MAGYAR TUDOMÁNYOS AKADÉMIA
KÖZGAZDASÁGTUDOMÁNYI INTÉZET

INSTITUTE OF ECONOMICS
HUNGARIAN ACADEMY OF SCIENCES

MŰHELYTANULMÁNYOK **DISCUSSION PAPERS**

MT–**DP.** 2004/15

WE CAN'T ARGUE FOREVER

KATA BOGNÁR

LONES SMITH

Institute of Economics
Hungarian Academy of Sciences

Budapest

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We Can't Argue Forever

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WE CAN'T ARGUE FOREVER

BY KATA BOGNÁR AND LONES SMITH

Abstract

We analyze time-costly decision-making in committees by privately informed individuals, such as juries, panels, boards, etc. In the spirit of the Coase Conjecture, we show that the decision is „almost instantaneous”when individuals entertain identical objectives. Delay can only be understood as the outcome of conflicting (biased) objectives.

Összefoglaló

A cikkben egy bizottság döntéshozási mechanizmusát elemezzük nem teljes információ esetén. Alapvető feltevésünk, hogy a bizottsági tagok privát információval rendelkeznek a döntés helyességét illetően, valamint hogy a folyamat procedurális költséget igényel, pontosabban szólva a tagoknak nem csak a helyes döntés, hanem egyúttal a gyors döntés is érdekében áll. A modell leírja tanácsadó testületek, orvos és egyéb szakértői csoportok működését, valamint jellemzi az amerikai igazságszolgáltatásban nagy jelentőséggel bíró esküdtszékek döntéshozási mechanizmusát. Megmutatjuk, hogy a tökéletesen egyező érdekek egy bizottságban majdhogynem azonnali döntést eredményeznek. Következtetésünk, hogy a valóságban megfigyelhető késleltetett döntés magyarázatához eltérő érdekek feltételezése szükséges.

1 INTRODUCTION

Decision-making in committees is quite common, e.g. juries, tenure cases, board of directors, professional panels of doctors or other experts. The purpose of deliberation is to aggregate the members' private information. In this paper we focus for simplicity on binary decisions (e.g. guilty or not guilty). One critical aspect of joint decision making we emphasize is its time cost.

We assume that individuals wish to make the best decision possible in the least time possible, but are unable to simply put their private information „on the table” as it were. (For example, in a group of experts in different fields understanding can be difficult due to different terminology.) That is, we wish to analyze costly committee decision-making by like-minded individuals. We assume in particular that individuals can only communicate their information by how and when they vote.

One stylized fact about joint decision-making is that the ultimate decision may be quite delayed. The classic movie „Twelve Angry Men” highlights this fact for jury decisions. In fact, the voting procedure in that film resembles the one that we adopt. We assume that every period a vote is taken, and the game ends with unanimity. For simplicity, we focus on just two-member panels. We show that contrary to the outcome of the movie, decision making should be quite fast, if in fact the jurors were only „interested in the truth”.

Our main result has the flavor of the Coase Conjecture.[1] If the time interval between votes diminishes, the probability that the final decision is realized in any given real time tends to 1. This paper thus suggests that it is impossible to reconcile delay in committee decision-making with rational, like-minded individuals. Further, it is clear that this result obtains despite our exclusion of all forms of non-voting communication. The only way to understand delay is by assuming that jury members entertain con°icting objectives. While the

[1] See Gul, Sonnenschein, and Wilson (1986).

movie has raced bias as a clear subtext, our result suggests that incoherent preferences invariably *must* lie at the heart of any observed delay.

Related literature. The paper relates to the work by Feddersen and Pesendorfer (1998) and also by Austen-Smith and Banks (1996), Coughlan (2000) and Duggan and Martinelli (2001) on voting rules in juries. Feddersen and Pesendorfer (1998) describe the decision making as one-round hence costless voting; they assume strategic behavior and show that the equilibrium outcome with unanimity rule is actually worse than the one with majority rule from a welfare point of view. Since the jurors are like-minded in this model, their result emphasizes the deficiency of aggregating information with unanimity rule.

A sequel of this literature by Austen-Smith and Feddersen (2002*b*), (2002*a*) provides parallel inferiority of unanimity rule even if some debate is allowed before the actually voting. In their setup pre-voting communication does not influence the outcome of the voting directly and cost-less. On the contrary, we see the deliberation as repeated voting where the process can end in any round. Hence, we assume less about the possible message space and also we perform the analysis with more general informational structure.

Our model is related to ’Agreeing to Disagree’ results as well. Aumann (1976) showed that if two people share the same prior, and if their posteriors are common knowledge, then those posteriors must coincide. Geanakoplos and Polemarchakis (1982) raises the issue of sequential *communication* of these posteriors. They show that this leads to the same beliefs in finitely many steps. Their result relies on the assumption that information is described by a finite partition of an underlying state space. By contrast, we make no restriction of the information partition, and do not allow agents to fully communicate their posteriors. Rather, the votes serve to communicate a binary signal of the posterior. Still, while our stages may last arbitrarily long, we do achieve an arbitrarily fast real-time agreement.

The structure of the paper is the following. In Section 2 we describe the model, in Section 3 we show that equilibrium exists and finally Section 4 is about the ’collapsing’ nature of the equilibrium. Section 5 concludes the paper.

2 The Model

In the following we will use juries as an example. We first describe the informational structure of the game and then we give the timing of the play as well as strategies and finally we define the payoffs.

