

Multiproduct Cost Function Estimation for American Higher Education: Economies of Scale and Scope

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1 Introduction¹

Institutions of Higher Education ('IHEs') usually produce a number of outputs: undergraduate and graduate education as well as research and public service. Moreover, these goods can greatly differ in quality. We have to take all this into account when trying to answer the question: are there economies of scale or scope in higher education production? If so, to what point?

In the last decade two pioneering studies, Cohn, Rhine and Santos (1989) and de Groot, McMahon and Volkwein (1991) estimated multiproduct cost functions for IHEs. The former used a sample of almost two thousand American IHEs, whereas the latter concentrated on a relatively homogenous set of 147 research universities whose research output was readily measurable thanks to an earlier study. Dundar and Lewis (1995) used the same approach to analyze the cost structures of different departments of 18 public research universities. Goudriaan and de Groot (1993) used the different regulative regimes of the different state governments and legislatures over their respective state universities as independent variables; they tried to uncover what efficiencies or inefficiencies these might cause. Finally, quite recently, Koshal and Koshal (1999) applied a methodology similar to ours to 158 private and 171 public comprehensive universities.

¹I thank an anonymous professor for his/her comments on the first version of this paper.

In this paper, we seek to extend the work of Cohn et al. (1989), concentrating on as large a set of American institutions of higher education as possible by

- introducing a proxy for the quality of educational output (like do Koshal and Koshal (1999) for their more limited sample),
- introducing the value of the endowment of an institution as a possible measure of the softness of the budget constraint an IHE faces,
- carrying out a sensitivity analysis of our specification w.r.t. the definition of costs (net, gross) or outputs (enrollment, degrees awarded; different proxies for research).
- choosing a different academic year to analyze.

2 Methodology

2.1 Theory

While the returns to education as an investment in human capital have been thoroughly discussed by economists (for a recent survey, see Psacharopoulos (1992)), the production side of the story has received much less attention.

So far, we do not have an accepted explicit analytical theory of how IHEs behave. In Rothschild and White’s words (Rothschild and White 1991) “The motivations of [...] senior administrators, regents and trustees [...] resist easy characterization”.

Estelle James (James 1989, James 1978) suggests that IHEs maximize an output-dependent utility function subject to a zero-profit constraint. Goudriaan and de Groot (1993) frame the behavior of state IHE decision-makers in a principal-agent framework, where the regulation can increase or decrease the (unobservable) cost function by an ad hoc multiplicative term, suggesting that the observed costs may well include inefficiencies.

Even though we are still uncertain about the exact nature of the behavior of IHEs, we know that they face a budget constraint (which may not be too ‘hard’– cf. Kornai (1986)). So they will be affected by the technological limits of producing educational and research output: a certain number of professors or rooms or equipment only allow a certain combination of outputs. Thus,

as Verry (1987a) notes, arguing along the same line, “it is not absurd” to assume that IHEs are cost minimizers.

Unfortunately, we do not know much about the shape of the technological production set of higher education, either. Lloyd (1994), building a cost function for Australian universities based on the technological characteristics of HE, suggests that “[the long run cost function] is not necessarily sub-homogeneous and not necessarily sub-additive in [the outputs]. [...] in the presence of common inputs which are fixed in supply, there may be economies or diseconomies of scale and economies or diseconomies of scope.”

Without any specific model to base our estimation on, we follow Cohn et al. (1989), in choosing a quadratic form with dummy-fixed costs. This is simple yet flexible enough a to be able to exhibit the different relevant multi-product characteristics (see (Baumol, Panzar and Willig 1982, pp.453–57)). (We also tried another cost function, a generalized Box-Cox translog function also mentioned by Baumol et al. (1982, pp.450–453) but it did not consistently improve the fit while the interpretation of its individual coefficients is rather cumbersome in this case.)

We did not find any study explicitly listing the different ways different states regulate their state universities for our time period comparable to the one used in Goudriaan and de Groot (1993) for the early eighties. Nevertheless, we differentiate between IHEs under private/public control and include the value of the endowment of the IHE, as a variable that might affect the behavior of decision-makers. This was considered to have a (negative) effect on the cost efficiency as early as in 1776, when Adam Smith (Smith 1976, Bk. V. Ch. I) wrote: “The endowments of schools and colleges have necessarily diminished more or less the necessity of application in the teachers” (cited by Verry (1987b)).

2.2 Multiproduct cost functions

The extension of the intuitive notion of returns to scale to multiproduct cost functions is not straightforward at all. We apply the analytical framework of Baumol et al. (1982) (‘BPW’).

Ray Economies of Scale

For a total cost function $C(y_1, y_2, \dots, y_n)$ of producing a vector of n products, $y = (y_1, \dots, y_n)$, BPW define The *degree of (ray) scale economies* at y , $S_N(y)$,

as

$$S_N(y) = \frac{C(y)}{\sum_{i=1}^n y_i C_i(y)}, \quad (1)$$

where $C_i(y) = \frac{\partial C(y)}{\partial y_i}$. The name of the measure refers to the ray connecting y to the origin in output-space. If $S_N(y)$ is larger than 1, then there exist scale economies at y . In other words, if $S_N(y)$ is more (less) than 1, the marginal cost of producing the composite output y is less (more) than the average cost, assuming that we hold the proportions within the output-bundle fixed. BPW suggest that “ $S_N(y)$ can be interpreted as the elasticity of the output of the relevant composite commodity with respect to the cost needed to produce it”.

