Determinacy with Capital Adjustment Costs and Sector-Specific Externalities*

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Abstract

This paper explores the stability properties of the steady state in the standard two-sector real business cycle model with a sector-specific externality in the capital-producing sector. When the steady state is stable then equilibrium is indeterminate and stable sunspots are possible. We find that capital adjustment costs of *any* size preclude stable sunspots for every empirically plausible specification of the model parameters. More specifically, we show that when capital adjustment costs of *any* size are considered, a necessary condition for the existence of stable sunspots is an upward-sloping labor demand curve in the capital-producing sector, which in turn requires an implausibly strong externality. This result contrasts sharply with the standard result that when we abstract from capital adjustment costs, stable sunspots occur in the two-sector model for a wide range of plausible parameter values.

Keywords: capital adjustment costs; determinacy; indeterminacy; sector-specific externality; sunspots.

JEL classification: E0; E3.

1 Introduction

It is well known that the steady state of one-sector growth model is unique and saddle-path stable and that the equilibrium paths near to the steady state are locally unique. We will summarize these properties by the term "determinacy". Even though these properties are typically considered standard, this model may have completely different properties when externalities are considered: the unique steady state may be stable, which means that a continuum of equilibrium paths converge to the steady and that the equilibrium near the steady state is indeterminate. In this case, changes in non-fundamental variables, usually called sunspots, can select the equilibrium path. We will summarize these properties by the term "stable sunspots".[1] Since both determinacy and stable sunspots are theoretically possible, the natural question to ask is which of the two will prevail for empirically plausible specifications of the parameters of the model economy, in particular, for the value of the externality. The goal of the present paper is to answer this question for real business cycle versions of the model, which abstract from steady state growth.

The literature on stable sunspots in real business cycle models can be divided into two broad groups. One group of papers studies one-sector versions of the real business cycle model and finds that stability requires strong externalities that are empirically implausible; see e.g. Benhabib and Farmer (1994), Farmer and Guo (1994), and Gali (1994). A second group of papers shows that when there are sector-specific externalities in the two-sector versions of the real business cycle model, the steady state can be stable for mild values of the externality that are empirically plausible; see e.g. Benhabib and Farmer (1996), Perli (1998), Weder (1998), Harrison (2000), and Weder (2000). The difference between these two strands of results comes from two of the different channels through which sunspots can affect the dynamic behavior of the model economies. The first one of these channels is the labor channel. It works through self-fulfilling changes in labor demand and can operate in both the one- and the two-sector model providing the labor demand curve slopes upwards. This requires implausibly strong externalities and has economic implications that are awkward; see Aiyagari (1995). The second channel is the capital channel and it operates through self-fulfilling changes in the allocation of capital across sectors and operates only in the two-sector model. The capital channel

[1] Classical contributions to the literature on sunspots include Azariadis (1981), Cass and Shell (1983), Kehoe and Levine (1985), Woodford (1991), and Howitt and McAfee (1992). A review of the literature on sunspots in the neoclassical growth model is Benhabib and Farmer (1999).

relies on capital gains, which can occur for mild sector-specific externalities that are empirically plausible and do not make the labor demand curve upward sloping.[2]

The project of this paper is to explore the robustness of the capital channel. We are motivated by the conjecture that the capital channel only functions as described in the literature if one abstracts from the costs of changing the allocation of capital across the two sectors. In order to prove this conjecture, we consider capital adjustment costs in a standard, two-sector real business cycle model with a sector-specific externality in the capital-producing sector. This modification of the standard model can be justified by the substantial empirical evidence on the existence of adjustment costs; see e.g. Hammermesh and Pfann (1996) for a review of this evidence. Here we employ the specification proposed by Huffman and Wynne (1999), which drastically improves the quantitative performance of the two-sector real business cycle model.

We obtain two results. First, we show that capital adjustment costs of *any* size shut down the capital channel and preclude the existence of stable sunspots for a wide range of model parameters that includes every empirically plausible specification. Specifically, we find that a necessary condition for stable sunspots is that the externality is so strong that the labor demand curve of the capital-producing sector slopes upward. In other words, if one considers capital adjustment costs of *any* size, then the difference between the stability properties of the one- and the two-sector real business cycle model disappears. Second, given a benchmark calibration of our model, we show that the unique steady state is saddle-path stable for every empirically plausible value of the externality in the capital-producing sector. In other words, given the benchmark calibration, we find not only hat stable sunspots are impossible but also that determinacy must occur. We show that this second result is robust to small changes in the calibrated model parameters.

The results of this paper are relevant for several reasons. To begin with, they contribute to the debate about whether or not optimal government policy should try to stabilize business cycles. In particular, if there are stable sunspots, then they can generate business cycles. This type of business cycles is inefficient and it has been argued that they should be stabilized. In contrast, if there is determinacy, then business cycles require stochastic shocks to total factor productivity or some other fundamental variable. This second type of business cycles is efficient and it has been

[2]For different versions of the neoclassical growth model, Boldrin and Rustichini (1994) and Benhabib, Meng and Nishimura (2000) find the same difference: indeterminacy is easier to obtain in two- than in one-sector versions.

argued that they should not be stabilized.[3] Second, there has been a renewed, recent interest in two-sector real business cycle models; see for example Fisher (1997), Huffman and Wynne (1999), or Boldrin, Christiano and Fisher (2000). Our results provide a better understanding of the stability properties of this important class of models. Last, but not least, this paper contributes to a recent debate about the robustness of multiple and indeterminate equilibria. Even though Adsera and Ray (1998), Morris and Shin (1998), Karp (1999), Frankel and Pauzner (2000), and Herrendorf, Valentinyi and Waldmann (2000) studied rather different environments with externalities, they all share a common theme with the present paper: multiple or indeterminate equilibria may well be a much less frequent phenomenon than it has previously been thought.

The rest of the paper is organized as follows. Section 2 lays out the economic environment. Section 3 defines the equilibrium, derives the reduced-form dynamics, and shows that the model has a unique steady state around which we can linearize the reduced-form dynamics. Section 4 discusses the calibration of the model while Section 5 reports the results of the stability analysis and the sensitivity analysis. Section 6 offers some intuition for our results and points out the related literature. Section 7 concludes the paper. The formal proofs and the results of our sensitivity analysis can be found in the Appendix.

2 Environment

Time is continuous and runs forever. There is no uncertainty, which simplifies matters but in no way affects the stability results derived. The economy is populated by a continuum of measure one of identical, infinitely-lived households, by a continuum of measure one of identical firms that own a technology with which a consumption good can be produced, and by a continuum of measure one of identical firms that own a technology with which new capital can be produced. The representative household is endowed with the initial capital stocks, with the property rights for the representative firm of each sector at time zero, and with time at each point in time.

There are sector-specific externalities in the capital-producing sector that are external to the representative firm producing there. Moreover, installed capital is sector specific and there are capital adjustment costs. Thus, at each point in time five commodities are traded: a perishable consumption good, a new capital good suitable for the production of consump-

[3] Of course, in both cases it is optimal to internalize the externalities, if possible.

tion goods, a new capital good suitable for the production of new capital goods, working time in the consumption-producing sector, and working time in the capital-producing sector. All trades take place in sequential markets, in which the representative household rents capital and time to the firms and uses the resulting income to buy from them consumption goods and new capital goods.

We now describe the programmes that are solved by the households and firms of our model economy. Note that since there are externalities here we cannot obtain the equilibrium allocation by solving the planner's problem but need to solve the decentralized problems.

2.1 Households

Formally, the representative household solves:

$$\max_{c_t,l_{ct},l_{xt},x_{ct},x_{xt}} \int_0^\infty e^{-\rho t}[\log c_t + (T - l_{ct} - l_{xt})]dt \tag{1a}$$

$$\text{s.t.} \quad c_t + p_{ct}x_{ct} + p_{xt}x_{xt} = \pi_{ct} + \pi_{xt} + w_{ct}l_{ct} + w_{xt}l_{xt} + r_{ct}k_{ct} + r_{xt}k_{xt}, \tag{1b}$$

$$\dot{k}_{ct} = x_{ct} - \delta_c k_{ct}, \tag{1c}$$

$$\dot{k}_{xt} = x_{xt} - \delta_x k_{xt}, \tag{1d}$$

$$0 \leq c_t, l_{ct}, l_{xt}, x_{ct}, x_{xt}, \tag{1e}$$

$$T \geq l_{ct} + l_{xt}, \tag{1f}$$

$$k_{c0}, k_{x0}, \pi_{ct}, \pi_{xt}, p_{ct}, p_{xt}, w_{ct}, w_{xt}, r_{ct}, r_{xt} \text{ given.} \tag{1g}$$

The notation is as follows: $\rho > 0$ is the constant discount rate; c_t denotes the consumption good at time t; the subscripts c and x indicate variables from the consumption- and the capital-producing sector, so e.g. l_{ct} and l_{xt} are the working times in the two sectors and w_{ct} and w_{xt} are the corresponding wages; $T > 0$ is the time endowment in each period implying that $(T - l_{ct} - l_{xt})$ is leisure; x_{ct} and x_{xt} represent the new capital goods and p_{ct} and p_{xt} represent their prices; k_{ct} and k_{xt} are the capital stocks and r_{ct} and r_{xt} are the real interest rates; δ_c and $\delta_x \in [0, 1]$ denote the depreciation rates and π_{ct} and π_{xt} denote profits (which will be zero in equilibrium). Note that in each period, the contemporaneous consumption good is taken to be the numeraire.

Several features of the representative household's programme deserve comment. First, the use of logarithmic utility in consumption implies not only analytical simplicity but also that the stability properties of the model become independent of whether or not there are increasing returns in the consumption-producing sector; see Harrison and Weder (1999) and Harrison (2000). Thus, our assumption of constant returns in the consumption-producing sector has no importance for the stability analysis. Second, the linearity of the utility in leisure results in an infinite wage elasticity of labor supply. Since it is typically harder to get saddle-path stability the higher the labor supply elasticity, this makes our results applicable for all labor supply elasticities.[4] Third, it is worth stressing that x_{ct} and x_{xt} are restricted to be non-negative because capital is assumed to be sector-specific here. Consequently, the only way in which the capital stock of a sector can be reduced is by not replacing depreciated capital.

