i)^2 = t\theta \frac{1}{y_i} \frac{1}{2} z_i^2 \quad (1)
\]

Notice that a decrease in reported income, \( x \), does not increase the probability that income is detected, as this probability is given. However, it increases the probability of paying a fine, as a fine is imposed on the difference between detected and declared income, if positive. In the Appendix we present a different detection technology resulting in expected fines equivalent to (1). Given the detection technology, the expected fraction of evaded income, \( z \), that is discovered in case of auditing is:

\[
\frac{1}{x_i} \frac{1}{y_i} \int_{x_i}^{y_i} (\hat{y}_i - x_i) d\hat{y}_i = \frac{1}{x_i} \frac{1}{y_i} \frac{1}{2} z_i^2 = \frac{1}{2} \frac{z_i}{y_i} \quad (1a)
\]

i.e. a fraction corresponding to half the ratio of evaded income over true product. The assumption is thus that it is relatively easy to get away with tax-evasion.

In the economy there is a minimum monthly wage \( \bar{w} \), with universal coverage. The enforcement of the minimum wage is not perfect, in the meaning
that it is possible to work for a lower wage, but workers cannot be legally employed at below that rate.

Below we first determine the equilibrium wage and evasion in case of no minimum wage, then with the minimum wage. Several implications of the interaction between minimum wage and underreporting are also explored. For convenience subscripts are suppressed where not necessary.

### 4.1 Equilibrium without minimum wage

The possible realizations of profits, \( \pi \), for a firm employing a worker with productivity \( y \), declaring \( x \), concealing \( z = y - x \), and paying a total \( w \) to the worker are given by:

\[
\begin{align*}
\text{Profit} & \quad \text{Probability} \\
\text{in case of non-auditing:} & \quad \pi^{na} = y - w & 1 - \gamma \\
\text{in case of auditing:} & \quad \pi^{a} = y - w - f \cdot \gamma
\end{align*}
\]

where \( f \) is given by (1).

Therefore, expected profits for the firm are:

\[
E\pi = (1 - \gamma)\pi^{na} + \gamma\pi^{a} = y - w - \gamma f \quad (2)
\]

Given an effective wage \( w \) and an official wage \( x = y - z \) a worker’s income \( I \) is given by:

\[
I = w - xt \quad (3)
\]

Firms compete for workers by offering an effective wage \( w \)-official wage \( x \) package, subject to expected profit being non-negative. Free entry implies that competition among firms will drive expected profits to zero and, using (2), this gives the wage equation:

\[
w = y - \gamma f \quad (4)
\]

Substituting (1) for \( f \) in (4), we get:
\[ w = w(z) = y - \gamma \theta t \frac{1}{y^2} z^2 \quad (5) \]

Therefore, the problem faced by firms is:

\[ \max_{z \in [0, y]} I \quad s.t. \quad w = y - \gamma \theta t \frac{1}{y^2} z^2 \quad (6) \]

Substituting (5) into (3) and taking into account that \( x = y - z \), we get:

\[ I = I(z) = y(1 - t) + zt - \gamma \theta t \frac{1}{y^2} z^2 \quad (3a) \]

We can then restate the problem of the firm as:

\[ \max_{z \in [0, y]} y(1 - t) + zt - \gamma \theta t \frac{1}{y^2} z^2 \quad (6a) \]

the f.o.c. for an interior solution is:

\[ t - \gamma \theta t \frac{1}{y} z = 0 \Leftrightarrow z = \frac{1}{\gamma \theta} y = \alpha y \]

where \( \alpha = \frac{1}{\gamma \theta} \) is the inverse of the expected penalty rate \( \gamma \theta \). As the penalty imposed or the probability of auditing increases, \( \alpha \) decreases.

The second-order condition is always satisfied as \( -\gamma \theta t \frac{1}{y} < 0 \)

To find a solution \( z^* \) to problem (6) the boundary conditions \( z \geq 0 \) and \( z \leq y \).

The condition \( z \geq 0 \) is always satisfied for \( \alpha = \frac{1}{\gamma \theta} \geq 0 \).

Thus, full compliance does not take place unless \( \gamma \theta \to +\infty \). In the appendix it is shown that evasion remains at all levels of income even with auditing probability dependent on reported income.

On the other hand, for \( \alpha = \frac{1}{\gamma \theta} > 1 \Leftrightarrow \gamma \theta < 1 \) evasion will be full.

To summarize,

\[
\begin{array}{llll}
\{ & z^* = \alpha y & x^* = (1 - \alpha)y & i f \quad \gamma \theta > 1 \Leftrightarrow \alpha < 1 \quad \text{case 1 - partial evasion} \\
& z^* = y & x^* = 0 & i f \quad 0 \leq \gamma \theta \leq 1 \quad \text{case 2 - full evasion} \\
\end{array}
\]

(7)
Thus, the model implies that, irrespective of the specific level of productivity, firms reveal a constant fraction of their production to fiscal authorities. As \( \frac{\partial \alpha}{\partial t} = -\frac{1}{y^2} < 0 \) and \( \frac{\partial \alpha}{\partial \theta} = -\frac{1}{\gamma y^2} < 0 \), in an interior solution the fraction of product that is evaded decreases with enforcement parameters (audit probability, penalty).