Information. There are two states of the world (θ): the defendant is either guilty (G) or innocent (I) and the prior belief of G is 0.5. Both jurors have some private information about the state of the world that is represented by a one-dimensional signal. We can 'skip' the updating phase and can summarize that private information immediately by p, the private posterior of the juror about state G. These posteriors are conditionally iid drawn from $F(p|\theta)$ with common support $co(supp(F)) = [\underline{p}, \overline{p}] \subseteq [0, 1]$ for $\theta = G, I$[2]. We also assume that there is no perfectly revealing signal. Furthermore, all these assumptions are common knowledge among the players. In what follows we refer to the type of a juror as p. As it is usual, in the literature we make the following assumptions about the underlying structure of information. First, the densities $f(p|\theta)$ exist and are bounded away from zero and infinity whenever $p \in (0, 1)$. Second, to represent the complementarity amongst private information we assume that the signal distribution has the strict monotone likelihood ratio property (MLRP)[3]. Also assume that the likelihood ratio is continuously differentiable.

Timing and Strategies. We consider an alternating move game.[4] Initially (at time zero), one of the juror decide on either conviction (C) or acquittal (A). In the next period the other has the right to agree or disagree; an agreement ends the game with the obvious verdict. In case of disagreement the first juror talks again; she either agrees with the other (changes her opinion) or disagrees, etc. We also assume a strictly positive flow cost of being in the decision process. Our intuition is that the less certain a juror originally is about the defendant's status, the earlier she is willing to change her mind during the decision process. A strategy of player 1 has the form (d, τ_D^1) and for player 2 (τ_D^2) where $d : [\underline{p}, \overline{p}] \rightarrow \{A, C\}$ describes the 'first' player's choice and $\tau_D^i : [\underline{p}, \overline{p}] \rightarrow \mathcal{N}$ where $D \in \{A, C\}$, gives the stopping time if the first choice is D.

Payoffs. Both jurors have a *common* interest in making the right decision, i.e. convict if the defendant is guilty and acquit if the defendant is innocent. As it is common in the literature, we normalize the payoff in case of good decision to zero while convicting an innocent costs q and acquitting a guilty costs $(1 - q)$ for both jurors. Formally, $u(C|G) = u(A|I) = 0$, $u(C|I) = -q$ and $u(A|G) = -(1 - q)$, and we refer them as *terminal costs.* We also have a positive *decision*

[2] Notice that $F(.)$ has to satisfy an extra so-called no introspection condition.

[3] This property often called affiliation in the auction literature.

[4] Allowing simultaneous moves is the subject of current research. The first formalization was a mixture of the two, so that at time zero there is a vote held and both jurors decide on either conviction or acquittal. In case of disagreement the jurors are involved in an alternating move game.

cost that is c per unit of time or Δc per time period where the length of the period is Δ. We choose unit cost since it is more intuitive here, on a relatively small time horizon than discounting. Finally, jurors are risk neutral and cost minimizing, so the 'preferred' verdict for a juror with low posterior is A and for a juror with high posterior is C.

3 Equilibrium Analysis

3.1 Description

We wish to show that in committee decisions with like-minded individuals there is no delay (there is no delay in the final subgame) in real time if the period length is vanishing. Our intuition in this setup is that jurors with more extreme signals will wait longer before giving in. Notice that the jurors are facing a tradeoff between acquiring information about the opponent's signal that helps their decision and paying the cost of the decision making. We think that due to the affiliated information structure, a juror with more extreme signal will valuate this information higher and so is willing to 'pay' more for it. Indeed we show that equilibrium exists and it is necessarily monotone.

First, we show that each player's best response is monotone. Results of this flavor usually follow from the single crossing property of the payoff function in action and type. We can phrase this property more intuitively: if players are arranged according their types then a player 'weakly prefers' an action to an other one implies that anyone with 'higher' type must strictly prefer the same action.

This is not quite obvious in our story. Consider a case when a juror prefers to stop at $n+2$ over stopping at n, i.e. her expected payoff increases by switching from n to $n+2$. By doing this, she changes the outcome of the voting with some probability and by assumption that change is favorable for her. On one hand, the same change in the outcome is more 'valuable' for a more extreme juror; on the other hand she might find it less likely that the change actually happens. Therefore, the overall effect is unclear. Fortunately, on balance we found a favorable effect.

The monotonicity of best response implies that all equilibria, if any, are necessarily monotone. Finally, we prove the existence of monotone equilibria. The best response is single valued and continuous what allows us to apply a fixed-point theorem.

3.2 Monotonicity

To characterize equilibria the following notation is useful. First, x (respectively y) is a proxy for the type of an A-juror (respectively C-juror). Second, given realizations x, y, the posterior of guilt is denoted by

$$\rho(x, y) = \frac{1}{1 + \ell(x)\ell(y)}$$

where $\ell(p)$ is he likelihood ratio $f(p|I)/f(p|G)$. The expected terminal costs from a verdict are $v_A(x,y) = (1-q)\rho(x,y)$ and $v_C(x,y) = q(1-\rho(x,y))$. In case of acquittal, the decision is appropriate and induces no costs if the defendant is innocent but costs $1-q$ if the defendant is guilty, an event that occurs with probability $\rho(x,y)$. In case of conviction, the decision is right if the defendant is guilty, but costs q if the defendant is innocent, that happens with probability $1-\rho(x,y)$. Finally, with some some abuse of notation $F(x|y)$ and $F(y|x)$ denote the conditional distributions[5]. Also, to handle alternating moves, we introduce the following definition: let $N_A = \{n|\text{A-juror moves}\}$ and $N_C = \{n|\text{C-juror moves}\}$.

