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DISCUSSION PAPERS

MT-DP. 2004/17

**INFLATION INERTIA AND
MONETARY POLICY SHOCKS**

JÚLIA LENDVAI

Institute of Economics
Hungarian Academy of Sciences

Budapest

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KTK/IE Discussion Papers 2004/17

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The paper selected for the 4th Budapest Summer Workshop for young economists, organised by the KTI/IE on 29–30 June 2004.

The Budapest Summer Workshops intend to bring together young economists with foreign PhD education, frequently still working or studying abroad.

Inflation Inertia and Monetary Policy Shocks

Author: Júlia LENDVAI, PhD Candidate. Department of Economics,
University of Namur. E-mail address: julia.lendvai@fundp.ac.be

I am grateful to Paul Reding for his following up this work, his advice and his encouragement. I would also like to thank Jordi Gali, Xavier Debrun and Michele Lenza for useful discussions, and participants of the Conference of the Scottish Economic Society in Perth, March 2004, of several doctoral seminars and of a seminar held at the National Bank of Hungary in Budapest, August 2004, for their comments. All remaining errors are mine.

HU ISSN 1785-377X

ISBN 963 9588 17 2

Published by the Institute of Economics Hungarian Academy of Sciences, Budapest, 2004.
With financial support from the Hungarian Economic Foundation

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INFLATION INERTIA AND MONETARY POLICY SHOCKS

BY JULIA LENDVAI

Abstract

This paper compares the New Keynesian Phillips curve with the hybrid Phillips curve for its ability to reproduce observed inflation and output dynamics. The analysis is based on impulse responses of a miniature general equilibrium model incorporating price and inflation inertia as the only friction. Impulse responses to a monetary policy shock for different intensities of inertia are compared with empirical impulse responses of the US economy estimated using a structural VAR. We find that the NKPC does a reasonably good job of reproducing the dynamic link between current output and future inflation. It fails however to reproduce the co-movement of current output with current and lagged inflation and it generates too low degrees of inflation and output persistence. The assumption of inflation inertia improves a model's performance of reproducing inflation persistence and the dynamic inflation-output link. A significant improvement requires high intensity of inflation inertia. The fit of observed output persistence can not be improved.

Keywords: Inflation inertia, Phillips Curve, Persistence, Dynamic link

LENDVAI JÚLIA

INFLÁCIÓS RAGADÓSSÁG ÉS MONETÁRIS POLITIKAI SOKKOK

Összefoglalás

Ez a cikk az Új Keynesi Phillips görbét és a hibrid Phillips görbét hasonlítja össze abból a szempontból, hogy a két specifikáció mennyire képes a megfigyelt inflációs és termelés dinamikát reprodukálni. Az elemzés egy olyan miniatúr általános egyensúlyi modell impulzus válaszain alapul, melyben a súrlódások kizárólagos forrása az árak és az infláció ragadóssága. A modell különböző intenzitású inflációs ragadósság mellett adott impulzus reakcióit az infláció és a kibocsátás valóságban megfigyelhető reakcióival hasonlítjuk össze. Az empirikus reakciókat strukturális VAR-ral becsüljük az USA gazdaságára.

Az elemzés során azt állapítjuk meg, hogy az Új Keynesi Phillips görbe viszonylag jól reprodukálja a jelenlegi kibocsátás és a jövőbeli infláció közötti dinamikus kapcsolatot. A jelenlegi kibocsátás együttmozgását az egyidejű illetve a múltbeli inflációval, valamint az infláció és az output gap perzisztenciáját ellenben ez a specifikáció nem képes reprodukálni. Az infláció ragadósságának feltételezése javítja a modell teljesítményét az infláció perzisztenciája és a dinamikus infláció-kibocsátás kapcsolat reprodukálása szempontjából. Szignifikáns javulás azonban csak az inflációs ragadósság erős intenzitásának feltételezésével érhető el. A megfigyelt kibocsátás perzisztencia illeszkedése nem javítható.

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1 INTRODUCTION

Recent monetary business cycles models most commonly use time-contingent staggered price setting of forward looking firms to specify nominal rigidities. This nominal rigidity specification builds on seminal work by Calvo (1983) and is usually referred to as the New Keynesian Phillips Curve (NKPC).

However, models using the NKPC have been subject to criticism, mainly on empirical grounds. Apart from being unable to explain real costs of disinflations¹, the Calvo model has been shown to have difficulties in reproducing the observed dynamics of both inflation and output when a plausible degree of price stickiness is assumed.² Critics claim that the NKPC fails to generate the observed persistence in inflation and output series. In addition, it has counterfactual implications for the dynamic link between inflation and output: while the output gap is observed to lead the inflation rate over the cycle in actual data, the NKPC is argued to imply the opposite of this. At the origin of these shortcomings, there is a common problem: while the NKPC is a sticky price model, it implies purely forward looking and fully flexible inflation dynamics in reaction to shocks. This, however, is contrary to the empirically observed slow adaptation of inflation to unexpected changes in economic conditions, which is referred to as 'inflation inertia'.

This observation has motivated the elaboration of alternative models which accounted for inflation inertia based on individual behavior. One class of those models is due to Gali, Gertler (1999), who introduce sticky inflation based on backward looking rule of thumb in the price adjustment of a fraction of firms. Recently, inflation inertia settings have been more and more incorporated into stochastic dynamic general equilibrium models. Yet, the precise implications of inflation inertia for business cycle dynamics have up to now remained unexplored.

This paper therefore seeks to clarify the influence of inflation inertia on inflation and output dynamics. To this aim, we formulate a simple dynamic general equilibrium model incorporating as the only friction the Gali, Gertler (1999) inflation inertia specification. This model is used to assess whether and to what extent the assumption of inflation inertia accounts for the observed inflation and output persistence and for the dynamic link between these variables. Our strategy is to compare the model's predictions for inflation and output responses to a single shock assuming different degrees of inflation inertia. We choose the shock to be a monetary policy shock. The empirical assessment of the theoretical predictions is based on estimated impulse responses of inflation and output to a monetary policy shock. The empirical responses are estimated for the United States over the sample period 1965Q3-1995Q2 using an identified vector autoregression (VAR). The question we ask is: Can a model incorporating the assumption of inflation inertia yield predictions for the dynamics of inflation and output responses to monetary shocks which

¹See e.g. Ball (1990).

²See e.g. Fuhrer, Moore (1995); Gali, Gertler (1999); Chari, Kehoe, McGrattan (2001).

come closer to the observed dynamics than predictions of a model assuming forward looking inflation?

The two features of output and inflation dynamics we are concerned with are the individual persistence of output and inflation on one hand, and dynamic association between them on the other hand. To measure persistence, we define a new indicator, which, as we show, is able to capture the persistence of both theoretical and empirical impulse responses more accurately than traditional measures. Our indicator is based on the time profile of a variable's conditional variance generated by an unexpected monetary shock.

The dynamic link between variables is evaluated on the basis of dynamic cross-correlations. Our study has the distinguishing feature of comparing the cross-correlation pattern between theoretical impulse responses with the estimated cross-correlation of empirical impulse responses instead of comparing it with observed cross-correlations in actual data.

The results we obtain allow us to reassess criticisms about the NKPC. In line with criticisms, we find that the Calvo model generates, by itself, relatively low degrees of inflation and output persistence. However, in contrast to some other criticisms, we show that the NKPC does a reasonably good job of accounting for the co-movement of the current output gap with future inflation rates. Its shortcoming lies rather in its failure to reproduce the dynamic link of current output with current and lagged inflation rates. It especially implies an excessively close contemporaneous co-movement between these variables.

Our comparison of the hybrid Phillips curve with the NKPC shows that inflation inertia and price stickiness have qualitatively different implications for the dynamics of inflation and output. Inflation inertia delays and prolongs the effect of a monetary shock on inflation compared to its effect on the output gap. In addition, higher degrees of inertia induce oscillations into the transition paths of impulse responses. At the same time, empirically plausible degrees of inflation inertia cannot generate an output response which is more persistent than the one generated by a model with fully flexible inflation.

The result of our empirical evaluation shows that the assumption of inflation inertia improves the model's fit for observed inflation persistence and for the dynamic link between inflation and output. A significant improvement can, however, only be observed for a relatively high degree of inertia. Moreover, the assumption of inflation inertia cannot make up for the NKPC's shortcoming regarding output persistence. It continues to fall short of its observed level when empirically plausible degrees of inflation inertia are assumed.

The remainder of the paper is organized as follows. Section 2 lays out the theoretical model and describes its impulse responses. Section 3 presents the empirical model. Section 4 describes and analyses the contribution of inflation inertia to reproducing observed inflation and output dynamics. Section 5 concludes.

2 Theoretical Model

This section presents a miniature general equilibrium model with a single source of friction which is inflation inertia. Our model is by no means meant to be realistic; we therefore do not expect it to reproduce observed business cycles. Instead, the model is kept as simple as possible in order to allow us to concentrate on the implications of inflation inertia.

We first discuss the concept and the model of inflation inertia in detail. Then we briefly present other parts of the general equilibrium model. Finally, we describe the model's calibration and its impulse responses focusing on the implications of inflation inertia.³

2.1 Inflation Inertia

To avoid confusions, it seems useful to start with the definition of our concepts of inertia and persistence which will be used throughout this paper.

By the notion of '*inertia*' we refer to the slow adaptation of a variable to unexpected changes in economic conditions. If a variable is not inertial, it will be said to be fully flexible. The notions of stickiness and inertia are used as synonyms. In contrast to inertia, the notion of '*persistence*' refers to the slow transition of a variable to its steady state after the initial impact of the unexpected shock. By this definition, a variable is inertial if and only if its past levels, or past expectations about its current level, have a direct influence on its current level. As opposed to this, a variable's persistence can be generated by various sources, only one of which is the inertia of the given variable. It is important to emphasize that a variable can theoretically be persistent even if it is not sticky.

The empirical persistence of inflation and output responses to exogenous economic shocks has been broadly documented in recent years.⁴ In addition, empirical analyses of the inflation process have given an important role to inflation inertia.⁵ Inflation inertia has however turned out to be difficult to rationalize on the basis of optimizing individual behavior. Standard inflation inertia models assume some kind of backward looking behavior. One strand of these settings follows Gali, Gertler (1999, hereafter GG) in specifying backward looking behavior by the assumption that a fraction of firms is backward looking and hence follows a rule of thumb in readjusting their prices. Another version of backward looking behavior models assumes that all firms apply indexation to lagged inflation in periods when they cannot readjust their prices. Such models are described by Christiano, Eichenbaum, Evans

³Throughout the entire paper, notations are as follows. The current level of an aggregate variable in period t is denoted by X_t . The current individual levels are denoted by lower case letters x_t . The steady state level of a variable is denoted by letters without time index X respectively x for aggregate and individual levels. Lower case letters with tilde will denote the variables' percentage deviation from their steady state level, i.e. $\tilde{x}_t \equiv \frac{X_t - X}{X}$. For the nominal interest rate, the inflation rate and the money supply growth rate $\tilde{x}_t \equiv \frac{X_t - X}{1 + X}$.

⁴See e.g. CEE (1999), Altig et al. (2002).

⁵See e.g. Fuhrer, Moore (1995).

(2001, henceforth CEE) and Smets, Wouters (2003). The two assumptions explaining inflation inertia are equivalent in that they yield the same aggregate relationship linking current inflation rate to lagged inflation, expected future inflation and to some measure of real economic activity.

As an alternative to bounded rationality, inflation inertia has been modeled on the basis of sticky information by Mankiw, Reis (2001), on the basis of adaptive expectations by Roberts (1997) and as a signal extraction problem by Erceg, Levin (2001). Although there is no direct mapping between these models and the bounded rationality models, they yield similar implications for inflation and output dynamics.

In this paper, we will follow Calvo (1983) in assuming that in any given period, each firm readjusts its price to innovations with probability $1 - \xi$, or equally, each firm keeps its price fixed with probability ξ . This probability is common across firms and constant over time. The time between two price readjustments for an individual firm follows hence a geometric distribution. The expected time between two price readjustments is therefore⁶ $(1 - \xi) \sum_{k=1}^{\infty} \xi^{k-1} k = \frac{1}{1-\xi}$.

Following GG (1999), we assume two types of firms: there is a fraction $1 - \omega$ of firms that readjust their prices in a forward looking way to maximize their share value. In contrast, the other fraction ω is assumed to follow a backward looking rule of thumb when having the possibility to readjust.

We will hence distinguish between *fixed* and *newly set* prices in each period, where the newly set prices may be set in either a *forward looking* or a *backward looking* manner.