Income for the worker in the two cases is given by:

\[
I^* = \begin{cases} 
    y(1-t) + \frac{1}{2} \alpha y t & \text{case 1 - partial evasion} \\
    y(1-\frac{1}{2} \alpha t) & \text{case 2 - full evasion}
\end{cases} 
\]

(8)

The expected fraction of concealed production that is discovered in case of auditing is, by substituting \( z^* = \alpha y \) into (1a), \( \frac{1}{\alpha} \). Consequently, expected fines in case of auditing and the effective wage of a worker are in an interior solution:

\[
\gamma f^* = \frac{\alpha}{2} ty \quad (8a) \quad w^* = (1 - \frac{\alpha}{2}) y \quad (8b)
\]

4.2 Effects of introducing the minimum wage

In this section we study what are the effects of introducing a minimum wage \( \bar{w} \) in this economy. We focus on the case where there is underreporting, i.e. \( \alpha \in (0, 1) \).

4.2.1 Effects on the distribution

With the introduction of a minimum wage, the problem the firm has to solve becomes:

\[
\max_{z \in \{0\} \cup [\bar{w},y]} y(1-t) + zt - \gamma \theta t^{1/2} z^2
\]

A worker-firm pair is unaffected by \( \bar{w} \) if productivity is such that income declared would have been greater than the minimum wage even in the absence of a minimum wage. The minimum wage is instead a binding constraint for worker-firm pairs that would have declared less, i.e. when productivity \( y_i \) is such that:

\[
(1 - \alpha) y_i < \bar{w} \Leftrightarrow y_i < \frac{\bar{w}}{1 - \alpha}
\]
Among this category, workers with productivity below the minimum wage, \( y_i < \bar{w} \), have no other choice but to withdraw from the official labour market, either working in the black market or being inactive.

The introduction of the minimum wage poses instead a non-trivial choice for worker-firm pairs when productivity is above \( \bar{w} \), but optimal declaration in case of no minimum wage regulation is less than \( \bar{w} \), i.e. in case of workers with \( y_i \) such that:

\[
(1 - \alpha)y_i < \bar{w} < y_i \Leftrightarrow \bar{w} < y_i < \frac{\bar{w}}{1 - \alpha} \quad (9)
\]

For these workers the declared wage will never be higher than the minimum wage. The choice is between declaring \( \bar{w} \), working in the black market and become inactive. The choice is made by comparing net income in the three cases.

Income in case of inactivity is 0.

Income in case of working in the black market is:

\[
I_{bm} = y_i (1 - \frac{1}{2\alpha}) \quad (10)
\]

Income in case of declaring the minimum wage is, by substituting \( z = y_i - \bar{w} \) in (3a):

\[
I_{mw} = y_i (1 - t) + (y_i - \bar{w}) t - t \frac{1}{\alpha y_i} \frac{1}{2} (y_i - \bar{w})^2
\]

We get that:

\[
I_{mw} \geq I_{bm} \Leftrightarrow y_i \geq \frac{1}{2(1 - \alpha)} \bar{w} \equiv y_{mw} \quad (11)
\]

As the choice between employment at the minimum wage and employment in the black market is relevant only for workers whose productivity \( y_i \) is s.t. \( \bar{w} < y_i < \frac{\bar{w}}{1 - \alpha} \) to determine the behaviour once a minimum wage is introduced it is necessary to position \( y_{mw} \) in the interval \( [\bar{w}, \frac{\bar{w}}{1 - \alpha}] \):

- \( y_{mw} < \bar{w} \Leftrightarrow \frac{1}{2(1 - \alpha)} \bar{w} < \frac{\bar{w}}{1 - \alpha} \), this condition is always satisfied.
- \( y_{mw} > \bar{w} \Leftrightarrow \frac{1}{2(1 - \alpha)} \bar{w} > \bar{w} \Leftrightarrow \alpha > \frac{1}{2} \)
We then have two possible states:

1. $\alpha > \frac{1}{2}$: then $y_{mw} \in [\bar{w}, \frac{w}{1-\alpha}]$ i.e. some of the workers affected by the minimum wage and with productivity higher than the minimum wage prefer to decrease evasion and declare the minimum, while others prefer to withdraw from the official labour market. For this to be the case the degree of underreporting has to be very high and the informal economy has to be at least as big as the formal economy. While this may be the case for some economies, we consider the other case to be more relevant.

2. $\alpha \leq \frac{1}{2}$: then $y_{mw} < \bar{w}$ i.e. all workers affected by the minimum wage and with productivity higher than the minimum wage prefer to decrease evasion and declare the minimum.

Worker-firm pairs may prefer to declare the minimum wage and pay the corresponding tax rather than declare nothing because the introduction of the minimum wage restricts the choice of the level of evasion and, taking into consideration enforcement parameters, some, possibly all, worker-firm pairs prefer to decrease evasion compared to their optimal level in case of unrestricted choice rather than increase it.

Notice that in case $\alpha < \frac{1}{2}$ workers with productivity below the minimum wage would withdraw from the labour market, as the expected return to work in the black market is negative and so no firm would employ them. Thus, for a given tax rate, in economies where enforcement is not very effective the minimum wage would push workers out of the formal labour market into the black market, while in economies with more effective enforcement the minimum wage pushes workers outside of the labour market.

The results are summarized in proposition below.

