

Learning, noise traders, the volatility and the level of bond spreads*

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Abstract

According to various studies, sovereign bond spreads often deviate from any "sensible" perception of default risk. It is usually attributed to behavioral effects (overreaction) or illiquidity. The former explanation imposes some irrationality or bounded rationality on investors; while the latter usually relies on some informational asymmetry or thin markets.

The paper presents a different source of liquidity risk: in a Diamond–Dybvig type model, where agents face a liquidity risk (becoming *more risk-averse* early consumers), changes in the speed of public learning about default risk may increase bond spreads. This effect operates through a link between future volatility and current levels: increased expected future price volatility (a volatility effect) leads to lower prices today (a level effect). Under reasonable parameter values, accelerated information revelation may increase spreads by 50%.

I also compare the welfare of the issuer and investors under different speeds of learning: revealing information may be good or bad for the issuer (issue prices may increase or decrease), and also for the investors (ex ante utility might be higher or lower).

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1 Introduction

What determines the price of a sovereign bond? It should reflect various risks investors face: default risk (you do not get the payment which was expected), exchange rate risk (the value of your payment changes because of exchange rate movements), interest rate risk (the short-term interest rates move thus the discount rate changes) and liquidity risk (when you sell, prices are low – either for exogenous reasons, or because your sale has a big price effects; or there might be some pure transaction costs). Looking at foreign currency denominated sovereign bond spreads, the exchange rate risk and the interest rate risk should be mostly eliminated: you get the payments in the "right" currency, and the payment is appropriately in excess of the benchmark (dollar) interest rate. Then all is left is default and liquidity risk.

Sovereign bond spreads often deviate too much from what one would think as reasonable default probability perceptions. Based only on default risk considerations, for example, crisis times should have much less effect on long-term than on short-term bond spreads – Broner and Lorenzoni (2000) documents the opposite. The empirical results in Benczur (2001) also suggest that bond spreads reflect many other factors besides pure default risk, illiquidity risk being a major extra risk component (in the form of expected future price volatility). This effect is in some sense also reminiscent of the excess volatility puzzle, documented first by Leroy and Porter (1981), and Shiller (1981): stock prices move too much relative to movements in dividends (a volatility effect), while for bonds, changes in default probability lead to too large movements in spreads (a level and a volatility effect).

One can attribute this to some form of bounded rationality of investors – instead, I want to maintain full rationality and look for a potential explanation within that framework. Then the natural candidate for the extra spread increase is the liquidity of the bond, which may come from many different sources. Maybe all potential buyers are without sufficient resources to buy the bond, so the price will have to drop a lot to attract a buyer. Or the market size is small, therefore each transaction will substantially reduce the price. A nearly mechanical though also canonical form of illiquidity is the presence of transaction costs (brokerage fees, bid-ask spread differentials). Another traditional source of illiquidity is asymmetric information: if you have any private information, then your sale might give a bad signal to the market, thus depress the prices even more than the true increase in

default risk.

These considerations are not new in the literature. For example, Bradley (1991) studies price differences among comparable Eurobond issues: after controlling for maturity, coupon size and default risk (proxied by credit rating, which is not necessarily a perfect measure), bonds from different issuer groups still have different prices, which can be attributed to liquidity. The approach, however, lacks a clear identification of a channel through which liquidity would operate.

Hirtle (1988) finds little *aggregate* liquidity effect in the junk bond markets: following certain aggregate shocks to market-specific liquidity, junk bond returns decrease only for a very short-term. These results are likely to be specific to the particular market and episode though: they are unlikely to hold after the East Asian and Russian crises; also, Broner and Lorenzoni (2000) show that the behavior of Latin American sovereign bonds can be explained by market-specific liquidity shocks.

Amihud and Mendelson (1991) show that T-notes are substantially less liquid than T-bills, which is attributable to a smaller number of potential buyers, and it also leads to higher fees and bid-ask spread differentials for T-notes than for T-bills. These differentials then clearly translate into price differentials, driven by liquidity. Redding (1999) picks up a related point: that paper analyzes the difference between currently issued thus heavily traded, and already existing but similar maturity US Treasury Bonds, and finds evidence for a liquidity premium. In both cases, the source of liquidity is the depth of the market: with many sellers, it is easier to do transactions, which forces fees to be smaller, and increases liquidity in general. However, unless one explicitly models systematic fluctuations in this depth, liquidity will be nearly constant, so spreads should move fully together with changes in default risk.

Instead of any of these particular explanations, I want to use a more general form of illiquidity: investors have a chance that they need to sell the bond before they were initially planning, and for *any* reason, the price may be particularly low at that time. The price may be low because the lack of aggregate resources, informational problems, thin markets or any other explanation – that does not change the overall interpretation that they are selling the bond at the wrong time. The general feature is that investors care not only for the terminal payoff (repayment probability, maybe even its variance), but the way uncertainty is resolved also matters. With many interim steps of the terminal payoff

lottery, pre-maturity prices will fluctuate a lot, and with a chance of early sale (liquidation), investors will value such a bond less.

This approach is somewhat similar to Grossman and Miller (1988): in their model, investors occasionally need to adjust their portfolios. Then they face the choice between selling immediately but maybe not for the most eager buyer, or searching longer but risking a price drop. Some intermediate liquidity providers may then take away some of this risk. It is not fully clear, however, how this framework relates to price volatility: with more volatility, waiting increases the chance of a price drop, but also of a recovery. Still, liquidity is also modeled in terms of potential price movements near times when investors may want to rebalance (or downsize) their portfolios.

In my framework, the reason why investors care for volatile pre-maturity prices is the following: they usually have uncertain investment horizons, so the probability is less than one that they can hold on to the particular asset as long as planned, desired or in some sense optimal. Thus there is also a possibility that they have to liquidate some or all of the investment at earlier stages, so fluctuating pre-maturity prices impose some risk. Therefore, I will try to link higher future (expected) volatility to lower current prices: thus a future volatility effect would lead to a current level effect.

If the source of future price fluctuations is different from a change in the *current* default risk, then this effect should be particularly clear: if something happens today that increases future volatility *without* increasing the current (expected) default probability, then one would see a drop in prices without an increase in perceived risk. The effect would also be present if a change in current default probabilities implies higher future price volatility: current prices would drop more than implied by the change in default risk.

This naturally leads to the conclusion that learning, or more precisely, changes in the speed of information revelation can give us the desired effect. Since sovereign bond defaults are rare, and they are usually driven by the unwillingness, and not just the inability of the country to pay, there is an inherent uncertainty about default. Investors can, however, observe the behavior of countries through time – which might change their beliefs about how bad fundamentals must be to have a default. This also means that investors would get much more new information about the country around crises – then they do see whether the country resisted a storm of a given size or had to bend.