Denoting an index of fixed prices P_t^{fix} and an index of newly set prices P_t^* , the aggregate price index P_t can be expressed as⁷:

$$P_t = [\xi(P_t^{fix})^{1-\varepsilon} + (1 - \xi)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}, \quad (1)$$

where $\varepsilon > 1$ is the consumers' elasticity of substitution between differentiated goods. The newly set price index can itself be described as a weighted average of backward looking prices P_t^b , and forward looking prices, P_t^f ⁸:

$$P_t^* = [\omega(P_t^b)^{1-\varepsilon} + (1 - \omega)(P_t^f)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \quad (2)$$

Following Yun (1996) we assume that *fixed prices* are updated by the target rate of inflation π , i.e. $P_t^{fix}(i) = P_{t-1}(i)(1 + \pi)$.

⁶As opposed to time dependent models, where the probability of readjustment is fixed and given exogenously, this probability might be explained endogenously by the state of the economy. For a description and a comparison see e.g. Dotsey, King, Wolman (1999). It is generally argued that the difference in outcomes is negligible for moderate inflation rates. See GG (2003).

⁷This formula can be deduced from consumers' optimal demand for differentiated consumer goods. Precisely, the index of fixed prices can be expressed as $P_t^{fix} \equiv \left(\frac{1}{\xi} \int_{fixed\ in\ t} P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ and the newly set prices as $P_t^* \equiv \left(\frac{1}{1-\xi} \int_{adj\ in\ t} P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

⁸The precise expressions are $P_t^f = \left(\frac{1}{1-\omega} \int_f P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$ and $P_t^b = \left(\frac{1}{\omega} \int_b P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$.

Forward looking firms set their price to maximize their share value, which can be determined as the present value of future dividends paid by the firm to the households. This can be derived of consumers optimization problem in a general equilibrium model. Denoting the production of a firm i by $Y_t(i)$; assuming that firms pay out their total profits to the households in form of dividends each period; and taking into account the symmetry of forward looking firms in equilibrium, any forward looking firm readjusting its price in period t will set P_t^f to maximize:

$$E_t \sum_{k=0}^{\infty} \phi_{t+k} (\beta\xi)^k \left[P_t^f (1 + \pi)^k - S_{t+k}^n \right] Y_{t+k}(i), \quad (3)$$

where S_t^n stands for the nominal marginal cost in period t , and $Y_{t+k}(i)$ denotes the production of firm i at $t+k$. Equation (3) shows that the price set at t influences the firm's profits and hence its dividends as long as it is not allowed to reoptimize, the probability of which is ξ^k with k denoting the number of successive fixed pricing periods. The marginal value of a currency unit to the households, ϕ_{t+k} is treated as exogenous by the firm.

The FOC of the reoptimization is:

$$E_t \sum_{k=0}^{\infty} \phi_{t+k} (\beta\xi)^k Y_{t+k}(i) \left[P_t^f (1 + \pi)^k - \frac{\varepsilon}{\varepsilon - 1} S_{t+k}^n \right] = 0. \quad (4)$$

Note that this relation reduces to the standard constant mark-up pricing rule of a flexible price environment, when $\xi = 0$.

It is instructive to rearrange this condition so as to express the percentage deviation of forward looking prices from the steady state as⁹:

$$\tilde{p}_t^f = \tilde{s}_t + \tilde{p}_t + E_t \sum_{k=1}^{\infty} (\beta\xi)^k \tilde{\pi}_{t+k} + E_t \sum_{k=1}^{\infty} (\beta\xi)^k (\tilde{s}_{t+k} - \tilde{s}_{t+k-1}), \quad (5)$$

where \tilde{s}_t stands for the deviation of *real* marginal cost from its steady state; the inflation rate π_{t+k} is the rate of change of the aggregate price level between the periods $t+k-1$ and $t+k$: $\pi_{t+k} \equiv \frac{P_{t+k}}{P_{t+k-1}} - 1$.

This relation shows, that forward looking prices are set higher than the current nominal marginal cost when agents are expecting fast increasing prices for the future and/or if they expect the real marginal cost of production to increase. This behavior is what CEE (2001) call '*front loading*'. Firms know that they might not be allowed to reoptimize their prices for a number of periods; the price they set today will then influence their future profits, too. Anticipating this, forward looking firms set their prices today to maximize their current and future expected profits. In addition to the current marginal cost, they thus need to take into account the future expected evolution of nominal marginal cost as well. This in turn depends on the future expected

⁹See Christiano et al. (2001).

inflation rate and on the future expected changes of the real marginal cost in our setting¹⁰.

Finally, *backward looking firms* are assumed to readjust their price according to the following rule of thumb:

$$P_t^b(i) = P_{t-1}^*(1 + \pi_{t-1}). \quad (6)$$

This shows, that a backward looking firm sets its price to the average of the newly set prices in the previous period updated by the previous period inflation rate of the aggregate price level. Although not very realistic, as discussed in GG (1999, p.13) this assumption has two appealing features: first, it implies no permanent deviations between the rule of thumb and the optimal behavior; second, P_t^b only depends on information known up to the period $t - 1$ but implicitly incorporates past expectations about the future at the same time.

Equations (1), (2), (4) and (6) imply the following loglinearized relationship¹¹:

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t(\tilde{\pi}_{t+1}) + \lambda_s \tilde{s}_t \quad (7)$$

where $\gamma_b \equiv \frac{\omega}{\varphi}$, $\gamma_f \equiv \frac{\beta\xi}{\varphi}$, $\lambda_s \equiv \frac{(1-\omega)(1-\xi)(1-\beta\xi)}{\varphi}$ with $\varphi = \xi + \omega[1 - \xi(1 - \beta)]$.

This relationship is what GG (1999) call the *hybrid Phillips curve* which displays inflation inertia if and only if $\gamma_b > 0$, i.e. when $\omega > 0$. Inflation inertia is hence introduced by the presence of backward looking firms in this model. To see how, note that inflation dynamics are determined by the evolution of newly set prices. Since newly set prices are purely forward looking in the NKPC setting, the inflation rate itself will be fully flexible. In contrast, when a fraction of firms follows a backward looking rule of thumb, newly set prices become sticky and thereby introduce inflation inertia. In terms of the model coefficients this means that the hybrid Phillips curve (7) trivially nests the NKPC for $\omega = 0$, i.e. when all firms are forward looking, which implies $\gamma_b = 0$. A rise in ω leads to a rise in the coefficient of the lagged inflation γ_b . On this basis, in what follows, the intensity of inflation inertia will be measured by the fraction of backward looking firms in the economy.

Two additional points are worth noting.

First, a larger fraction of backward looking producers also implies a lower weight of the currently expected future inflation, γ_f , as well as a lower value of the coefficient of the current real wage λ_s . This is because only forward looking firms react contemporaneously to current market conditions. The importance of current variables in the determination of inflation dynamics is hence lower when the fraction of forward looking firms, $1 - \omega$ is smaller. In the limiting case, where all firms are backward looking, the inflation rate would

¹⁰Note, that frontloading is not the only source of the forward looking price's deviation from the flexible price, i.e. the price level prevailing without rigidities. Forward looking price deviates from the flexible price also and especially because the current nominal wage's path under sticky prices is different from its path under flexible prices.

¹¹For a derivation of this relationship see e.g. GGL (2001), Appendix A.

not at all react to current revision of inflation expectations or to changes in the current real marginal cost. At this point, we simply rule out the case of $\omega = 1$, since this degree of backward lookingness seems to be empirically implausible.¹²

Finally, throughout this paper, we assume an exogenously given constant fraction of backward looking firms. This assumed irrational price setting behavior might be considered as a shortcut for the rational decision of reoptimization, when firms face costs of information gathering or of decision making for example.¹³

2.2 General Equilibrium

In order to examine the implications of backward looking price for the model's dynamic properties, we will use the following miniature general equilibrium model consisting of three sectors: firms, households and monetary authority. For sake of simplicity, we assume no capital goods and no public expenditure in the economy.

2.2.1 Firms

There is a continuum of firms in the economy, each of which produces a differentiated good, $Y_t(i)$, using labor as the only input. The production technology is given by:

$$Y_t(i) = L_t(i),$$

where $L_t(i)$ is the labor employed by firm $i \in [0, 1]$.

On the aggregate level, the following relationship applies:

$$L_t^d \cong zY_t,$$

with $Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ and total labor demand $L_t^d \equiv \int_0^1 L_t(i) di$ and z standing for a constant scalar¹⁴. Price setting is characterized by the above described specification. Following from this technology, the real marginal cost of a firm $S_{t+k} = W_t$, with W_t denoting real wage.

¹²As shown by Steinsson (2003), this degenerate result can be ruled out by a minor change in the rule of thumb of backward looking producers. Steinsson assumes that backward looking prices react to the deviation of the output gap from its natural level in addition to the lagged newly set price level and lagged inflation rate. By this modified rule of thumb, the hybrid Phillips curve would nest the traditional Phillips curve at the limiting case $\omega = 1$. For estimations of ω see e.g. GG (1999), Gali, Gertler, Lopez-Salido (2001).

¹³See e.g. CEE (2001). Note also that, although the credibility problem is not directly treated in this model, the costs of reoptimization can be expected to depend on the credibility of the monetary policy. Despite these considerations, to our knowledge, backward looking price setting behavior has not yet been modelled on the basis of optimising behavior.

¹⁴The exact relationship between aggregate labor demand and aggregate output would be $L_t^d = \int_0^1 Y_t(i) di = Y_t Z_t$, with $Z_t = \int_0^1 \frac{Y_t(i)}{Y_t} di$. However, as shown in Gali, Monacelli (2002) Appendix 3 for instance, the percent deviations of Z_t around its steady state are of second order. For the purpose of the following first order approximation of the model's solution, it is hence sufficient here to consider Z_t constant.

2.2.2 Demand Side

The demand side of the model can be deduced from infinitely-lived households' optimization problem as discussed e.g. in Jeanne (1998). The optimal equilibrium aggregate consumption path of households is characterized by the following consumption Euler equation:

$$Y_t^\sigma = \beta E_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} Y_{t+1}^\sigma \right], \quad (8)$$

where i_t stands for the riskless nominal interest rate.

For the money demand, we choose the simplest specification:

$$M_t = Y_t,$$

with M_t standing for the real money supply. This equation, which is identical to the quantity-theory equation can be rationalized by a version of Cash-in-Advance constraints faced by the households as described in Jeanne (1998, p.1014). The modification allows to abstract from the inflation tax effect on aggregate labor supply.

2.2.3 Monetary Policy

Money is injected into the economy by the government via lump sum transfers to households.

Monetary policy is specified as the exogenous path of nominal money supply growth rate, given by the AR(1) process:

$$\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t, \quad (9)$$

where $\tilde{\mu}_t$ denotes the percent deviation of the money supply growth rate from its steady state target value. The autocorrelation coefficient is denoted by $\rho_\mu \in [0, 1[$. The monetary shock ϵ_t follows an i.i.d. white noise process with a standard deviation of σ_e .

2.2.4 Solution

After clearing of all markets, the equilibrium processes of the nominal interest rate, the inflation rate and the output gap can be expressed by the following log-linearized equation system:

$$\sigma \tilde{y}_t = \tilde{u}_t - E_t \tilde{\pi}_{t+1} + \sigma \tilde{y}_{t+1}, \quad (10)$$

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t (\tilde{\pi}_{t+1}) + \lambda \tilde{y}_t, \quad (11)$$

$$\tilde{y}_t = \tilde{y}_{t-1} - \tilde{\pi}_t + \tilde{\mu}_t \quad (12)$$

where the money supply growth rate, $\tilde{\mu}_t$ follows the exogenous law of motion given by equation (9).

Equation (10) is a first order approximation of the consumption Euler equation (8) taking into account the goods market equilibrium condition. The hybrid Phillips curve (11) uses the linear correspondence between output

and real wage, with¹⁵ $\lambda \equiv \lambda_w \left(-\frac{Y}{1-Y} \vartheta - \sigma \right)$. Finally, equation (12) is a first order approximation of the money supply growth process in equilibrium.

The solution for this dynamic equation system can be found by the method of undetermined coefficients as described in McCallum (1983). With monetary policy defined as in equation (9), the solution for the nominal interest rate is recursive. This allows us to subsequently concentrate on the solutions of the inflation rate and/or the output gap only¹⁶. A derivation of the solution is described in Appendix A.