**Proposition 1** Given a population where productivity $y_i$ is distributed according to $f(y)$ and $F(y)$ on the support $[y, \bar{y}] \subseteq [0, +\infty)$ and where fiscal and enforcement parameters are such that $\gamma \theta > 1$, so that declared earnings are $(1-\alpha) y_i$, where $\alpha = \frac{1}{\gamma \theta}$, the introduction of a minimum wage $\bar{w} > y$ implies that:

1) Workers with productivity $y_i < \bar{w}$ are pushed into the black market if $\alpha \geq \frac{1}{2}$ or withdraw from the labour market if $\alpha < \frac{1}{2}$, in any case they declare nothing;
2) Worker-firm pairs with productivity $y_i$ s.t. $\bar{w} < y_i < \frac{\bar{w}}{1-\alpha}$ :

2a) If $\alpha \leq \frac{1}{2}$ : declare the minimum wage;

2b) If $\alpha > \frac{1}{2}$ then,

if $y_i$ s.t. $\bar{w} < y_i < \frac{1}{2(1-\alpha)} \bar{w}$ : declare nothing

if $y_i$ s.t. $\frac{1}{2(1-\alpha)} \bar{w} < y_i < \frac{\bar{w}}{1-\alpha}$ : declare the minimum wage

3) Worker-firm pairs characterized by productivity $y_i > \frac{\bar{w}}{1-\alpha}$ are unaffected and continue to declare $(1-a)y_i$

The distribution of declared earnings $x$ before the minimum wage is given by:

$$f_x(x) = \begin{cases} f(\frac{x}{(1-a)\bar{y}}) & y(1-\alpha) < x < \bar{y}(1-\alpha) \\ 0 & \text{otherwise} \end{cases}$$

after the introduction of the minimum wage it is given by:

$$f_{mw}(x) = \begin{cases} \int_{\frac{\bar{w}}{1-a}\bar{y}}^{\frac{1}{2}(1-a)\bar{w}} f_x(x) dx & x = 0 \\ \int_{\frac{\bar{w}}{1-a}\bar{y}}^{\frac{1}{2}(1-a)\bar{w}} f_x(x) dx & \bar{w} < x < \bar{y}(1-\alpha) \\ f_x(x) & \text{otherwise} \end{cases}$$

or equivalently:

$$f_{mw}(x) = \begin{cases} \int_{\frac{\bar{w}}{1-a}\bar{y}}^{\frac{1}{2}(1-a)\bar{w}} f(y) dy & x = 0 \\ \int_{\frac{\bar{w}}{1-a}\bar{y}}^{\frac{1}{2}(1-a)\bar{w}} f(y) dy & \bar{w} < x < \bar{y}(1-\alpha) \\ f(\frac{x}{(1-a)}) & \text{otherwise} \end{cases}$$

Thus, a "smooth" distribution of productivity is associated with a "smooth" distribution of declared earnings without a minimum wage. However, with the introduction of the minimum wage, two spikes appear at the minimum wage level and at zero.
4.2.2 Effects on fiscal quantities

The minimum wage divides worker-firm pairs into three categories: those declaring nothing, those declaring the minimum wage, and the unaffected. Here we first determine payments to fiscal authorities for each category. Then, we use the analysis of the distribution of declared earnings done above to find out the effects of the minimum wage on fiscal revenues.

Payments to fiscal authorities Total payments, $P$, to the fiscal authorities include taxes, $T$, and expected fines, $F$. For worker-firm pairs not affected by the minimum wage these quantities are:

$$P_1 = (1 - \frac{\alpha}{2})ty$$
$$T_1 = (1 - \alpha)ty$$
$$F_1 = \frac{\alpha}{2}ty$$

Underreporting gives to worker-firm pairs with relatively high productivity the opportunity to reduce the tax rate by a factor $\frac{\alpha}{2}$.

For worker-firm pair declaring the minimum wage fiscal payments are given by:

$$P_2 = t\bar{w} + t\frac{1}{2\alpha} \frac{(y-\bar{w})^2}{y}$$
$$T_2 = t\bar{w}$$
$$F_2 = \frac{1}{2\alpha} \frac{(y-\bar{w})^2}{y}t$$

The remaining category is represented by worker-firm pairs that are either in the black economy (when $\alpha \geq \frac{t}{2}$) or do not participate to the labour market (when $\alpha < \frac{t}{2}$).

For workers in the black market fines are the only type of payment, so:

$$P_3 = F_3 = t\frac{1}{2\alpha}y$$

Workers who withdraw from the labour market do not contribute to public finances, so:

$$P_4 = F_4 = 0$$

Notice that $\frac{P_2}{y} \geq \frac{P_3}{y} \forall y$, $\frac{P_2}{y} \geq \frac{P_4}{y} \forall y$, $\frac{P_2}{y} \geq \frac{P_3}{y} \Leftrightarrow y \geq \frac{w}{2(1-\alpha)}$. As only workers with productivity $y_i \geq \max(\bar{w}, \frac{w}{2(1-\alpha)})$ will declare the minimum wage, then $\frac{P_2}{y} \geq \frac{P_3}{y}$ for the relevant interval.

---

6In particular, $\frac{P_2}{y} \geq \frac{P_3}{y} \forall y$, $\frac{P_2}{y} \geq \frac{P_4}{y} \forall y$, $\frac{P_2}{y} \geq \frac{P_3}{y} \Leftrightarrow y \geq \frac{w}{2(1-\alpha)}$. As only workers with productivity $y_i \geq \max(\bar{w}, \frac{w}{2(1-\alpha)})$ will declare the minimum wage, then $\frac{P_2}{y} \geq \frac{P_3}{y}$ for the relevant interval.
Effects of the minimum wage on revenues  The way the working population splits into the three categories analyzed above has been established in proposition 1. We analyze the "high underreporting" case, \( \alpha \geq \frac{t}{2} \), and the "low underreporting" case, \( \alpha < \frac{t}{2} \), separately.