Faster learning, the revelation of more precise information will in general lead to more

volatile prices. This is documented in various empirical papers: for example, Ederington and Lee (1993), and Harvey and Huang (1991) show that announcements lead to increased price volatility of US Treasuries and also in foreign exchange futures markets. Jones, Kaul and Lipson (1994) also find that public information is the major source of short-term volatility in the stock market. Krebs (1999) provides a theoretical model supporting the same argument.

Expecting an increase in the speed of learning, thus anticipating more volatile prices in the near future may cause large expected wealth fluctuations – and if there is a chance that an investor has to liquidate her portfolio exactly in those volatile weeks, that increases the risk of the bond. Even for the same levels of volatility and risk further away in the future, the bond becomes more risky, which then drives the price down, potentially much more than what the increase or uncertainty about default probabilities would imply. This effect might dominate for long-term bonds: with any new twist of a crisis, their default risk changes slightly, leading to higher current and expected price volatility. Increased expected volatility then amplifies the initial level effect of higher risk, leading to a much larger price drop.

The aim of the paper is to establish this channel in a model with rational agents who face a Diamond-Dybvig (1983) type taste shock. Under certain assumptions, I will establish that getting more information before maturity may decrease issue prices, together with (or intuitively: driven by) increased pre-maturity price volatility. This effect can be quite important quantitatively: under reasonable parameter assumptions, the spread may increase by 50% in response to accelerated learning. In this sense, releasing information may hurt the issuer of the bond (getting a lower issue price), but it may also hurt investors (decreasing their welfare). This latter result is similar to Hirshleifer (1971), where a release of some information eliminates the possibility of self-insurance, thus decreases total welfare. My result is driven by the following mechanism. Suppose agents are subject to the same wealth constraints (hence, the same bond price, and they must buy 1 unit per head) but they can self-insure among themselves. For a common price level p , early information revelation is unambiguously good: it enables fully contingent consumption plans. If, however, the price levels are different for the two learning scenarios, the wealth effect (through the budget constraints) and this contingency effect might have opposite signs and the comparison is ambiguous. Moving to the no self-insurance setup, there is a welfare

loss: the joint budget constraint of early and late consumers might be sub-optimally split into two separate constraints. Whether this loss is bigger for early revelation than for late revelation is again ambiguous – so the comparison of no self-insurance welfare levels is ambiguous, fast learning can increase or decrease investor welfare.

In order to have learning at all, I need to have a positive default probability – otherwise no interesting information could be ever released. Let me emphasize that learning is completely mechanical here, it simply refers to the revelation of some information, getting or not some (perfect) public information – there are neither noisy signals, nor a need to estimate probabilities based on past realizations of events. That kind of a more sophisticated learning also has very important effects on pricing behavior, as explored and surveyed in Cassano (1999), among others.

The key ingredient for my results is that early consumers are more risk-averse than late consumers: if late consumers are risk-neutral while early consumers show any small degree of risk-aversion, the result applies. For risk-averse late consumers, there must be a sufficient difference in the risk aversion of the two types of consumers, but for any concave utility function u of late consumers, it is possible to find a set of appropriate utility functions v for early consumers in a way that the results hold.

In reality, this entire learning process is obscured by the very important fact that default is ultimately a deliberate action of the country. This means that countries might try to strategically alter their behavior in order to decrease the market's risk perception. And it is indeed the case that developing countries have mostly refrained from bond defaults ever since the debt crises. In my simple framework, I will not be able to address this issue – I assume that there is a reduced form of the country's behavior, so these strategic elements are already taken into account.

The paper is organized as follows. The next section describes the model, and derives the first order conditions of investor maximization. Section 3 contains the results: first the price and welfare comparison of slow and fast learning, then a more continuous mixture of these two extremes. Finally, Section 4 concludes.

2 The model

2.1 Ingredients

I want to model learning as getting a more precise signal about the default realization before maturity. In one extreme, you do not get any signal, so you do not learn anything new before maturity – late, or slow learning case; the other extreme is a perfect signal, or complete information revelation: you learn before maturity whether repayment will be made – early, or fast learning case.

There is a continuum of agents, who face a potential (unobservable and idiosyncratic) taste shock: with some probability, they become early consumers, who value consumption (or in general: asset returns) at an early period. The model thus must have three periods: an issue period with a fixed (inelastic) supply of the bond (period 0), a pre-maturity period 1, when some investors are hit by idiosyncratic and unobservable taste shocks, plus the uncertainty is potentially resolved; finally, a maturity period, when repayment is realized, and all investors get their returns.

In period 0, investors buy the risky bond at p_0 , and store the rest of their initial wealth (one to one technology). There is a fixed (inelastic) supply of the bond, which gives 1 in period 2 with probability θ , and 0 with probability $1 - \theta$. In period 1, they learn whether they are early consumers (in which case they cash all their assets and eat only in period 1) or not (consume only in period 2). In the case of slow learning, they trade after this was realized. There is a chance θ of becoming an early consumer, but there is no aggregate uncertainty about it (idiosyncratic risk, continuum of agents) – so there will be one price level.

In the case of early learning, agents also learn whether the bond will be paid back or not (same probabilities as ex ante: θ and $1 - \theta$), and then they can trade based on both pieces of information – so there will be two price levels: $p = 1$ if there is repayment, $p = 0$ if no repayment. Finally, in period 2, repayment is realized, and late consumers consume all of their wealth.

The utility of consumers is given by $E[u_1(c_1) + u_2(c_2)]$. For late consumers, $u_1 = 0$, $u_2 = u$; for early consumers (hit by liquidity shock), $u_1 = v$, $u_2 = 0$: I assume that agents are not risk-lovers (either risk-neutral or risk-averse), and that early consumers are at least as risk-averse as late consumers. A special case is when late consumers are risk-neutral,

but early consumers are risk-averse.

2.2 Solving the model – slow learning case

In period 0, all agents are identical, and have a wealth of 2. There is some fixed (positive) quantity of the risky bond offered by the issuer – it must be less than 2, otherwise there would not be enough aggregate resources to buy all the bonds. For simplicity, I assume that the supply of bond is 1. Each agent maximizes her expected utility by buying some amount b_0 of bonds and keeping x_0 in cash. There is an equilibrium price p_0 at the bond market.

In period 1, early consumers sell all their bonds at the equilibrium price p_1 and eat all of their wealth, thus $c_1 = x_0 + p_1 b_0$. Late consumers buy b_1 bonds and keep $x_1 = x_0 + p_1 b_0 - p_1 b_1$ in cash.

In period 2, late consumers eat $c_{2G} = x_1 + b_1$ if bond is repaid (probability π), or only $c_{2B} = x_1$ if no repayment (probability $1 - \pi$).