Before we proceed we wish to emphasize some 'regularity properties' of the signal distribution and the posterior, those are straight consequences of the underlying informational structure. First, the posterior $\rho(x,y)$ is monotone increasing in both arguments. This result is quite intuitive, the higher is the private signal, the higher is the posterior of guilt holding the opponent's signal realization constant. Second, the posterior has finite first derivatives whenever $x, y \in (0,1)$ And finally, the 'conditional' density $f(a|b)$ is bounded away from zero and infinity whenever $a, b \in (0,1)$.

Recall that a strategy assigns an integer to each of the continuum possible types. Therefore, any strategy defines a set structure on the type space that we can characterize in the following way. Given a strategy τ_A, let $\chi(n) = \{x|\tau_A(x) = n\}$ and $\chi^+(n) = \{x|\tau_A(x) > n\}$, i.e. all the types $x \in \chi(n)$ are stopping at n and also all the types $x \in \chi^+(n)$ are holding out longer than n according to the strategy at hand. Similarly, for τ_C, let $\lambda(n) = \{y|\tau_C(y) = n\}$ and $\lambda^+(n) = \{y|\tau_C(y) > n\}$.

Using the notation above we can formalize the expected cost for a C-juror with posterior y stopping at period n given that the opponent plays according to τ_A

$$V(n, y; \tau_A) = \sum_{i \in N_A, i<n} \int_{\chi(i)} [v_C(x,y) + i\Delta c]\, f(x|y)dx + \int_{\chi^+(n)} [v_A(x,y) + n\Delta c]\, f(x|y)dx.$$

Assume that the opponent has a private posterior x then τ_A and n together determines the verdict. If that particular x stops before n then the verdict is conviction what leads to terminal costs $v_C(x,y)$ and decision costs $\tau_A(x)$. Hence, the first part of the expression above refers to states when the opponent gives in earlier so the verdict is conviction and the decision cost is determined by the stopping time of the A-juror. The second part gives the expected costs if the opponent gives in later so the verdict is acquittal and the decision cost is according to the C-juror. The expected cost is analogous for an A-juror given that the opponent plays according to τ_C

[5]Notice that in case of disagreement at time-zero each player acquires relevant information about the opponents type, i.e. the support of that is restricted to either the lower 'half' or the upper 'half' of $[\underline{p}, \overline{p}] \subseteq [0,1]$. Therefore the conditional distributions have support of $[\underline{p}, \kappa]$ and $[\kappa, \overline{p}]$ respectively.

$$V(n,x;\tau_C) = \sum_{i\in N_C, i<n} \int_{\lambda(i)} [v_A(x,y) + i\Delta c]\, f(y|x)dy + \int_{\lambda^+(n)} [v_C(x,y) + n\Delta c]\, f(y|x)dy.$$

Again, the first part refers to states when the C-juror gives in earlier so the verdict is acquittal and the decision cost is determined by the stopping time of the C-juror. The second part gives the expected costs if the opponent gives in later so the verdict is conviction and the decision cost is according to the A-juror. Finally, we can state the main result of this section.

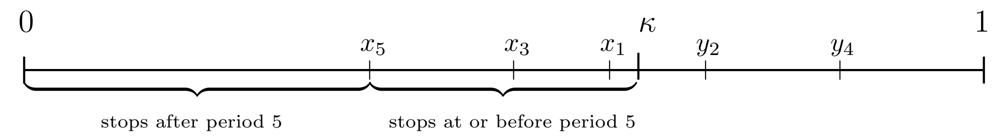
Proposition 1 *For any strategy of the opponent the best response exists and is monotone.*

Proof See the appendix. □

Corollary *All equilibria, if any, are in monotone strategies.*

3.3 Existence of Equilibrium

Having monotonicity, the sets $\chi(i), \lambda(i) =$ are intervals so we can conveniently refer to strategies by an infinite vector containing endpoints of those intervals. Let $\mathbf{x}$ be a vector where $x_n = \inf\{x|\tau_A(x) \le n\}$ if there is some $n' > n$ such that $\tau_A(x) = n'$ and $x_n = 0$ otherwise. Similarly, for a C-juror a strategy is represented by $\mathbf{y}$ where $y_n = \sup\{y|\tau_C(y) \le n\}$ if there is some $n' > n$ such that $\tau_C(y) = n'$ and $y_n = 1$ otherwise (see, for a similar construction, Athey (2001)). Later we will refer elements of these vectors as 'indifferent types'. It is useful to emphasize that $\chi(i) = [x_i, x_{i-2}], \lambda(i) = [y_{i-2}, y_i]$ and $\chi^+(i) = [0, x_i]$ decreasing set and $\lambda^+(i) = [y_i, 1]$ increasing set. Notice that $\mathbf{x}$ does not specify behavior for the cutoff type, but since there is no atom in the distribution of types this will not affect the best response of the opponent. The following graph helps to understand this construction (recall that κ denoted the cutoff type at time-zero):



To prove existence we would like to apply a fixed point theorem for the best response correspondence. The problem arises; however, that the strategy space is infinite (i.e. the vectors representing strategies are infinite dimensional) and does not satisfy the conditions of Kakutani's Theorem. Instead, we will use Schauder's Fixed Point Theorem to prove existence of monotone equilibrium.