2.3 Propagation of Monetary Shocks

Since there exists no closed form solution to our model we proceed as follows. The model's structural parameters will be calibrated based on the results of existing literature. We then study the reaction of the model to a monetary shock under this baseline calibration for different levels of $\omega \in [0, 1[$, everything else unchanged.

2.3.1 Calibration

As in most NK models, one period equals a quarter of a year. Setting the subjective discount factor $\beta = 0.99$ hence implies an annual real interest rate of 4.04% in the steady state.¹⁷ The parameters of the utility function are set to $\sigma = \vartheta = -1$, which corresponds to a log-utility for both consumption and leisure.¹⁸ The elasticity of substitution between consumption and leisure, θ_l is set to imply a perfect competition steady state of labor equal to 0.33; that is, in the steady state a household is assumed to spend one third of its total disposable time on working. The elasticity of substitution between differentiated goods ε is set to 6, implying a steady state markup of 20%. This lies within the range of calibrations suggested in related literature.¹⁹ The steady state labor's share implied by this mark-up is equal to 0.833.²⁰ The probability for a firm of not being able to reoptimize its price (hereafter

¹⁵Note that the parameter λ is positive since $\sigma, \vartheta < 0$ and because the specification of the utility function and the equilibrium conditions imply $0 < Y < 1$.

¹⁶Strictly speaking \hat{y}_t is the deviation of output from its steady state level, whereas the output gap is usually defined as the deviation of output from the level that would prevail under flexible prices. However, with monetary shocks only, the flexible price output corresponds to its steady state level, which allows us to call \hat{y}_t the output gap. See Gali (2003).

¹⁷This assumption is common in NK literature. See e.g. Walsh (1998), p.74.

¹⁸For a discussion of this calibration see Gali (2003).

¹⁹Gali, Monacelli (2002) set $\varepsilon = 6$ as well. Gali (2003) sets $\varepsilon = 11$. Christiano et al. (2001) have estimated $\varepsilon = 3$ for the US. Benigno, Lopez-Salido (2002) calibrate ε to imply a steady state mark-up of 20%. Gali, Gertler, Lopez-Salido (2001, p.1254) discuss the choice of markups in the empirically plausible range of 10-40%: they find that any choice within the interval yielded similar results.

²⁰This value may seem high. However, it could be reduced by assuming decreasing marginal labor productivity. This modification of basic assumptions would slightly complicate the derivation of the Phillips curve, it would however not qualitatively modify our results. For a derivation of the hybrid Phillips curve under decreasing returns to scale see e.g. Gali, Gertler, Lopez-Salido (2001).

probability of fixed price) ξ is set to 0.75. This implies an average price duration of 1 year, which is in line with several empirical estimations.²¹ The benchmark value of the money growth rate's autoregression coefficient will be set to $\rho_\mu = 0.5$; this value corresponds to empirical estimates.²² The standard deviation of the money shock, σ_e will be normalized to 1 percent.

As discussed in Section 2.1, the degree of inflation inertia is measured by the fraction ω of backward looking firms in the economy. Three threshold values of ω are chosen. First, the $\omega = 0$ case, which corresponds to the NKPC. Second, $\omega = 0.3$, which is estimated by GG (1999) and Gali, Gertler, Lopez-Salido (2001, henceforth GGL) for the US and the European economies.²³ Finally, $\omega = 0.7$ corresponds to the calibration of Fuhrer-Moore (1995). This is also the value implied by the estimations of the Christiano et al. (2001) specification.²⁴

2.3.2 Theoretical Impulse Responses

Figure 1 displays the impulse responses of the price level, inflation and output under the benchmark calibration for different levels of ω .²⁵ The unexpected one standard deviation expansionary monetary shock takes place in the first period. The economy is supposed to have been in steady state up to the shock.²⁶

Inspection of the graphs suggests the following two effects of backward looking behavior:

First, the impact of a monetary shock on nominal variables is lower the higher the fraction of backward looking firms in the economy. Second, for high enough levels of ω , the series of price level, inflation rate and of the output gap responses become oscillating.²⁷

The mechanism by which backward looking behavior modifies the propagation of a monetary shock can be seen as follows. Under any sticky price assumption, monetary shocks are transmitted onto the real economy by the deviation of the sticky price from the flexible price level, i.e. the level which would prevail if prices were fully flexible. The effect of backward looking behavior on the responses given to a monetary shock are hence exerted via the

²¹See e.g. GG (1999) and Gali, Gertler, Lopez-Salido (2001). Christiano et al. (2001) find a somewhat lower value, but the standard deviation of their estimate is relatively high. See also discussions in Rotemberg, Woodford (1998) and Smets, Wouters (2003).

²²See e.g. Walsh (1998), Yun (1996), Gali (2003).

²³GG (1999) and GGL (2001) estimate values of ω in the interval of 0.2 – 0.4.

²⁴Setting $\omega = 0$ implies $\gamma_b = 0$, $\gamma_f = 0.99$, $\lambda = 0.12$. When $\omega = 0.3$, $\gamma_b = 0.286$, $\gamma_f = 0.709$, and $\lambda = 0.061$. Finally, $\omega = 0.7$ implies $\gamma_b = 0.485$, $\gamma_f = 0.514$, and $\lambda = 0.019$. Note that, while Christiano et al.'s (2001) specification implies values for γ_b and γ_f close to those implied by setting $\omega = 0.7$, their estimate of λ is much higher.

²⁵The impulse responses have been simulated by means of a MATLAB code we have written based on the solution given in Appendix A.

²⁶Note, that the steady state of any price index changes in response to a shock on the money supply growth rate. The following results are displayed and interpreted with respect to the initial steady state.

²⁷Under the baseline calibration, the series are oscillating for $\omega \geq 0,31$. The threshold value ω is a function of the underlying parameters.

changes it implies in the evolution of the aggregate price index. It should at the same time be noted, that the changes in the \tilde{p}_t path when ω increases, reflect the result of complex simultaneous interactions among all firms in the economy. Figure 2 shows the impulse responses of different price categories to the monetary shock.

The smaller impact reaction of nominal variables, when ω is higher, is due to the fact that backward looking firms behave like fixed pricing firms in the period of the shock: they do not react contemporaneously to the unexpected shock. Although this is anticipated by forward looking firms, their stronger reaction does not offset the effect of backward looking firms to lower the contemporaneous impact of the shock on the aggregate price index.

The oscillations of the series for higher levels of ω are due to the evolution of backward looking prices in the transition towards their new steady state. After the initial low level, the rule of thumb of backward looking firms implies that \tilde{p}_t^b increases relatively fast to relatively high levels a couple of periods after the shock. Large values of ω therefore tend to imply a relatively high average of newly set prices and thereby a high aggregate price index in the given periods. The aggregate price index, \tilde{p}_t hence starts to increase slower after the shock but would then increase at a faster pace and to higher levels, the more so, the higher ω . The aggregate price index therefore reaches the flexible price level faster when the fraction of backward looking firms is larger. With ω high enough, the aggregate price level will overshoot the flexible price level in the medium run. This can be observed in the first panel of Figure 1.

The way the *output* reaction to a monetary shock is modified by backward looking firms, follows directly from their influence on the aggregate price level. Note in particular that oscillations around the steady state can also be observed in the output transition path when ω is high enough.

3 Empirical Model

In this section, we describe the empirical impulse responses of inflation and output to a monetary shock in the US economy from 1965Q3 to 1995Q2.²⁸ The dynamic properties of the estimated impulse responses will serve as a benchmark for the evaluation of the theoretical model's predictions. We begin by describing how the monetary policy shock is estimated. Then we discuss the empirical impulse responses and the decomposition of the forecasting error variance of output and GDP. Last, we discuss the comparability of different monetary policy equations.

Estimating the monetary policy shock, we follow CEE (1999) to characterize the monetary policy reaction function by:

$$R_t = f(\Omega_t) + \epsilon_t. \quad (13)$$

Here, R_t is the instrument of the monetary policy measured by the Federal

²⁸This dataset along with sample RATS codes is available on the website of Lawrence J. Christiano at www.faculty.econ.northwestern.edu/faculty/christiano. Our estimations of this section have been conducted by RATS programs based on the available sample codes.

Funds rate (FF_t), $f(\cdot)$ is a linear function, Ω_t is an information set and ϵ_t is an orthogonal shock.

The variables included in the estimation are denoted by Y_t , which vector is partitioned as $Y_t = [Y_{1t}, R_t, Y_{2t}]'$. The subvector Y_{1t} is composed of variables whose contemporaneous elements are contained in the information set. The subvector Y_{2t} is composed of variables which figure only with their lagged values in Ω_t . The variables in Y_{1t} are HP detrended GDP, inflation rate of the GDP deflator, change of commodity prices. The variables in Y_{2t} are National Bank reserves, total reserves and the money stock (M2).²⁹

This dataset is slightly modified with respect to data included in CEE (1999) estimation: first, we have chosen to use the output gap as measured by HP detrended GDP instead of actual output series; second, we have directly included the inflation rate instead of the price level. The motivation of these choices was to make our estimation results directly comparable to the theoretical impulse responses. These particular choices did not modify the key properties of the impulse responses to a monetary policy shock.

The identification of the monetary policy shock is based on the following recursiveness assumptions. First, variables in Y_{1t} do not respond contemporaneously to the monetary policy shock. Second, systematic monetary policy is assumed to react contemporaneously to changes in Y_{1t} , while it reacts only with a lag to changes in Y_{2t} .³⁰

The VAR contains four lags of each variable. The following system's coefficients have been estimated using OLS equation by equation:

$$Y_t = A_1 Y_{t-1} + \dots + A_4 Y_{t-4} + v_t.$$

We have used Cholesky decomposition to identify the 7 dimensional vector η_t of orthogonal, serially uncorrelated structural shocks.³¹ Corresponding to our ordering, the monetary policy shock ϵ_t is the 4th element of η_t .

Based on these estimates we have computed the path of Y_t after a one standard deviation expansionary monetary policy shock in the first period. The impulse response functions of the Federal Funds rate, the money stock M2, the output gap, and the inflation rate are displayed in Figure 3. Solid lines display the point estimates, 90% error bands are indicated by dashed lines³².

The impulse responses show much of what is conventional wisdom. After an expansionary monetary policy shock:

²⁹Variables included in levels have been transformed to logarithms. Inflation rate is computed as the annualized percentage rate.

³⁰This ordering follows CEE (1999) who also give a comprehensive discussion of different ordering assumptions as well as of different identification schemes.

³¹Denoting the covariance matrix of estimation residuals by $\Sigma = E v_t v_t'$, the Cholesky decomposition of Σ yields $\Sigma = CDC'$, where D is a 7×7 diagonal matrix and C is a 7×7 lower triangular matrix with its diagonal elements equal to 1. The structural shocks are then: $\eta_t | v_t = C \eta_t$.

³²Error bands have been calculated by bootstrapping methods. The number of draws has been set to 500.

- the interest rate falls for several quarters and the money stock increases progressively to its new steady state;
- the response of the output gap is hump shaped and highly persistent;
- the inflation response is hump shaped as well and seems to peak about 3 quarters after the output response.

The decomposition of the forecasting error variance shows that actual series are only partly driven by monetary policy shocks. The results are shown in Table 1. Note that the monetary policy shock is estimated to explain in the long run about 10 percent of both the output gap's and the inflation rate's variance. Although the error bands are relatively large, according to our estimations, the monetary shock does not explain more than about 20 percent of either of these variables' long run variance at a 10 percent significance level. This supports our strategy to use empirical impulse responses instead of actual data as a benchmark for the evaluation of theoretical impulse responses.

Finally, it should be noted, that the empirical model's monetary policy differs from the theoretical monetary policy equation in two ways: first, monetary policy is described as money supply growth rate rule in the theoretical model, while an interest rate rule is assumed in the VAR; second, the reaction function has a much simpler specification in the theoretical model than in the VAR: the money supply growth in the theoretical model is specified by an exogenous law of motion, while in the VAR, the Federal Funds rate is assumed to react to changes in endogenous variables. Nevertheless, the two monetary policy rules and the monetary policy shock are comparable. CEE (1998) show that there is a mapping between the two types of rules.³³ The authors also estimate both the endogenous interest rate rule and the exogenous money growth rate rule on the same dataset as we have used. They find that the exogenous money growth rate rule can be reasonably well approximated by an AR(1) process $\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t$, with $\rho_\mu = 0.5$. This is precisely the specification of our theoretical model.