The expected utility at period 0 is:

$$\begin{aligned} U &= v(c_1) + (1 - \pi) [u(c_{2G}) + (1 - \pi)u(c_{2B})] \\ &= v(x_0 + p_1 b_0) \\ &\quad + (1 - \pi) [u(x_0 + p_1 b_0 - p_1 b_1 + b_1) + (1 - \pi)u(x_0 + p_1 b_0 - p_1 b_1)] \end{aligned}$$

Agents want to maximize (this period 1]3 TDaint is already substituted in) period 0 utility subject to the budget constraint. Let λ be the Lagrange multiplier of the budget constraint, then the first order conditions for x_0 and b_0 are:

$$\begin{aligned} \frac{\partial U}{\partial x_0} &: \\ &= v'(x_0 + p_1 b_0) + (1 - \pi) [u'(x_0 + p_1 b_0 - x_1 - p_1 b_1) \\ &\quad + (1 - \pi)(1 - \pi)u'(x_0 + p_1 b_0 - p_1 b_1)] \end{aligned}$$

$$\frac{\partial}{\partial b_0} : \\ p_0 = p_1 v'(x_0 + p_1 b_0) + (1 - \lambda) p_1 u'(x_0 + p_1 b_0 - x_1 - p_1 b_1) \\ + (1 - \lambda)(1 - \lambda) p_1 u'(x_0 + p_1 b_0 - p_1 b_1)$$

Since the marginal utilities are strictly positive, these two first order conditions imply that in equilibrium, we must have $p_0 = p_1 = p$: The intuition and interpretation of this result is clear: since there is no uncertainty about the period 1 price, it cannot be different from the period 0 price, because then everyone would want to buy infinite amounts of the bond or nobody would be ready to buy at all. For this constant price level, individuals are indifferent between bonds and cash in period 0, since the period 1 returns of the two are equal. So $b_0 = 1$ can be achieved (market clearing condition for bonds at period 0).

Using this, the objective function can be rewritten as

$$U = v(2) + (1 - \lambda) [u(2 - pb_1 + b_1) + (1 - \lambda) u(2 - pb_1)] :$$

This is an unconstrained maximization, and the first order condition becomes

$$\frac{\partial}{\partial b_1} : u'(2 - pb_1 + b_1)(1 - \rho) = (1 - \lambda) p u'(2 - pb_1)$$

In equilibrium, we must have $b_1 = \frac{1}{1-\lambda}$, since there are $1 - \lambda$ agents buying bonds, and the total holding must be equal to the amount of bonds, which is 1. This gives us an equation for $p_{slow}(\lambda)$:

$$u' \left(2 + \frac{1-\rho}{1-\lambda} \right) (1 - \rho) = (1 - \lambda) u' \left(2 - \frac{\rho}{1-\lambda} \right) \rho \\ \frac{1-\rho}{1-\lambda} \frac{1-\rho}{\rho} = \frac{u' \left(2 - \frac{\rho}{1-\lambda} \right)}{u' \left(2 + \frac{1-\rho}{1-\lambda} \right)} \quad (1)$$

Compared to the "no taste shock, no learning" benchmark ($\lambda = 0$), one can see an "excess supply" effect in this equation: the price level is determined through market equilibrium in period 1, when there are only $1 - \lambda$ agents as buyers. If they are risk-averse, the price must fall in order to make them willing to hold that much of the risky asset.

2.3 The fast learning case

At period 0, the equilibrium price is p_0 , and agents choose to hold x_0 in cash and b_0 in bonds. In period 1, if the news were good, then $p_1 = 1$ (the bond becomes a perfect substitute for money, since it is perfectly safe), the wealth of consumers is $x_0 + b_0$. Early consumers thus eat $x_0 + b_0$, late consumers then eat $x_0 + b_0$ in the second period. In period 1, if the news were bad, then the bond becomes worthless, $p_1 = 0$, early consumers eat x_0 , late consumers choose $c_2 = x_0$. So the objective function of investors is:

$$U = (1 - \beta)u(x_0 + b_0) + \beta v(x_0 + b_0) + (1 - \beta)(1 - \beta)u(x_0) + (1 - \beta)\beta v(x_0):$$

Using the budget constraint, I can eliminate x_0 from the objective function: $x_0 = 2 - p_0 b_0$, thus

$$\begin{aligned} U = & (1 - \beta)u(2 - p_0 b_0 + b_0) + \beta v(2 - p_0 b_0 + b_0) \\ & + (1 - \beta)(1 - \beta)u(2 - p_0 b_0) + (1 - \beta)\beta v(2 - p_0 b_0): \end{aligned}$$

First order condition:

$$\begin{aligned} & (1 - \beta)(1 - \beta)u'(2 - p_0 b_0 + b_0) + \beta(1 - \beta)v'(2 - p_0 b_0 + b_0) \\ = & (1 - \beta)(1 - \beta)p_0 u'(2 - p_0 b_0) + (1 - \beta)\beta p_0 v'(2 - p_0 b_0) \end{aligned}$$

In equilibrium, we must have $b_0 = 1$, which gives us a single equation defining $p_{fast}(\beta)$:

$$\begin{aligned} & (1 - \beta)(1 - \beta)u'(3 - \beta) + \beta(1 - \beta)v'(3 - \beta) \\ = & (1 - \beta)(1 - \beta)\beta u'(2 - \beta) + (1 - \beta)\beta v'(2 - \beta) \end{aligned}$$

$$\frac{1 - \beta}{1 - \beta} \frac{1 - \beta}{\beta} = \frac{(1 - \beta)u'(2 - \beta) + \beta v'(2 - \beta)}{(1 - \beta)u'(3 - \beta) + \beta v'(3 - \beta)} \quad (2)$$

Assume that $v(x) = f(u(x))$. Later I will specify what assumptions are necessary about this function f – in general, it should be strictly concave, which makes v more concave than u , so early consumers are more risk-averse than late consumers. Then $v'(x) = f'(u(x))u'(x)$:

Substituting this into (2):

$$\frac{1 - \rho}{1 - \rho} = \frac{u'(2 - \rho)}{u'(3 - \rho)} \underbrace{\frac{(1 - \rho) + f'(u(2 - \rho))}{(1 - \rho) + f'(u(3 - \rho))}}_{\text{Adjustment term } K} \quad (3)$$

Here there is no "excess supply" effect: the argument of the marginal utilities corresponds to period 0, where each agent must end up holding one unit of the bond. There is, however, the wealth fluctuation term K . This reflects the fact that the uncertainty is resolved already at period 1, and some of the period 1 investors (the early consumers) are different from the period 2 investors (late consumers): if f' is not constant, i.e., early consumers have a different risk-aversion from late consumers, then period 0 prices will reflect this difference. With $\rho > 0$ and f' decreasing, K is greater than one, which decreases the price, compared to the "no taste shock, no learning" benchmark ($\rho = 0$).

Intuitively, fast learning makes the risky asset riskless before maturity, so its issue price will be determined by equilibrium at the time of issue – when demand is high (all investors are buyers). However, fast learning also leads to pre-maturity wealth fluctuations, while slow learning keeps wealth constant. At the time of issue, since all investors have a chance to get the taste shock, this makes bonds more risky, which works to lower issue prices for fast learning. Depending on the balance of these two effects, accelerated future learning may increase or decrease current bond prices.