Proposition 2 *There is an equilibrium in monotone strategies.*

Proof See the appendix. □

4 Characterization

In the previous section we argued that there is equilibrium in monotone strategies, now we want to step further and describe the nature of this equilibrium. We aim to estimate the length of deliberation, i.e. the length of time elapses before the conclusion is made. What we find matches with Coase Conjecture type results, that as the time-interval shrinks the delay vanishes too.

This result leans on monotonicity, implying that the longer the deliberation, the stronger the opponent's private information. Hence, after each period a juror can 'learn' about the the lowest possible type of the opponent. We show that equilibrium strategies must be 'embedded' in the sense that types whose private information offsets each other should give in in consecutive periods. Intuitively, there is no reason to hold out if it is sure that the opponent has an extreme signal enough to make her preferred decision better overall. The flip side of this coin is that costly deliberation induces jurors to stop before a 'sufficient' amount of information is acquired for doing so. To maintain the equilibrium, each juror should provide enough incentive for the opponent not to give in too early. This intuition is formalized below.

Definition 1 (Signal Strength) *A signal x is at least as A-strong as y or $x \succeq_s y$, if $\rho(x, y) \leq q$. In other words, if x is stronger than y the jurors prefer acquittal ex post.*

Notice that equivalence in this ordering gives us the types those are offsetting each other. We show that in equilibrium the cutoff type at period n cannot be stronger, in the sense of the definition above, than the one in period $n+1$. Not having this, allows for profitable deviation, the indifferent type at n will strictly prefer to undercut. The argument is the following, waiting for $n+2$ increases both the terminal and decision cost for the juror, so it is strictly dominated. The following lemma proves this. Since we address the problem of vanishing time periods we need to extend our notation that indicates this length. The vectors $\mathbf{x}^{\Delta}$ and $\mathbf{y}^{\Delta}$ will describe monotone equilibrium strategies for Δ period length.

Lemma 1 *For any Δ, in equilibrium $x_n^{\Delta} \succ_s y_{n-1}^{\Delta}$ $\forall n \in N_x$ and $y_n^{\Delta} \succ_s x_{n-1}^{\Delta}$ $\forall n \in N_y$.*

Proof (by contradiction) We only prove that $y_n^{\Delta} \succ_s x_{n-1}^{\Delta}$, the other is similar. Assume $y_n^{\Delta} \preceq_s x_{n-1}^{\Delta}$ where $n \in N_y$ and show that a type y_n^{Δ} strictly prefers stopping at n to $n+2$. Consider the indifference condition:

$$\int_{x_{n+1}^{\Delta}}^{x_{n-1}^{\Delta}} \left[\rho(x, y_n^{\Delta}) - q\right] f(x|y_n^{\Delta})dx - \Delta c\left[F(x_{n-1}^{\Delta}|y_n^{\Delta}) + F(x_{n+1}^{\Delta}|y_n^{\Delta})\right] = 0$$

By monotonicity, $y_n^{\Delta} \preceq_s x_{n-1}^{\Delta}$ implies $y_n^{\Delta} \prec_s x_{n+1}^{\Delta}$, so for any $x \in [x_{n-1}^{\Delta}, x_{n+1}^{\Delta}]$, $\rho(x, y_n^{\Delta}) \leq q$. Therefore, the LHS is strictly negative, so y_n^{Δ} strictly prefers stopping at n. Contradiction. □

To support the previous result, we need a large mass of types giving in each period, for almost all the periods. To prove this, we will show that the marginal benefit for the indifferent C-juror is proportional to $(x_{n-1}^{\Delta} - x_{n+1}^{\Delta})^2$, while the marginal cost is proportional to Δ. Hence, the indifference condition forces $x_{n-1}^{\Delta} - x_{n+1}^{\Delta}$ to approach zero slower than Δ. Clearly, a similar argument holds for an A-juror.

Lemma 2 *For all $\alpha > 0$ and $\eta > 0$ there exists $\bar{\Delta} > 0$ such that*

1. $\frac{|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|}{\Delta} > \alpha$ *for all $\Delta < \bar{\Delta}$, if $n \in N_y$ and $x_{n+1}^{\Delta} > \eta$*

   and

2. $\frac{|y_{n+1}^{\Delta} - y_{n-1}^{\Delta}|}{\Delta} > \alpha$ *for all $\Delta < \bar{\Delta}$, if $n \in N_x$ and $y_{n+1}^{\Delta} < 1 - \eta$.*

Proof We only prove part 1 as the other is similar. By Lemma 1 we know that in equilibrium $x_{n+1}^{\Delta} \succeq_s y_n^{\Delta} \succeq_s x_{n-1}^{\Delta}$, $\forall n \in N_y$. So there exists $\xi_n^{\Delta} \in (x_{n+1}^{\Delta}, x_{n-1}^{\Delta})$ such that $\rho(\xi_n^{\Delta}, y_n^{\Delta}) = q$. By the definition of y_n^{Δ}, the following must hold:

$$\int_{x_{n+1}^{\Delta}}^{x_{n-1}^{\Delta}} \left[\rho(x, y_n^{\Delta}) - q\right] f(x|y_n^{\Delta})dx = \Delta c\left[F(x_{n-1}^{\Delta}|y_n^{\Delta}) + F(x_{n+1}^{\Delta}|y_n^{\Delta})\right]. \tag{1}$$