Denoting the current value multipliers by μ_{ct} and μ_{xt}, the first-order conditions are (1b)–(1f) and

$$\frac{p_{ct}}{c_t} = \mu_{ct}, \tag{2a}$$

$$\frac{p_{xt}}{c_t} = \mu_{xt}, \tag{2b}$$

$$c_t = w_{ct} = w_{xt}, \tag{2c}$$

$$\dot{\mu}_{ct} \leq \mu_{ct}(\delta_c + \rho) - \frac{r_{ct}}{c_t} \quad \text{(with equality if } x_{ct} > 0\text{)}, \tag{2d}$$

$$\dot{\mu}_{xt} \leq \mu_{xt}(\delta_x + \rho) - \frac{r_{xt}}{c_t} \quad \text{(with equality if } x_{xt} > 0\text{)}, \tag{2e}$$

$$\lim_{t\to\infty}(\mu_{ct}k_{ct} + \mu_{xt}k_{xt}) \leq 0. \tag{2f}$$

Note that (1e), (2a), (2b), and (2f) imply the standard terminal conditions:

$$\lim_{t\to\infty}\frac{p_{ct}k_{ct}}{c_t} = \lim_{t\to\infty}\frac{p_{xt}k_{xt}}{c_t} = 0. \tag{2g}$$

2.2 Firms

Consistent with the evidence reported by Basu and Fernald (1997), we assume that there are constant returns in the consumption-producing sector. The representative firm of the consumption-producing sector solves:

$$\max_{c_t,k_{ct},l_{ct}} \quad \pi_{ct} \equiv c_t - r_{ct}k_{ct} - w_{ct}l_{ct} \tag{3a}$$

$$\text{s.t.} \quad c_t = k_{ct}^{a} l_{ct}^{1-a}, \tag{3b}$$

$$c_t, l_{ct}, k_{ct} \geq 0, \tag{3c}$$

$$w_{ct}, r_{ct} \text{ given.} \tag{3d}$$

[4]An economic justifications for linear utility in leisure is the lottery argument put forth by Hansen (1985).

The first-order conditions are (3b), (3c), and

$$r_{ct} = ak_{ct}^{a-1} l_{ct}^{1-a}, \tag{4a}$$
$$w_{ct} = (1-a)k_{ct}^{a} l_{ct}^{-a}. \tag{4b}$$

The representative firm of the capital-producing sector solves:

$$\max_{x_{xt}, x_{ct}, l_{xt}, k_{xt}} \quad \pi_{xt} \equiv p_{xt}x_{xt} + p_{ct}x_{ct} - r_{xt}k_{xt} - w_{xt}l_{xt} \tag{5a}$$
$$\text{s.t.} \quad [\phi x_{ct}^{\eta} + (1-\phi)x_{xt}^{\eta}]^{\frac{1}{\eta}} = B_t k_{xt}^{b} l_{xt}^{1-b}, \tag{5b}$$
$$x_{xt}, x_{ct}, k_{xt}, l_{xt} \geq 0, \tag{5c}$$
$$B_t, p_{xt}, p_{ct}, r_{xt}, w_{xt} \text{ given}, \tag{5d}$$

where $\phi \in (0,1)$ and $\eta > 1$ are constants. Before we will discuss the roles played by B_t, ϕ, and η, we derive the first-order conditions of (5). Denoting the multiplier attached to (5b) by λ_t, the first-order conditions are (5b), (5c), and

$$r_{xt} = \lambda_t b B_t k_{xt}^{b-1} l_{xt}^{1-b}, \tag{6a}$$
$$w_{xt} = \lambda_t (1-b) B_t k_{xt}^{b} l_{xt}^{-b}, \tag{6b}$$
$$p_{ct} \leq \lambda_t \phi x_{ct}^{\eta-1} [\phi x_{ct}^{\eta} + (1-\phi)x_{xt}^{\eta}]^{\frac{1-\eta}{\eta}} \quad \text{(with equality if } x_{ct} > 0), \tag{6c}$$
$$p_{xt} \leq \lambda_t (1-\phi) x_{xt}^{\eta-1} [\phi x_{ct}^{\eta} + (1-\phi)x_{xt}^{\eta}]^{\frac{1-\eta}{\eta}} \text{(with equality if } x_{xt} > 0). \tag{6d}$$

Note that if $\eta > 1$ the optimal investments x_{ct} and x_{xt} are interior, $x_{ct} > 0$ and $x_{xt} > 0$. Thus, we can restrict attention to interior solutions for which the first-order conditions (6c) and (6d) hold with equality. [5]

The left-hand side of constraint (5b) together with the sector-specificity of capital implies the existence of capital adjustment costs. There are several reasons to consider adjustment costs in real business cycle models. First, there is substantial microevidence that firms' adjustment to stochastic disturbances exceeds by far the length of one year, and hence the maximal length of a period in real business cycle models [Hammermesh and Pfann (1996)]. For this reason, models of firms' investment behavior typically feature convex costs of changing the capital stock; see Abel (1990).

[5] To see the interiority suppose to the contrary that $\eta > 1$ and e.g. that $x_{ct} = 0$. If $x_{ct} = 0$, the first-order condition (6c) implies that $p_{ct} \leq 0$, and thus $p_{ct} = 0$. The household's first-order condition (2a) then shows that $\mu_{ct} = 0$ too. Furthermore, the household's first-order condition (2d) immediately gives that $\dot{\mu}_{ct} < 0$. Since μ_{ct} is zero already it must become negative, which is a contradiction.

Second, multi-sector business cycle models with costless adjustment have counterfactual properties in that consumption, aggregate labor productivity, labor productivity in the consumption-producing sector, and investment in the capital-producing sector are all countercyclical. Huffman and Wynne (1999) show that all of these variables become procyclical when the above specification of capital adjustment costs is introduced into two-sector real business cycle model that is identical to the one used here except for the fact that it has no externalities.[6]

Capital adjustment costs affect the equilibrium allocation by affecting the curvature of the production possibility frontier. Here we capture this effect by using the simplest CES functional form with only two parameters. This specification has been fairly popular in the literature on adjustment costs; see, among others, Fisher (1997) and Huffman and Wynne (1999). The weight parameter $\phi \in (0, 1)$ can be thought of capturing a choice of units. We will show below that it will not affect the stability properties. The curvature parameter $\eta > 1$ can be thought of as introducing a cost of changing the composition of the output of new capital goods.[7] We interpret this CES functional form as a local approximation at the steady state. While it is clearly inappropriate for other purposes, there are several reasons why it serves us well here. First, it gives rise to a concave (to the origin) production possibility frontier in (x_{ct}, x_{xt}) space, and so it generates the curvature to which any type of capital adjustment costs would give rise.[8] Second, it is homogeneous of degree one, implying that there are constant returns from all firms' perspectives. Consequently, equilibrium profits will be zero in both sectors, $\pi_{ct} = \pi_{xt} = 0$, and can be suppressed from now on.[9] Third, as demonstrated by Huffman and Wynne, the two parameters ϕ and η can be calibrated.

There is empirical evidence for the presence of positive externalities in manufacturing durables [Basu and Fernald (1997)]. Consistent with it, our specification of B_t implies sector-specific, positive externalities in the capital-producing sector:

$$B_t = k_{xt}^{\theta b} l_{xt}^{\theta(1-b)}, \tag{7a}$$

where $\theta \in [0, (1-b)/b)$. Substituting (7a) back into the capital-producing

[6]Fisher (1997) made a related point for a model with a household and a market sector.

[7]Recall that installed capital is assumed to be sector specific; otherwise part of the capital adjustment costs could be avoided by reallocating capital across sectors.

[8]Below we will demonstrate this for other standard forms of capital adjustment costs.

[9]Note that zero profits are consistent with the evidence that there are no significant pure profits.

sector's production function (5b), we obtain aggregate capital output:

$$x_t = k_{xt}^{\beta_1} l_{xt}^{\beta_2}, \tag{7b}$$

where $\beta_1 \equiv (1+\theta)b$ and $\beta_2 \equiv (1+\theta)(1-b)$.

We end this section with some remarks on the way in which externalities are introduced here. First, as is standard the externality is not taken into account by the firms operating in the capital-producing sector. For this reason, a competitive equilibrium exists and the capital and labor shares in total output of the capital-producing sector are the usual ones: $r_{xt}k_{xt}/k_t = b$ and $w_{xt}l_{xt}/k_t = 1-b$. Second, the upper bound $(1-b)/b$ on θ is imposed to exclude the possibility of endogenous growth and guarantee stationarity. For plausible parameter values it will never be binding. Third, we assume the externality θ to be the same on capital and labor in the capital-producing sector. The main reason is that separate estimates for the strength of the increasing returns do not exist. The results of Harrison and Weder (1999) suggest that imposing this constraint does not a ffect the stability properties in an important way.

3 Equilibrium Dynamics

Definition 1 (Competitive equilibrium) *A competitive equilibrium are positive, initial capital stocks k_{c0} and k_{x0}, prices $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_{ct}, p_{xt}\}_{t=0}^{\infty}$, an allocation $\{l_{ct}, l_{xt}, x_{ct}, x_{xt}, c_t\}_{t=0}^{\infty}$, $\{k_{ct}, k_{xt}\}_{t>0}^{\infty}$, and a path $\{B_t\}_{t=0}^{\infty}$ such that:*

(i) given k_{c0} and k_{x0} and $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_{ct}, p_{xt}\}_{t=0}^{\infty}$, the allocation $\{l_{ct}, l_{xt}, x_{ct}, x_{xt}, c_t\}_{t=0}^{\infty}$, $\{k_{ct}, k_{xt}\}_{t>0}^{\infty}$ solves the problem of the representative household, (1)*;*

(ii) given $\{w_{ct}, r_{ct}\}_{t=0}^{\infty}$, $\{c_t, l_{ct}, k_{ct}\}_{t=0}^{\infty}$ solves the problem of the representative firm of the consumption-producing sector, (3)*;*

(iii) given $\{B_t, p_{xt}, p_{ct}, w_{xt}, r_{xt}\}_{t=0}^{\infty}$, $\{x_{xt}, x_{ct}, l_{xt}, k_{xt}\}_{t=0}^{\infty}$ solves the problem of the representative firm of the capital-producing sector, (5)*;*

(iv) B_t is determined consistently, that is, (7a) *holds.*

Note that since we have two sectors here, market clearing is automatically satisfied when the firms' production constraints are satisfied. Thus, we do not need to specify an economy-wide resource constraint.