3 Results

3.1 Comparing slow and fast learning

My object of interest is the price and welfare difference between slow and fast learning. If it is possible to have $\rho_{slow} > \rho_{fast}$, then an increase in the speed of information revelation may decrease bond prices, thus increase bond spreads – even without those news being bad in expectations. Similarly, I am interested in welfare: is more information (faster learning) always good for investors or for the issuer?

The first result is about the "no taste shock" benchmark: it is easy to see that the speed of learning will have no effect on prices or welfare. If agents care only about their terminal payoffs, it does not matter how uncertainty is resolved; thus the "no taste shock"

case is identical to the "no taste shock, no learning" benchmark.

Proposition 1 *If $\sigma = 0$, then $\rho_{fast} = \rho_{slow}$.*

Proof. In this case, the two first order conditions (1) and (3) become identical, so period 0 prices are the same. In period 1, there are no trades no matter whether new information is released or not, since everyone is identical. Therefore, the level of utility is also the same in the two cases. ■

This means that without the chance of some investors becoming early consumers ("noise traders"), a bond with earlier information revelation will always sell at the same price than a bond with late information, and investors are unaffected by way uncertainty is resolved. The reason is that both effects ("excess supply" and "wealth fluctuations") are reduced to zero: without early consumers, no investor cares for period 1 wealth fluctuations, and there is no difference between period 0 and period 1 demand for the bond.

In order to establish conditions under which it is possible to have $\rho_{fast} < \rho_{slow}$, I will use the monotonicity of certain terms from the first order conditions. These properties are established in the following lemmas.

Lemma 2 *For any concave function u ; $\frac{u'(2-px)}{u'(2+(1-p)x)}$ is increasing in x :*

Proof. If $x_1 > x_2$, then $2 - px_1 < 2 - px_2$; so $u'(2 - px_1) > u'(2 - px_2)$: Similarly, $u'(2 + (1 - p)x_1) < u'(2 + (1 - p)x_2)$: This means

$$\frac{u'(2 - px_1)}{u'(2 + (1 - p)x_1)} > \frac{u'(2 - px_2)}{u'(2 + (1 - p)x_2)}.$$

■

Lemma 3 *For any concave u with nonincreasing coefficient of absolute risk aversion, $\frac{p}{1-p} \frac{u'(2-p)}{u'(3-p)}$ is increasing in p :*

Proof. The term $\frac{p}{1-p}$ is increasing in p : when p increases, the numerator goes up and the denominator goes down. So all I need is that $\frac{u'(2-p)}{u'(3-p)}$ is increasing. Its derivative is

$$\frac{-u''(2-p)u'(3-p) + u''(3-p)u'(2-p)}{(u'(3-p))^2} = \frac{u'(2-p)}{u'(3-p)} \left(\frac{-u''(2-p)}{u'(2-p)} - \frac{-u''(3-p)}{u'(3-p)} \right);$$

fast

Which is positive if u has no increasing coefficient of absolute risk aversion. Note that this marginal utility ratio term will be decreasing if the coefficient of absolute risk aversion is increasing – but the $\frac{p}{1-p}$ might be still increasing enough to make the product also increasing. So the statement is true for a moderately increasing absolute risk aversion case as well. ■

Proposition 4 *It is possible to have $p_{fast}(\cdot) < p_{slow}(\cdot)$. In particular, if late consumers are risk-neutral ($u(x) = x$) and $\lambda > 0$, then accelerated learning decreases period 0 bond prices. Moreover, for any concave u , v has to be "sufficiently more concave" than u in order to have $p_{fast} < p_{slow}$. Having $u = v$ (standard Diamond-Dybvig case) is not sufficient to generate this effect.*

As the probability of a liquidity shock becomes positive, early information revelation (fast learning) might cause the period 0 price to drop – the effect of period 1 wealth fluctuations on early consumers becomes dominant and makes the bond less desirable. In the risk-neutral late consumer case, the "excess supply" channel is simply reduced to zero: with risk-neutral buyers at period 1, demand for the bond is flat.

In general, the wealth fluctuations effect is large if early consumers are sufficiently hurt by wealth fluctuations, so if v is concave enough. The "excess supply" effect depends on how much risky asset each late consumer must end up buying ($\frac{1}{\lambda}$) and how much premium they require to compensate for the riskiness of the bond (the concavity of the u). Thus, in order to have the wealth fluctuations effect dominate the excess supply effect, the adjustment term K in (3) has to be large, so f' must be sufficiently decreasing, which means that v must be sufficiently more concave than u .

Proof. When $u(x) = x$, the argument is straightforward: in the late revelation case, the period 1 price is determined by risk neutral traders, so it is equal to the expected value α ; and period 0 prices must be equal to period 1 prices. In the early revelation case, period 0 trade involves risk-averse traders (the component v of their utility function), so the bond must be traded below its expected value. More formally: when u' is constant, (1) implies

$$\frac{\alpha}{1-\alpha} = \frac{p_{slow}}{1-p_{slow}}: \text{ If } \lambda > 0 \text{ then } p_{fast} < p_{slow}$$

If $u = v$ (so f is simply linear), then the adjustment term in (3) is 1. So

$$\begin{aligned} \frac{1 - \rho_{slow}}{1 - \rho_{fast}} &= \frac{\rho_{slow}}{1 - \rho_{slow}} \frac{u' \left(2 - \frac{\rho_{slow}}{1-\lambda} \right)}{u' \left(2 - \frac{1-\rho_{slow}}{1-\lambda} \right)} \\ &= \frac{\rho_{fast}}{1 - \rho_{fast}} \frac{u' (2 - \rho_{fast})}{u' (3 - \rho_{fast})} \end{aligned}$$

According to Lemma 2, since $\lambda > 0$;

$$\frac{u' (2 - \rho)}{u' (3 - \rho)} < \frac{u' \left(2 - \frac{\rho}{1-\lambda} \right)}{u' \left(2 + \frac{1-\rho}{1-\lambda} \right)}$$

so

$$\frac{u' (2 - \rho_{slow})}{u' (3 - \rho_{slow})} \frac{\rho_{slow}}{1 - \rho_{slow}} < \frac{u' (2 - \rho_{fast})}{u' (3 - \rho_{fast})} \frac{\rho_{fast}}{1 - \rho_{fast}}.$$

From Lemma 3, $g(\rho) = \frac{u'(2-p)}{u'(3-p)} \frac{p}{1-p}$ is increasing, which shows that $\rho_{fast} > \rho_{slow}$ holds.