Product-specific economies of scale

The *degree of product specific economies of scale* with respect to product i at y , $S_i(y)$, is defined as:

$$S_i(y) = \frac{C(y) - C(y_{n-i})}{y_i C_i(y)}, \quad (2)$$

where $C(y_{n-i})$ is the cost of producing all but the i th component of y . If $S_i(y)$ is greater (less) than one, we say that there are economies (diseconomies) of scale for producing y_i . This measure tells us whether there are returns to scale in increasing the production of one individual product in our output vector.

Economies of Scope

The *degree of economies of scope* at y relative to the product set T is defined as:

$$SC_T(y) = [C(y_T) + C(y_{n-T}) - C(y)] / C(y), \quad (3)$$

where $C(y_T)$ stands for the costs of producing only the products in y that are in the subset T , and $C(y_{n-T})$ means the costs of producing the complement set. If SC_T is positive at y , then the fragmentation of production to T and its complement would increase costs. Hence we can say that C

exhibits economies of scope w.r.t. T at y . If $SC_T(y) < 0$ then splintering production would reduce costs.

BPW give a detailed analysis of this concept; Let us just cite a result that connects this notion with the more familiar notion of (weak) *cost complementarities*. ($C(y)$ exhibits (weak) cost complementarities up to y if, for all $0 \leq \hat{y} \leq y$, all the cross partials $C_{ij}(\hat{y})$ ($i \neq j$) are non-positive².)

BPW show that a cost function which exhibits weak cost complementarities up to y exhibits economies of scope at y with respect to all partitions of the product set.

It is important to note that cost complementarities are an inherently local notion, whereas the degree of economies of scope takes the global perspective; among other things, it is sensitive to the presence of product-specific ('dummy'-) fixed costs. Economies of scope can be present despite some degree of anticomplementary.

Another result connecting ray- and product specific returns to scale that might be helpful in interpreting section 4.2's findings is the following:

$$S_N(y) = \frac{\sum_{i=1}^n \beta_i S_i(y)}{\frac{\sum_{i=1}^n (C(y) - C(y_{n-i}))}{C(y)}} \quad (4)$$

where $\beta_i = \frac{y_i C_i(y)}{\sum_{j=1}^n y_j C_j(y)}$

Invoking (3), we can rewrite the denominator to obtain

$$S_N(y) = \frac{\sum_{i=1}^n \beta_i S_i(y)}{\frac{\sum_{i=1}^n C(y_i)}{C(y)} - \sum_{i=1}^n SC_i(y)} \quad (5)$$

Note that the numerator is a weighted average of the product specific returns to scale, whereas the denominator contains the effect of the interrelatedness of the costs. If the cost function were additively separable ($C(y) = \sum_{i=1}^n C(y_i)$), then the denominator would be exactly 1, therefore ray economies of scale could be decomposed as the weighted average of the product-specific economies of scale.

For a general cost function, however, the denominator is not easy to characterize. One way to interpret it is to say that if the cost of producing y in $n - 1$ identical full multiproduct plants is more (less) than the aggregate cost of producing all but the i th product in plant i ($i = 1, 2, \dots, n$), then the denominator is greater (less) than one.

²Assuming that C is twice-differentiable.

Equations

On theoretical grounds, like de Groot et al. (1991) but unlike Cohn et al. (1989) and Koshal and Koshal (1999)³, we decided not to include wage terms (the price of the single most important input) among the independent variables. Since the market for academic labor is quite competitive, variation in wages will only reflect variation in productivity, not exogenous factors. There is no point in calculating, say, returns to scale ‘at a given wage level.’ By excluding the wage terms, we also assure that S_N , S_i and SC are independent of wage, which amounts to making sure that the cost function be linearly homogeneous in input prices.

Theoretical considerations suggest that public and private IHEs are operating under different incentive systems, so their observable cost functions (including different production inefficiencies) ought to be different. In all specifications of our regression, Chow tests rejected the hypothesis that public and private IHEs have the same cost function ($p < 0.001$); therefore we estimated separate cost functions for private and public IHEs. Next we tested over the sub-sample for which we had a quality proxy whether an F-test rejects the restriction that our quality measure (including quadratic and all interaction terms containing it) has no effects on costs.

In the case of private IHEs, quality did matter, in the case of public IHEs, we could not reject the hypothesis that our quality proxy does not explain any of the costs. In the former case, the \bar{R}^2 improved when we dropped the interaction and quadratic terms involving the quality proxy and left just the linear term.

We also carried out a Chow-like F-test of the hypothesis that within the set of public and private IHEs, graduate-degree granting institutions produce undergraduate education and research with a different cost function than graduate-degree granting institutions. In the specification we present, we could not reject the hypothesis that the presence of a graduate school does not affect the shape of the cost function for the other two products.