4 Inflation Inertia and Business Cycle Dynamics

This section evaluates the extent to which the assumption of inflation inertia can improve a model's fit of observed inflation and output dynamics. This study allows us at the same time to reassess criticisms about the NKPC. The discussion focuses on two features of the dynamics. First, the persistence of inflation and output gap responses are examined. Next, the dynamic link between these variables is discussed.³⁴

³³More precisely, when trying to implement some predefined relationship between endogenous variables, the policy maker must allow the money stock to respond in a given way to exogenous economic shocks. For a detailed analytical discussion see CEE (1998).

³⁴The effect of inflation inertia on the generated volatility of these series is not subject of this paper for two reasons: first, the NKPC does not seem to have problems generating observed volatility of the series. Second, it is difficult to precisely estimate the volatility

4.1 Persistence

One of the major criticisms about the NKPC concerns its ability to reproduce observed persistence of the inflation rate and of the output gap.

Fuhrer, Moore (1995) compare a model incorporating a version of the NKPC with an estimated VAR of the US economy and find that the persistence of inflation implied by this setting is 'radically different' from the persistence implied by the autoregression. Chari, Kehoe, McGrattan (2001, hereafter CKM) use a general equilibrium model to examine whether small nominal frictions can account for a high degree of endogenous inflation and output persistence. They find that it cannot. They claim that 'staggered price-setting, by itself, does not generate business cycles driven by monetary shocks'. Both these papers can however be criticized for comparing the persistence of series generated by a single source of exogenous disturbances to the persistence observable in actual data.

CEE (2003) adopt a different approach. They lay out a general equilibrium model incorporating various types of frictions. They examine the model's response to a monetary policy shock which they compare to empirical reactions of variables to monetary policy shocks estimated with an identified VAR. Two of their results are of interest for our concern. On one hand, the authors find that a modest degree of price and nominal wage rigidities 'does a very good job of accounting quantitatively for the estimated [inflation and output] responses of the US economy to a policy shock'. On the other hand, CEE (2003) also analyze the importance of different types of frictions in their model's performance in generating observed output persistence. The authors conclude that the assumption of price and inflation inertia does not play an important role: a version of their model with flexible prices and inflation does almost as well as the model with price and inflation inertia. They emphasize the role of nominal wage rigidities and of frictions in the real economy.

At this point, we try to clarify this debate by examining the following questions. First, we ask how much persistence can be accounted for by a model incorporating Calvo type staggered price setting as the *only* friction? Second, we study whether the assumption of inflation inertia can improve the model's fit with respect to empirically observed persistence.

We begin this analysis by defining a new persistence indicator which in our view is a more accurate measure of oscillating time series' persistence than both the traditional measure used by CKM (2001) and the alternative measure proposed by CEE (2003). We then use our indicator to measure theoretical and empirical persistence. Last, we discuss in what way and to what extent inflation inertia can improve our model's performance.

of the monetary policy shock which would be necessary to evaluate the performance of the theoretical model. Note, that the standard deviation of the shock in the theoretical model has been normalized to 1.

4.1.1 Persistence Indicator

Persistence is traditionally measured by the half-life of a shock's initial impact³⁵. Apart from the difficulty of this measure to indicate persistence of impulse responses which are restricted to 0 on impact, as discussed in CEE (2003), the half-life indicator also fails to capture the persistence of oscillating series. We therefore define a new persistence indicator that can capture the intertemporal distribution of a shock's impact on a variable, while being independent of the sign of the variable's deviations from its steady state in response to the shock.

To discuss persistence, uncertainty will be restricted to a single period t . That is, all information about the economy's state is known up to period $t - 1$. An unexpected temporary shock to the money supply growth rate may then hit the economy in period t according to equation (9). After this period, the money supply evolves in a perfectly foreseen manner according to its exogenous law of motion with an autoregression coefficient $\rho_\mu \in [0, 1]$. The volatility of the shock is denoted by σ_ϵ .

The difference between a variable's realization s periods after the shock, \tilde{x}_{t+s} and its expected value based on information up to the shock, $E_{t-1}(\tilde{x}_{t+s}|I_{t-1})$ can then be expressed as:

$$\tilde{x}_{t+s} - E_{t-1}(\tilde{x}_{t+s}|I_{t-1}) = \delta_s \epsilon_t, \quad \forall s,$$

with $I_{t-1} = \{\tilde{y}_{t-2}, \tilde{y}_{t-3}, \dots, \epsilon_{t-1}, \epsilon_{t-2}, \dots\}$. The coefficient δ_s is a function of ρ_μ and of the model's other structural parameters.

The conditional variance of variable \tilde{x} s periods ahead, generated by the stochastic shock in period t , would then be equal to:

$$V_{t-1}(\tilde{x}_{t+s}) = E_{t-1} \left[(\tilde{x}_{t+s} - E_{t-1}(\tilde{x}_{t+s}|I_{t-1}))^2 \right] = \delta_s^2 \sigma_\epsilon^2, \quad \forall s.$$

Let $ISV_{t-1}(\tilde{x}_{t+s})$ denote the intertemporal sum of variances, i.e. the across-time sum of a variable's conditional variances between the periods t and $t + s$, caused by the stochastic shock of period t :

$$ISV_{t-1}(\tilde{x}_{t+s}) \equiv \sum_{j=0}^s V_{t-1}(\tilde{x}_{t+j}) = \sigma_\epsilon^2 \sum_{j=0}^s \delta_j^2.$$

Let us define a variable's intertemporal total volatility, ITV_x as the across-time sum of conditional variances caused by the shock over indefinite time:

$$ITV_x \equiv \sum_{j=0}^{\infty} V_{t-1}(\tilde{x}_{t+j}) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \delta_j^2.$$

³⁵For a description of the half-life of a shock as a measure of persistence see Chari, Kehoe, McGrattan (2000) or Woodford (2003, Ch.3).

Considering non-explosive solution paths only, the intertemporal total volatility is finite³⁶. Intertemporal total volatility is hence the finite limit to which the across-time sum of variances converges when the number of observed periods s increases, i.e. when $s \rightarrow \infty$, $ISV_{t-1}(\tilde{x}_{t+s}) \rightarrow ITV_x$.

Having said this, we shall define the indicator of a variable's persistence as:

$$\Psi_s(\tilde{x}) \equiv \frac{ITV_x - ISV_{t-1}(\tilde{x}_{t+s})}{ITV_x},$$

where $0 \leq \Psi_s(\tilde{x}) \leq 1$, by construction.

On the basis of the above described considerations, the persistence indicator $\Psi_s(\tilde{x})$ expresses the *percentage of the variable's intertemporal total volatility generated by a shock in period t , which is to take place later than s periods after the shock*. Is the variable's response not persistent at all, this fraction would be equal to 0 in the period of the shock. That is, the total volatility of the variable in response to a monetary shock would take place in the period of the shock, and no more variability would follow. The *more persistent* the effect of the shock on the variable, the greater a fraction of the variable's intertemporal total volatility takes place in periods further away from the shock. This implies a *higher value* for $\Psi_s(\tilde{x})$ in any given period $t+s$. Should the variable's persistence unambiguously increase for greater fractions of backward looking firms, one would observe $\Psi_s(\tilde{x} | \omega_1) \geq \Psi_s(\tilde{x} | \omega_2)$ when $\omega_1 > \omega_2$ for $\forall s$.

It is possible to define the half-life of a shock on the basis of this indicator as the number of periods needed for the half of the intertemporal total volatility to take place. Formally, the half life is s such that $\Psi_s(\tilde{x}) = 0.5$. Note, that the half-life definition based on our indicator has no difficulty with responses which are restricted to be 0 for the period of the shock. (In this case $\Psi_0(\tilde{x}) = 1$).

4.1.2 Persistence of Impulse Responses

The persistence of theoretical and empirical impulse responses is measured by our persistence indicator.

The values of the persistence indicator for the *theoretical impulse responses* are listed in Table 2. Panel 1 displays $\Psi_s(\tilde{\pi})$ as a function of ω for different periods s under the baseline calibration. Panel 2 contains the values of $\Psi_s(\tilde{y})$. The results are the following. First, as expected, a higher degree of inflation inertia, as measured by ω , implies a higher degree of inflation persistence. This is shown by $\Psi_s(\tilde{\pi} | \omega_1) \geq \Psi_s(\tilde{\pi} | \omega_2)$ for $\omega_1 > \omega_2$ for $\forall s$.

Second, and in contrast, inflation inertia tends to decrease the output persistence for plausible levels of backward lookingness. For some positive fractions of backward looking firms, the value of $\Psi_s(\tilde{y} | \omega)$ is lower than $\Psi_s(\tilde{y} |$

³⁶Note that in the case when all fluctuations come from a single source, the unconditional variance σ_x^2 generated by an independent white noise process of consecutive shocks will be equal to the intertemporal total volatility ITV_x caused by a single stochastic shock in period t over indefinite time.

$\omega = 0$). Note especially, that the degree of output persistence is strictly lower with $\omega = 0.3$, i.e. the value estimated by GG (1999) and GGL (2001), than in the purely forward looking case. To generate a degree of output persistence similar to the degree implied by the NKPC, the fraction of backward looking producers needs to be set as high as $\omega = 0.6$ or 0.7 , i.e. approximately the values implied by the Christiano et al. (2001) specification.

As the level of $\Psi_s(\tilde{y})$ and $\Psi_s(\tilde{\pi})$ depends on the persistence of the monetary shock ρ_μ , we checked for the robustness of the shape of $\Psi_s(\omega)$ across ρ_μ . We found that the above described pattern of the indicators as a function of ω is robust across different degrees of money shock persistence: $\Psi_s(\tilde{\pi})$ is monotonously increasing in ω , and the $\Psi_s(\tilde{y})$ displays a U shaped schedule as a function of ω , independently of the level of ρ_μ .

The pattern of $\Psi_s(\tilde{y})$ as a function of ω reflects the dynamic impact of backward lookingness described in section 2. The increasing fraction of backward looking firms tends to shorten the time necessary for the output to reach its steady state level after the shock; this then has a decreasing effect on the output persistence, $\Psi_s(\tilde{y})$ for lower levels of ω . However, when the fraction of backward looking firms is large enough, cyclical fluctuations in the output path would set in. This then induces larger deviations of the output from its steady state in later periods, and thereby increases the persistence of those deviations as measured by $\Psi_s(\tilde{y})$.

Figure 4 compares the persistence generated by the *theoretical* model for three different values of ω to the persistence of *empirical* impulse responses. The grey dashed lines around the point estimates show 90 % confidence intervals³⁷. The comparison of the theoretical and the empirical persistence indicators yields the following conclusions.

First, the degrees of persistence generated by the NKPC are relatively low for both inflation and output compared to the estimated empirical persistence.

Second, the assumption of inflation inertia significantly improves the model's fit of observed inflation persistence compared to the NKPC. Note that for $\omega = 0.7$, our price setting assumption can, by itself, generate degrees of inflation persistence which lie within the 90 percent error bands of the empirical persistence for the first couple of periods following the shock.

Third, inflation inertia does not significantly change the model's performance in reproducing the observed persistence of output when an empirically plausible intensity of inflation inertia is assumed. The persistence of the output gap is below its empirical persistence for any plausible level of ω .³⁸

³⁷To calculate Ψ_s for the empirical series we have used our estimates of the impulse responses of the identified VAR presented in Section 3. The error bands have been computed by a RATS code we have written. We use bootstrapping methods. We have run the VAR estimation for 500 different draws of estimated residuals. At each reestimation we computed the impulse responses, the conditional variances of the responses and the Ψ_s itself. This yielded a distribution for Ψ_s the 5 % resp. 95 % fractiles of which constitute the displayed error bands.

³⁸Output persistence for the empirically implausible level of $\omega = 0.9$ would lie within the 90 percent error bands. This is not displayed in the figure.

4.2 Dynamic Link

Another shortcoming of the NKPC as discussed among others by Fuhrer, Moore (1995) and GG (1999) is its failure to reproduce the empirically observed link between inflation and output. As emphasized by GG (1999), 'the NKPC implies that the inflation rate should lead the output gap over the cycle in the sense that a rise (decline) in current inflation rate should signal a subsequent rise (decline) in output gap. Yet, exactly the opposite can be found in the data.' This, they argue, is shown by the dynamic cross-correlation patterns between inflation and the output gap³⁹: current output gap is positively correlated with leads of inflation and negatively correlated with lags of inflation. This is displayed in Figure 5, which is computed for the output gap and inflation series included in our VAR estimations.