Consider the case when f is concave, then the adjustment term of (3) is greater than 1. The same argument as for $u = v$ gives us that $g(\rho_{slow}) < g(\rho_{fast})K$ for some $K > 1$: If $\rho_{slow} > \rho_{fast}$ holds, then we also have $g(\rho_{slow}) > g(\rho_{fast})$; using Lemma 3 again. So the adjustment term $K = \frac{(1-\lambda)+\lambda f'(u(2-p))}{(1-\lambda)+\lambda f'(u(3-p))}$ must be large enough to enable $\rho_{slow} > \rho_{fast}$. ■

Notice that the variance of period 1 prices is zero in the slow learning case, and it is positive in the fast learning case – so a bond with the same default probability but higher interim price volatility will have a lower price than with the same default probability but smaller interim price volatility. However, this is not always the case – for other parameter values, the higher volatility bond would have a higher price. Still, this is a potential explanation of my empirical findings from the first chapter – without any correlation with the rest of the market, under certain conditions, a bond with more volatile future prices is less attractive today.

The next example shows that the risk-neutrality of late consumers is not necessary for the previous result. Moreover, for any u , one can find an appropriate v which leads to $\rho_{slow} > \rho_{fast}$ (by choosing an appropriately decreasing function f').

Example 5 *For some concave u , it is possible to have $\rho_{slow} > \rho_{fast}$:*

Choose $u = \frac{1}{\sigma} x^\sigma$, $v = \frac{1}{\beta} x^\beta$. For $\lambda = 0.5$; $\rho = 0.1$; $\rho_{fast} = 0.9$; $\rho_{slow} = 0.1$, solving numerically the first order conditions (1) and (2), one gets $\rho_{slow} = 0.4858\dots$; $\rho_{fast} = 0.4811\dots$, so $\rho_{fast} < \rho_{slow}$. For $\lambda = 0.95$; $\rho = -3$, $\rho = 0.5$, $\rho_{fast} = 0.95$, we have an even bigger difference: $\rho_{fast} = 0.9180\dots < \rho_{slow} = 0.9426\dots$. In this second example, the spread has increased by near 3 percentage points in response to faster learning – in other words, it has increased by nearly 50%, and faster learning is "responsible" for almost one third of the total spread.

Proposition 6 *For any concave u and $\lambda > 0$; it is possible to have v such that for any $1 > \rho > \rho_{fast}$, $\rho_{slow}(\rho) > \rho_{fast}(\rho)$ holds.*

Proof. First I show that $\rho_{slow}(\rho)$ is decreasing. When ρ increases, the right hand side of the first order condition (1) increases (using lemma 2). In order to restore equality, $\frac{\rho}{1-\rho} \frac{u'(2-\frac{\rho}{1-\lambda})}{u'(2+\frac{1-\rho}{1-\lambda})}$ must decrease. A straightforward modification of lemma 3 shows that this implies a decrease in ρ ; so ρ_{slow} is really decreasing.

So there is some declining function $\rho_{slow}(\rho)$. Now specify an arbitrary but also declining "target" path for ρ_{fast} , always being below ρ_{slow} (with the exception of 0 and 1 – in 0, the two functions must coincide; in 1, ρ_{slow} is undefined since nobody is willing to hold any bonds if there is nobody to buy them in period one). This implies a target path for the adjustment term K in (3). Since ρ_{fast} is decreasing, then $\frac{\rho_{fast}}{1-\rho_{fast}} \frac{u'(2-\rho_{fast})}{u'(3-\rho_{fast})}$ is also decreasing, so K must be increasing.

Now I will construct a concave function f ($f' > 0$ but decreasing) that will achieve the target path of K – so $v = f \circ u$ will implement the target path of ρ_{fast} . I need to define f consistently in the intervals $[u(2 - \rho_{fast}(0)); u(2)]$ and $[u(3 - \rho_{fast}(0)); u(3)]$: for any ρ , I have a condition on f' at $2 - \rho_{fast}(\rho)$ and $3 - \rho_{fast}(\rho)$, plus f' has to be decreasing. Choose a value for $\rho = 0$, and then a decreasing path in the first interval – from the monotonicity of ρ_{fast} , this can be done in an arbitrary fashion. All I need to ensure is that $f'(u(2)) > f'(u(3 - \rho_{fast}(0)))$ holds. Later I will restrict the choice of f' in the first interval to imply monotonicity in the second interval.

Let $f'(u(2 - \rho(\rho))) = g(\rho)$ and $h(\rho) = f'(u(3 - \rho(\rho)))$, as implied by the target path of K . It is already ensured that f' is decreasing in $[u(2 - \rho_{fast}(0)); u(2)]$, and its value in $3 - \rho_{fast}(0)$ is even smaller. So if I show that $h(\rho)$ is decreasing, then f' is decreasing,

and the proof is done. Recall the definition of K :

$$K(\theta) = \frac{1 - \theta + g(\theta)}{1 - \theta + h(\theta)},$$

so

$$h(\theta) = \frac{(1 - \theta)(1 - K(\theta)) + g(\theta)}{K(\theta)}$$

and its derivative is

$$A^2 [(-1 + K - (1 - \theta)K' + g + g')K - ((1 - \theta)(1 - K) + g)(K + K')]$$

for some constant A . Expanding the bracket term:

$$H(\theta) = A^2 (\theta^2 K g' - K - K' + K^2 + \theta^2 K' - \theta^2 g K'):$$

Here $\theta^2 K g' < 0$, $(\theta^2 - \theta)K' < 0$, so I need to ensure that $K^2 - K - \theta^2 g K'$ is not too positive. But I can multiply g with any positive constant, which will still maintain the monotonicity of f' in the first interval and the starting point of the second interval. Given that $K^2 - K$ has a maximum in $[\bar{\theta}; 1]$; and $\theta^2 K'$ has a (strictly positive) minimum, I can choose g in a way that $H' < 0$ holds. ■

So far, I have shown that if agents face a chance of future taste shocks (becoming early consumers, or "noise traders") and they are aware of this possibility, then an increase in the speed of information revelation decrease the current price of a bond. Its mechanism was through an increase in period 1 (pre-maturity) wealth fluctuations, which, if agents might have to consume in that period due to the taste shock, makes the bond less attractive ex ante. In a more general interpretation, it means that the way uncertainty is resolved will affect the price of a risky asset.

Notice that the issue price is negatively related to the welfare of the *issuer*: for the same expected repayment, the issuer gets smaller funds. So it might be in the interest of the issuer to try to restrict information about its future repayment behavior. Let me now turn to the welfare of investors: are they better-off with faster learning? I will show that the welfare loss from period 1 wealth fluctuations might cause their period 1 expected

utility level to decrease; but it is not necessarily the case.

Proposition 7 $U_{late}(\cdot) < U_{early}(\cdot)$ and $U_{late}(\cdot) > U_{early}(\cdot)$ are both possible. Further, it is possible to have any combinations of utility and price rankings; but in the risk-neutral case ($u(x) = x$), we always have $U_{late}(\cdot) < U_{early}(\cdot)$.