**High underreporting**  When \( \alpha \geq \frac{t}{2} \) total revenues \( R \) are given by:

\[
R = \max(\tilde{w}, \frac{\tilde{w}}{2(1-\alpha)}) \int_0^\tilde{w} \frac{1}{2\alpha} y f(y)dy + \int_{\max(\tilde{w}, \frac{\tilde{w}}{2(1-\alpha)})}^{\frac{\tilde{w}}{1-a}} \left[ t\tilde{w} + t \frac{1}{2\alpha} \frac{(y-\tilde{w})^2}{y} \right] f(y)dy + \int_{\frac{\tilde{w}}{1-a}}^{\tilde{w}} (1-\frac{\alpha}{2})ty f(y)dy
\]

The effects of increasing the minimum wage on total revenues depends on whether \( \alpha \leq \frac{1}{2} \). In case \( \alpha > \frac{1}{2} \) a worker with productivity equal to the minimum wage prefers being employed in the black market than declaring the minimum wage. Then,

\[
\frac{\partial R}{\partial \tilde{w}} = \int_{\frac{\tilde{w}}{1-a}}^{\tilde{w}} \left[ 1 - \frac{1}{\alpha} \frac{(y-\tilde{w})}{y} \right] tf(y)dy
\]

As \( 1 - \frac{1}{\alpha} \frac{(y-\tilde{w})}{y} > 0 \leftrightarrow y < \frac{\tilde{w}}{1-a} \), then \( \frac{\partial R}{\partial \tilde{w}} > 0 \). The variation is due to more payment by workers affected by minimum wage.

In case \( \alpha \leq \frac{1}{2} \) a worker with productivity equal to the minimum wage prefers declaring the minimum wage than being employed in the black market. Then,

\[
\frac{\partial R}{\partial \tilde{w}} = \left[ \frac{1}{2\alpha} - 1 \right] t\tilde{w} f(\tilde{w}) + \int_{\tilde{w}}^\infty \left[ 1 - \frac{1}{\alpha} \frac{(y-\tilde{w})}{y} \right] tf(y)dy
\]

As \( 1 - \frac{1}{\alpha} \frac{(y-\tilde{w})}{y} > 0 \leftrightarrow y < \frac{\tilde{w}}{(1-\alpha)} \) and \( \frac{1}{2\alpha} - 1 > 0 \leftrightarrow \alpha < \frac{1}{2} \), then \( \frac{\partial R}{\partial \tilde{w}} > 0 \). In this case there is an additional term, representing the effect of pushing into the black market worker-firm pairs previously in the official economy.
Low underreporting  When $\alpha < \frac{t}{2}$ total revenues are given by:

$$R = \int_{\bar{w}}^{\bar{w}} [t\bar{w} + t \frac{1}{2\alpha} \frac{(y-\bar{w})^2}{y}] f(y)dy + \int_{\frac{\bar{w}}{1-a}}^{\bar{w}} (1-\frac{\bar{w}}{y})ty f(y)dy$$

Then,

$$\frac{\partial R}{\partial \bar{w}} = -t\bar{w} f'(\bar{w}) + \int_{\bar{w}} \left[ 1 - \frac{1}{\alpha} \frac{(y-\bar{w})}{y} \right] ty f(y)dy$$

The first term represents the loss due to the withdrawal of workers from the labour market. In this case the net effect of an increase in the minimum wage depends on the shape of the distribution.

We can then state the following proposition:

**Proposition 2**  When underreporting is high revenues increase with the minimum wage, i.e. $\frac{\partial R}{\partial \bar{w}} \geq 0$. When underreporting is low the effect of increasing the minimum wage on revenues depends on the productivity distribution.

The intuition is straightforward: maximization of workers' net income is equivalent to minimization of transfers to the government. Choice is limited to the possible declaration space $\{0\} \cup [\bar{w}, +\infty)$. Increasing the minimum wage shrinks the possible declaration space, so that the newly chosen compliance after the increase in the minimum wage cannot make workers better off. When the increase in the minimum wage does not have a negative impact on production, i.e. it does not "shrink the pie", this implies that the government cannot be made worse off, i.e. revenues cannot decrease. This can be counterbalanced by a decrease in revenues due to reduced total production when an increase in the minimum wage pushes low productivity workers out of the labour market.
4.3 The link between underreporting and the spike at minimum wage level

Both the size of the spike at the minimum wage level and the size of the underground economy relative to the economy as a whole are determined by the interplay of the productivity distribution, the fiscal enforcement parameters as summarized by $\alpha$, and the minimum wage, $\bar{w}$. In this section we study the link between underreporting and the size of the spike.

The spike at the minimum wage  The size of the spike at the minimum wage level is given by:

$$S = \int_{\max(\frac{\bar{w}}{1-a}, \bar{w})}^{\bar{w}} f(y) dy$$

A decrease in enforcement parameters initially increases unambiguously the size of the spike, but as the process goes on the effect depends on the shape of the distribution:

$$\frac{\partial S}{\partial \alpha} = \begin{cases} \frac{\bar{w}}{(1-a)^2} f\left(\frac{\bar{w}}{1-a}\right) > 0 & \text{if } 0 < \alpha \leq \frac{1}{2} \\ \frac{\bar{w}}{(1-a)^2} f\left(\frac{\bar{w}}{1-a}\right) - \frac{\bar{w}}{2(1-a)^2} f\left(\frac{\bar{w}}{2(1-a)}\right) & \text{if } \frac{1}{2} < \alpha < 1 \end{cases}$$

A decrease in enforcement parameters induces to declare the minimum wage workers previously declaring more, increasing the size of the spike. If enforcement is weak enough, however, some workers previously declaring the minimum wage prefer to go into the black economy, reducing the size of the spike. The necessary condition for the size of the spike to increase as enforcement parameters decrease in this case is:

$$\frac{1}{2} < \alpha < 1 \implies \frac{\partial S}{\partial \alpha} > 0 \iff f\left(\frac{\bar{w}}{1-a}\right) > \frac{1}{2} f\left(\frac{\bar{w}}{2(1-a)}\right)$$

Assuming that the distribution of productivity is single peaked, if the minimum wage is binding for workers with productivity lower than the mode, then the necessary condition is satisfied. As $S$ is continuous the eventual switch of the economy from $\alpha \leq \frac{1}{2}$ to $\alpha > \frac{1}{2}$ as $\alpha$ increases is not a problem.