In this subsection, we try to assess this criticism about the NKPC. We first compute dynamic cross-correlation patterns between the responses of inflation and output to a monetary policy shock in our theoretical model assuming different degrees of inflation inertia. Then we compute the analogous indicators for the empirical impulse responses. As we have already argued, this is a better benchmark for evaluating the performance of the theoretical model to reproduce the joint dynamics of inflation and output.

Bold lines in Figure 6 display the *theoretical cross-correlations* of the current output gap with lags and leads of inflation, $Corr(\tilde{y}_t, \tilde{\pi}_{t+j})$, generated by an independent white noise process of consecutive monetary shocks in our model. These dynamic cross-correlations have been evaluated under the baseline calibration for three different values of ω . The formulae used in this section are described in Appendix B.

Two features indicated in this figure are worth noting. First, the cross-correlation pattern generated by the NKPC is positive for all leads and lags of inflation, with the highest correlation between contemporaneous inflation and output gap. Second, the assumption of inflation inertia shifts this pattern to the right in the sense that higher levels of ω imply higher correlations of current output with leads of inflation and lower correlation with lagged inflation. However, the value of $\omega = 0.3$ does not generate significant changes. Only the high value of $\omega = 0.7$ can produce the highest positive correlation between current output and leads of inflation rather than between contemporaneous variables. This level of backward lookingness can also generate negative correlations between current output and lags of inflation rate.

The fact that inflation inertia implies higher positive correlations between current output and leads of inflation than the NKPC follows from the inertia's effect on the persistence of these variables. By increasing inflation persistence and decreasing output persistence, inflation inertia delays the effect of a monetary shock on the inflation rate compared to the output gap, which explains the change in the correlation pattern.

The implied negative correlation between current output and lagged in-

³⁹Output gap is measured as HP detrended real GDP.

flation, on the other hand, is due to inflation inertia's effect to introduce oscillations into the impulse responses as discussed in Section 2. Both inflation and output move in the same direction in response to a monetary shock in the sense that an expansionary (restrictive) monetary shock implies a rise (decline) in both output and inflation. Therefore, as long as the series are not oscillating no negative correlations can be reproduced between these series at any lead or lag.

Figure 6 also displays the dynamic cross-correlations we have computed between the *empirical* output and inflation responses to a monetary policy shock. Thin solid lines show our point estimates. Dashed grey lines correspond to the 90 percent error bands.⁴⁰ Notice, that the dynamic cross-correlation pattern of empirical impulse responses is similar to the pattern between actual series: current output gap co-moves positively with leads of inflation and negatively with lags of inflation in response to a monetary policy shock. The closest relationship can be observed between the current output gap and inflation 4 quarters ahead. At the same time, the confidence intervals around the point estimates are relatively large.

Comparing the empirical cross-correlation pattern with the theoretical pattern for three different levels of ω , we find, that the NKPC does a reasonably good job in reproducing correlations between current output and leads of inflation: these values lie all within the 90 percent error bands of our estimation. The shortcomings of the NKPC appear to be first, that it generates an excessively close contemporaneous co-movement of inflation and output and second, that it misses to reproduce negative cross-correlations between current output and lagged inflation.

The assumption of inflation inertia can improve the fit of the empirical cross-correlation pattern only for a high degree of backward lookingness. While at the low level of $\omega = 0.3$ no significant improvement of the empirical fit is observed, the cross-correlation generated with our model for $\omega = 0.7$ lies within or relatively close to the error bands of the estimated pattern. It should be noted however, that even this high degree of inflation inertia implies a relatively strong contemporaneous relationship between inflation and the output gap.

4.3 Sticky Prices vs. Sticky Inflation

So far, we have been discussing the impact of increasing the intensity of inflation inertia keeping everything else unchanged. In this section, we relate our findings to the effect of increasing the degree of price stickiness. This comparison will show what the assumption of backward looking behavior qualitatively adds to the baseline Calvo model. We discuss the substitutability of price inertia and inflation inertia for the impact response of inflation and output to a monetary policy shock as well as for the persistence of and dynamic link between these responses. The degree of price stickiness is measured by the

⁴⁰Error bands have been computed by bootstrapping methods in a similar way as described in footnote 35.

probability of fixed prices ξ while we continue to measure the intensity of inflation inertia by ω .

As shown by Jeanne (1998), assuming higher price stickiness by increasing ξ decreases the contemporaneous impact of the monetary policy shock on the inflation rate and increases its effect on the output gap. At the same time, the higher degree of price stickiness also implies higher degree of inflation and of output persistence.

Hence, price and inflation inertia are substitutes for shaping the impact of the shock. This is obvious, since, as discussed in Section 2, in the impact period fixed pricing firms and firms resetting their prices in a backward looking way react in the same way. The substitutability of ξ and ω also holds for their influence on the persistence of the inflation response to the monetary shock. However, inflation inertia has different implications for the persistence of the output response. Thereby, inflation inertia has also qualitatively different implications for the dynamic cross-correlation pattern between the inflation and output responses. In figure 7, we show theoretical cross-correlations we have computed for the impulse responses generated by the Calvo model for different levels of ξ . An increase in ξ increases the correlations of current output with both leads and lags of the inflation rate. The contemporaneous correlation remains however highest for any level of ξ . The change in the pattern is the result of the increasing persistence of both series due to higher levels of ξ .

The main gain of the inflation inertia specification with respect to the baseline Calvo model seems hence to lie in its ability to generate asymmetries in the time profile of inflation and output gap responses to a monetary policy shock and to explain oscillations.

5 Conclusion

In this paper, we have analyzed the effect of inflation inertia on a model's ability to reproduce observed dynamics of inflation and output. We have compared the hybrid Phillips curve, as specified by Gali, Gertler (1999), to the standard New Keynesian Phillips curve in a theoretical framework. We have discussed the way inflation inertia modifies the model economy's reaction to a monetary policy shock. We have then discussed how these modifications change the model's prediction for the persistence of inflation and output responses and the dynamic association between them. We have compared our theoretical results with the dynamic properties of observed inflation and output responses to a monetary policy shock. The empirical responses, used as benchmark, were estimated using an identified vector autoregression for US data. This strategy allowed us to assess whether the assumption of inflation inertia can improve a model's fit of observed dynamics.

The principal findings of our analysis are as follows. First, we confirm that the Calvo type staggered price setting generates, by itself, relatively modest degrees of inflation and output persistence. However, we do not confirm that the NKPC implies a dynamic link between inflation and output which is

exactly the opposite what can be observed in the data as claimed e.g. by Fuhrer, Moore (1995) and Gali, Gertler (1999). Indeed, we show that the Calvo model does a reasonably good job of reproducing the co-movement of current output with leads of the inflation rate. The shortcoming of the NKPC in this respect is that it produces a too strong contemporaneous relationship between inflation and output and that it is unable to explain the negative cross-correlations found between current output and lagged inflation.

Second, inflation inertia has implications for the dynamics of inflation and output which are qualitatively different from the implications of price stickiness. Inflation inertia delays and prolongates the effect of a monetary shock on inflation compared to its effect on the output gap. In addition, higher degrees of inflation inertia induce oscillations into the transition paths of impulses. Empirically plausible levels of inflation inertia do, however, not generate more output persistence than the flexible inflation specification.

By these changes, the assumption of inflation inertia improves the model's fit of observed inflation persistence and its fit of the dynamic link between inflation and output. A significant improvement of these properties can however only be observed for a relatively high degree of inertia. As for the persistence of the output gap, the assumption of inflation inertia cannot make up for the NKPC's shortcoming. As we have shown, the generated output persistence continues to fall short of its observed level when empirically plausible degrees of inflation inertia are assumed.

While the hybrid Phillips curve is hence a useful tool in modelling inflation dynamics, dynamics of the output seem to be governed by factors different from inflation or price inertia. One plausible explanation of output persistence might be frictions in the economy which go beyond price and inflation rigidity. Jeanne (1998) e.g. makes a case for real rigidities. In his paper he shows that a low degree of real wage rigidity can account for a significant degree of output persistence. Gali, Gertler (1999) also stress the importance of labor market rigidities in explaining the short run dynamics of the output gap and the inflation rate. In addition, Christiano et al. (2003) emphasize the importance of frictions in capital adjustment in explaining output persistence. Future research should be devoted to further investigate this set of hypotheses.

Another interesting question worth investigating would be the empirical evaluation of the hybrid Phillips curve on the basis of real marginal cost instead of detrended output. This is related to an ongoing debate in the literature concerning the appropriate choice of the real activity variable in the Phillips curve.⁴¹ Microeconomic foundations of the model imply inflation to be directly linked to the real marginal cost instead of the output gap. The evolution of real marginal cost as measured by unit labor cost is at the same time documented to be more synchronized with the inflation rate than detrended output. As we have shown, the NKPC and the hybrid Phillips curve with low degrees of inflation inertia generate a relatively close contempora-

⁴¹See e.g. GG (1999), Sbordone (2002).

neous co-movement of inflation with the real activity variable. We might therefore expect that impulse responses of models with low intensities of inflation inertia yield a better fit of the observed co-movement of inflation with real marginal cost than they do account for the inflation - detrended output relationship.

6 Appendix A

In this Appendix we describe the solution of the log-linearized model for output. The solution for inflation can be found in a similar way.

The solution for output can be found from equations (11) and (12):

$$\tilde{\pi}_t = \gamma_b \tilde{\pi}_{t-1} + \gamma_f E_t(\tilde{\pi}_{t+1}) + \lambda \tilde{y}_t, \quad (\text{A.1})$$

and

$$\tilde{\pi}_t = \tilde{y}_{t-1} - \tilde{y}_t + \tilde{\mu}_t, \quad (\text{A.2})$$

knowing the exogenous process governing the money growth $\tilde{\mu}_t = \rho_\mu \tilde{\mu}_{t-1} + \epsilon_t$.

Substituting out for $\tilde{\pi}_t$, $\tilde{\pi}_{t-1}$ and $E_t(\tilde{\pi}_{t+1})$ using equation (14) gives a third order stochastic difference equation linking output and money growth:

$$\gamma_f E_t \tilde{y}_{t+1} - (1 + \lambda + \gamma_f) \tilde{y}_t + (1 + \gamma_b) \tilde{y}_{t-1} - \gamma_b \tilde{y}_{t-2} = \gamma_f E_t \tilde{\mu}_{t+1} - \tilde{\mu}_t + \gamma_b \tilde{\mu}_{t-1}. \quad (\text{A.3})$$

This equation reduces to the solution presented in Jeanne (1998) when $\omega = \gamma_b = 0$.

The conjectured solution is

$$\tilde{y}_t = \nu_1 \tilde{y}_{t-1} + \nu_2 \tilde{y}_{t-2} + \nu_{\mu 1} \tilde{\mu}_t + \nu_{\mu 2} \tilde{\mu}_{t-1}. \quad (\text{A.4})$$

Making use of this conjecture to rewrite the difference equation (14) and equating the coefficients yields:

$$\begin{aligned} \nu_1 &= \frac{1 + \gamma_b + \gamma_f \nu_2}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_2 &= -\frac{\gamma_b}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_{\mu 1} &= \frac{1 - \gamma_f(1 - \nu_{\mu 1})\rho_\mu + \gamma_f \nu_{\mu 2}}{1 + \gamma_f(1 - \nu_1) + \lambda} \\ \nu_{\mu 2} &= -\frac{\gamma_b}{1 + \gamma_f(1 - \nu_1) + \lambda}. \end{aligned}$$

Rearranging the expressions for ν_1 and ν_2 implies the following third degree polynomial for ν_1 :

$$\gamma_f^2 \nu_1^3 - 2(1 + \gamma_f + \lambda) \gamma_f \nu_1^2 + [(1 + \gamma_f + \lambda)^2 + \gamma_f(1 + \gamma_b)] \nu_1 - (1 + \gamma_f + \lambda)(1 + \gamma_b) + \gamma_f \gamma_b = 0. \quad (\text{A.6})$$

Under any plausible calibration this polynomial has a unique stable root⁴², which will then be chosen as the coefficient ν_1 . The remaining parameters can be calculated recursively.

The coefficients ν_2 and $\nu_{\mu 2}$ are found to be equal and can be expressed as

$$\nu_2 = \nu_{\mu 2} = \frac{-\gamma_b}{1 + \lambda + \gamma_f - \gamma_f \nu_1}.$$

⁴²For a comprehensive discussion of the stability conditions see Hamilton (1994 Ch.1)

Finally, the coefficient of impact $\nu_{\mu 1}$ is given by:

$$\nu_{\mu 1} = \frac{1 - \gamma_f \rho_\mu + \gamma_f \nu_{\mu 2}}{1 + \lambda + \gamma_f - \gamma_f \nu_1 - \gamma_f \rho_\mu}.$$

Note that in the purely forward looking case, the coefficients ν_2 and $\nu_{\mu 2} = 0$, and the solution of the output gap thus reduces to

$$\tilde{y}_t = \nu_1 \tilde{y}_{t-1} + \nu_{\mu 1} \tilde{\mu}_t.$$

This result corresponds to Jeanne's (1998) solution in the case of perfect competition in the labor market.