So fast learning might increase or decrease the welfare of consumers. Further, it is possible that fast learning increases the price and decreases utility (the issuer is happy to give the information, but the market would prefer not to listen); it might increase both the price and utility (everyone is happy with the information); $p_{fast} < p_{slow}$ and $U_{early} > U_{late}$: in this case, investors would be willing to incur some cost for fast learning, but that would decrease the price. This means that the issuer of the bond would lose – it gets a smaller amount for the same expected repayment. In this case, the issuer may even try to limit the speed of revealing information – a current piece of news describes a similar event, though the explanation is not necessarily restricted to the effect I am exploring: China restricted posting news to websites without government approval (Financial Times, November 7 and 8, 2000). Finally, faster learning might decrease the price and the level of utility, making it undesirable for both parties.

Proof. Again, choose $u(x) = \frac{1}{\sigma} x^\sigma$; $v(x) = \frac{1}{\beta} x^\beta$, $\sigma = 0.5$: Table 1 gives four different combinations of β , σ and ρ , for which the relative ranking of utilities and prices shows all possible variations.

Table 1: Utility and price comparison

			p_{fast}		p_{slow}		U_{early}		U_{late}
0.1	0.9	0.1	0.4811::	<	0.4858::		2.9463::	>	2.9445::
0.1	0.7	0.1	0.4574::	<	0.4583::		3.1711::	<	3.1774::
0.1	0.4	0.1	0.4238::	>	0.4190::		4.0650::	<	4.0671::
0.9	-1	-1.01	0.2844::	>	0.0222::		-0.468::	>	-0.474::

For the risk-neutral late consumers case, it is clear that $p_{slow} = 2 > p_{fast}$: As for the utility levels,

$$U_{late} = v(2) + (1 - \rho) \cdot 2$$

and

$$\begin{aligned}
 u_{early} &= (1 - \rho)(2 - \rho) + \rho v(3 - \rho) + (1 - \rho)(1 - \rho)(2 - \rho) + (1 - \rho) \rho v(2 - \rho) \\
 &= (1 - \rho) \cdot 2 + (1 - \rho)(\rho - \rho) + \rho v(3 - \rho) + (1 - \rho) \rho v(2 - \rho) :
 \end{aligned}$$

The difference of these two expressions is

$$u_{late} - u_{early} = -(1 - \rho)(\rho - \rho) + [\rho v(2) - \rho v(3 - \rho) - (1 - \rho) \rho v(2 - \rho)] : \quad (4)$$

The first order condition for ρ_{fast} is

$$\frac{1 - \rho}{1 - \rho} \frac{1 - \rho}{\rho} = \frac{(1 - \rho) + \rho v'(2 - \rho)}{(1 - \rho) + \rho v'(3 - \rho)} = \frac{(1 - \rho) + c_1}{(1 - \rho) + c_2} ,$$

which yields

$$\rho = \frac{-c_1 + c_2}{1 - \rho + c_1 - c_1 + c_2} : \quad (5)$$

Note that this is not a "real" closed form of the solution, since c_1 and c_2 also depend on ρ , but this will be sufficient. Using the notations from Figure 1, $v(2) = v(2 - \rho) + \overrightarrow{BC} + \overrightarrow{CF} =$

Figure 1: Utility comparison for the risk-neutral case

$v(2 - \rho) + \rho c_1 + \overrightarrow{CG}$, $v(3 - \rho) = v(2 - \rho) + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DF} = v(2 - \rho) + \rho c_1 + \overrightarrow{CD} + (1 - \rho)c_2$, since the slope of AC is c_1 , and the slope of DE is c_2 . Substituting these into (4), one gets

$$\begin{aligned} u_{late} - u_{early} &= - + + \rho - \rho + \left(\rho c_1 + \overrightarrow{CG} - \rho c_1 - \overrightarrow{CD} - c_2 + \rho c_2 \right) \\ &= \underbrace{\rho (c_1 - c_1 + c_2 + 1 -) + - - c_2}_{\text{cancels}} + (\overrightarrow{CG} - \overrightarrow{CD}): \end{aligned}$$

Plugging in the expression for ρ from (5), the underbracketed term cancels, so

$$\begin{aligned} u_{late} - u_{early} &= (\overrightarrow{CG} - \overrightarrow{CD}) \\ &= (\overrightarrow{CG} - \overrightarrow{CG} - \overrightarrow{GD}) = ((1 -)\overrightarrow{CG} - \overrightarrow{GD}): \end{aligned}$$

Now I need to use the concavity of u : since G is $v(2)$, so it must be below the tangent from $2 - \rho$. It implies that $\overrightarrow{CG} < 0$: Moreover, G is also below the tangent from $3 - \rho$, which means $\overrightarrow{GD} > 0$: Putting these two together shows that $u_{late} - u_{early} < 0$: ■

Let me give some interpretation to this result. Given a certain price level ρ_0 , I define a constrained efficient allocation: it corresponds to the case when agents can write contracts based on the taste shock, but they are still subject to the same bond prices and the same uncertainty about terminal repayment. This can be thought as a coalition of agents investing their total wealth, and agreeing on an allocation rule between late and early consumers (self-insurance). I am imposing the same investment decision on this coalition (buying one unit of the bond, at the same price ρ_0) – I want to focus only on the allocation of consumption within the coalition, without any changes in its total wealth. Let c_{ED} be the level of consumption of early consumers if the bond issuer defaults, c_{ER} under repayment; c_{LD} and c_{LR} are the consumption levels for late consumers. The optimal arrangement for early information revelation then maximizes

$$v(c_{ER}) + (1 -)v(c_{ED}) + (1 -)(u(c_{LR}) + (1 -)u(c_{LD}))$$

subject to the budget constraints:

$$\begin{aligned} c_{ER} + (1 -)c_{LR} &= 3 - \rho_0 \\ c_{ED} + (1 -)c_{LD} &= 2 - \rho_0 \end{aligned}$$

Under slow information revelation, we have a similar optimization problem, but with a potentially different price level p_0 and the extra constraint $c_{ED} = c_{ER}$, since the repayment uncertainty is not yet resolved when early consumers need to get their consumption.

Moving from this second best allocation to the one without self-insurance (which is the market outcome), the investor choice problem in fact means maximizing the same objective function, subject to separated budget constraints: the total endowment $3 - p_0$ is forced to be split evenly in the fast learning case ($c_{ER} = c_{LR}$, $c_{ED} = c_{LD}$); and in another fixed way in the slow learning case ($c_{ED} = c_{ER} = 2$, $c_{LR} = \frac{3-p_0-2\lambda}{1-\lambda} = 2 + \frac{1-p_0}{1-\lambda}$, $c_{LD} = 2 - \frac{p_0}{1-\lambda}$).