The effect on the size of the spike of an increase of the minimum wage depends in general on the shape of the distribution:
As \( \bar{w} \) increases some workers previously declaring the minimum wage are pushed out of the formal labour market, thus decreasing the size of the spike, while some previously declaring more declare the minimum wage, increasing the size of the spike.

The necessary conditions for the size of the spike to increase as the minimum wage increases are in this case:

\[
0 < \alpha \leq \frac{1}{2} \implies \frac{\partial s}{\partial \bar{w}} > 0 \Leftrightarrow f\left(\frac{\bar{w}}{1-a}\right) > (1-a) f(\bar{w})
\]

\[
\frac{1}{2} < \alpha < 1 \implies \frac{\partial s}{\partial \bar{w}} > 0 \Leftrightarrow f\left(\frac{\bar{w}}{1-a}\right) > \frac{1}{2} f(\bar{w})
\]

Also in this case the necessary conditions are satisfied if the minimum wage is binding for workers with productivity lower than the mode and the distribution of productivity is single peaked.

The informal economy

To investigate the impact of \( \alpha \) and \( \bar{w} \) on the size of the informal economy it is necessary to distinguish between the high and low underreporting cases.

High underreporting

The size of the underground economy is given by:

\[
U = \int_{\bar{y}}^{\max(\frac{1}{2(1-a)}, \bar{w})} y f(y) dy + \int_{\max(\frac{\bar{w}}{1-a}, \bar{a})}^{\bar{w}} (y - \bar{w}) f(y) dy + \alpha \int_{\bar{w}}^{\bar{y}} y f(y) dy
\]

(12)

As the size of the economy for a given distribution of productivity is fixed at \( Y = \int_{\bar{y}}^{\bar{y}} y f(y) dy \) the derivatives of \( U, \frac{U}{Y}, \frac{U}{Y-U} \) (size of informal economy relative to formal economy) all have the same sign, so we focus only on the effects of \( \alpha \) and \( \bar{w} \) on \( U \).

A decrease in enforcement, i.e. an increase in \( \alpha \), increases the size of the informal economy:

\[
\frac{\partial U}{\partial \alpha} = \left\{ \begin{array}{ll}
\int_{\bar{y}}^{\bar{w}/(1-a)} y f(y) dy > 0 & \text{if } \frac{1}{2} \leq \alpha \leq \frac{1}{2}\\
\int_{\bar{w}/(1-a)}^{\bar{w}/(1-a)} \bar{w} f\left(\frac{\bar{w}}{2(1-a)}\right) + \int_{\bar{w}}^{\bar{y}} y f(y) dy > 0 & \text{if } \frac{1}{2} < \alpha < 1
\end{array} \right.
\]
This is due to the fact that workers unaffected by the minimum wage evade more. Moreover, when enforcement is already low, i.e. $\frac{1}{2} < \alpha < 1$, some workers previously declaring the minimum wage go into the black economy. As $U$ is continuous the eventual switch of the economy from $\alpha \leq \frac{1}{2}$ to $\alpha > \frac{1}{2}$ as $\alpha$ increases is not a problem.

The effect of an increase in the minimum wage on the size of the informal economy depends in general on the shape of the distribution:

$$\frac{\partial U}{\partial \bar{\alpha}} = \begin{cases} \bar{\alpha} f(\bar{\alpha}) - F(\frac{\bar{\alpha}}{1-a}) + F(\bar{\alpha}) & \text{if } \frac{1}{2} \leq \alpha \leq \frac{1}{2} \\ \frac{1}{2(1-a)} \bar{\alpha} f(\frac{\bar{\alpha}}{2(1-a)}) - F(\frac{\bar{\alpha}}{2(1-a)}) + F(\bar{\alpha}) & \text{if } \alpha > \frac{1}{2} \end{cases}$$

An increase in the minimum wage pushes some workers previously declaring the minimum wage into the black economy, thus increasing informality, but also forces workers continuing to declare the minimum to declare more of their true income, thus reducing informality. Which effect prevails depends on the shape of the distribution.

**Low underreporting** When $0 < \alpha < \frac{1}{2}$ the size of the underground economy is given by:

$$U = \int_{\bar{\alpha}}^{\alpha} (y - \bar{\alpha}) f(y) dy + \alpha \int_{\frac{\bar{\alpha}}{1-a}}^{\alpha} y f(y) dy \quad (13)$$

The derivative w.r.t. $\alpha$ is the same as in the high underreporting case, but, as it is evident by comparing (12) with (13), there is a discontinuity in the size of the informal economy at $\alpha = \frac{1}{2}$. When enforcement parameters increases (i.e. $\alpha$ decreases) so that there is a switch in the economy from the high underreporting status to the low underreporting status, the size of the informal economy drops discretely as workers previously in the black market withdraw from the labour market. This jump goes in the same direction as the derivative, so that we can state that the size of the informal economy always decreases as enforcement parameters increase.