The reduced-form equilibrium dynamics must contain the two states of the model, k_{ct} and k_{xt}, and two controls. We use μ_{ct} and μ_{xt} as the controls. The next proposition shows that the reduced-form dynamics can be represented in terms of $k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}$.

Proposition 1 (Reduced-form dynamics) *In equilibrium, all endogenous variables are functions of $k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}$. The reduced-form dynamics of $k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}$ can be represented by:*

$$\dot{k}_{ct} = F_{kc}(k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}) \tag{8a}$$

$$\equiv \left[\frac{(1-b)\mu_{ct}}{\phi}\right]^{\frac{\beta_2}{1-\beta_2}} \left[\phi + (1-\phi)\left(\frac{\mu_{ct}}{\mu_{xt}}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_{xt}^{\frac{\beta_1}{1-\beta_2}} - \delta_c k_{ct},$$

$$\dot{k}_{xt} = F_{kx}(k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}) \tag{8b}$$

$$\equiv \left[\frac{(1-b)\mu_{xt}}{1-\phi}\right]^{\frac{\beta_2}{1-\beta_2}} \left[\phi\left(\frac{\mu_{ct}}{\mu_{xt}}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_{xt}^{\frac{\beta_1}{1-\beta_2}} - \delta_x k_{xt},$$

$$\dot{\mu}_{ct} = F_{\mu c}(k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}) \equiv (\rho + \delta_c)\mu_{ct} - \frac{a}{k_{ct}}, \tag{8c}$$

$$\dot{\mu}_{xt} = F_{\mu x}(k_{ct}, k_{xt}, \mu_{ct}, \mu_{xt}) \equiv (\rho + \delta_x)\mu_{xt} \tag{8d}$$

$$- \frac{b}{1-b}\left[\frac{(1-b)\mu_{xt}}{1-\phi}\right]^{\frac{1}{1-\beta_2}} \left[\phi\left(\frac{\mu_{ct}}{\mu_{xt}}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right]^{\frac{\eta-1}{\eta(1-\beta_2)}} k_{xt}^{\frac{\beta_1+\beta_2-1}{1-\beta_2}}.$$

Proof. See the Appendix A.

Proposition 2 (Existence and uniqueness of steady state) *There is a unique steady state, (k_c, k_x, μ_c, μ_x), in which all variables are constant.*

Proof. See the Appendix B.

To study the dynamic properties of our economy close to the steady state, we linearize the reduced-form dynamics around it. Indicating variables in steady state by dropping the time subscript, the result can be written as:

$$\begin{bmatrix} \dot{k}_{ct} \\ \dot{k}_{xt} \\ \dot{\mu}_{ct} \\ \dot{\mu}_{xt} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{kc}}{\partial k_c} & \frac{\partial F_{kc}}{\partial k_x} & \frac{\partial F_{kc}}{\partial \mu_c} & \frac{\partial F_{kc}}{\partial \mu_x} \\ 0 & \frac{\partial F_{kx}}{\partial k_x} & \frac{\partial F_{kx}}{\partial \mu_c} & \frac{\partial F_{kx}}{\partial \mu_x} \\ \frac{\partial F_{\mu c}}{\partial k_c} & 0 & \frac{\partial F_{\mu c}}{\partial \mu_c} & 0 \\ 0 & \frac{\partial F_{\mu x}}{\partial k_x} & \frac{\partial F_{\mu x}}{\partial \mu_c} & \frac{\partial F_{\mu x}}{\partial \mu_x} \end{bmatrix} \begin{bmatrix} k_{ct} - k_c \\ k_{xt} - k_x \\ \mu_{ct} - \mu_c \\ \mu_{xt} - \mu_x \end{bmatrix}. \tag{9}$$

It is well-known that given that our dynamical system has two states and two controls, the steady state is saddle-path stable if and only if the matrix in (9) has two stable and two unstable roots, it is stable if and only if the matrix in (9) has at least three stable roots, and it is unstable if and only if the matrix in (9) has at least three unstable roots.[10] If the steady state is saddle-path stable then the steady state equilibrium is determinate, that is, given any pair (k_{c0}, k_{x0}) close to (k_c, k_x) there is a unique pair (μ_{c0}, μ_{x0}) such that starting from $(k_{c0}, k_{x0}, \mu_{c0}, \mu_{x0})$ the economy converges to the steady state. If the steady state is stable, then the steady state equilibrium is indeterminate, that is, given any pair of capital stocks close to the steady state pair there exists a continuum of pairs of shadow prices such that the economy converges to the steady state. In this case, sunspots can select the equilibrium. If one assumes that the sunspots follow certain stochastic processes they can then also generate business cycles.

Note that all we can achieve here are local results close to steady state, and so we are not able to study the implications of the transversality condition. Thus, saddle-path stability does not rule out the possibility that there are pairs $(\tilde{\mu}_{c0}, \tilde{\mu}_{x0})$ such that starting from $(k_{c0}, k_{x0}, \tilde{\mu}_{c0}, \tilde{\mu}_{x0})$ the economy evolves along a dynamic path that does not converge to the steady state but nonetheless is consistent with the equilibrium conditions. Since business cycles are typically understood as small deviations from steady state, this possibility is not interesting from the point of view of business cycle research.

4 Benchmark calibration

Except for the increasing returns parameter θ, we use the parameter values of Huffman and Wynne (1999) for our benchmark calibration. Huffman and Wynne calibrate a two-sector model similar to our's to quarterly, postwar, one-digit US data. The difference to our model is that Huffman and Wynne have constant returns in both sectors. As can be checked from the above formulas, the choice of θ does not affect the calibration of any other parameter. Huffman and Wynne count a sector as a capital-producing sector if more than fifty percent of its output is capital goods or intermediate goods, otherwise it is counted as a consumption-producing sector. This gives depreciation rates of $\delta_c = 0.018$ and $\delta_x = 0.020$ and labor shares of $a = 0.41$ and $b = 0.34$. Moreover, they set the rate of time preference to $\rho = 0.01$.

[10] A root of the matrix in (9) is called stable if it has a negative real part and unstable if it has a positive real part.

There is an issue of how appropriate Huffman and Wynne's ad-hoc categorization of one-digit sectors as consumption- or capital-producing sector is. For example, the "more-than-fifty-percent rule" implies that all manufacturing is counted in the capital-producing sector. The reason for using this rather coarse assignment rule is that although the national income accounts report labor, capital, investment, and depreciation by sector, they do not give these statistics by consumption or capital goods produced by each sector. Given that most sectors produce both goods, these quantities somehow need to be allocated between consumption and capital production. A second reason for Huffman and Wynne's categorization is that it is consistent with the existence of capital adjustment costs across, and not within, sectors. This is more in the spirit of our capital adjustment costs function. To get an idea of how robust their calibration is to changes in the categorization, we report the labor shares that result from two alternative ways of proceeding. First, if one disaggregates more and uses two-digit instead of one-digit industries but the same assignment rule, the 1992 benchmark of the NIPAs implies labor shares in consumption and capital of 0.39 and 0.29. Second, one could also compute the labor shares in each sector's outputs of consumption goods and of investment plus intermediate goods and then take the average across sectors.[11] Using the input-output tables of the NIPA, 1987 benchmark, Chari, Kehoe and McGrattan (1997) report shares of 0.39 and 0.31. Since these estimates of share parameters are very close to those of Huffman and Wynne, we have some confidence in using their other parameter values. Nonetheless, we will conduct some sensitivity analysis below.

Huffman and Wynne (1999) calibrate the adjustment costs parameters ϕ and η from data on the real and the nominal investment for the two sectors. To see how this can be done, divide (6c) by (6d) (both with equality) and rearrange to find:

$$\frac{p_{ct}x_{ct}}{p_{xt}x_{xt}} = \frac{\phi}{1-\phi}\left(\frac{x_{ct}}{x_{xt}}\right)^{\eta}. \tag{10}$$

Taking first differences and solving for η, this implies that

$$\eta = \frac{\log \dfrac{p_{ct}x_{ct}}{p_c x_c} - \log \dfrac{p_{xt}x_{xt}}{p_x x_x}}{\log \dfrac{x_{ct}}{x_c} - \log \dfrac{x_{xt}}{x_x}}. \tag{11}$$

Using postwar data on real and nominal sectoral investment, Huffman and Wynne obtain $\eta = 1.1$ and $\eta = 1.3$, depending on the exact procedure.

[11] Note that this procedure does not work for assigning a sector's total investment and depreciation to its production of consumption and capital goods.

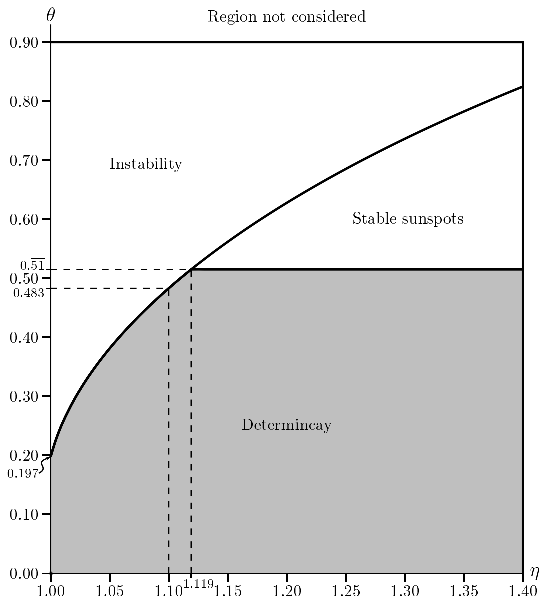
Given η, choosing ϕ is essentially a choice of units and does not affect the stability properties of the system.[12] It is convenient for the derivation of some of the analytical results below to set ϕ such that the relative price of both investment goods becomes one in steady state. Using (B.1c) and imposing $\mu_c = \mu_x$, this gives:

$$\phi = \left\{ 1 + \left[\frac{\rho + \delta_x(1-b)}{b\delta_x} \right]^{\eta - 1} \right\}^{-1}. \qquad (12)$$

The evidence on increasing returns is mixed. However, it is non-controversial that Hall's (1988) initial estimates of $\theta \approx 0.5$ were upwardly biased. More recent empirical studies have instead come up with estimates between constant returns and more mild increasing returns up to 0.3; see e.g. Bartelsman, Caballero and Lyons (1994), Burnside, Eichenbaum and Rebelo (1995), or Basu and Fernald (1997). Another piece of evidence due to Basu and Fernald (1997) is that non-durable manufacturing is estimated to have constant returns, whereas durable manufacturing is found to have increasing returns up to 0.36.