When one compares the second best welfare levels of the different information revelation scenarios, we have two effects in general: a "wealth" effect, coming from a potentially different p_0 , and the additional constraint $c_{ED} = c_{ER}$ (not allowing fully contingent consumption plans). If $p_{fast} < p_{slow}$, this comparison is unambiguous: with fast learning, you have higher wealth and less constraints. If $p_{fast} > p_{slow}$, the two effects work against each other, so the comparison is ambiguous.

Moving to the market outcomes, one cannot determine which scenario implies a larger welfare loss relative to second best – so we can have any combinations of price and welfare comparisons. Note that if $p_{fast} < p_{slow}$, the second best level is higher for the fast revelation case, so to have this order reversed in the market outcome, the distortion due to the lack of self-insurance must be large. With risk-neutral late consumers, we always have $p_{fast} < p_{slow}$, so in a second best world, it is always desirable to get new information, and the distortion can never be large enough to reverse this conclusion. As the examples show, with risk-averse late consumers, it is possible that fast learning is desirable under the second best allocation of consumption ($p_{fast} < p_{slow}$), but without self-insurance, $u_{early} < u_{late}$. This is also the case when both the issuer and the investors are worse-off.

3.2 Convexification of the speed of learning

In the previous subsection, I have compared two extremes: one case when there was no new information revealed at period 1, the other when there was full revelation in period 1. This comparison also gave me a two-point relationship between issue (period 0) prices and interim (period 1) price volatility. Now I want to do a more convex version of the same comparison. One potential way would be to assume that even slow learning involves getting some new information, and fast learning implies much more but not perfect revelation.

Under this modification, the same two effects would be present as before: the "excess supply" channel and the "wealth fluctuations" channel, yielding similar, though maybe less clear results (one may need stronger assumptions on parameters, utility functions etc.).

In some sense, fast learning would be a mean-preserving spread relative to slow learning – in the risk-neutral late consumer case, it is definitely true for period 1 prices: Slow learning means that the ex ante binary outcome (repayment or default) is refined into some interim positions, each with a binary outcome but different probabilities. Period one prices are then equal to these probabilities. Under fast learning, we get the same refinement as with slow learning, and some extra: each node is further decomposed into binary nodes. So at each node, period 1 prices become a lottery with the same expected value as the slow learning period 1 price at that node, which is a mean-preserving spread.

I will instead mix the fast and the slow learning cases in a probabilistic sense: assume that there is a chance α that in period 1, investors learn the repayment behavior (fast learning case), and $1 - \alpha$ that there is no news revelation. This in general defines $\rho_0(\alpha)$ and $V_0[\rho_1](\alpha)$. I will give conditions under which $V'(\alpha) > 0$ and $\rho'(\alpha) < 0$ – so with a bigger chance of getting new information, prices actually go down, together with increased future price volatility.

This can be an explanation for crisis periods giving much bigger movements in long-term bond prices than any "sensible" estimates of changes in default probability: in crises, you expect to learn quite much about the country's ability and willingness to repay, so this mechanism would increase near-future price volatility and decrease current prices, even without any change in α (default risk) or σ ("noise trader" risk). We would see, however, no extra effect on trade volumes – noise traders sell inelastically, so the quantity traded must remain the same if α remained the same. Once α also fluctuates, one can get quantity effects as well.

Proposition 8 $V'(0) > 0$; so for small values of α , the variance term is increasing. If $u(x) = x$, then $V' > 0$ for all α . If $u(x) = -\frac{1}{\theta}e^{-\theta x}$ (CARA), then under some conditions on α , σ , and β , V is increasing.

Proof. The period 1 price level is 1 with probability α , 0 with probability $(1 - \alpha)$,

and ρ_1 with probability $(1 - \lambda)$. So its variance is

$$\begin{aligned} V(\rho_1) &= \lambda \rho_1^2 + (1 - \lambda) \rho_1^2 - (\lambda \rho_1 + (1 - \lambda) \rho_1)^2 \\ &= (\lambda - \lambda^2) + (\lambda - \lambda^2) (\rho_1 - \rho_1)^2: \end{aligned}$$

In general, ρ_1 is coming from the period-one, slow learning case maximization problem and market clearing. The objective function is

$$u(x_1 + b_1) + (1 - \lambda) u(x_1) = u(3 - \rho_0 + b - \rho_1 b) + (1 - \lambda) u(3 - \rho_0 - \rho_1 b):$$

and the first order condition is

$$u'(3 - \rho_0 + b(1 - \rho_1))(1 - \rho_1) = (1 - \lambda) \rho_1 u'(3 - \rho_0 - b\rho_1):$$

Market clearing implies that $b = \frac{1}{1-\lambda}$, so ρ_1 is the solution of

$$u' \left(3 - \rho_0 + \frac{1 - \rho_1}{1 - \lambda} \right) (1 - \rho_1) = (1 - \lambda) \rho_1 u' \left(3 - \rho_0 - \frac{\rho_1}{1 - \lambda} \right): \quad (6)$$

It is clear that ρ_1 in general also depends on ρ_0 : though ρ_0 itself does not appear in the equation, but ρ_0 depends on ρ_1 . So there is some function $\rho_1(\rho_0)$. Then

$$V'(\rho_1) = \lambda - 2\lambda \rho_1 + (1 - 2\lambda) (\rho_1 - \rho_1)^2 - 2(\lambda - \lambda^2) (\rho_1 - \rho_1) \rho_1':$$

For $\rho_1 = 0$, this value is $\lambda - 2\lambda \rho_1 + (\lambda - \lambda^2) \rho_1^2 > 0$, so V is increasing for small values of ρ_1 .

If $u(x) = x$, then $\rho_1 = \rho_0$, and $V(\rho_1) = (\lambda - \lambda^2) \rho_1^2 > 0$. Finally, if $u(x) = -\frac{1}{\theta} e^{-\theta x}$, then the solution of the first order condition (6) is $\rho_1 = \frac{\frac{\alpha}{1-\alpha} e^{-\frac{\theta}{1-\lambda}}}{1 - \frac{\alpha}{1-\alpha} e^{-\frac{\theta}{1-\lambda}}}$, which is a constant. Then $V'(\rho_1) = \lambda - 2\lambda \rho_1 + (1 - 2\lambda) (\rho_1 - \rho_1)^2$ is decreasing in ρ_1 . In order to have $V'(\rho_1) > 0$ for all ρ_1 , it is enough that $V'(1) > 0$ holds. That requires $\rho_1 > 1 - \sqrt{1 - \lambda^2}$, which is equivalent to

$$1 < \sqrt{\frac{1 - \lambda}{1 - \lambda}} + e^{-\frac{\theta}{1-\lambda}} + \sqrt{\frac{1 - \lambda}{1 - \lambda}} e^{-\frac{\theta}{1-\lambda}}: \quad (7)$$

So if λ ; α and θ are such that this condition is satisfied, then $V'(\rho_1) > 0$: ■

Intuitively, one would expect the variance to be increasing in λ : with a higher chance of fast learning, there is higher chance of a random price next period, which means more variance. This effect clearly dominates initially. Later, however, as λ becomes even larger, $p_1(\lambda)$ might be significantly different from p_1 , which is the expected price in the fast learning realization, so the slow learning realization may end up contributing more to the total variance than the fast learning case. If this is the case, an increase in λ might decrease the variance. With constant absolute risk aversion, $p_1(\lambda)$ is constant – but it still may be too far from p_1 . Condition (7) ensures that $p_1(\lambda)$ is close enough to p_1 :

Next I turn to the behavior of prices. I will show that prices are always monotonic in λ (the chance of fast learning, or more generally, the speed of learning) – so if $p(0) > p(1)$, then $p(\lambda)$ is decreasing. For this I first establish a relationship between p_0 and p_1 (slow-learning period 1 prices).