In the low underreporting case the size of the economy for a given distribution of productivity is not fixed anymore, as it is given by $Y = \int_{\bar{\alpha}}^{\alpha} y f(y) dy$, with $\frac{\partial Y}{\partial \bar{\alpha}} = -\bar{\alpha} f(\bar{\alpha}) < 0$. As the minimum wage increases, workers with productivity below the minimum wage withdraw from the labour market into non-activity, lowering total production.
Given the sign of \( \frac{\partial U}{\partial \bar{w}} \), the sign of the derivative of \( \frac{U}{Y} \) w.r.t. \( \bar{w} \) is given by the sign of \( \frac{\partial U}{\partial \bar{w}} Y - \frac{\partial Y}{\partial \bar{w}} U = \frac{\partial U}{\partial \bar{w}} Y + \bar{w} f(\bar{w}) U \). The derivatives of \( \frac{U}{Y} \) and \( \frac{U}{Y-U} \) (size of informal economy relative to formal economy) have the same sign.

In case of low underreporting \( \frac{\partial U}{\partial \bar{w}} \) is given by:

\[
\frac{\partial U}{\partial \bar{w}} = -[F(\frac{\bar{w}}{1-a}) - F(\bar{w})] < 0
\]

An increase in the minimum wage decreases the absolute size of the informal economy. When workers with productivity lower than the minimum wage withdraw from the labour market, an increase in the minimum wage has the only effect to increase compliance by active workers, thus shrinking the size of the informal economy. However, in this case the economy as a whole also shrinks. The sign of the derivative of the relative size of the informal economy w.r.t. the minimum wage is given by:

\[
\text{sign}[\frac{\partial (\frac{U}{Y})}{\partial \bar{w}}] = \text{sign}[-[F(\frac{\bar{w}}{1-a}) - F(\bar{w})]Y + \bar{w} f(\bar{w}) U]
\]

which depends on the shape of the distribution.

Given the analysis above, it is possible to state the following proposition:

**Proposition 3**

1. The size of the informal economy increases as enforcement decreases.

2. The effect of an increase in the minimum wage on the size of the informal economy depends on the shape of the distribution.

3. Assuming that the distribution of productivity is single peaked, a minimum wage binding for workers with productivity lower than the mode is a sufficient condition for the size of the spike at the minimum wage level to increase as enforcement decreases and as the minimum wage increases.

Thus, given a single peaked distribution of productivity and provided that the minimum wage is not too high, an increase in \( \alpha \) increases both the spike at the minimum wage level and the size of the informal economy, inducing a positive correlation between the two.

### 5 A numerical exercise

In this section...
6 Extentions

In this section the robustness of the mechanism proposed is discussed and some extentions are proposed.

6.1 Alternative structures of the labour market

The model presented assumes a specific structure of the labour market, where the equilibrium distribution of wages with and without the minimum wage is easily characterized as a worker’s earnings are independent of other workers’ earnings. The mechanism presented is however more general. As far as workers maximize net income, firms maximize expected profits, and the product generated by a firm-worker pair is independent from the reporting decision, then there is an incentive to minimize the expected total payment (taxes plus expected fines) to tax authorities.

The mechanism can be extended to other models of the labour market by recognizing that the tax system introduces a wedge between net take-home pay to the worker and labour cost for the firm and that the decision on how much to report aims at minimizing such wedge, irrespectively of how savings from tax evasion are then distributed. In some models of the labour market the problem can be most naturally framed as minimization of expected total labour cost given a net wage; in other models, like in the one developed here, as a maximization of net wage given expected total labour cost; in still other models, like the ones with bargaining, as the maximization of surplus net of payments to the fiscal authorities.

Ignoring general equilibrium effects on the distribution of wages, the introduction of the minimum wage poses a binding constraint for whose firms that would have reported a lower wage in its absence. Given the trade-off beneath the reporting decision, it is likely that a mass point at the minimum wage level will emerge in the distribution of declared earnings due to the interaction between underreporting and minimum wage alone. However, in models of endogenous wage dispersion like Burdett and Mortensen (1998) or Bhaskar and To (2003), where a worker’s wages depends on other workers’ wage, the general equilibrium effects of the introduction of a minimum wage make the analysis more complex.

6.2 The black economy
The model presents no discontinuity when a firm-agent pair leaves the formal economy and goes completely underground. It may however be argued that being completely in the black economy is substantially different than being part of the official economy. In particular, we analyze the implication of possible discontinuities in two key variables: productivity and expected fines. In the analysis we assume that enforcement parameters are such that there is underreporting.

6.2.1 Productivity discontinuity

While it seems unlikely that the product generated by a firm-worker pair is dependent on the reporting behavior in case of simple underreporting, it is more plausible that entering completely into the black economy may have an effect. More difficult access to the legal protection system to enforce contracts and property rights, inability to tap formal credit, restricted possibility to advertise, no access to support programs (like training schemes, subsidies to R&D) for enterprises are some of the factors that may cause a decrease in the surplus once a firm goes underground. On the other side, the avoidance of official regulation and red tape may boost the product of firms fully in the underground economy (see Loayza, 1996, for a review). The relative relevance of the pros and cons depends on the specific situation of a country. For instance, an ineffective court system and a credit market that is not accessible for some types of enterprises (like SME) even if registered may decrease the disadvantage of being underground.

To extend the model to take into account this potential discontinuity is straightforward. Assume that productivity is:

\[
\begin{align*}
    f(y_i) & \quad \text{if } x_i > 0 \\
    y_i + d & \quad \text{if } x_i = 0 \\
\end{align*}
\]

In case \( d < 0 \) or \( \eta < 1 \) the cons of being in the black market outweigh the pros. When there is no minimum wage nothing changes. When there is a minimum wage \( \bar{w} \), then worker-firm pair have a greater incentive to increase compliance to the minimum wage level, instead of going into the black market, thus reinforcing the tendency to show a spike at the minimum wage level.