The LHS of (1) gives us the marginal benefit from waiting, i.e. how much the terminal cost is decreasing in expectation while the RHS of (1) describes the expected increase in decision cost. Next, we compare them and find upper and lower bounds. Notice that for low x, the terminal cost is actually increasing, so that the marginal benefit is negative, while for high x, the opposite holds. So it seems convenient to separate these effects. Using the definition of ξ_n^{Δ}, the marginal benefit $\int_{x_{n+1}^{\Delta}}^{x_{n-1}^{\Delta}} \left[\rho(x, y_n^{\Delta}) - q\right] f(x|y_n^{\Delta})dx$ equals:

$$\begin{aligned}
&\int_{x_{n+1}^{\Delta}}^{\xi_n^{\Delta}} \left[\rho(x, y_n^{\Delta}) - \rho(\xi_n^{\Delta}, y_n^{\Delta})\right] f(x|y_n^{\Delta})dx \quad + \quad \int_{\xi_n^{\Delta}}^{x_{n-1}^{\Delta}} \left[\rho(x, y_n^{\Delta}) - \rho(\xi_n^{\Delta}, y_n^{\Delta})\right] f(x|y_n^{\Delta})dx \\
&= \int_{\xi_n^{\Delta}}^{x_{n-1}^{\Delta}} \left[\int_{\xi_n^{\Delta}}^{x} \rho_x(\xi, y_n^{\Delta})d\xi\right] f(x|y_n^{\Delta})dx \quad - \quad \int_{x_{n+1}^{\Delta}}^{\xi_n^{\Delta}} \left[\int_{x}^{\xi_n^{\Delta}} \rho_x(\xi, y_n^{\Delta})d\xi\right] f(x|y_n^{\Delta})dx \\
&< \int_{\xi_n^{\Delta}}^{x_{n-1}^{\Delta}} \left[\int_{\xi_n^{\Delta}}^{x} \rho_x(\xi, y_n^{\Delta})d\xi\right] f(x|y_n^{\Delta})dx \quad < \quad \int_{\xi_n^{\Delta}}^{x_{n-1}^{\Delta}} \bar{\rho}_x(x_{n-1}^{\Delta} - \xi_n^{\Delta}) f(x|y_n^{\Delta})dx \\
&< \bar{\rho}_x \bar{f}(x_{n-1}^{\Delta} - \xi_n^{\Delta})^2
\end{aligned}$$

where first we used $\rho_x(x,y) > 0$ and then that $\rho_x(x,y)$ and $f(x|y)$ are bounded above for $x, y \in [\eta, 1-\eta] \in (0,1)$, with bounds $\bar{\rho}_x$ and $\bar{f}$, respectively. Also, for the marginal cost (RHS of (1))

$$\Delta c \left[F(x_{n-1}^{\Delta}|y_n^{\Delta}) + F(x_{n+1}^{\Delta}|y_n^{\Delta})\right] > \Delta c 2\bar{F}$$

where $\underline{F}$ is a lower bound for $F(x|y)$ if $x, y \in [\eta, 1-\eta]$. Therefore,

$$\bar{\rho_x}\bar{f}(x_{n-1}^{\Delta} - \xi_n^{\Delta})^2 > \Delta c 2\bar{F}.$$

Taking the square root of both sides after some manipulation, we find that

$$\frac{|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|}{\Delta} > \frac{|x_{n-1}^{\Delta} - \xi_n^{\Delta}|}{\Delta} > \frac{A}{\sqrt{\Delta}}, \tag{2}$$

where $A^2 = \frac{2c\bar{F}}{\bar{f}\bar{\rho}_x} > 0$. The first inequality follows by Lemma 1. □

Finally, we can prove our main proposition about vanishing delay in decision. Formally, we will show that by any real time T, the decision has been made by this time with probability near one if the period length is small enough.

Proposition 3 *Fix $T > 0$ and $\varepsilon > 0$. There exists $\Delta > 0$ such that if $N(\Delta, T) = T/\Delta$ then*

$$|y_N^{\Delta} - x_{N-1}^{\Delta}| > 1 - 2\varepsilon.$$

Hence, the decision process ends by time T with probability at least $1 - 2\varepsilon$.

Proof (by contradiction) If $y_N^{\Delta} > 1-\varepsilon$, and $x_N^{\Delta} < \varepsilon$ then it is obvious. Assume $\exists\, T, \varepsilon > 0$ such that for all $\Delta > 0$

$$|y_N^{\Delta} - x_{N-1}^{\Delta}| < 1 - 2\varepsilon$$

and $y_N^{\Delta} < 1 - \varepsilon$ and/or $x_{N-1}^{\Delta} > \varepsilon$. Without loss of generality, assume $x_{N-1}^{\Delta} > \varepsilon$. Then by Lemma 2, there exists $\overline{\Delta} > 0$ such that for all $n \in N_C$ and $n \leq N$

$$\frac{|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|}{\Delta} > \frac{2(1-2\varepsilon)}{T}$$

for all $\Delta < \overline{\Delta}$. Therefore, for small enough Δ

$$\begin{aligned} 1 - 2\varepsilon &> |y_N^{\Delta} - x_{N-1}^{\Delta}| = \sum_{i \in N_C, i \leq N} |y_i^{\Delta} - y_{i-2}^{\Delta}| + \sum_{i \in N_A, i < N} |x_{i-2}^{\Delta} - x_i^{\Delta}| \\ &> \Delta \frac{N}{2} \frac{2(1-2\varepsilon)}{T} = 1 - 2\varepsilon. \end{aligned}$$

Contradiction. □

5 Conclusion

We analyzed costly dynamic voting as a decision making process for committees of two members. We assumed private information and common interest of the decision makers. Our research question was whether delay can be explained in deliberation solely by the problems of like minded individuals trying to communicate their information, but with an inability to do so except by persistence in voting. We found a negative answer, i.e. if the time interval between votes shrinks, the real time elapse of the delay vanishes too. Immediate decision is quite counterfactual so our result suggests that delay in decisions can only be explained by different preferences among committee members, that is subject of future research.