Since it is difficult to draw a sharp line between empirically plausible and implausible values for η and θ, we do not choose a calibration for these two parameters but report the results for a range of different values. More specifically, we restrict attention to parameter pairs $(\eta, \theta) \in (1.000, 0.000) \times (1.400, 0.900)$ and put a grid of size 0.001 on this rectangle. Note that we need to be careful with $\eta = 1.000$ because the above first-order conditions are not defined. We approximate $\eta = 1.000$ by $\eta = 1.000000001$. Note too that given the calibration of $b = 0.34$ all increasing returns of $\theta < 1.942$ are possible without leading to endogenous growth. However, since increasing returns up to 1.942 are not of interest empirically we draw a line at $\theta \leq 0.9$, which allows for much larger values of θ than are typically thought to be realistic.

Figure 1: Local Stability Results for $\delta_c = 0.018$, $\delta_x = 0.020$, $a = 0.41$, $b = 0.34$, $\rho = 0.01$.



5 Stability Properties

5.1 Results for the benchmark calibration

Our findings for the benchmark calibration are reported in Figure 1 and can be summarized as follows. For all moderate values $\eta \in (1.000, 1.119)$ there is a threshold value of increasing returns at which the model's properties change from "determinate" to "unstable". So, for such parameter values the steady state cannot be stable and there is no scope at all for stable sunspots. It should be pointed out that in this case there may exist unstable sunspots. The reason is that when the steady state is unstable the eigenvalues are complex for many choices of η and θ, implying that the equilibrium path can "spiral out off the steady state" and end somewhere other than at the steady state. Since our analysis is local in nature we cannot say anything about this type of unstable sunspots, except that they are not interesting from the point of view of business cycle research.

[12]To see this formally, one needs to substitute the steady state expressions for k_c, k_x, μ_c, and μ_x, (B.4d), (B.4b), (B.4c), and (B.4a), into the matrix of expression (9). One can then show that all elements of a given row depend on ϕ through the same factor. Specifically, these factors are $\phi^{-1/\eta}(1-\phi)^{-\beta_1/[\eta(1-\beta_1)]}$, $(1-\phi)^{-1/[\eta(1-\beta_1)]}$, $\phi^{1/\eta}(1-\phi)^{\beta_1/[\eta(1-\beta_1)]}$, and $(1-\phi)^{1/[\eta(1-\beta_1)]}$, respectively. Since the determinant of a matrix is to be multiplied by a number if all elements of one of its rows are multiplied by that number, the choice of ϕ will not affect the sign of the real parts of the eigenvalues.

For more sizable capital adjustment costs, $\eta \in [1.119, 1.400]$, the properties change from determinacy to stable sunspots at a first threshold of increasing returns equal to $0.\overline{51}$ and from stable sunspots to instability at a second, larger threshold value of θ, which increases in η. Put differently, for this range of capital adjustment costs, stable sunspots are possible but they require degrees of increasing returns that are generally considered implausible. To understand the significance of the number $0.\overline{51}$, note that given that the labor share in the capital-producing sector is $1 - b = 0.66$, the labor demand of the capital-producing sector is upward sloping for $\theta > 0.\overline{51}$. Since the labor supply elasticity is infinite here, an upward sloping labor demand curve would imply the stability of the steady state also in the standard one-sector model [Benhabib and Farmer (1994)].

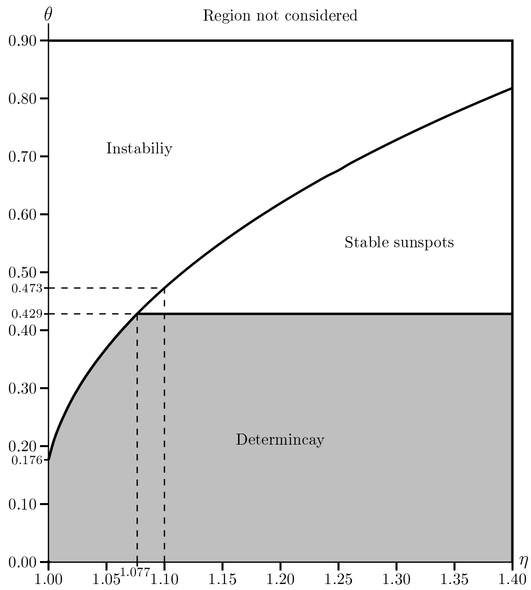
In sum, given our benchmark calibration, we find that (i) a necessary condition for stable sunspots is that the externality is strong enough to make the labor demand curve of the capital-producing sector upward sloping; (ii) a necessary condition for determinacy is that this labor demand curve is downward sloping. So, when we consider adjustment costs, stable sunspots no longer occur through the capital channel but through the labor channel, implying that the stability properties of the two-sector model with capital adjustment is like that of the one-sector model and unlike that of the two-sector model without adjustment costs. Another way of putting this result is that there is a bifurcation at $\eta = 1$. We will see in the next section that this first result is very robust to changes in the parameter values.

A second result or our analysis is that given the benchmark calibration and capital adjustment costs within the range calibrated by Huffman and Wynne, $\eta \in [1.1, 1.3]$, the steady state is determinate if the increasing returns do not exceed 0.483. The range $\theta \in [0, 0.483]$ includes all values of increasing returns that are usually considered reasonable. So, given $\eta \in [1.1, 1.3]$, the properties of the benchmark calibration can be summarized by determinacy for every empirically plausible specification of θ.

We also explore the stability properties of our model for the *annual* parameter values that Benhabib and Farmer (1996) choose: $\rho = 0.05$, $a = b = 0.3$, and $\delta_c = \delta_x = 0.1$.[13] Figure 2 summarizes the results: they are very similar to those of the benchmark calibration. An interesting detail to appreciate about the figure is that for $\eta = 1.000000001$, the stability

[13]It should be mentioned that we do not have available a calibration of η to annual data, and so we will not make any statements about empirically plausible or implausible values of η for this calibration. While simple intuition suggests that calibrating η to annual data should produce smaller values than calibrating it to quarterly data, it is unclear how large this effect is quantitatively.

Figure 2: Local Stability Results for $\rho = 0.05$, $a = b = 0.3$, $\delta_c = \delta_x = 0.1$.



properties change at $\theta = 0.176$ from determinacy to instability. Harrison and Weder (1999, page 13) show that without capital adjustment costs, the stability properties also change at $\theta > 0.176$ to instability. For $\theta < 0.176$, however, they find determinacy only for $\theta \in [0, 0.064]$ and stable sunspots for $\theta \in (0.064, 0.176]$. This detail illustrates how *arbitrarily* small capital adjustment costs shut down the capital channel. [14]

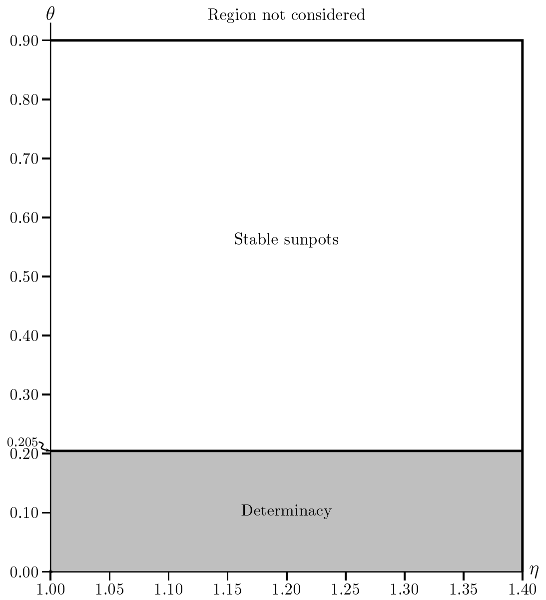
5.2 Sensitivity Analysis

In order to explore the robustness of the results found so far we conduct some sensitivity checks. To begin with, we depart from our benchmark calibration in the following ways: we keep η fixed at 1.000000001 or at 1.1 and vary instead θ and one of the other parameters, i.e. a, b, δ_c, δ_x, or ρ. The results are summarized by Figures C.1 to C.6, which can be found in the Appendix C.

The most important outcome of these sensitivity checks is that our main result is very robust in that stable sunspots always require $\theta > 0.\overline{51}$

[14] In terms of the roots, the following happens. The economy with $\eta = 1.000000001$ and with sector-specific capital has the roots $-0.019117659 \pm 0.485906504i$ and $0.075639397 \pm 1489.090568804i$. The economy with $\eta = 1$ and without sector-specific capital has the roots $-0.019117647 \pm 0.485906476i$. These numerical results suggest that the imaginary part of the two unstable roots converges to infinity whereas the two stable roots converge to the roots of the two-dimensional system. Thus, $\eta = 1$ is a bifurcation point.

Figure 3: Local Stability Results for $\rho = 0.005$, $a = 0.41$, $b = 0.17$, $\delta_c = 0.036$, $\delta_x = 0.01$.



except when we change b. The reason for the qualifier is obvious: changing b changes the value of increasing returns that make the capital-producing sector's labor demand upwards sloping. Our second result that given the benchmark calibration and given $\eta \in [1.1.1.3]$, determinacy results for all empirically plausible values of the externality is not as robust as the previous one. Specifically, if we increase ρ and δ_x and decrease δ_c and b sufficiently, then stable sunspots or instability result. Note that the common effect of all of these changes is that they decrease the amount of steady state capital in the capital-producing sector. Finally, note that the value for a does not affect the stability results. This reflects the general fact that the stability properties of the two-sector model are independent of the properties of the production function in the consumption sector.