Thus the basic equation that we estimate is:

$$C_i = \alpha_0 \text{CONSTANT}_i + \alpha_{f1} \text{DBAC}_i + \alpha_{f2} \text{DDOC}_i + \alpha_{f3} \text{DRES}_i + \\ \alpha_1 \text{BAC}_i + \alpha_{11} \text{BAC}_i^2 + \alpha_2 \text{DOC} + \alpha_{22} \text{DOC}_i^2 + \alpha_3 \text{RES}_i + \alpha_{33} \text{RES}_i^2 + \\ \alpha_{12} \text{BAC}_i \text{DOC}_i + \alpha_{13} \text{BAC}_i \text{RES}_i + \alpha_{23} \text{DOC}_i \text{RES}_i + \alpha_4 \text{QUA}_i +$$

³They reported that their results did not change when dropping the wage terms.

$$\alpha_5 END_i + \alpha_6 DHOSP_i + v_i, \tag{6}$$

where *BAC*, *DOC* and *RES* are variables measuring undergraduate and graduate education and research output, respectively. *DBAC*, *DDOC* and *DRES* are dummies for the respective variables not being 0; *QUA* is the quality proxy, *END* stands for the value of the endowment of the IHE and *DHOSP* is a dummy that equals 1 if a hospital is affiliated with the IHE.

For the three samples (6) is appropriately modified:

- For the samples, both private and public, where no IHEs that do not grant at least a bachelor’s degree are included, $DBAC_i = CONSTANT_i$, thus *DBAC* is dropped.
- For public IHEs we estimate the equation without QUA_i .
- Since quality, as measured by our proxy, does not affect costs in public IHEs, we also estimate the equation for public IHEs from the larger sample (containing many sub-bachelor-degree IHEs) of IHEs that we don’t have a quality proxy for. In this specification we retain *DBAC*.

In the regression, we used a measure called ‘educational and general expenditures,’ net of scholarship and public service expenditures as the cost variable. This was first suggested by de Groot et al. (1991), who note that transfers are not part of the production process and that public service is a joint output we don’t have any proxy for.

In the version we present below, as customary, we used full-time-equivalent⁴ undergraduate and graduate enrollments to measure undergraduate- and graduate education output. Arguably, the number of degrees awarded better measures the output quantity. We used that measure in our sensitivity analysis.

The research output is proxied by the amount of federal science and engineering research and development grants awarded to the IHE. This measure, although gives a large weight to costly (medical, engineering) research, has the merit of being awarded according to a more or less uniform standard, thus it is, we hope, a measure of the quality of research output as well. We think such a measure is superior to the input-based approach that simply uses IHE research expenditures as a proxy for research output as do (Koshal

⁴FTE is a weighted sum of full-time and part-time enrollments.

and Koshal 1999) and other. Could it be composed, an aggregate citation index for our sample would be probably an even better proxy.

We use the IHE-level undergraduate ‘grades’ of the Gourman report (Gourman 1993), a rating of undergraduate programs. as a proxy for university quality. Unfortunately, the SAT scores of students admitted (used by Koshal and Koshal (1999)) were not available for our sample.

While the method by which the Gourman score is formed is not public⁵, it is claimed to aggregate ‘objective’ variables (that may already be present in our set of independent variables (e.g. student/faculty ratios) and questionnaire-based subjective evaluations. It covers a large number of undergraduate-degree granting IHEs (1283, after imputation) and it is presented in a convenient numerical form (a number in the 2.24–4.95 range). The very fact that the ranking sells on the market suggests that it may be correlated with the ‘true’ quality.

In order to test how well this proxy measures ‘true’ quality, we matched the Gourman scores with the celebrated U.S. News & World Report ranking⁶ (US News & World Report 1994) for the 249 IHEs that the US News & World Report ranks and found a correlation of 0.79 (significant at the 0.001 level) despite the integer nature of the ranking of the weekly.

An explicit criticism of Webster (1984) is that the Gourman score disfavors non-doctorate-granting and non-public IHEs. Our analysis confirms this with respect to the private/public institutions and rejects it with respect to the presence of graduate education: in a regression of the Gourman score on the US News & World Report-score and the dummies for doctorate-granting and private control, the coefficient of the dummy for private control is -0.2582 (St. E.: 0.0481) whereas the hypothesis that the effect of the presence of graduate education on the Gourman score, once controlled for ‘true’ quality, is zero, cannot be rejected. This, however, is no problem for our purposes, since we estimate private and public cost functions separately.

For the econometric calculations were done in *GAUSS*[©]; for calculating the different measures of economies of scale and scope, the software *Mathematica*[©] was used.

⁵For a devastating critique of the Gourman rating, see (Webster 1984).

⁶We transformed the partly qualitative ranking of the US News & World Report in the following manner: we assigned the numbers 1,2,3,4 for the IHEs ranked as being in the fourth, third, second and first tier of IHEs, respectively and we scaled the scores of the ‘best schools’ (i.e. those above the first tier) to the range 4.5–5.5.

3 Data

Our data set is constructed by matching three data files. The enrollment and cost variables for the academic year 1994-95 come from the Integrated Postsecondary Education Data System of the National Center for Education Statistics and comprises practically all IHEs granting at least an associate's degree. The source of federal research grants data is the National Science Foundation's Division of Science Resources Studies' *Federal obligations for science and engineering research and development to universities and colleges* file, financial year 1993 (the closest year for which data were available). Finally, the Gourman quality proxies are taken from Gourman (1993). In matching these files, we first formed a file of 2110 IHEs granting at least an associate's degree or receiving a positive amount in federal research grants. Then we formed the Gourman-file to obtain a set of 1114 IHEs, all of which grant at least a bachelor's degree, thus excluding two-year IHEs (community colleges)⁷.