A Monotonicity: Proof of Proposition 1

We work out the proof only for C-jurors. To prove that the best response exists and monotone we will use a theorem by Milgrom and Shannon (1994)[6]. Recall that $f : X \times T \to \mathbb{R}$ has the single crossing property in $(x; t)$ if for $x'' > x'$ and $t'' > t'$, $f(x'', t') - f(x', t') > 0$ implies that $f(x'', t'') - f(x', t'') > 0$.

Theorem (Milgrom-Shannon) *Let $g : X \times X \to \mathbb{R}$, where $X \subset \mathbb{R}$ and $S \subset X$. Then $\arg\max_{s \in S} g(x, t)$ is monotone nondecreasing in (t, S) iff g satisfies the single crossing property in $(x; t)$.*

The theorem states that if the objective function has the single crossing property in choice variable and type then the optimal action increases in type. Notice that the theorem is stated for maximization problems but in our model the players are cost minimizers. Hence, we need to show that the negative expected costs satisfies the assumption of this theorem. Fix the real-time elapses between two periods and fix a strategy of the opponent. Since we fixed the opponent's strategy we abuse notation and leave out τ_A. Recall the value function for a C-juror of type y, stopping at $n \in N_C$ is:

$$V(n, y) = \sum_{i \in N_A, i < n} \int_{\chi(i)} [v_C(x, y) + i\Delta c]\, f(x|y)dx + \int_{\chi^+(n)} [v_A(x, y) + n\Delta c]\, f(x|y)dx.$$

Hence, the change in value function (decrease in expected costs) for a C-juror of type y who stops at m instead of n so that $n < m$ is:

$$\Delta V(n, m, y) = \sum_{i \in [n,m)} \left(\int_{\chi(i)} [\rho(x, y) - q] f(x|y)dx - \Delta c \int_{\chi^+(i)} f(x|y)dx \right).$$

where we used the expressions $v_A(x, y) = (1 - q)\rho(x, y)$ and $v_C(x, y) = q(1 - \rho(x, y))$. Changing the time of stopping changes both the terminal and the decision cost if the opponents realization is such that $\tau_A(x) \in (n, m)$. First, the terminal cost varies since the verdict changes from acquittal to conviction, i.e. the costs decreases by $v_A(x, y) - v_C(x, y)$. Second, the decision cost clearly increases.

The core of the proof is to realize that the positivity of ΔV implies that it is increasing in type. In our model we can show that ΔV itself is a lower bound for ΔV_y, so the single crossing property is implied.

Lemma 3 *The ratio f_y/f has upper bound.*

Proof Define $K = \frac{F(\kappa|I)}{F(\kappa|G)}$. After some algebra

[6] An earlier, less general version is in Topkis (1978).

$$f(x|y) = \frac{f(x|G)}{F(\kappa|G)} \left(\frac{1 + \ell(x)\ell(y)}{1 + K\ell(y)} \right)$$

where $\ell(p) \equiv \frac{f(p|I)}{f(p|G)}$. Hence,

$$f_y(x|y) = \frac{f(x|G)}{F(\kappa|G)} \left(\frac{K - \ell(x)}{y^2(1 + K\ell(y))^2} \right).$$

Since $\ell(x) > 0$

$$\frac{f_y(x|y)}{f(x|y)} = \frac{K - \ell(x)}{y^2(1 + K\ell(y))(1 + \ell(x)\ell(y))} < \frac{K}{y^2(1 + K\ell(y))} \equiv C.$$

□

Lemma 4 *The function $\psi(x, y) = [\rho(x, y) - q]f(x|y)$ is increasing in y. Define $X'(y) \subset [0, 1]$ so that $\psi(x, y) > 0$. The ratio of ψ_y/ψ has lower bound on $X'(y)$.*

Proof After some algebra

$$\psi(x, y) = \frac{f(x|G)}{F(\kappa|G)} \left(\frac{(1 - q) - q\ell(x)\ell(y)}{1 + K\ell(y)} \right).$$

Hence,

$$\psi_y(x, y) = \frac{f(x|G)}{F(\kappa|G)} \left(\frac{q\ell(x) + (1 - q)K}{y^2(1 + K\ell(y))^2} \right).$$

The expression ψ_y is always positive. Moreover on $X'(y)$:

$$\frac{\psi_y(x, y)}{\psi(x, y)} = \frac{q\ell(x) + (1 - q)K}{y^2(1 + K\ell(y))((1 - q) - q\ell(x)\ell(y))} > \frac{(1 - q)K}{y^2(1 + K\ell(y))(1 - q)} = C.$$

□

Lemma 5 *For any $n_2 > n_1$, if $\Delta V(n_1, n_2, y) > 0$ for some y then $\Delta V(n_1, n_2, y') > 0$ for any $y' > y$.*

Proof The derivative of ΔV w.r.t y exists

$$\Delta V_y(n, m, y) = \sum_{i \in [n,m)} \left(\int_{\chi(i)} \psi_y(x, y)dx - \Delta c \int_{\chi^+(i)} f_y(x|y)dx \right).$$