Since we have only explored how the stability properties change as we change a, b, δ_c, δ_x, or ρ, it is in principle possible that our results would change if we changed all of them together. To counter this objection, we conduct a final robustness check; if according to the previous results increasing (decreasing) a parameter of our benchmark calibration makes stable sunspots easier to obtain then we double (halve) the value that this parameter takes in our benchmark calibration. This produces a fairly unrealistic set of parameter values: $a = 0.41$, $b = 0.17$, $\delta_c = 0.036$, $\delta_x = 0.01$, and $\rho = 0.005$.[15] The stability results for these parameters are reported in Figure 3. Again, it turns out that stable sunspots require an upward sloping labor demand of the capital-producing sector, which is ensured if $\theta \geq 0.205$. So, our main result is robust also to this rather "crazy" change of parameters.

6 Discussion

6.1 Intuition

The previous section has shown that stable sunspots are much harder to obtain with capital adjustment costs and sector-specific capital than without these features. Here we seek to develop economic intuition for this result. We start by demonstrating that as η goes to one, the steady states of the economies with capital adjustment costs and sector-specific capital converge to the steady state of the economy without these features. This means that the explanation for our results cannot be that capital adjustment costs introduce a discontinuity at the steady state prices and allocation.

Proposition 3 (Existence and uniqueness of steady state for $\eta = 1$) *The economy without capital adjustment costs and sector-specific capital has a unique steady state, in which all variables are constant.*

Proof. See the Appendix D.

Proposition 4 (Convergence of steady states) *Suppose that ϕ is chosen such that $p_c = p_x$ in steady state. As η converges to one from above, the steady states of the economies with capital adjustment costs and sector-specific capital indexed by η converge in the supremum norm to that of the economy without capital adjustment costs and sector-specific capital.*

Proof. See the Appendix E.

Proposition 4 also implies that the sector-specificity of capital does not matter for the equilibrium allocation at the steady state. The intuitive reason is that there is positive depreciation of capital in both sectors, so at the steady state any desired reduction in capital can be achieved by not replacing depreciated capital. Note that Christiano (1995) finds a related result for a discrete time version of the two-sector model: making installed capital sector-specific for one period does not change at all the stability properties of the steady state.

[15] a remains unchanged because its value does not a ffect the stability properties.

We will now argue that the difference in the stability properties of the economies without and with capital adjustment costs comes from the behavior of the relative price between the two capital goods. In particular, when capital adjustment costs are abstracted from, this relative price is constant, see (C.1). In contrast, when capital adjustment costs are considered this relative price changes when the ratio of the two capital goods changes. To see this formally, note that from (A.2g), (A.2h), and (A.3a) it follows that

$$\frac{p_{xt}}{p_{ct}} = \frac{1-\phi}{\phi}\left(\frac{x_{xt}}{x_{ct}}\right)^{\eta-1}. \tag{13}$$

In order to explain why changes in p_{xt}/p_{ct} make a difference, it is useful to explain first how stable sunspots can be consistent with equilibrium for mild sector-specific externalities in the capital-producing sector. This amounts to describing how the capital works in the two-sector model when capital adjustment costs are abstracted from.[16] So, suppose that the economy is on an equilibrium path when a sunspot makes individuals believe in a temporarily higher return on capital. They will then allocate more capital to the capital-producing sector today and reverse that decision tomorrow, which will increase capital output today and decrease it tomorrow. Since there are sector-specific, positive externalities in the capital-producing sector, the relative price of capital in terms of consumption decreases today and increases tomorrow. In other words, the initial change in the allocation of capital produces a capital gain that makes the sunspot self-fulfilling and consistent with equilibrium.

When capital adjustment costs of any size are considered, then the relative price of the two capital goods changes when the two capital stocks change. To see why this precludes capital gains in equilibrium, note first that since installed capital is sector specific the two capital stocks can only be changed by changing the quantities of the new capital goods that are invested in the two sectors. Specifically, a temporary increase in the capital stock of the capital-producing sector requires a temporary increase in x_{xt}/x_{ct}. While there will still be a capital gain on both capital goods (their relative price in terms of consumption decreases today and increases tomorrow), it is no longer optimal to collect it by temporarily holding more capital for the capital-producing sector and less for the consumption-producing sector. The reason is that, as is evident from expression (13), p_{xt}/p_{ct} is higher today than tomorrow, so x_{xt} is relatively *expensive* today and relatively *cheap* tomorrow. Thus, optimizing households will wish to collect the capital gain by holding more x_{ct} and less x_{xt} today, implying

[16] The arguments presented here are close to those of Benhabib and Farmer (1999).

that the initial increase in x_{xt}/x_{ct} cannot be consistent with equilibrium. As a result, the capital channel is not operative when capital adjustment costs are considered. Note that this effect prevails independent of the size of the capital adjustment costs.

The intuitive argument just provided suggests that our main result would go through for all specifications of capital adjustment costs that have the same qualitative implications for the relative price ratio p_{xt}/p_{ct} as the specification used so far. It is easy to show this for the case in which installed capital is sector specific and there are convex costs of changing the capital stocks, an assumption that is widely made in the literature; see e.g. Abel and Blanchard (1983) and Ortigueira and Santos (1997). Expression (5b) would then change to

$$x_{ct}\left[1+m_c\left(\frac{x_{ct}}{k_{ct}}\right)\right]+x_{xt}\left[1+m_x\left(\frac{x_{xt}}{k_{xt}}\right)\right]=B_t k_{xt}^b l_{xt}^{1-b}, \tag{14}$$

where m_c and m_x are increasing, non-negative, and convex functions. [17] It is straightforward to show that the equilibrium relative price of the two capital goods would be:

$$\frac{p_{xt}}{p_{ct}}=\frac{1+m_x\left(\frac{x_{xt}}{k_{xt}}\right)+\frac{x_{xt}}{k_{xt}}m_x'\left(\frac{x_{xt}}{k_{xt}}\right)}{1+m_c\left(\frac{x_{ct}}{k_{ct}}\right)+\frac{x_{ct}}{k_{ct}}m_c'\left(\frac{x_{ct}}{k_{ct}}\right)}. \tag{15}$$

So, given the assumed properties of m_c and m_x and given that the installed capital stocks are the states, a change in x_{xt}/x_{ct} affects p_{xt}/p_{ct} in the same way as above.

6.2 Related literature

Our results are related to several existing papers that explore the implications of capital adjustment costs for the stability properties of dynamic models. To begin with, Kim (1998) and Wen (1998b) study this issue in the standard one-sector neoclassical growth model. More specifically, Kim (1998) demonstrates analytically that convex costs of investment raise

[17] Note that to be consistent with the above model structure we assume that the capital adjustment costs are paid by the firms that produce the new capital goods. It is well known that the results would not change if we assumed that the capital adjustment costs are paid by the owners of capital (here households) or by the firms that actually install the new capital (here the firms in either sectors); see for example the discussion in Kim (1998).

the minimal value of increasing returns for which the steady state becomes stable and Wen (1998b) identifies quantitatively the value of a convex cost of changing investment that ensures the saddle-path stability of the steady state of the calibrated model. Another related paper is Matsuyama (1991), who employs an overlapping generations model with sector-specific externalities and sector-specific labor. One of his results is that it is harder to get equilibrium sunspots the larger are the costs that individuals incur when they change sector.[18] The main differences between these papers and the present one are: (i) there exist calibrated values for our capital adjustment costs, and so our results are not only qualitative but also quantitative in nature; (ii) we do not need a minimum threshold value of capital adjustment costs for our main result to hold, rather it holds for *any* value of capital adjustment costs. Note that the difference between the results for the one- and the two-sector model suggests that the labor channel is much more robust to the introduction of capital adjustment costs than the capital channel.

Our paper is also related to a recent literature that investigates the robustness of multiple equilibria. A first contribution in this spirit is Adsera and Ray (1998). Employing a stripped down-version of Matsuyama (1991), they show that an arbitrarily small departure from the assumption of instantaneous payoffs can introduce a free-riding problem that eliminates multiple equilibria. A second contribution in this spirit is Morris and Shin (1998), who demonstrate that arbitrary small departures from the assumption of common knowledge can be sufficient to eliminate multiple equilibria in models of speculative currency attacks.[19] Here, we have shown here that an arbitrary small departure from the assumption of costless adjustments in capital has the same effect in a two-sector real business model with sector-specific externalities.

7 Conclusion

This paper has explored the conditions under which stable sunspots exist in the standard two-sector real business cycle model with a sector-specific externality in the capital-producing sector. We have found that capital adjustment costs of *any* size preclude stable sunspots for every empirically plausible specification of the model parameters. More specifically, we have shown that when capital adjustment costs of any size are considered,

[18]In this particular model, the costs are captured by the frequency with which individuals can change sector.

[19]Karp (1999) applies this idea to the model of Matsuyama (1991).

a necessary condition for the existence of stable sunspots is an upward-sloping labor demand curve in the capital-producing sector, which in turn requires implausibly strong externalities. This result contrasts sharply with the standard result that when we abstract from capital adjustment costs, stable sunspots occur in the two-sector model for a wide range of plausible parameter values.

The results of this paper imply that the occurrence of stable sunspots in the two-sector real business cycle model with sector-specific externalities is not robust to the introduction of capital adjustment costs. Since we have argued above that this result is unlikely to depend on the particular features of the model version or on the form of the capital adjustment costs specification, we are led to conclude that proponents of stable sunspots will have to demonstrate the plausibility of their point in other versions of the neoclassical growth model. One possibility is opened by the recent work of Wen (1998a), who discovers a third channel through which stable sunspots can occur in real business cycle models, namely, variable capital utilization. Specifically, it turns out that in a one-sector version of the real business cycle model with variable capital utilization, stable sunspots require only mild increasing returns that are empirically defendable. Exploring the robustness of this third channel is an interesting topic, which we leave for future research.

Appendix

A Proposition 1

Proof. (1c), (1d), and (2c)–(2e) imply

$$\dot{k}_{ct} = x_{ct} - \delta_c k_{ct}, \tag{A.1a}$$

$$\dot{k}_{xt} = x_{xt} - \delta_x k_{xt}, \tag{A.1b}$$

$$\dot{\mu}_{ct} = \mu_{ct}\left[\delta_c + \rho - \frac{r_{ct}}{p_{ct}}\right], \tag{A.1c}$$

$$\dot{\mu}_{xt} = \mu_{xt}\left[\delta_x + \rho - \frac{r_{xt}}{p_{xt}}\right]. \tag{A.1d}$$

To represent the economy as a dynamical system in k_{ct}, k_{xt}, μ_{ct}, and μ_{xt}, we need to express all endogenous variables, i.e. (x_{ct}, x_{xt}, l_{ct}, l_{xt}, r_{ct}, r_{xt}, p_{ct}, p_{xt}, w_{ct}, w_{xt}), as functions of these four variables.