A summary of the descriptive statistics of the variables for the three relevant subsets we use for estimation: private IHEs with Gourman scores, public IHEs with Gourman scores and public IHEs in the whole sample, are presented in table 1.

4 Results

We estimated simple OLS-regressions, then tested for heteroscedasticity: the Breusch-Pagan (Breusch and Pagan 1979) test revealed that the hypothesis of homoscedasticity could be rejected at a significance level 0.001 in all three samples.

Since there is no obvious pattern to the heteroscedasticity we could think of, we chose two ways to handle it.

⁷In matching federal grants with the IPEDS data, where the federal research grant data were given for sub-institutions that are recorded as one unit in IPEDS, we aggregated them; where the institutional unit in the research grant data was coarser than the IPEDS file, we divided up the grant with the research outlays of the IHEs as weights. This, however, only affected 42 observations. Similarly, where the Gourman score referred to the whole of a school whose separate branches consider themselves separate units, we assigned the same score to all of these. In a number of cases it was impossible to identify IHEs from the different data sets: we dropped these.

Table 1: Description, Mean and St. Dev. of Variables

Variable	Description	sub-sample w/ quality proxy				whole sample	
		Private		Public		Public	
		Mean	St. Dev.	Mean	St. Dev.	Mean	St. Dev.
<i>COST</i>	Educational and General Expenditures, net of scholarships and outlays for public service (million dollars)	47.82	119.65	117.68	159.83	70.494	123.38
<i>DBAC</i>	1 if <i>BAC</i> > 0, 0 otherwise	1.00	0.00	1.00	0.00	0.9927	0.0853
<i>DDOC</i>	1 if <i>DOC</i> > 0, 0 otherwise	0.6671	0.4716	0.9115	0.2845	0.4890	0.50002
<i>DRRES</i>	1 if <i>RES</i> > 0, 0 otherwise	0.4151	0.4931	0.7786	0.4157	0.4573	0.4985
<i>BAC</i>	number of FTE undergraduate enrollments (1000)	1.936	2.0584	7.898	6.0656	5.2879	5.3176
<i>BACSQR</i>	<i>BAC</i> squared	7.979	33.88	99.07	156.47	56.204	119.58
<i>DOC</i>	number of FTE graduate enrollments (1000)	0.5285	1.3724	1.4663	2.0156	0.7482	1.5760
<i>DOCSQR</i>	<i>DOC</i> squared	2.160	11.87	6.202	16.773	3.0406	11.911
<i>RES</i>	Federal research grants awarded (billion dollars)	0.0055	0.0313	0.0114	0.0299	0.0061	0.0220
<i>RESSQR</i>	<i>RES</i> squared	0.0010	0.0118	0.0010	0.0051	0.0005	0.0036
<i>BACDOC</i>	<i>BAC</i> X <i>DOC</i>	2.943	12.80	21.882	46.5715	10.416	33.696
<i>BACRES</i>	<i>BAC</i> X <i>RES</i>	0.0346	0.1979	0.2038	0.6730	0.0961	0.4712
<i>DOCRES</i>	<i>DOC</i> X <i>RES</i>	0.0316	0.2124	0.0646	0.2516	0.0315	0.1752
<i>QUA</i>	Gourman score -2.24	0.8108	0.4751	1.1458	0.4862	—	—
<i>END</i>	Endowment (million dollars)	92.51	356.0	36.67	203.6	19.053	140.76
<i>DHOSP</i>	1 if a hospital is affiliated, 0 otherwise	0.0247	0.1552	0.0599	0.2376	0.0488	0.2155
Number of observations		730		384		820	

Since OLS estimates [referred to as \mathbf{b} below], although far from efficient, are still unbiased in the presence of homoscedasticity, we present the OLS results. Standard errors, however, need to be adjusted for homoscedasticity. In calculating a consistent estimate of the covariance matrix, we used White's (White 1980) estimator, based on the OLS residuals $[e_i]$:

$$\text{Est. Var}[\mathbf{b}] = (\mathbf{X}'\mathbf{X}) \sum_i e_i^2 \mathbf{x}_i \mathbf{x}_i' (\mathbf{X}'\mathbf{X}) \quad (7)$$

Since scatterplots and different regressions of the OLS residuals on combinations of right hand side variables suggest that much of the heteroscedasticity may be related to the size of the IHEs which can be measured as some combination of the right-hand side variables, we repeated the estimation, using a two-step feasible generalized least squares approach suggested by Greene (Greene 1993, p.400). The estimation is based on the OLS regression of squared OLS residuals on the whole set of independent variables⁸, and using the $\hat{\sigma}_i$ s estimated in this manner to estimate $\mathbf{\Omega}$ that, in turn, is used in a standard FGLS estimation⁹. This, as shown by Amemiya (1985), provides us with an asymptotically efficient estimator of the coefficients.

While our sample is reasonably large, we would still certainly want to know the finite-sample properties of these estimators. Unfortunately Greene (1993) suggests that there is no definitive answer as to what estimator performs best in small samples. Rilstone's (Rilstone 1991) Monte Carlo estimates for samples of 20 and 50 suggest that correctly specified FGLS¹⁰ dominates OLS for moderate and high levels of heteroscedasticity; he also finds that the White heteroscedasticity-consistent standard errors for OLS tend to be downward biased.