By the Lemma (3) and (4) for any A, B and appropriate subsets A', B' those satisfy the positivity restrictions above

$$\begin{aligned}\int_B f_y(x|y)dx &< C\int_B f(x|y)dx \\ \int_A \psi_y(x,y)dx &\geq \int_{A'} \psi_y(x,y)dx > C\int_A \psi(x,y)dx\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta V_y(n,m,y) &= \sum_{i\in[n,m)}\left(\int_{\chi(i)} \psi_y(x,y)dx - \Delta c\int_{\chi^+(i)} f_y(x|y)dx\right) \\ &> \sum_{i\in[n,m)}\left(C\int_{\chi(i)} \psi(x,y)dx - C\Delta c\int_{\chi^+(i)} f(x|y)dx\right) = C\Delta V(n,m,y) > 0\end{aligned}$$

□

Corollary *The negative expected costs has the single crossing property in $(n;y)$, so the condition for Milgrom-Shannon Theorem is satisfied. The best response is monotone increasing.*

For an A-juror we can repeat the same argument except that we need to redefine the stopping time $n = -m$ and show the single crossing property of the negative expected costs for A in $(m;x)$.

B Existence: Proof of Proposition 2

Theorem (Schauder) *If M is a compact, convex and non-empty subset of a Banach space, and $T : M \to M$ is a continuous mapping, then T has a fixed point.*[7]

To complete the proof we have to check whether the conditions for Schauder Theorem are satisfied. Hence, in the rest of the section we need to show an appropriate Banach space of which the space of monotone strategies is a compact, convex and nonempty subset, as well as to prove that the best response correspondence is single-valued and continuous in the defined norm.

Definition *Define the strategy space $S \equiv X \times Y$ where*[8]

$$\begin{aligned}X &\equiv \{x : \mathbb{N} \to [0,1] \mid x(i) \geq x(i+1)\}, \qquad \forall i \in \mathbb{N} \\ Y &\equiv \{y : \mathbb{N} \to [0,1] \mid y(i) \leq y(i+1)\}, \qquad \forall i \in \mathbb{N}.\end{aligned}$$

Lemma 6 *There exists a Banach space B such that $S \subset B$*

[7] See e.g. Granas and Dugundji (2003).

[8] Alternatively, we can see a strategy as a vector in $[0,1]^{\mathbb{N}} \times [0,1]^{\mathbb{N}}$. We are going to alternate these two interpretations conveniently.

Proof First, find the appropriate Banach space. Notice that $\{\mathbb{N}^2, 2^{\mathbb{N}^2}, \mu\}$ is a measure space with

$$\mu(E) = \sum \chi_E(n_1, n_2) \frac{1}{2^{n_1}} \frac{1}{2^{n_2}}.$$

Then $L^1(\mathbb{N}^2)$ is the set of measurable functions $z : \mathbb{N}^2 \to \mathbb{R}^2$ such that

$$||z|| = \int_{\mathbb{N}^2} |z| d\mu < \infty.$$

It is easy to see that $||.||$ is a norm and completeness implied by Riesz-Fischer Theorem[9]. Therefore, $L^1(\mathbb{N}^2)$ is a Banach space. Finally, the strategy set $S \subset L^1(\mathbb{N}^2)$ since for any $s \in S$, $||s|| < 2$, so exists. □

Before we proceed to prove the properties of the strategy space we show equivalence between topologies.

Lemma 7 *On S the norm $||.||$ implies convergence that is equivalent to pointwise convergence in Euclidian norm.*

Proof Notice that S is a space of bounded functions.

($\Rightarrow$) Pick i and $\varepsilon > 0$ and find N_i such that $\forall n > N_i$, $|z^n(i) - z(i)| < \varepsilon$. By convergence in $||.||$ there is N such that $\forall n > N$, $||z^n - z|| < \frac{\varepsilon}{2^i}$. Then, $N_i = N$ is appropriate since

$$|z^n(i) - z(i)| \leq 2^i ||z^n - z|| < 2^i \frac{\varepsilon}{2^i} = \varepsilon.$$

($\Leftarrow$) Pick $\varepsilon > 0$ and find N such that $\forall n > N$, $||z^n - z|| < \varepsilon$. Let I be such that $\frac{1}{2^I} < \frac{\varepsilon}{2}$ and $\varepsilon_I < \frac{\varepsilon}{2} \frac{1}{1-(1/2)^I}$. By pointwise convergence $\forall i \leq I$, there is N_i such that $\forall n > N_i$, $|z^n(i) - z(i)| < \varepsilon_I$. Then, $N = \max\{N_i\}$ is appropriate since

$$\begin{aligned} ||z^n - z|| &= \sum_{i=1}^{I} |z^n(i) - z(i)| \frac{1}{2^i} + \sum_{i=I+1}^{\infty} |z^n(i) - z(i)| \frac{1}{2^i} \\ &< \varepsilon_I \sum_{i=1}^{I} \frac{1}{2^i} + 1 \sum_{i=I+1}^{\infty} \frac{1}{2^i} < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

□

Lemma 8 *The strategy space is convex and compact.*

Proof Convexity follows conveniently from the monotonicity of the strategies. Hence, we only need to show that S is compact. Using the vector interpretation, notice that the strategy space is the product of $[0, 1]$ intervals those are compact in Euclidian norm. Tychonov Theorem[10] implies compactness for the product topology what is equivalent to 'our' topology by Lemma 7. □

[9] See e.g. DiBenedetto (2002).
[10] See e.g. DiBenedetto (2002).