We start by deriving the prices as functions of the real variables and the shadow prices. The first useful fact to notice is that (2c), (3b), and (4b) imply that labor in the consumption-producing sector is constant:

$$l_{ct} = 1 - a. \tag{A.2a}$$

This together with (2c) and (3b) gives a reduced-form for consumption and both wages:

$$c_t = w_{ct} = w_{xt} = c(k_{ct}) \equiv (1 - a)^{1-a} k_{ct}^{a}. \tag{A.2b}$$

Moreover, dividing (4a) by (4b) and (6a) by (6b) and using (A.2a), we can express the relative factor prices as functions of the corresponding factors:

$$\frac{r_{ct}}{w_{ct}} = \frac{a}{k_{ct}}, \tag{A.2c}$$

$$\frac{r_{xt}}{w_{xt}} = \frac{b}{1 - b} \frac{l_{xt}}{k_{xt}}. \tag{A.2d}$$

Using (A.2b), these two equations can be solved for the real rates of return on the two capital goods:

$$r_{ct} = r_c(k_{ct}) \equiv a(1 - a)^{1-a} k_{ct}^{a-1}, \tag{A.2e}$$

$$r_{xt} = \frac{(1 - a)^{1-a} b}{1 - b} \frac{l_{xt} k_{ct}^{a}}{k_{xt}}. \tag{A.2f}$$

Note that the second equation is not a reduced form because it still depends on l_{xt}. Finally, combining (2a), (2b), and (A.2b), we obtain reduced form expressions for the prices of the two investment goods:

$$p_{ct} = p_c(k_{ct}, \mu_{ct}) \equiv (1 - a)^{1-a} \mu_{ct} k_{ct}^{a}, \tag{A.2g}$$

$$p_{xt} = p_x(k_{ct}, \mu_{xt}) \equiv (1 - a)^{1-a} \mu_{xt} k_{ct}^{a}. \tag{A.2h}$$

The remaining task is to find labor in the capital-producing sector and the two new capital goods as functions of the two capital stocks and the two shadow prices. The first step is to write the investment ratio as a function of the shadow price ratio. Note that (2a) and (2b) imply that $p_{ct}/p_{xt} = \mu_{ct}/\mu_{xt}$. Dividing (6c) by (6d) (both with equality) and using this, we get:

$$\frac{x_{ct}}{x_{xt}} = \left(\frac{1 - \phi}{\phi} \frac{\mu_{ct}}{\mu_{xt}} \right)^{\frac{1}{\eta - 1}}. \tag{A.3a}$$

Substituting this into (6c) and (6d), both with equality, one arrives at:

$$p_{ct} = \lambda_t \phi \left[\phi + (1-\phi) \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{1-\eta}} \right]^{\frac{1-\eta}{\eta}}, \tag{A.3b}$$

$$p_{xt} = \lambda_t (1-\phi) \left[\phi \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{\eta-1}} + (1-\phi) \right]^{\frac{1-\eta}{\eta}}. \tag{A.3c}$$

Now, from (2a), (2b), and (2c) we know that $\mu_{ct} = p_{ct}/w_{xt}$ and $\mu_{xt} = p_{xt}/w_{xt}$; using this and (7a) after dividing (A.3b) and (A.3c) by (6b), we obtain the reduced form for labor in the capital-producing sector:

$$l_{xt} = l_x(k_{xt}, \mu_{ct}, \mu_{xt}) \equiv \left\{ \frac{(1-b)\mu_{ct}}{\phi} \left[\phi + (1-\phi) \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{1-\eta}} \right]^{\frac{\eta-1}{\eta}} k_{xt}^{\beta_1} \right\}^{\frac{1}{1-\beta_2}} \tag{A.3d}$$

$$= \left\{ \frac{(1-b)\mu_{xt}}{1-\phi} \left[\phi \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{\eta-1}} + (1-\phi) \right]^{\frac{\eta-1}{\eta}} k_{xt}^{\beta_1} \right\}^{\frac{1}{1-\beta_2}}.$$

Next, we derive expressions for each type of investment. Substituting (7a) and (A.3a) into (5b) gives

$$x_{ct} \left[\phi + (1-\phi) \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{1-\eta}} \right]^{\frac{1}{\eta}} = k_{xt}^{\beta_1} l_{xt}^{\beta_2}, \tag{A.3e}$$

$$x_{xt} \left[\phi \left(\frac{\mu_{ct}}{\mu_{xt}} \frac{1-\phi}{\phi} \right)^{\frac{\eta}{\eta-1}} + (1-\phi) \right]^{\frac{1}{\eta}} = k_{xt}^{\beta_1} l_{xt}^{\beta_2}. \tag{A.3f}$$

To eliminate l_{xt} from these expressions, we can use (A.3d). After rearranging, the result is:

$$\begin{aligned} x_{ct} &= x_c(k_{xt}, \mu_{ct}, \mu_{xt}) \\ &\equiv \left[\frac{(1-b)\mu_{ct}}{\phi}\right]^{\frac{\beta_2}{1-\beta_2}} \left[\phi + (1-\phi)\left(\frac{\mu_{ct}}{\mu_{xt}}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{1-\eta}}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_{xt}^{\frac{\beta_1}{1-\beta_2}}, \\ x_{xt} &= x_x(k_{xt}, \mu_{ct}, \mu_{xt}) \\ &\equiv \left[\frac{(1-b)\mu_{xt}}{1-\phi}\right]^{\frac{\beta_2}{1-\beta_2}} \left[\phi\left(\frac{\mu_{ct}}{\mu_{xt}}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_{xt}^{\frac{\beta_1}{1-\beta_2}}. \end{aligned}$$

Substituting the above reduced forms for x_{ct}, x_{xt}, r_{ct}, r_{xt}, p_{ct}, and p_{xt} into (A.1) and rearranging, (8) follows. □

B Proposition 2

Proof. Representing variables in steady state by dropping the time index t, (8b) and (8d) in steady state change to

$$\delta_x k_x^{\frac{1-\beta_1-\beta_2}{1-\beta_2}} = \left[\frac{(1-b)\mu_x}{1-\phi}\right]^{\frac{\beta_2}{1-\beta_2}} \left[\phi\left(\frac{\mu_c}{\mu_x}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}}, \tag{B.1a}$$

$$(\rho+\delta_x) k_x^{\frac{1-\beta_1-\beta_2}{1-\beta_2}} = \frac{b}{(1-b)\mu_x}\left[\frac{(1-b)\mu_x}{1-\phi}\right]^{\frac{1}{1-\beta_2}} \left[\phi\left(\frac{\mu_c}{\mu_x}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right]^{\frac{\eta-1}{\eta(1-\beta_2)}}. \tag{B.1b}$$

Dividing the second equation by the first one leads to

$$\frac{\rho+\delta_x}{\delta_x} = \frac{b}{1-\phi}\left[\phi\left(\frac{\mu_c}{\mu_x}\frac{1-\phi}{\phi}\right)^{\frac{\eta}{\eta-1}} + (1-\phi)\right], \tag{B.1c}$$

which can be solved for the ratio of the shadow value of the capital stocks in the consumption-producing and capital-producing sectors,

$$\frac{\mu_c}{\mu_x} = \left[\frac{\rho+\delta_x(1-b)}{b\delta_x}\right]^{\frac{\eta-1}{\eta}} \left(\frac{1-\phi}{\phi}\right)^{-\frac{1}{\eta}}. \tag{B.1d}$$

Substituting this relationship into equations (8a) and (8b) evaluated at the steady state and using (8c), we obtain

$$\delta_c k_c = [(1-b)\mu_c]^{\frac{\beta_2}{1-\beta_2}} \phi^{-\frac{1}{\eta(1-\beta_2)}} \left[\frac{\rho+\delta_x}{\rho+\delta_x(1-b)}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_x^{\frac{\beta_1}{1-\beta_2}}, \tag{B.2a}$$

$$\delta_x = [(1-b)\mu_x]^{\frac{\beta_2}{1-\beta_2}} (1-\phi)^{-\frac{1}{\eta(1-\beta_2)}} \left[\frac{\rho+\delta_x}{\delta_x b}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_x^{\frac{\beta_1+\beta_2-1}{1-\beta_2}}, \tag{B.2b}$$

$$k_c = \frac{a}{\rho+\delta_c}\frac{1}{\mu_c} \tag{B.2c}$$

To show uniqueness, we derive explicitly k_c, μ_x, μ_c as a function of k_x and then supply an analytical formula for k_x. Dividing (B.2a) by (B.2b) and rearranging yield

$$\frac{\delta_c k_c}{\delta_x} = \left(\frac{\mu_c}{\mu_x}\right)^{\frac{\beta_2}{1-\beta_2}} \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\eta(1-\beta_2)}} \left[\frac{\delta_x b}{\rho+\delta_x(1-b)}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_x.$$

Taking into account (B.2c), this equation can be rewritten as

$$\frac{a\delta_c}{\rho+\delta_c}\frac{1}{\mu_x\delta_x} = \left[\frac{\mu_c}{\mu_x}\left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\eta}}\right]^{\frac{1}{1-\beta_2}} \left[\frac{\delta_x b}{\rho+\delta_x(1-b)}\right]^{\frac{\eta\beta_2-1}{\eta(1-\beta_2)}} k_x \tag{B.3}$$

Using (B.1d), we obtain

$$\mu_x = \frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]} k_x^{-1} \tag{B.4a}$$

Substituting this into equation (B.2b) and rearranging leads to

$$k_x = (1-b)^{\frac{\beta_2}{1-\beta_1}} \delta_x^{\frac{1-\eta}{\eta(1-\beta_2)}} (1-\phi)^{-\frac{1}{\eta(1-\beta_1)}} \cdot \left[\frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]}\right]^{\frac{\beta_2}{1-\beta_1}} \left[\frac{b}{\rho+\delta_x}\right]^{\frac{1-\eta\beta_2}{\eta(1-\beta_1)}}, \tag{B.4b}$$