In table 2 we present estimates and standard errors, both OLS and 2FGLS. In the case of OLS, the White heteroscedasticity-consistent standard errors are shown.

⁸Greene also mentions a multiplicative specification for the regression of the squared OLS residuals; we could not see any reason for this kind of pattern to heteroscedasticity; also we have a lot of zero values on the right hand side. Moreover, Rilstone (1991) suggests that, at least for small and middle-sized samples and not very high heteroscedasticity, 2FGLS estimates misspecified in this way (i.e. if the true pattern of the heteroscedasticity were multiplicative) are as good as the true 2FGLS estimates.

⁹in the case of a handful observations, the estimated variance was negative. In these cases we replaced these values with their respective absolute values.

¹⁰I.e. with respect to additive vs. multiplicative heteroscedasticity.

Table 2: Quadratic Cost Function

Variable	Private		Public		Public (whole sample)	
	OLS	2FGLS	OLS	2FGLS	OLS	2FGLS
<i>CONSTANT</i>	-1.850 (2.160)	-0.324 (0.8566)	2.875 (3.754)	-4.987 (3.227)	7.772 (13.72)	1.727 (1.876)
<i>DBAC</i>	— (—)	— (—)	— (—)	— (—)	-0.6949 (14.12)	-3.0705 (1.7054)
<i>DDOC</i>	-0.0264 (0.901)	-0.9425 ^b (0.3866)	2.0771 (2.942)	7.491 ^a (1.250)	4.639 (2.424)	4.987 ^b (1.990)
<i>DRES</i>	3.4379 ^a (0.970)	2.6594 ^a (0.3771)	-1.914 (2.214)	2.137 (1.369)	-0.6110 (2.210)	0.3467 (1.7276)
<i>BAC</i>	7.689 ^a (1.266)	6.811 ^a (0.3821)	5.480 ^b (2.180)	6.4488 ^a (0.8386)	2.6752 ^b (1.102)	7.279 ^a (0.5478)
<i>BACSQR</i>	-0.1266 (0.0674)	-0.0226 (0.0620)	0.0314 (0.1859)	-0.1202 (0.0714)	0.1809 ^b (0.0718)	-0.1456 ^a (0.0343)
<i>DOC</i>	9.270 ^a (3.298)	9.475 ^a (1.4557)	22.17 ^a (5.690)	8.6098 ^b (3.886)	33.46 ^a (6.652)	12.823 ^a (2.667)
<i>DOCSQR</i>	0.6586 (1.0306)	2.555 ^a (0.6561)	-1.760 (1.1667)	-1.0752 (1.1026)	-2.931 ^a (1.1217)	-2.195 ^a (0.7003)
<i>RES</i>	2610.1 ^a (465.78)	2730.8 ^a (329.40)	2424.4 ^a (545.95)	3500.5 ^a (437.32)	2145.5 ^a (660.55)	3016.9 ^a (438.90)
<i>RESSQR</i>	-1649.3 ^a (509.22)	-324.79 (436.49)	-7240.8 ^b (2847.8)	-3861.0 (3939.3)	-11049.5 ^a (4112.1)	-5578.1 (3631.3)
<i>BACDOC</i>	1.8642 (0.9770)	0.7369 (0.7030)	0.5208 (0.6485)	1.563 ^a (0.4779)	-0.0510 (0.3978)	1.515 ^a (0.1854)
<i>BACRES</i>	-56.63 (64.85)	44.53 (50.048)	1.8758 (33.576)	-23.47 (38.08)	6.3026 (36.070)	-14.54 (24.47)
<i>DOCRES</i>	-26.75 (83.48)	-265.49 ^a (53.03)	214.48 ^b (94.617)	18.34 (138.00)	347.73 ^a (120.63)	100.26 (101.21)
<i>QUA</i>	5.634 ^b (2.849)	3.822 ^a (0.752)	— (—)	— (—)	— (—)	— (—)
<i>END</i>	0.0736 ^a (0.0105)	0.1186 ^a (0.0056)	-0.0534 ^a (0.01943)	-0.0404 (0.0251)	-0.0319 (0.0218)	-0.0263 (0.0226)
<i>DHOSP</i>	52.44 ^a (19.147)	53.149 ^a (9.150)	40.90 ^a (11.889)	45.37 ^b (18.07)	65.71 ^a (14.38)	79.59 ^a (12.93)
Sample size	730		384		820	
R^2	0.980	0.963	0.974	0.953	0.960	0.947
\bar{R}^2	0.979	0.962	0.973	0.951	0.959	0.946

Numbers in parentheses are standard errors

^aSignificant at the 1% level or better

^bSignificant at the 5% level or better

4.1 The Regression Coefficients

From table 2, we infer the following:

- The fits are good: R^2 s are fairly high in all three specifications.
- On the whole, more coefficients are significant in the two larger samples (private IHEs in the sub-sample and public IHEs in the whole sample) than in the second regression.
- The dummy for graduate education is positive and significant in public IHEs and negative (and for 2FGLS, significant at the 5% level) in private IHEs. Since the presence of graduate education may require a certain quality of education, we considered the possibility that the strange negative sign might just show that the effect of this dummy was picked up by the quality proxy in the private regression. So we re-run the private regressions without the quality dummy: *DDOC* turned out to be positive but not significant for the White-corrected OLS and still negative and significant for 2FGLS. We do not have any ready explanation for the negative sign.
- The dummy for research is positive in all the three 2FGLS regressions, but only in the private one is it significant.
- The coefficients of the linear output variables are positive and almost everywhere significant. The magnitudes (6000-8000 dollars per year for an undergraduate, 8000–13000 dollars per year for a graduate student and 2-3 dollars for one dollar in federal research grant) are reasonable.
- The squared output terms are negative but not everywhere significant for undergraduate education and research. In the case of *RESSQR*, the uniform loss in significance as we move from OLS to 2FGLS is caused by the fact that federal research grants are skewed: a handful of larger universities (with large variances) get the most federal grants, while most small IHEs get very little. Thus, since observations with large variances are weighted down in 2FGLS, the standard error is relatively large. This speculation is corroborated by the fact that in estimating variances for 2FGLS the coefficients of *RES* and *RESSQR* are significant. It is important to note that, for private higher education, we

cannot reject the hypothesis that (not considering cross terms) the effect of undergraduate enrollments on costs is simply linear.

The coefficient of the number of graduate students squared constitutes an important difference: it is positive in the case of private IHEs and negative for public IHEs.

- The strange positive cross-effect between undergraduate and graduate enrollments, positive and significant in public schools, also noted by Cohn et al. (1989), suggests that there is anticomplementarity between the two outputs.

This anticomplementarity is contrary to the insight that graduate students can be conveniently employed to teach undergraduates.

In de Groot et al. (1991)'s homogeneous sample this cross-effect was negative, in (Koshal and Koshal 1999) insignificant; in our private IHE regression, where we could use a proxy for quality, the term is not significant, but it was positive and significant at the 1% level in the corrected OLS when we dropped *QUA*. This might suggest that the positivity of the coefficient is a tell-tale sign of misspecification due to a lack of being able to control for quality.

The cross-effect between undergraduate education and research is nowhere significant; the cross-effect between graduate-level education and research, another term that we expect to display complementarity, is indeed negative and significant in the private regression while positive in the case of public schools. As this coefficient, too, was significant but had a puzzling positive sign in Cohn et al. (1989)'s regressions (both private and public), we are again lead to believe that the positivity of this cross effect is another sign of misspecification. On dropping *QUA* from the private regression, *DOCRES*, while still negative, loses significance.

- The quality of education is costly but the price is not exorbitant: it costs an average private college about \$ 382 200 to increase its Gourman rating by one tenth of a grade point.
- Just as Adam Smith suggested, the value of the endowment has a significantly positive effect on private IHEs, but the mechanism of this relationship can be manifold. One suggestion is that with more own

resources, there can be more slack in the organization; another that endowment is correlated with age and old universities are more fossilized and less efficient. A third possibility is that endowment is just a stock result of an omitted but costly output: fund-raising.

In a puzzling way, however, for public IHEs the effect of endowment on costs is negative, though not significant. Perhaps public IHEs with an endowment are the ones that are more exposed to the forces of the market?

- Finally, IHEs with a hospital and a medical school incur large extra costs.

4.2 Economies of scale and scope

Because of the presence of cross and quadratic terms, the individual coefficients don't reveal a lot about economies of scale and scope. To get to know more, we calculated the BPW indices presented above: the degree of scale economies, the degree of product-specific economies of scale and the degree of economies of scope with respect to the most likely alternative to joint production, i.e. separating undergraduate teaching from the rest. We do this by simply calculating those measures according to definitions (1)–(3), substituting the estimated quadratic multiproduct cost function (6) for $C(\cdot)$. More specifically, we used the 2FGLS coefficients from table 2 to calculate S_N , $S_{i,s}$ and SC. We present our results for different multiples of the average output vectors (as listed in table 1), assuming there is no hospital affiliated to the school ($DHOSP = 0$), assuming average endowment, and, for private IHEs, fixing the quality proxy at its mean, $QUA = 0.8108$.

From tables 3, 4 and 5, the following emerge:

For an average private IHE, proportionate expansion brings economies of scale with it up to the point where the IHE grows to almost four times¹¹ its original size. At the point where economies of scale in this direction are exhausted, it has an FTE enrollment of 7 700 undergraduates, 2 100 graduates and research attracting 22 million dollars in federal research grants. This returns to scale does not come from expanding undergraduate or graduate education (product-specific economies of scale are below 1 for both); it comes from the positive returns to scale from more research, and, more importantly,

¹¹ $S_N = 1$ at 379% of the mean.

Table 3: Scale and scope economies for private IHEs, at different multiples of the average output, sub-sample

% of output means	Ray Economies	Product-specific Economies			Economies of Scope w.r.t. BAC
		BAC	DOC	RES	
50%	1.886	1.003	0.608	1.358	0.415
100%	1.407	1.006	0.742	1.182	0.252
150%	1.239	1.009	0.762	1.123	0.163
200%	1.147	1.011	0.758	1.094	0.102
300%	1.049	1.016	0.734	1.065	0.022
400%	0.990	1.019	0.710	1.051	-0.036
500%	0.948	1.023	0.688	1.043	-0.081

$DHOSP = 0$, $END = 92.51$ (mean), $QUA = 0.8108$ (mean)
The output mean is: $\overline{BAC} = 1.936$, $\overline{DOC} = 0.528$, $\overline{RES} = 0.0055$

from economies of scope that are above zero up to 334% of the average. Above this point (at which the IHE has 6500 FTE undergraduate students) economies of scope are negative, i.e., the IHE would be better off splitting its production to a separate college and a separate graduate school–research center. The denominator of the right hand side in (5) ranges from 0.58 [at 50% of the mean] to 0.96 [at 500% of the mean], suggesting that the interrelatedness of costs at this combination of outputs is, in fact, pushing ray economies of scale up. If we change the quality of the school, while holding output proportions and endowment constant, the point to which there exist returns to scope moves outward: at a Gourman grade of 4.9, ray economies of scale are only exhausted at 488% of the mean.