In case \( d > 0 \) or \( \eta > 1 \) (and \( \alpha > \frac{\eta}{2} \)), being in the black market provides an advantage compared to being in the official economy. In case of an addictive productivity difference, when there is no minimum wage worker-firm pairs
characterized by low productivity, i.e. with $y_i < d \frac{(2\alpha-t)}{(1-\alpha)^2}$, will go into the black market, for higher productivity pairs instead nothing changes. When there is a minimum wage, a positive productivity advantage of being in the black market reduces the incentive for firms to declare the minimum wage level instead of going into the black economy, but as far as the minimum wage is high enough compared to the productivity differential, in particular for $\frac{\bar{w}}{d} > \frac{2\alpha-t}{t(1-\alpha)}$, then there is still a spike at the minimum wage level. In case the productivity difference is multiplicative, for the no minimum wage case, a productivity advantage low enough, i.e. $\eta < 1 + \frac{t(1-\alpha)^2}{2n-t}$, is necessary to avoid that all agents go into the black market. In such circumstances, the incentives to declare the minimum wage are reduced, but do not disappear. In particular, a spike at the minimum wage level will anyway be present.

### 6.2.2 Discontinuity in expected fines

A discontinuity at zero declaration may also exist with regard to the expected fine. Again, it is not a priori obvious in which direction such a discontinuity may work. On one side, the non-existence of a company in official registers may make more difficult to localize it and perform an audit. On the other side, once an audit is performed, to prove underreporting is much more difficult than proving non-reporting, as in the latter case the operation of a firm without registration constitutes evidence in itself. Discontinuities may also exist in the fine applied in case of detection, with complete underreporting likely to be punished more harshly than partial underreporting. Assume that the expected fine is:

$$
\begin{align*}
\{ & \gamma f & \text{if } x_i > 0 \\
& \rho \gamma f & \text{if } x_i = 0
\end{align*}
$$

where $f$ is given by (1).

In case $\rho > 1$ being in the black market gives rise to higher expected fines due to higher probability of auditing or higher fines imposed in case of detection. Without minimum wage, nothing changes. With a minimum wage, the incentive to declare the minimum are stronger.

In case $\rho \in (0, 1)$ being in the black market gives rise to lower expected fines due to lower probability of auditing. Unless the advantage of being in the black market is not too high, every agent goes underground. In particular for $\rho > (2 - \alpha) \alpha$ the equilibrium without minimum wage will not change, while in case of minimum wage, the incentives to declare the minimum wage
instead of going into the black economy are reduced, but do not disappear, with a spike at the minimum wage level remaining.

6.3 Entitlements from social security

Social security contributions usually provide entitlements in the form of pensions, unemployment benefits, health insurance, maternity benefits and so on. If workers value such entitlements, then their existence represents an incentive to contribute and should be taken into account when analyzing the evasion decision. Entitlements are usually partly linked to contributions and partly independent of them. Below, the implications for the model for each case are analyzed.

6.3.1 Proportional transfers

Suppose that workers receive from social security institutions a transfer proportional to their declared wage, $\vartheta x$. In theory the value for workers of this could be more than its cost, i.e. $\vartheta > t$. This may be the case when social security funds run a deficit or are subsidized by the general budget (and thus by fiscal imposition on a different tax base) or when workers highly value these transfers (for instance because they provide some insurance, that, due to some market failure, cannot be purchased separately.) In this case, however, there is no reason to evade taxes, so we assume, more realistically, that $\vartheta < t$.

Equation (3) becomes:

$$I = w - tx + \vartheta x$$

In case also equation (1) is modified, so that fines are paid only on the amount of evasion net of foregone benefits, then the model is simply modified by substituting $(t - \vartheta)$ to $t$. In case fines continue to be paid on evaded taxes, then, the solution to (6) becomes:

$$z = (1 - \frac{\vartheta}{t})\alpha y$$

Not surprisingly, evasion declines, while a positive correlation between the tax rate and the portion of income that is evaded appears. This is consistent with the results reported by Alm et al. (1990) in their study about Jamaican employees tax evasion and avoidance. They find that "the
tax base rises with higher benefit for payroll tax contributions and falls with higher marginal tax rates", albeit estimated elasticities are small. As for the effects of the minimum wage, the productivity threshold above which workers prefer to declare the minimum wage is lower in case of transfers proportional to contributions, thus possibly increasing the size of the spike.

6.3.2 Lump-sum transfers

Here the case of a lump-sum transfer $\delta$ is analyzed. The transfer is assumed to be conditional on formal working status. In absence of a minimum wage, the only effect of a lump-sum transfer is to displace complete evasion emerging when enforcement is weak with a minimal declaration, as to qualify for the transfer by being formally part of the workforce. More interestingly, in case of minimum wage, a transfer conditional on formal working status represents a further incentive to declare the minimum instead of going into the black market and thus reduces the productivity threshold above which workers prefers to declare the minimum wage. In particular the threshold becomes:

$$y_{mw} = \frac{1}{2(1-\alpha) + 2\alpha \frac{\tilde{w}}{w}}$$

The lump-sum transfer $\delta$ should be intended as the difference between transfers conditional on being employed and transfers conditional on being not employed (unemployment benefits or other forms of social support.) In case $\delta < 0$ then the threshold would be higher as being formally employed would mean giving up some net transfer, but the effects of the minimum wage will not disapper as far as the monetary loss in case of official employment status is low enough compared to the minimum wage, in particular for $\frac{\delta}{w} < \frac{(1-\alpha)}{2\alpha}$.