Combining (B.1d) with (B.4a) yields

$$\mu_c = \left[\frac{\rho+\delta_x(1-b)}{b\delta_x}\right]^{\frac{\eta-1}{\eta}} \left(\frac{1-\phi}{\phi}\right)^{-\frac{1}{\eta}} \frac{\delta_c a}{\rho+\delta_c}\frac{b}{\rho+\delta_x(1-b)} k_x^{-1}, \tag{B.4c}$$

Finally, the previous equation and (B.2c) together imply

$$k_c = \left[\frac{\rho + \delta_x(1-b)}{b\delta_x}\right]^{\frac{1-\eta}{\eta}} \left(\frac{1-\phi}{\phi}\right)^{\frac{1}{\eta}} \frac{\rho + \delta_x(1-b)}{\delta_c b} k_x, \tag{B.4d}$$

which proves that the steady state is unique. □

C Sensitivity Analysis

Figure C.1: Varying ρ while $\delta_c = 0.018$, $\delta_x = 0.020$, $a = 0.41$, $b = 0.34$

$\eta = 1.000000001$ $\eta = 1.1$

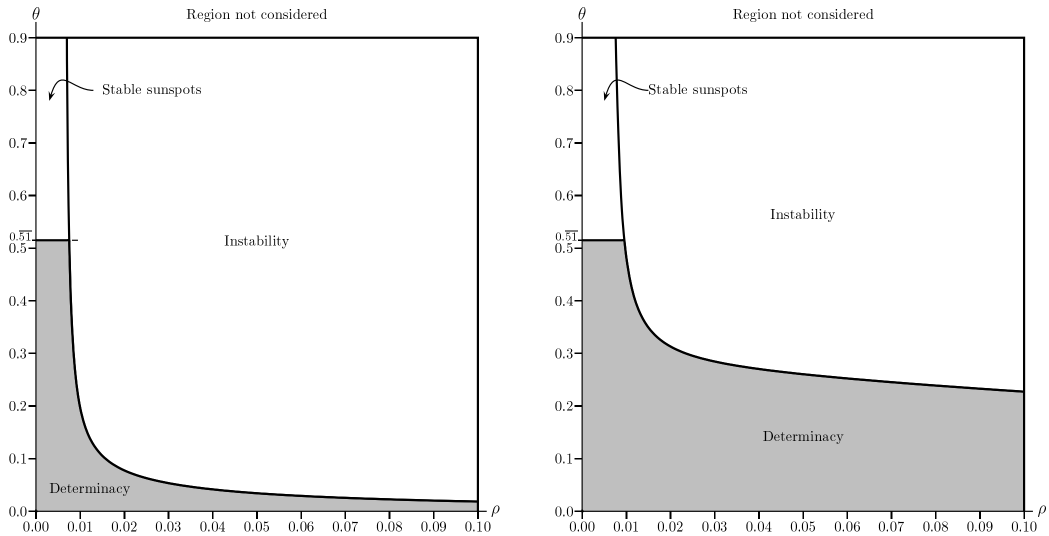


Figure C.2: Varying $\delta_c = \delta_x$ while $a = 0.41$, $b = 0.34$, $\rho = 0.01$

$\eta = 1.000000001$ $\eta = 1.1$

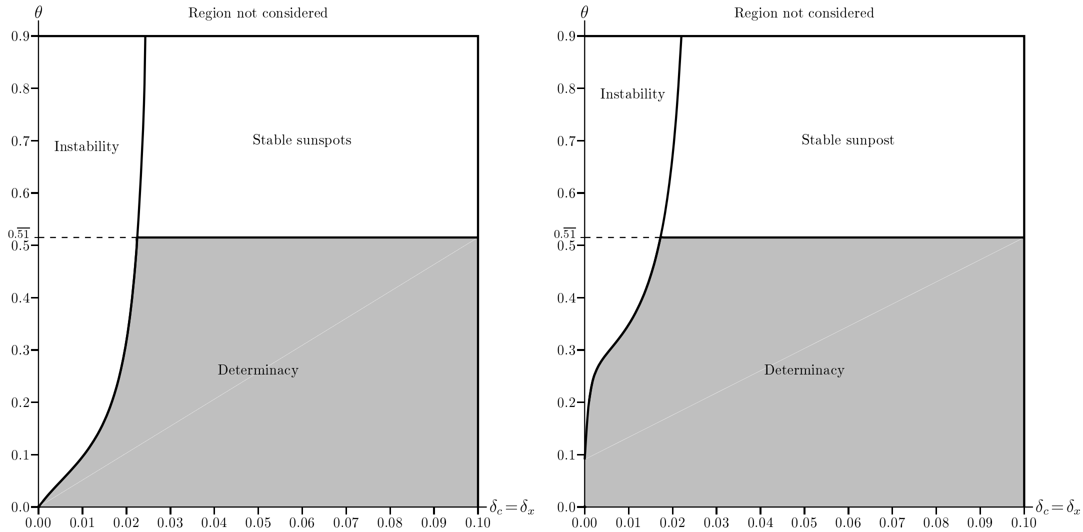


Figure C.3: Varying δ_x while $\delta_c = 0.018$, $a = 0.41$, $b = 0.34$, $\rho = 0.01$

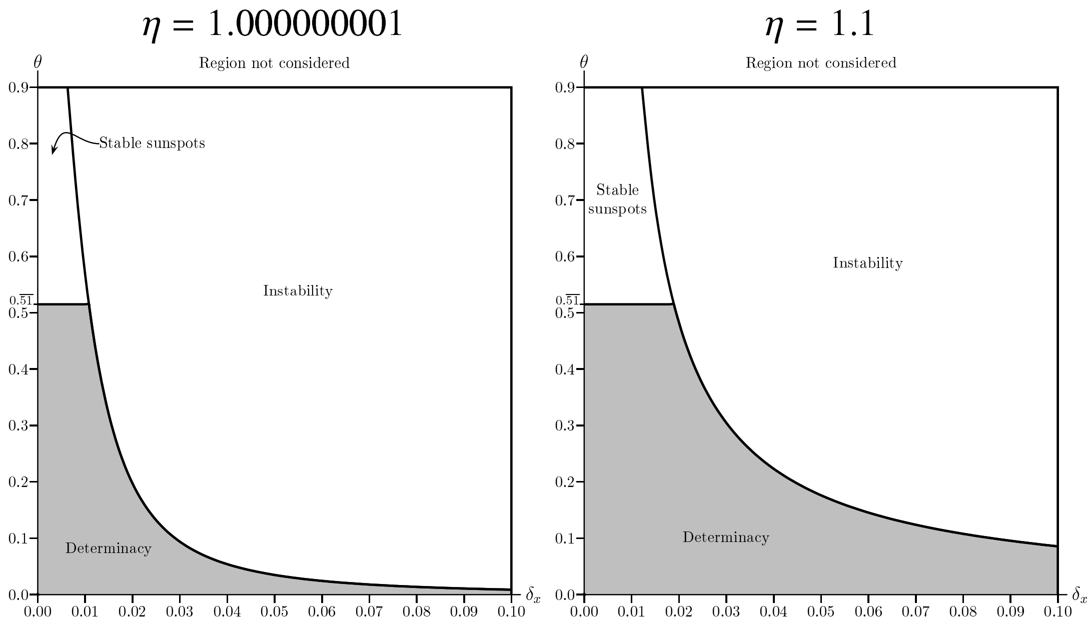


Figure C.4: Varying δ_c while $\delta_x = 0.020$, $a = 0.41$, $b = 0.34$, $\rho = 0.01$

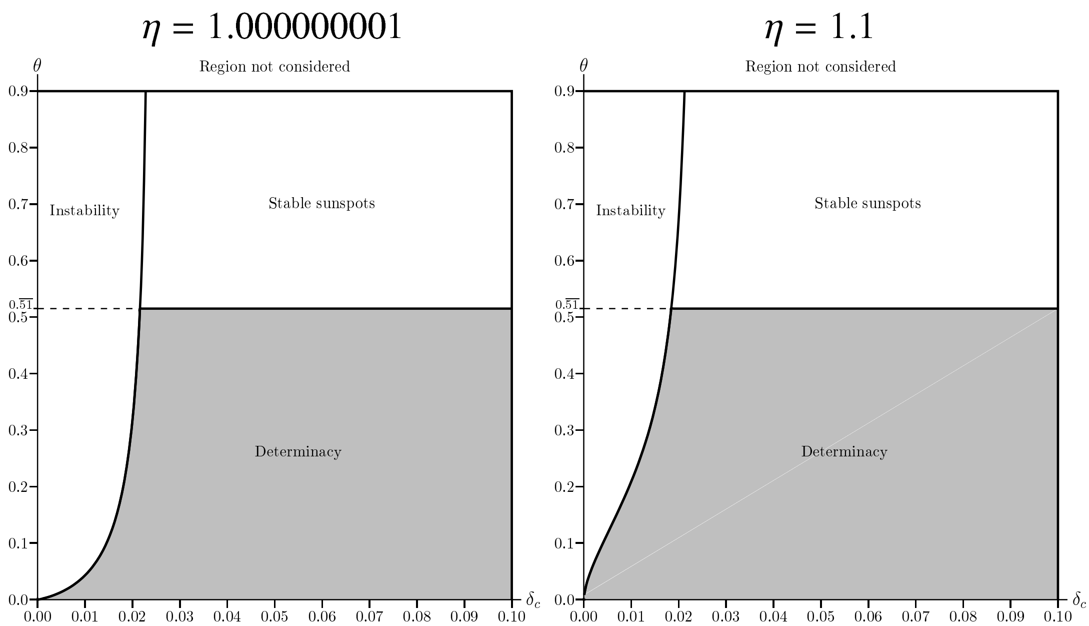


Figure C.5: Varying a while $\delta_c = 0.018$, $\delta_x = 0.020$, $b = 0.34$, $\rho = 0.01$