Since S_N is almost horizontal where it reaches 1, a slight shift in it will cause a large shift outward or inward.

The results concerning economics of scope are similar to Koshal and Koshal (1999)’s findings about private comprehensive universities that exhibit positive economies of scope up to about 300% of the mean; they, however, obtain positive ray economies of scale at or above mean size. Their findings coincide with ours with respect to the high, although decreasing economies of scale, at any level, as regards research.

It is interesting to compare the average private IHE with the average pri-

Table 4: Scale and scope economies for public IHEs, at different multiples of the average output in the sub-sample

% of output means	Ray Economies	Product-specific Economies			Economies of Scope w.r.t. BAC
		BAC	DOC	RES	
50%	1.055	1.073	1.827	1.118	-0.158
100%	0.987	1.144	1.371	1.071	-0.185
150%	0.951	1.215	1.254	1.061	-0.237
200%	0.924	1.283	1.208	1.060	-0.294
300%	0.882	1.417	1.174	1.070	-0.404
400%	0.849	1.546	1.164	1.089	-0.508
500%	0.822	1.671	1.160	1.114	-0.603

$DHOSP = 0$, $END = 36.66$. (mean)

The output mean is: $\overline{BAC} = 7.8981$, $\overline{DOC} = 1.4663$, $\overline{RES} = 0.0114$

vate *college*, i.e., the average non-doctorate-granting private IHE. Assuming that the quality and endowment value is the same as above, this college has ray economies of scale at any multiple of its average size of approx. 1 200 undergraduates.

This last insight is similar to Koshal and Koshal (1999)'s finding quoted above, and to Cohn et al. (1989)'s results for private IHEs in general, who find that there are ray returns to scale for any multiple of the average private IHE.

In the case of public IHEs that have been ranked by the Gourman report (table 4), we find a dramatically different picture. Our model suggests that ray returns to scale are exhausted at 87% of the average, i.e. at 6 850 undergraduates, 1 270 graduates and 9.8 million in federal research grants. What is more striking though is that there would be returns to scale to reap if only the IHE expanded in one direction and did not stick to its composite output. The denominator of the decomposition (5) is more than 1 for all the different multiples of the average output, equalling 1.18 at 100% of the mean. The degree of economies of scope is negative along the ray: at any multiple of the average it would reduce costs if the public IHE split up. We find a similar pattern if we consider the whole universe of public IHEs, not just those in the Gourman sample (table 5);

Table 5: Scale and scope economies for public IHEs, at different multiples of the average output; whole sample

% of output means	Ray Economies	Product-specific Economies			Economies of Scope w.r.t. BAC
		BAC	DOC	RES	
50%	1.101	0.890	1.913	1.044	-0.004
100%	1.045	1.028	1.457	1.031	-0.061
150%	1.024	1.117	1.332	1.031	-0.107
200%	1.012	1.199	1.282	1.034	-0.151
300%	0.996	1.365	1.248	1.043	-0.234
400%	0.985	1.554	1.241	1.055	-0.315
500%	0.976	1.777	1.241	1.068	-0.394

$DHOSP = 0$, $END = 19.05$ (mean)

The output mean is: $\overline{BAC} = 5.2879$, $\overline{DOC} = 0.7482$, $\overline{RES} = 0.0061$

the only difference is that economies of scale are not exhausted up to 270% of the average (14 000 undergraduates, 2 000 graduates, \$16 million in grants). However, above 47% of the average output bundle (2 500 undergraduates, 350 graduates and \$2.9 million in grants) splintering production into a separate college and graduate school decreases costs.

Cohn et al. (1989) also found negative returns to scope but only for public IHEs smaller than about 170% of the average. They found that ray economies of scale for public IHEs were exhausted around the mean.

While positive, the economies of scope calculated by Koshal and Koshal (1999) for public comprehensive universities are also rather low and decreasing in size; they, however, find positive ray returns to scale above the average size.

Since de Groot et al. (1991) found economies to scale way above their sample mean of research universities, we calculated returns to scope for the average graduate-degree granting public IHE: the results, however, were essentially similar to those presented.

These results suggest that the segment of private higher education sector that is below or around average size (2000 FTE undergraduate students) could benefit from economies of scale. Also, in private higher education there are economies of scope present: a splintering of production would increase

costs.

Naturally, the lack of useful quality proxy for this sector makes our specification rather questionable. Our findings about public higher education suggest that public universities that jointly produce IHEs graduate education, research and undergraduate education tend to be costlier than specialized public colleges and research universities that concentrate separately on graduate education and research. Also, public IHEs above 300% of the average size may well be in the region where there are diseconomies of scale present.