The coefficients of autoregression of the *inflation rate* turn out to be equal to those of the output gap, i.e. $\nu_1 = \kappa_1$ and $\nu_2 = \kappa_2$. The contemporaneous impact of a monetary shock on the inflation rate is $\kappa_{\mu 1} = 1 - \nu_{\mu 1}$. This is a consequence of the CIA specification. The coefficient of lagged money growth in the solution of the inflation rate is $\kappa_{\mu 2} = \frac{-\gamma_f \lambda \rho_\mu}{(1 + \lambda - \gamma_f \kappa_1)(1 + \lambda + \gamma_f - \gamma_f \kappa_1 - \gamma_f \rho_\mu)}$.

It follows from these results, that $\kappa_2 = 0$, when all firms are forward looking. In contrast, $\kappa_{\mu 2}$ needs not necessarily be zero when $\omega = 0$. Instead, this coefficient is zero, when the monetary shock is not persistent⁴³. The solution of the inflation rate in the purely forward looking case is hence

$$\tilde{\pi}_t = \kappa_1 \tilde{\pi}_{t-1} + \kappa_{\mu 1} \tilde{\mu}_t + \kappa_{\mu 2} \tilde{\mu}_{t-1}.$$

7 Appendix B

In this appendix we describe the formulae to compute dynamic cross-correlations between impulse responses.

The solution of the theoretical and the empirical model allows us to represent inflation and output as a moving average of contemporaneous and past shocks. Let us denote the Wold representation of output and inflation as:

$$\tilde{y}_t = \sum_{j=0}^{\infty} \delta_j \epsilon_{t-j} \text{ respectively } \tilde{\pi}_t = \sum_{j=0}^{\infty} \varphi_j \epsilon_{t-j}.$$

By definition, the contemporaneous covariance is:

$$Cov(\tilde{y}_t, \tilde{\pi}_t) = E(\tilde{y}_t \tilde{\pi}_t) = E\left(\sum_{j=0}^{\infty} \delta_j \epsilon_{t-j} \sum_{j=0}^{\infty} \varphi_j \epsilon_{t-j}\right)$$

Using the fact that ϵ_{t-j} is a white noise sequence of serially uncorrelated shocks, we can write the contemporaneous covariance as:

$$Cov(\tilde{y}_t, \tilde{\pi}_t) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \delta_j \varphi_j.$$

⁴³To be precise, this coefficient can also be zero when all firms are backward looking, which would imply that the inflation rate does not react to the output gap, i.e. $\lambda = 0$. This case has, however, been ruled out.

To get the correlation, this needs to be divided by the standard deviations of output and inflation:

$$Corr(\tilde{y}_t, \tilde{\pi}_t) = \frac{Cov(\tilde{y}_t, \tilde{\pi}_t)}{\sigma_y \sigma_\pi}$$

where $\sigma_y = \sqrt{\sum_{j=0}^{\infty} \delta_j^2}$ and $\sigma_\pi = \sqrt{\sum_{j=0}^{\infty} \varphi_j^2}$.

The formula for the covariance of current output with leads of inflation is:

$$Cov(\tilde{y}_t, \tilde{\pi}_{t+k}) = E\left(\sum_{j=0}^{\infty} \delta_j \epsilon_{t-j} \sum_{j=0}^{\infty} \varphi_j \epsilon_{t+k-j}\right) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \delta_j \varphi_{j+k},$$

the covariance of current output with lags of inflation is:

$$Cov(\tilde{y}_t, \tilde{\pi}_{t-k}) = E\left(\sum_{j=0}^{\infty} \delta_j \epsilon_{t-j} \sum_{j=0}^{\infty} \varphi_j \epsilon_{t-k-j}\right) = \sigma_\epsilon^2 \sum_{j=0}^{\infty} \delta_{j+k} \varphi_j.$$

References

- [1] Altig, David; Lawrence J. Christiano; Martin Eichenbaum; Jesper Linde (2002): "Technology Shocks and Aggregate Fluctuations", mimeo., Northwestern University.
- [2] Angeloni, Ignazio; Guenter Coenen; Frank Smets (2003), "Persistence, the Transmission Mechanism and Robust Monetary Policy", European Central Bank Working Paper No.250.
- [3] Ball, Laurence (1990): "Credible Disinflation with Staggered Price Setting", NBER Working Paper No. 3555.
- [4] Benigno, Pierpaolo; J.David Lopez-Salido (2002), "Inflation Persistence and Optimal Monetary Policy in the Euro Area", ECB Working Paper No. 178.
- [5] Calvo, Guillermo (1983), "Staggered Prices in a Utility Maximizing Framework" *Journal of Monetary Economics* 12, 383-398.
- [6] Chari, V. V.; Patrick J. Kehoe, Ellen R. McGrattan (2000), "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?", *Econometrica* pp.1151-80.
- [7] Christiano, Lawrence J.; Martin Eichenbaum; Charles L. Evans (1998), "Modeling Money", NBER Working Paper Nr. 6371.
- [8] Christiano, Lawrence J.; Martin Eichenbaum; Charles L. Evans (1999), "Monetary Policy Shocks: What Have We Learned, and to What End?" in Taylor, Woodford (eds.): *Handbook of Macroeconomics*, North Holland.
- [9] Christiano, Lawrence J.; Martin Eichenbaum; Charles L. Evans (2001), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Working Paper 0107, Federal Reserve Bank of Cleveland.
- [10] Christiano, Lawrence J.; Martin Eichenbaum; Charles L. Evans (2003), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", forthcoming in *Journal of Political Economy*.
- [11] Coenen, Guenter; Volker Wieland (2003), "A Small Estimated Euro Area Model with Rational Expectations and Nominal Rigidities", European Central Bank Working Paper Series, Research Paper No.30.
- [12] Coenen, Guenter (2003), "Inflation Persistence and Robust Monetary Policy Design", European Central Bank Working Paper No.290.
- [13] Dotsey, Michael; Robert G. King; Alexander L. Wolman (1999), "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output", *Quarterly Journal of Economics* 114, 655-690.
- [14] Erceg, Christopher J.; Andrew T. Levin (2001), "Imperfect Credibility and Inflation Persistence", *Journal of Monetary Economics* (forthcoming).
- [15] Fuhrer, Jeffrey; George C. Moore (1995), "Inflation Persistence", *Quarterly Journal of Economics*, No. 440 February, pp 127-159.
- [16] Gali, Jordi (2003): "New Perspectives on Monetary Policy, Inflation, and the Business Cycle" in *Advances in Economic Theory*, edited by: M.

- Dewatripont, L. Hansen, and S. Turnovsky, vol. III, 151-197, Cambridge University Press 2003.
- [17] Gali, Jordi; Mark Gertler (1999), "Inflation Dynamics: A Structural Econometric Analysis", *Journal of Monetary Economics* 44, 195-222.
 - [18] Gali, Jordi; Mark Gertler; J. David Lopez-Salido (2001), "European Inflation Dynamics", *European Economic Review* 45. pp. 1237-1270.
 - [19] Gali, Jordi; Mark Gertler (2003), "Inflation Dynamics: Combining Measurement with Theory", *NBER Reporter*, Summer 2003.
 - [20] Gali, Jordi; Tommaso Monacelli (2002), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *CEPR Discussion Paper No. 3346*.
 - [21] Hamilton, James D. (1994), *Time Series Analysis*, Princeton University Press.
 - [22] Jeanne, Olivier (1998), "Generating Real Persistent Effects of Monetary Shocks: How Much Rigidity Do We Really Need?", *European Economic Review* 42, pp 1009-1032.
 - [23] Kollmann, Robert (2001), "The Exchange Rate in a Dynamic-Optimizing Business Cycle Model With Nominal Rigidities: a Quantitative Investigation", *Journal of International Economics* 55, 243-262.
 - [24] McCallum, Bennett T. (1983), "On Non-Uniqueness in Rational Expectations Models", *Journal of Monetary Economics* vol. 11. pp. 139-168.
 - [25] Nelson, Edward (1998): "Sluggish Inflation and Optimizing Models of the Business Cycle", *Journal of Monetary Economics*, 42, no. 2, 303-322.
 - [26] Obstfeld, Maurice; Kenneth Rogoff (1999), *Foundations of International Macroeconomics*, MIT Press.
 - [27] Roberts, John M. (1997), "Is Inflation Sticky?", *Journal of Monetary Economics* 39, p.173-196.
 - [28] Rotemberg, Julio J.; Michael Woodford (1998), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version", *NBER Technical Working Paper 233*.
 - [29] Sbordone, Argia (2002), "Prices and Unit Labor Costs: A New Test of Price Stickiness", in *Journal of Monetary Economics* vol. 49 (2), pp 265-292.
 - [30] Smets, Frank; Raf Wouters (2003), "An Estimated Stochastic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association* Vol.1, Issue 5, pp.1123-1175.
 - [31] Steinsson, Jon (2003), "Optimal Monetary Policy in an Economy with Inflation Persistence", *Journal of Monetary Economics* 50:1425-1456.
 - [32] Stock, James H.; Mark W. Watson (1999), "Business Cycle Fluctuations in U.S. Macroeconomic Time Series" in *Handbook of Macroeconomics*, edited by J. Taylor and M. Woodford, North Holland, Vol. 1a, 1999, pages 3-64.
 - [33] Uhlig, Harald (1999), "A Toolkit for Analyzing Dynamic Stochastic Models Easily" in *Computational Methods for the Study of Dynamic Economies*, editors R. Marimon and A. Scott, Oxford: Oxford Univer-

sity Press.

- [34] Walsh, Carl E. (1998), *Monetary Policy and Theory*, MIT Press.
- [35] Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- [36] Yun, Tack (1996), "Nominal Rigidity, Money Supply Endogeneity and Business Cycles", *Journal of Monetary Economics*, 32(2), pp 345-370.

Table 1: Variance Explained by Monetary Shock

		Output gap		
Periods ahead		4	10	20
FEV(s)		0.042	0.093	0.085
		(0.014;0.15)	(0.031;0.207)	(0.031;0.207)

		Inflation Rate		
Periods ahead		4	10	20
FEV(s)		0.029	0.102	0.128
		(0.012;0.102)	(0.036;0.19)	(0.039;0.195)