$\eta = 1.000000001$ $\eta = 1.1$

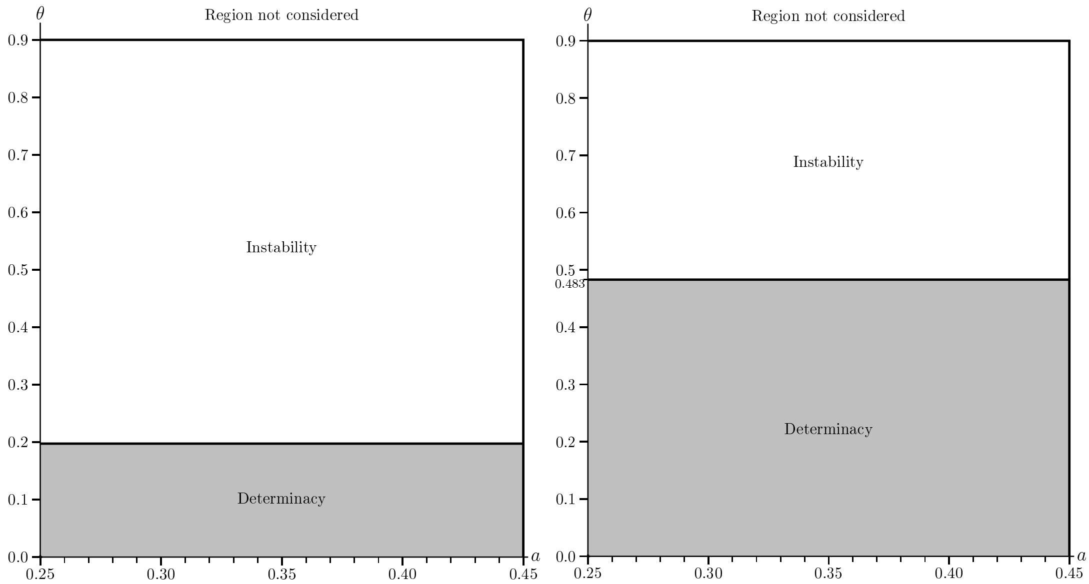
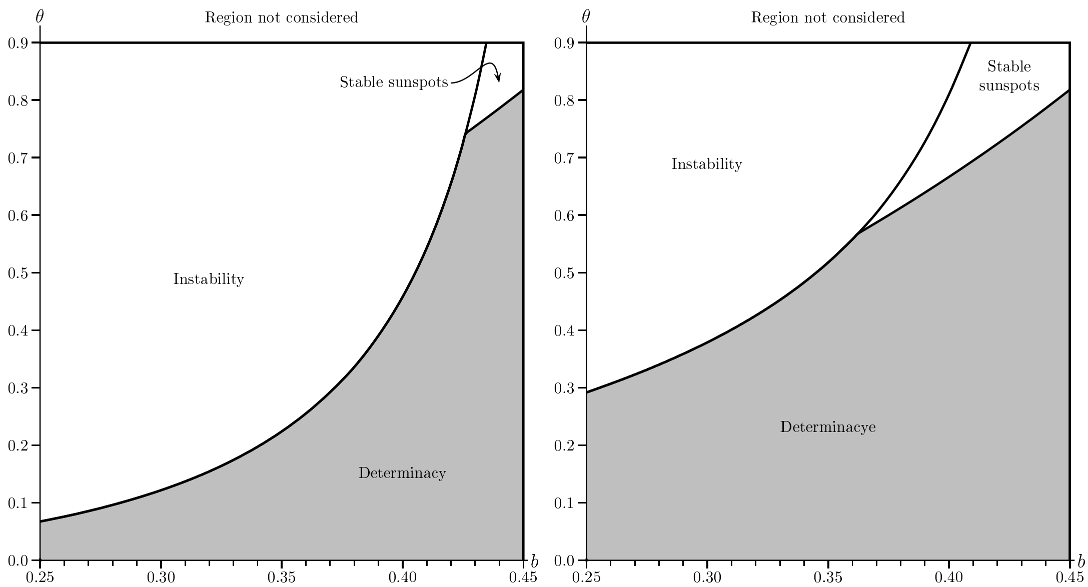


Figure C.6: Varying b while $\delta_c = 0.018$, $\delta_x = 0.020$, $a = 0.41$, $\rho = 0.01$

$\eta = 1.000000001$ $\eta = 1.1$



D Proposition 3

Proof. For $\eta = 1$ and $\phi = 1/2$, the economy studied above reduces to the economy without capital adjustment costs and sector-specific capital. As a result, we do not need to make many modifications to the above first-order conditions. More specifically, since from the households point of view the only novelty is that investment can now be negative, all equations in (2) with the exception of (2d) and (2e) are still appropriate. These two equations hold now with equality. Furthermore, there is no di fference from the point of view of the firms of the consumption-producing sector, so (4) are still the relevant first-order conditions. Third, the problem of the firms of the investment sector needs to be modified: from (6) only (6a) and (6b) are still relevant, whereas (6c) and (6d) change to

$$p_{ct} = p_{xt} = \frac{1}{2}\lambda_t. \tag{C.1}$$

Combining the modified first-order conditions with the steady state condition that all time derivatives are to be zero, it is straightforward to show that given k the steady state $(k_c, k_x, l_c, l_x, x, c, r_c, r_x, w_c, w_x, p, \mu)$ is characterized by the following equations:

$$p = \mu c, \tag{C.2a}$$

$$c = w_c = w_x, \tag{C.2b}$$

$$\rho = \frac{r_c}{p} - \delta_c = \frac{r_x}{p} - \delta_x, \tag{C.2c}$$

$$r_c = \frac{ac}{k_c}, \tag{C.2d}$$

$$r_x = \frac{2pbx}{k_x}, \tag{C.2e}$$

$$w_c = \frac{(1-a)c}{l_c}, \tag{C.2f}$$

$$w_x = \frac{2p(1-b)x}{l_x}, \tag{C.2g}$$

$$c = k_c^a l_c^{1-a}, \tag{C.2h}$$

$$x \equiv \delta_c k_c + \delta_x k_x = k_x^{\beta_1} l_x^{\beta_2}, \tag{C.2i}$$

$$k = k_c + k_x, \tag{C.2j}$$

where $\beta_1 \equiv (1+\theta)b$, $\beta_2 \equiv (1+\theta)(1-b)$, $\mu \equiv \mu_c = \mu_x$, and $p \equiv p_c = p_x$.

To see that there is a unique steady state, we first reduce (C.2) to a system of three equations in k_c, k_x, and μ, which can be solved uniquely. It is then easy to determine the remaining steady state variables. To begin with, equations (C.2b) and (C.2g) imply $l_x = 2(1-b)\mu x$. Plugging this into (C.2i) gives total investment in steady state:

$$x = 2[(1-b)\mu]^{\frac{\beta_2}{1-\beta_2}} k_x^{\frac{\beta_1}{1-\beta_2}}. \tag{C.3}$$

Using this together with (C.2a), equations (C.2c), (C.2d), (C.2e), and (C.2f) can be rewritten as

$$\rho + \delta_c = \frac{a}{\mu k_c}, \tag{C.4a}$$

$$\rho + \delta_x = b[2(1-b)\mu]^{\frac{\beta_2}{1-\beta_2}} k_x^{\frac{\beta_1+\beta_2-1}{1-\beta_2}}, \tag{C.4b}$$

$$\delta_c k_c + \delta_x k_x = [2(1-b)\mu]^{\frac{\beta_2}{1-\beta_2}} k_x^{\frac{\beta_1}{1-\beta_2}}. \tag{C.4c}$$

These equations can explicitly be solved for k_c, k_x, and μ. To see this, substitute (C.4a) into (C.4c) for k_c and divide the result by (C.4b). This gives:

$$\mu = \frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]} k_x^{-1}. \tag{C.5}$$

After substituting this back into (C.4) and solving for k_c, k_x, and μ, the unique steady state turns out to be:

$$k_c = 2^{-\frac{1}{1-\beta_1}} \frac{a(1-b)^{\frac{\beta_2}{1-\beta_1}}}{\rho+\delta_c} \left[\frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]}\right]^{\frac{\beta_1+\beta_2-1}{1-\beta_1}} \left[\frac{b}{\rho+\delta_x}\right]^{\frac{1-\beta_2}{1-\beta_1}}, \tag{C.6a}$$

$$k_x = 2^{-\frac{1}{1-\beta_1}} (1-b)^{\frac{\beta_2}{1-\beta_1}} \left[\frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]}\right]^{\frac{\beta_2}{1-\beta_1}} \left[\frac{b}{\rho+\delta_x}\right]^{\frac{1-\beta_2}{1-\beta_1}}, \tag{C.6b}$$

$$\mu = 2^{\frac{1}{1-\beta_1}} (1-b)^{-\frac{\beta_2}{1-\beta_1}} \left[\frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]}\right]^{\frac{1-\beta_1-\beta_2}{1-\beta_1}} \left[\frac{b}{\rho+\delta_x}\right]^{-\frac{1-\beta_2}{1-\beta_1}}. \tag{C.6c}$$

□

E Proposition 4

Proof. Imposing the steady state conditions and the condition that ϕ is chosen so that $\mu = \mu_{ct} = \mu_{xt}$ in steady state, (B.1d) implies

$$\frac{1-\phi}{\phi} = \left[\frac{\rho+\delta_x(1-b)}{\delta_x b}\right]^{\eta-1}. \tag{D.1}$$

The equations that characterize the steady state, (B.4a)-(B.4d), become

$$k_c = \frac{\rho + \delta_x(1-b)}{\delta_c b} k_x, \tag{D.2a}$$

$$\mu = \frac{\delta_c a}{\rho + \delta_c} \frac{b}{\rho + \delta_x(1-b)} k_x^{-1} \tag{D.2b}$$

$$k_x = (1-b)^{\frac{\beta_2}{1-\beta_1}} \delta_x^{\frac{1-\eta}{\eta(1-\beta_2)}} (1-\phi)^{\frac{1}{\eta(1-\beta_1)}} \cdot \left[\frac{ab\delta_c}{(\rho+\delta_c)[\rho+\delta_x(1-b)]}\right]^{\frac{\beta_2}{1-\beta_1}} \left[\frac{b}{\rho+\delta_x}\right]^{\frac{1-\eta\beta_2}{\eta(1-\beta_1)}}, \tag{D.2c}$$

Since $\lim_{\eta\to 1}\phi = 1/2$, it is straightforward to show that as $\eta \to 1$ each equation converges to the corresponding equation in (C.6). Since there are finitely many steady state variables, this means that convergence is in the supremum norm. □

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