5 Sensitivity Analysis

In addition to the specification presented above, we tested a number of alternative ways of estimating equation 6. Below we briefly report our findings.

In all specifications we tried, Chow tests rejected the hypothesis that private and public IHEs have the same cost-function. Moreover, the finding that the quality proxy works for private IHEs also proved to be general. In every specification mentioned below, endowment had a positive effect on costs in private IHEs and negative in public IHEs.

We first experimented with replacing costs net of scholarship transfers and public service outlays with gross costs. Scholarships probably do play a role in increasing quality. (Rothschild and White (1991), among others, point out that students (especially talented students) are an input in educating the others too. Financing these students can be a part of the production process. The expenditures earmarked for ‘public service’ may well have been spent for activities that also produce research or increased enrollment. This specification resulted in a similar set of coefficients. In the private IHE equation, Only *DOCRES* changed signs, but it was also highly insignificant. The only notable difference was that for the public IHEs in the whole sample *BACSQR* had a positive significant coefficient.

For private IHEs the shapes of the different degrees of scale and scope curves are the same as in the presented regression. All that happened is that S_N shifted up a bit, therefore, in this specification ray returns to scales persist up to 1200% of the mean.

For public IHEs, however, the functions radically changed shape. For this specification we obtained results similar to those of Cohn et al. (1989): ray returns to scale were greater than 1 for any size and returns to scope were positive.

Next we changed our proxy for research output: in this specification we used the portion of expenses reported as used for research as a proxy. This probably measures research input reasonably well but does not reflect research quality. In this version the quality proxy proved significant in both the private and the public specification, presumably because the new research variable was uncorrelated with quality research quality, which, in turn, is correlated with teaching quality.

For the private case, again we obtained similar coefficients and the same pattern of returns to scale and scope, with ray economies of scale up to 380% of the mean. For public IHEs, now controlled for quality, we get a different pattern again: this time the degree of returns to scale is less than 1 for any fraction or multiple of the average, but the returns to scope are positive for any size, too.

Finally we attempted to use degrees granted, rather than FTE enrollments, as output variables. Arguably, the actual output is better measured if we don't take drop-outs into account. Naturally, if we relate degrees to cost data from the same year, we are disregarding the delay in production. If higher education production is not stationary, then our estimates are biased; if the growth rate of different IHEs depends on the independent variables, the bias will be even worse.

Nevertheless, we did estimate OLS/2FGLS regressions for the four outputs: associate's degrees, bachelor's degrees, graduate degrees and research, while controlling for quality. In this specification, however, it turned out that there is another structural division in the sample. After having tested for a structural difference between private and public IHEs, and having ascertained that the difference was indeed significant, we also tested the hypothesis that undergraduate production (associate's and bachelor's degrees) and research had different costs in graduate-degree-granting institutions than in colleges. Unlike in all the earlier specifications, here we could reject the hypothesis that the two sets have the same cost function. Thus we estimated four sub-samples: Private-Graduate, Private-no-Graduate, Public-Graduate, Public-no-Graduate. For all four groups we could reject the hypothesis that quality does not affect costs at the 5% level. While R^2 s are above 0.97, few coefficients are significant on their own (with the associate's degrees have 19 independent variables). We sum up our findings as to ray returns to scale and scope¹² in table 6.

¹²We calculate returns to scope with respect to the alternative of grouping the pro-

Table 6: Ray returns to scale and returns to scope: output in degrees awarded

Sample	Interval s.t. $S_N > 1$	Interval s.t. $SC > 0$	ASS	BAC	DOC	RES
PRIV-GRAD	$[0, \infty)$	$[0, 140\%]$	0.012	0.776	0.154	0.026
PRIV-NO-GRAD	$[0, 174\%*]$	$[0, 417%*]$	0.0074	0.146	0	0.0001
PUB-GRAD	$[0, 1400\%]$	$[0, 141\%]$	0.034	1.632	0.164	0.027
PUB-NO-GRAD	$[0, 875\%]$	$[0, \infty)$	0.0195	0.47	0	0.0009

* The function approaches infinity

The table does not suggest any simple conclusions. The fact that the degree of returns to scale approaches infinity at a point well within the sample in the PRIV-NO-GRAD case casts doubt on the specification. Nevertheless, it is worth noting that, in this instance, IHEs with graduate schools show a similar pattern. This is parallel to de Groot et al. (1991)'s finding that amongst research IHEs there is no structural difference between private and public schools.

6 Concluding Comments

We estimated a quadratic multiproduct cost function for higher education, using a large sample of institutions. The following are our main findings.

Private and public IHEs have different cost functions. Private IHEs have a consistent cost function, robust to different versions of the regression; the estimated cost functions of public IHEs change wildly on minor respecifications of the model.

In private IHEs we find that economies of scope are present throughout. Also, there are economies of scale up to a point that is above the average size of an average private IHE. In private IHEs the marginal cost of educating undergraduates is decreasing in the number of undergraduates while the cost of educating graduate students is increasing. The value of the private IHE's endowment is positively correlated with its costs.

About public IHEs we can say rather little. Our regression has found that there are no economies of scope whatsoever for public IHEs larger than 50% of their mean, but this result is not robust at all.

duction of Associate's and Bachelor's degrees apart from producing graduate degrees and research.

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