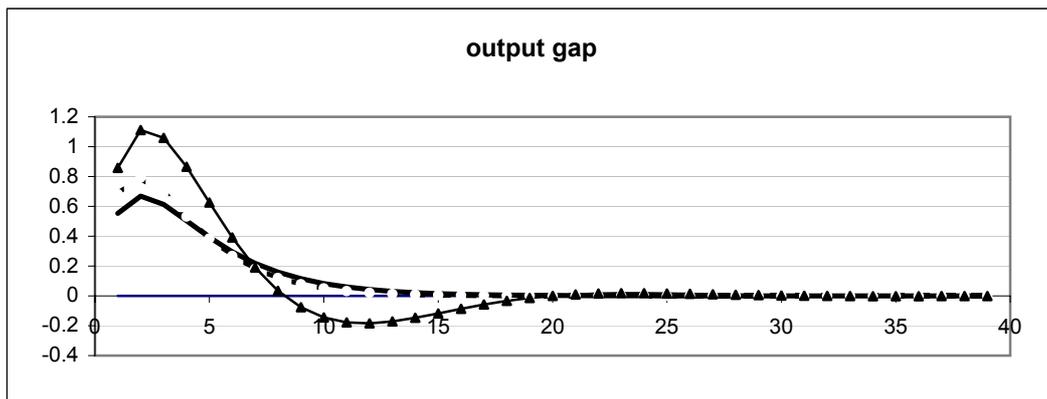
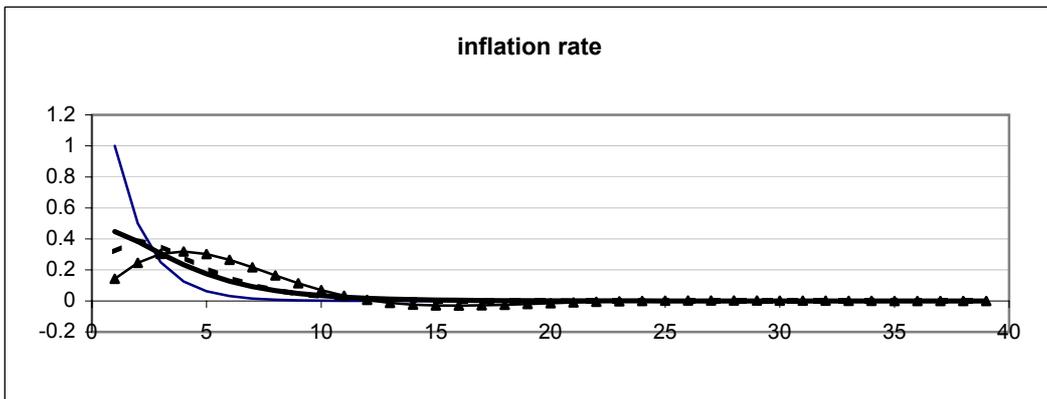
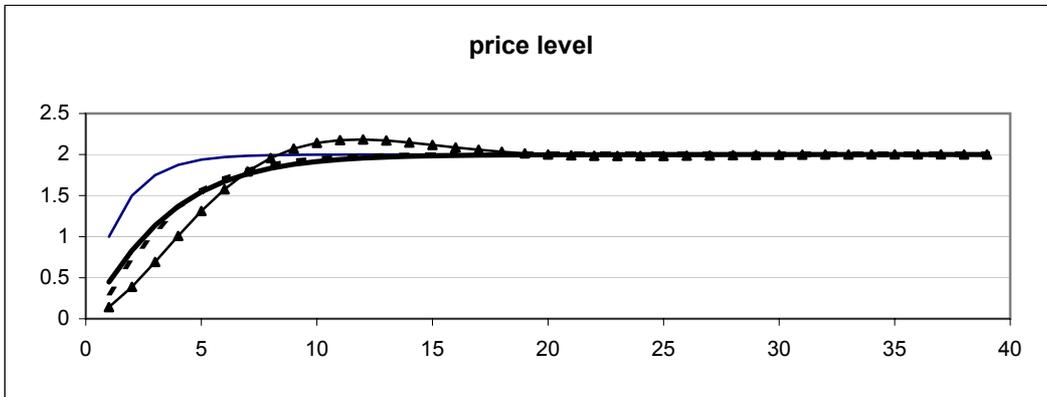
Table 2: Persistence of Inflation and Output

PSIs(π)		ω									
s		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		0.64	0.70	0.76	0.81	0.86	0.90	0.93	0.96	0.98	1.00
2		0.21	0.23	0.26	0.30	0.36	0.45	0.55	0.68	0.81	0.93
4		0.06	0.06	0.07	0.08	0.09	0.13	0.19	0.31	0.51	0.79
8		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.09	0.39
12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.14
16		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.09

PSIs(y)		ω									
s		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		0.82	0.80	0.79	0.78	0.78	0.79	0.81	0.84	0.88	0.94
2		0.35	0.32	0.29	0.26	0.24	0.24	0.26	0.33	0.45	0.66
4		0.11	0.09	0.08	0.06	0.04	0.03	0.04	0.08	0.18	0.42
8		0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.04	0.11	0.25
12		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.24
16		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.16

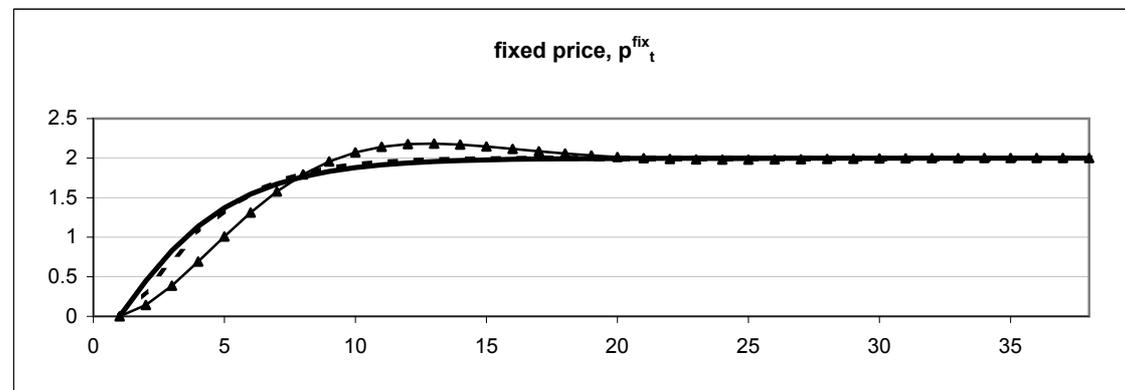
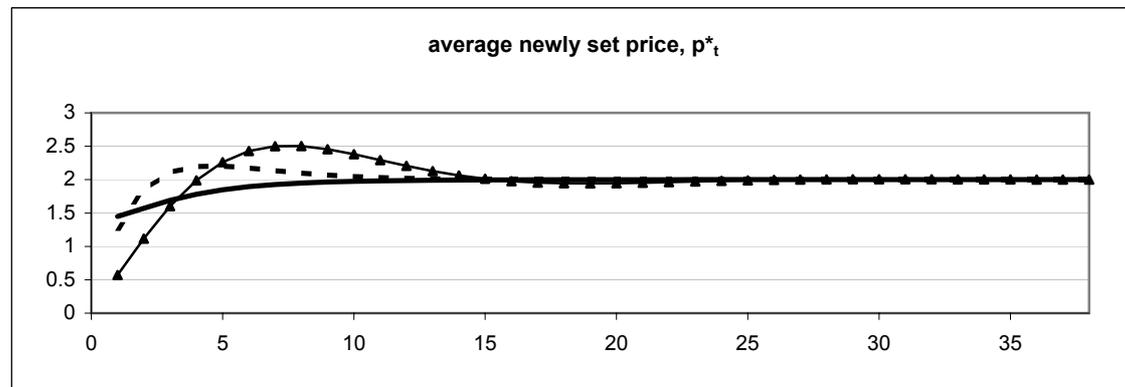
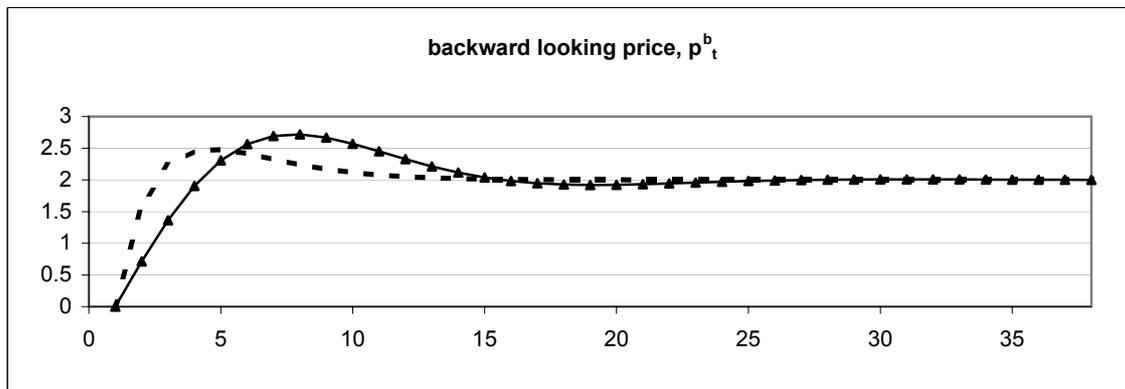
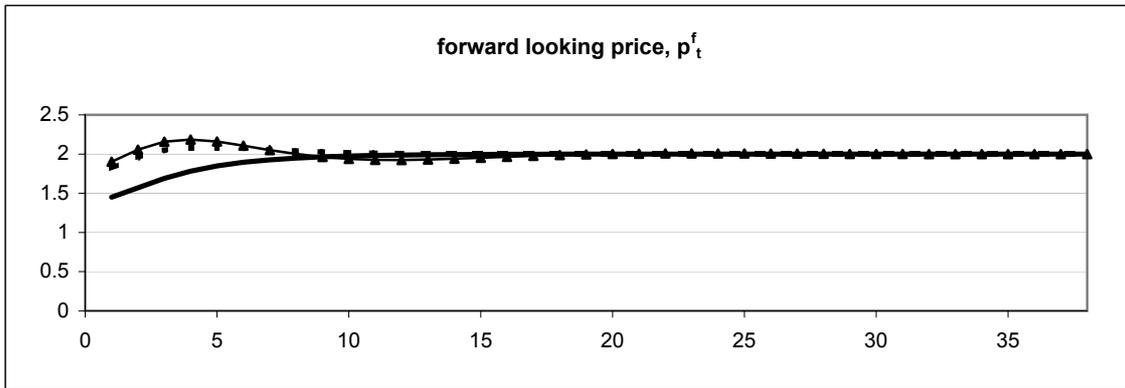
Persistence of inflation and output measured by $PSis(x|\omega) = (ITV(x) - ISVt-1(x_{t+s})) / ITV(x)$ for different periods s , and different levels of ω .
 Increasing persistence is indicated by $PSis(x|\omega_1) \geq PSis(x|\omega_2)$ when $\omega_1 > \omega_2$ for all s .

Figure 1: Impulse Responses - Theoretical Model



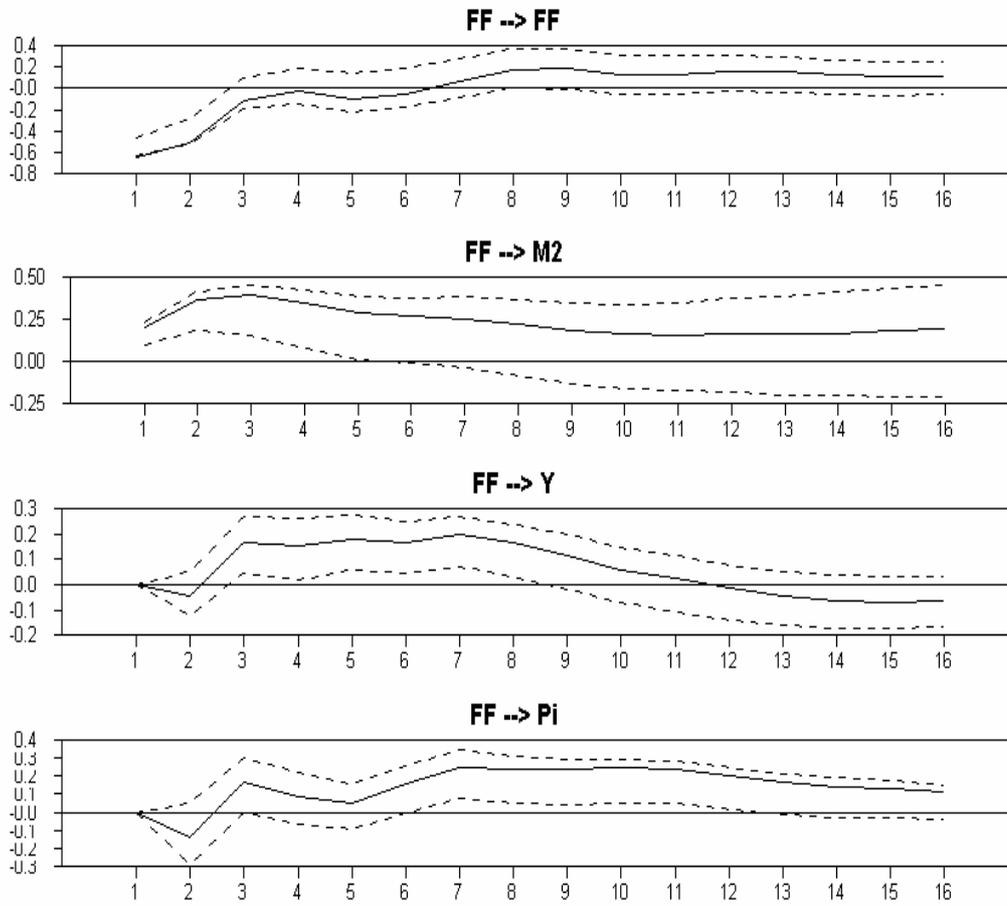
thin lines - flexible price
 bold lines - NKPC ($\omega=0$)
 dashed lines - $\omega=0.3$
 lines marked with triangles - $\omega=0.7$