Before we describe the properties of the best response function we show the continuity of the expected costs function in type and the opponent's strategy. As usual we work this out only for a C-juror.

Lemma 9 *The expected cost function $V(n, y; \boldsymbol{x})$ is continuous in y and $\boldsymbol{x}$.*

Proof Recall the expected cost function:

$$V(n, y; \mathbf{x}) = \sum_{i \in N_A, i<n} \int_{x_i}^{x_{i-2}} [v_C(x, y) + i\Delta c]\, f(x|y)dx + \int_0^{x_i} [v_A(x, y) + n\Delta c]\, f(x|y)dx.$$

The continuity in type is obvious since the both the terminal values and the conditional density are continuous in y. We prove continuity in the opponent's strategy. For an $\varepsilon > 0$ we have to find appropriate δ such that $||\mathbf{x}' - \mathbf{x}|| < \delta$ implies $|V(n, y; \mathbf{x}') - V(n, y; \mathbf{x})| < \varepsilon$. Notice that $V(n, y; \mathbf{x}') = V(n, y; \mathbf{x})$ whenever $x'_i = x_i$ for all $i \leq n$. For any $\mathbf{x}', \mathbf{x}$ define $\tilde{\mathbf{x}}^j$

$$\tilde{x}_i^j = \begin{cases} x_i, & \text{if} \quad i \leq j \\ x'_i, & \text{otherwise.} \end{cases}$$

Let B an upper bound for $[v_C(x, y) + n\Delta c]\, f(x|y)$ and $[v_A(x, y) + n\Delta c]\, f(x|y)$, B exists. Hence, for any j, $|V(n, y; \tilde{\mathbf{x}}^{j-1}) - V(n, y; \tilde{\mathbf{x}}^j)| < 2B|x'_j - x_j|$. Then, $\delta < \frac{\varepsilon}{2B2^n}$ is appropriate since

$$\begin{aligned} |V(n, y; \mathbf{x}') - V(n, y; \mathbf{x})| \;&\leq\; \sum_{j=1}^{n} |V(n, y; \tilde{\mathbf{x}}^j - 1) - V(n, y; \tilde{\mathbf{x}}^j)| \;<\; 2B \sum_{j=1}^{n} |x'_j - x_j| \\ &<\; 2B2^n \sum_{j=1}^{n} |x'_j - x_j| \frac{1}{2^j} \;<\; 2B2^n \sum_{j=1}^{\infty} |x'_j - x_j| \frac{1}{2^j} \;<\; \varepsilon. \end{aligned}$$

□

Lemma 10 *The best response correspondence $BR : S \to S$ is single valued and continuous.*[11]

Proof BR is single-valued since a generic element of $BR(s)$ is defined by

$$\sup\{y\, |V(n, y; \mathbf{x}) \leq V(n + 2, y; \mathbf{x})\}$$

or

$$\inf\{x\, |V(n, x; \mathbf{y}) \leq V(n + 2, x; \mathbf{y})\}.$$

To show continuity of the best response we will prove that for two sequences of strategies $\mathbf{x}^k \to \mathbf{x}$ and $\mathbf{y}^k \to \mathbf{y}$ if $\mathbf{y}^k = BR_y(\mathbf{x}^k)$ then $\mathbf{y} = BR_y(\mathbf{x})$. Intuitively, if for some i, y_i^k is an indifferent type, i.e. $V(i, y_i^k; \mathbf{x}^k) = V(i + 2, y_i^k; \mathbf{x}^k)$ then these values should be equal in the limit as well.

[11] Alternatively $BR = BR_x \times BR_y$.

Assume that there is y_i such that $|V(i, y_i; \mathbf{x}) - V(i+2, y_i; \mathbf{x})| = \varepsilon > 0$. By Lemma 9 there exist N_1, N_2, N_3, N_4 such that $|V(i, y_i; \mathbf{x}) - V(i, y_i^k; \mathbf{x})| < \varepsilon/4$, $|V(i, y_i^k; \mathbf{x}) - V(i, y_i^k; \mathbf{x}^k)| < \varepsilon/4$, $|V(i+2, y_i^k; \mathbf{x}^k) - V(i+2, y_i^k; \mathbf{x})| < \varepsilon/4$, $|V(i+2, y_i^k; \mathbf{x}) - V(i+2, y_i; \mathbf{x})| < \varepsilon/4$ respectively if $k > N_i$. Hence, for k high enough

$$
\begin{aligned}
\varepsilon = |V(i, y_i; \mathbf{x}) - V(i+2, y_i; \mathbf{x})| &\leq |V(i, y_i; \mathbf{x}) - V(i, y_i^k; \mathbf{x})| + |V(i, y_i^k; \mathbf{x}) - V(i, y_i^k; \mathbf{x}^k)| + \\
|V(i+2, y_i^k; \mathbf{x}^k) - V(i+2, y_i^k; \mathbf{x})| &+ |V(i+2, y_i^k; \mathbf{x}) - V(i+2, y_i; \mathbf{x})| \\
< \frac{\varepsilon}{4} + \frac{\varepsilon}{4} &+ \frac{\varepsilon}{4} + \frac{\varepsilon}{4} = \varepsilon
\end{aligned}
$$

Contradiction. The best response is continuous. □

Corollary *The conditions of Schauder Theorem hold for the best response function. Hence, equilibrium exists.*

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