7 Conclusions

8 Appendix A

8.1 An alternative setting for imperfect detection

Tax authority devotes an exogenously given $\gamma \geq 0$ units of "auditing resources" to every firm-worker pair. The more resources are used, the more
income is discovered in expectations. In particular, if $\gamma$ unit of resources are used, then discovered income $\hat{y}$ is distributed with uniform probability in the interval $[(1-a^{-\gamma})y, y]$ where $a > 1$ measures the effectiveness of auditing.

- if $\gamma = 0$ (no resources) the interval is $[0, y]$ (even with no resources there is the possibility of discovering - may be interpreted as emergence of evidence from other investigations or receiving denunciation or other costless way of getting evidence)

- if $\gamma \to +\infty$ the (degenerated) interval is $[y, y] = \{y\}$ i.e. the full income is discovered

The pdf of the distribution over the interval is $f(\hat{y}) = \frac{1}{y-(1-a^{-\gamma})y} = \frac{a^\gamma}{y}$, so:

$$f(\hat{y}) = \begin{cases} \frac{a^\gamma}{y} & \hat{y} \in [(1-a^{-\gamma})y, y] \\ 0 & \text{otherwise} \end{cases}$$

Provided the tax authority devotes resources $\gamma$ to a taxpayer characterized by true income $y$ and declared income $x$ then the expected fine is:

$$F = \begin{cases} \int_x^y (\hat{y} - x) f(\hat{y}) d\hat{y} & \text{if } x \geq (1-a^{-\gamma})y \\ ((1-a^{-\gamma})y - x) t \theta + \theta \int_x^y (\hat{y} - x) f(\hat{y}) d\hat{y} & \text{if } x < (1-a^{-\gamma})y \end{cases}$$

as the part of undeclared income below $(1-a^{-\gamma})y$ is discovered with certainty and a fine is imposed on it, then it will never be the case that $x < (1-a^{-\gamma})y$, provided the taxpayer knows the detection technology and $\gamma$.

Thus, concentrating on $x \geq (1-a^{-\gamma})y$ we have:

$$F = t \theta \int_x^y (\hat{y} - x) f(\hat{y}) d\hat{y} = t \theta \frac{a^\gamma}{y} \frac{1}{2} (y - x)^2 = t \theta \frac{a^\gamma}{y} \frac{1}{2} z^2$$

Then,

$$I = y(1-t) + zt - a^\gamma \theta \frac{1}{y} \frac{1}{2} z^2$$
that is equivalent to (5a), where the probability of an auditing being performed $\gamma \in [0, 1]$ is substituted by the coefficient $a^\gamma \geq 0$, where $\gamma$ is the amount of resources devoted to auditing and $a$ indicated how fast the amount of discovered income increases with auditing effort.

8.2 Audit conditional on report $x$

Probability of performing an audit can be conditioned on declared income $x$, so $\gamma = \gamma(x)$

**Proposition 4** As far as $\gamma \theta < +\infty$ it is impossible to induce any taxpayer to fully comply.

**Proof.** Given an income $y$ and a probability of audit $\gamma(x) \in [0, 1]$ a taxpayer prefers to declare $y$, i.e. to fully comply, than declaring $x \in [0, y)$ iff

$$(1 - t)y \geq y - xt - \gamma(x)t\theta \frac{1}{2}(y - x)^2 \iff \theta \gamma(x) \geq \frac{2}{1 - x^2} = \gamma_{x,y}^*$$

As $\lim_{y \to 1-} \gamma_{x,y}^* = \lim_{x \to y-} \gamma_{x,y}^* = +\infty$ then as far as $\gamma \theta < +\infty$ there is a neighborhood of $y$ at which the above condition cannot hold and thus taxpayers prefer to declare $x$ than $y$.

In the alternative setting proposed in this appendix the equivalent condition not to have full compliance even in case of devoted "auditing resources" conditional on declared income is $\alpha^{\gamma(x)} \theta < +\infty$.

The above proposition implies that whatever auditing policy is implemented, at any income level there will be some evasion. So, for any auditing policy there is room for the minimum wage to exert its influence. However, a fixed cost for the taxpayer of being subject to an audit, together with a higher probability of being audited in case of non-compliance than in case of full compliance, would undo the result.

9 Appendix B

The relative size of the shadow economy is estimated by Schneider (2005) using a DYMIMIC (Dynamic Multiple Indicator, Multiple Causes) approach, where the size of the hidden economy is a latent variable. For transition countries cause variables used in the structural model are: share of direct
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<th>GDP 2001/02</th>
<th>GDP 2002/03</th>
<th>Proportion of full-time employees with earnings on the minimum wage (%)</th>
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1. minimum wage is fixed on an hourly base;  
2. part-time workers included in the data on earnings;  
3. earnings at or below the minimum wage;  

Source: Schneider (2005)  
taxation, share of indirect taxation (both in % of GDP); share of public administrative employment in % of total employment as a proxy for burden of state regulation or state interference; unemployment rate and GDP per capita. For highly developed OECD countries additional cause variables used are the burden of social security payments, the tax morale, quality of state institutions and an index of the regulation of the labor market. Employment rate (% of the population between 18 and 64), annual growth rate of GDP, and annual growth rate of local currency per capita are used as indicator variables in the measurement model. The absolute size of the shadow economy is calculated combining the estimates for the relative size obtained through the above mentioned method with available estimates for the size of the informal economy obtained through a currency demand approach and available for some countries. For details on the method see Schneider (2005).

10 Bibliography


Schneider, F. 2002. The size and development of the shadow economies of 22 transition and 21 OECD countries. (IZA DP 514.)