Figure 2: Impulse Response: Price Setting



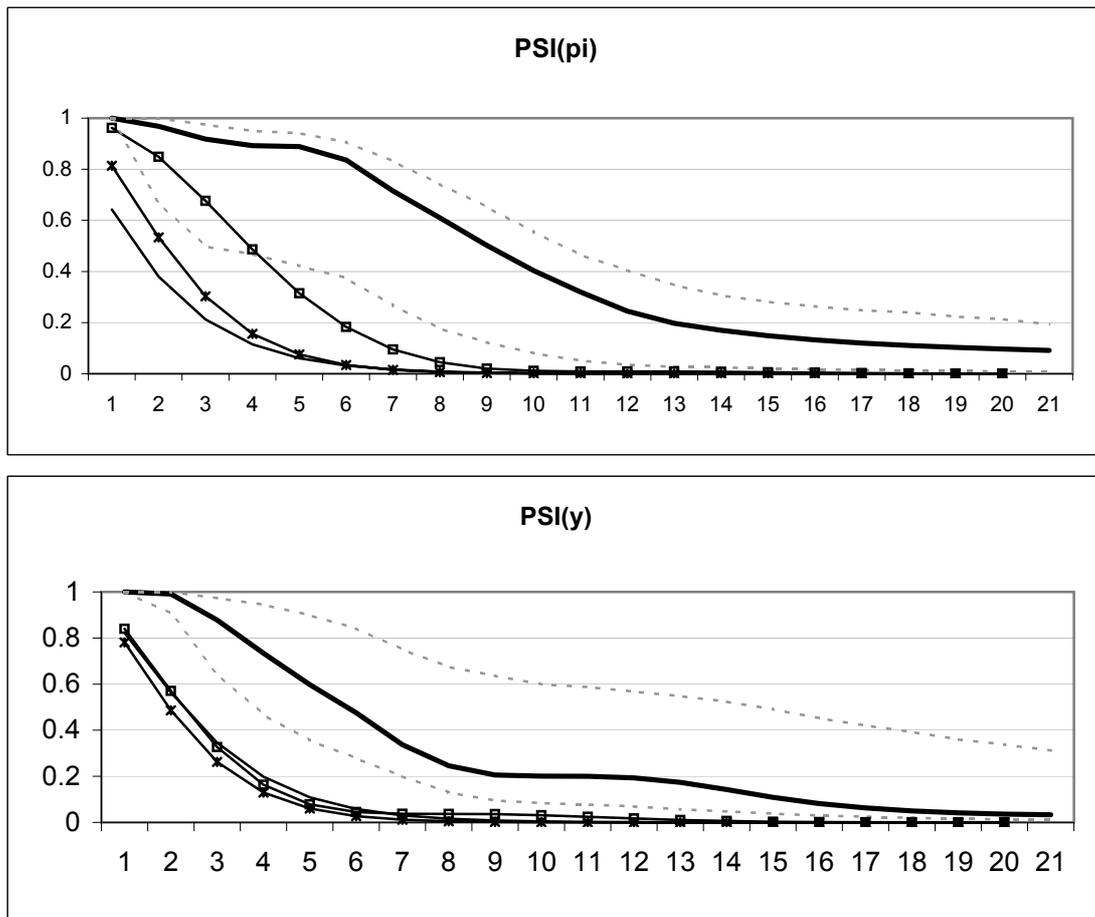
bold lines - NKPC ($\omega=0$)
dashed lines - $\omega=0.3$
lines marked with triangles - $\omega=0.7$

Figure 3: Impulse Responses - SVAR
 One standard deviation expansionary monetary policy shock



FF = Federal Funds rate, M2 = money stock, Y = output gap, Pi = inflation rate
 solid lines - point estimates, dashed lines - 90% error bands

Figure 4: Persistence - Theory vs. Data



bold lines - point estimates
 grey dashed lines - 90% error bands
 solid lines - theoretical persistence NKPC ($\omega=0$)
 lines marked with stars - theoretical persistence ($\omega=0.3$)
 lines marked with bullets - theoretical persistence ($\omega=0.7$)

Figure 5: Cross-correlations - Actual Data

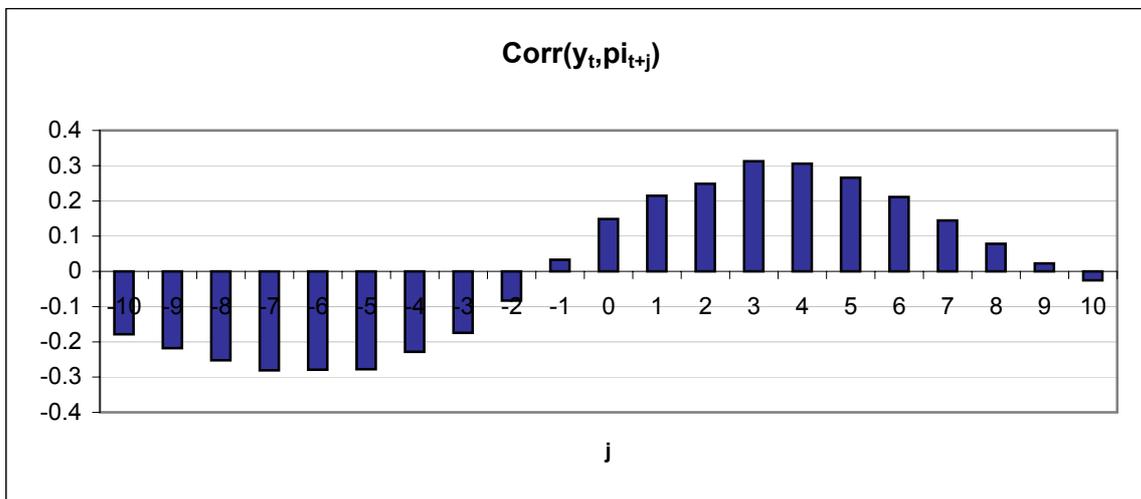
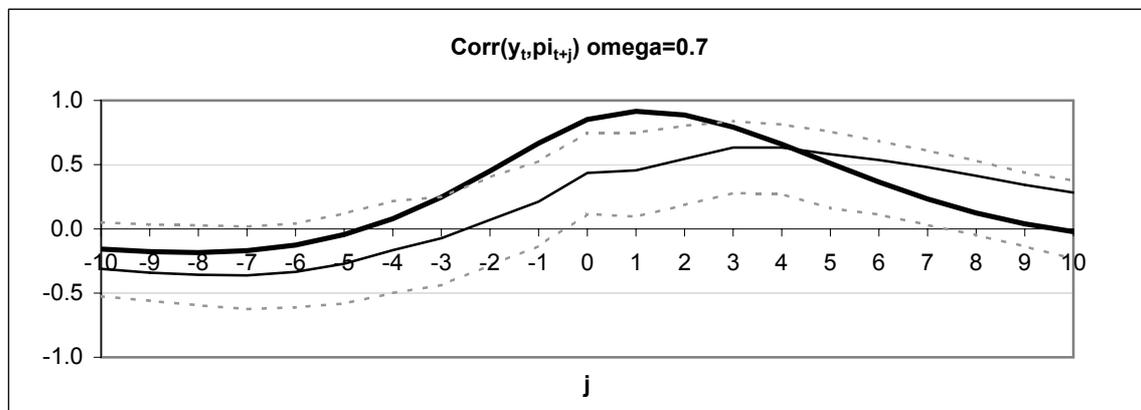
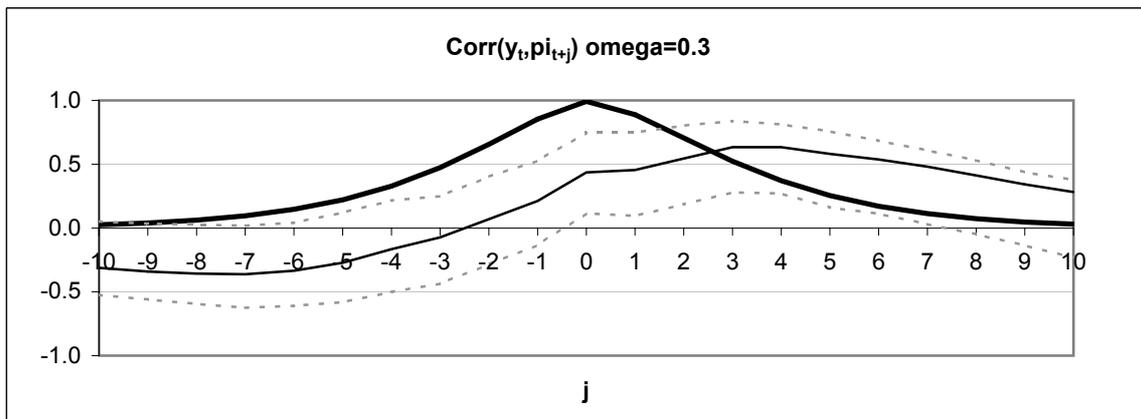
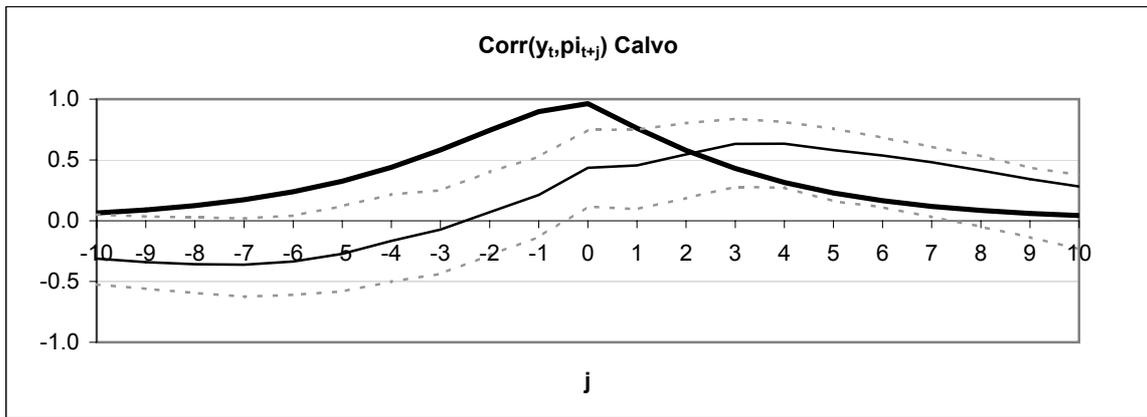


Figure 6: Dynamic Cross-Correlations

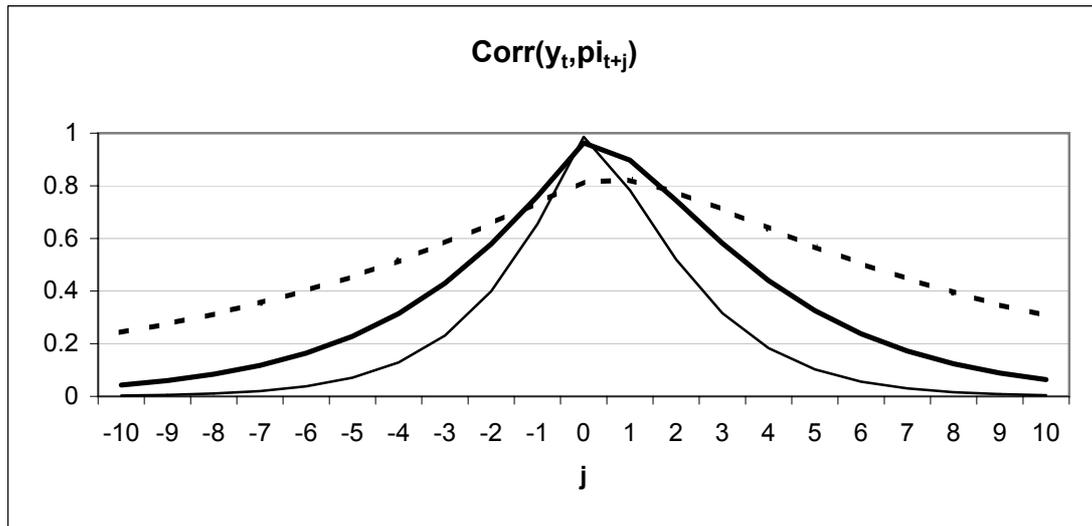


solid lines - point estimates

grey dashed lines - 90% error bands

bold lines - cross-correlations of theoretical impulse responses

Figure 7: Cross-correlation for different degrees of price stickiness in the Calvo model



$\omega=0$ for all simulations

solid line: $\kappa=0.5$

bold line: $\kappa=0.75$ (benchmark)

dashed line: $\kappa=0.9$