Lemma 9 *If u has nonincreasing absolute risk aversion and $p_0 > \tilde{p}_0$, then $p_1 < \tilde{p}_1$:*

Proof. With the slow learning realization, wealth in period 1 is $W = x_0 + p_1 b_0 = 2 - p_0 b_0 + p_1 b_0 = 2 - p_0 + p_1$: Suppose $\tilde{p}_0 < p_0$. Then at the same period 1 price level p_1 , $W < \tilde{W}$: Given that u has nonincreasing absolute risk aversion, lower wealth implies lower demand for the risky asset (the bond). So with \tilde{W} , there would be excess demand for bonds at period 1 (since the market clears with wealth W), which implies $\tilde{p}_1 > p_1$: ■

Proposition 10 *The period 0 price is always a monotonic function of λ . In particular, if $p(0) = p_{slow} > p_{fast} = p(1)$, then $p(\lambda)$ is decreasing.*

Proof. Consider the period 0 objective function:

$$U = \underbrace{[\lambda (v(x+b) + (1-\lambda)u(x+b)) + (1-\lambda)(v(x) + (1-\lambda)u(x))]}_{\text{fast learning}} + \underbrace{(1-\lambda)[(1-\lambda)v(x+p_1b) + \max\{u(x_2+b_2) + (1-\lambda)u(x_2)\}]}_{\text{slow learning}}$$

The indirect utility of the slow learning realization depends only on wealth at period 1 (since the price level is fixed at the equilibrium level), so it can be rewritten as $g(x + p_1 b) = g(2 - \lambda b + p_1 b)$: It is clear that $g' > 0$: For the early learning realization, utility is a function

of both x and b , but $x = 2 - pb$, so the utility can be written as $h(b)$: So the first order condition is

$$g'(2 - pb + p_1 b)(p_1 - p) = [g'(2 - pb + p_1 b)(p_1 - p) - h'(b)] :$$

From market clearing, $b = 1$, so p satisfies

$$g'(2 - p + p_1)(p_1 - p) = [g'(2 - p + p_1)(p_1 - p) - h'(b)] :$$

Suppose that $\beta' > \beta$: If $p(\beta') < p(0)$, then from lemma 9, $p_1(\beta') > p_1(0) = p(0)$: Look at the first order condition for $p = p(\beta')$ and β' : Since $g' > 0$ and $p_1 > p$, the right hand side must also be positive. So if one replaces β with β' , at the same price levels p and p_1 , the first order condition becomes negative. So with β' , at the same p (thus p_1), investors would want to hold less bonds than 1 – so the equilibrium price, $p(\beta')$, must be smaller.

Similarly, if $p(\beta') > p(0)$, then p is increasing at β' . So if p is increasing at zero, then it goes above $p(0)$, and it has to stay above and further increase; if p is decreasing at zero, then it is decreasing everywhere. ■

This is quite an intuitive result (though the argument was a bit complicated): as β goes from zero to one, there is an increasing chance to get the realization with fast information revelation. Therefore, the importance of the wealth fluctuation effect is increasing, and the role of the excess supply effect is decreasing. At any point, the infinitesimal change in $p(\beta)$ is determined by whether we are "subtracting more" from the supply effect or "adding more" from the fluctuations effect. If the full fluctuation effect is bigger than the full supply effect ($p(0) > p(1)$), then this infinitesimal change is negative, and $p(\beta)$ is decreasing; and vice versa.

4 Conclusions

The paper presented a Diamond–Dybvig type model in which agents face a liquidity risk (becoming *more risk-averse* early consumers), and the speed of learning about default risk may also change. The two extremes of learning were no information revelation between issue and maturity, versus full information revelation at some point before maturity. Later I also considered a more continuous form of the speed of learning: learning is either fast

or slow, with some probability α and $1 - \alpha$.

In general, there are two effects at work when one compares price and welfare levels of the two extremes: one is an "excess supply", the other is a "wealth fluctuation" effect. The excess supply effect applies to the slow learning case: without new information, period 0 and period 1 prices must be the same, so the issue price will be determined by asset market equilibrium in period 1, when demand is low and supply is high (only the late consumers are ready to hold the asset, and all the early consumers want to sell). When compared to the "no learning, no taste shock" benchmark, this decreases issue prices: in order to make risk-averse investors hold more of the risky bond, the price must fall.

The wealth fluctuation channel operates under fast learning: with new information in period 1, the period 1 price level becomes a random variable. This generates wealth fluctuations, which hurts early consumers: they consume from their period 1 wealth, and if they are more risk-averse when hit by the taste shock, these fluctuations make them worse-off. Incorporating the chance of a taste shock in the period 0 maximization problem then makes issue prices go down in equilibrium.

So the price comparison of fast and slow learning depends on the relative strength of these two effects: if wealth fluctuations matter more, then fast learning will lead to lower issue prices. Under certain conditions (for example, risk-neutral late consumers but risk-averse early consumers), it is true that the wealth effect dominates the excess supply effect. This result carries through to the continuous case: under the same conditions, the period 0 price level was decreasing in the probability of the fast learning realization.

I also compared the welfare of the issuer W_0 and W_1 for fast and slow learning. For fast learning, the welfare of the issuer is $W_0 = \frac{1}{1+r} [p_1 + (1-p_1)W_1]$ and for slow learning, the welfare of the issuer is $W_0 = \frac{1}{1+r} [p_1 + (1-p_1)W_1]$.

way uncertainty is resolved. With many interim steps of the terminal payoff lottery, pre-maturity prices will fluctuate a lot, and with a chance of early sale (liquidation), investors value such a bond less.

Faster learning, more precise information will in general lead to more volatile prices. Expecting such an event in the future may lead to important price changes already today: if there is a chance that an investor has to liquidate her portfolio exactly in those volatile weeks, that increases the risk of the bond, driving its price down, potentially much more than what the increase in or the uncertainty about default probabilities would imply. Moreover, this effect should be more profound for long-term bonds: their default risk might change only slightly, but price volatility still increases during a crisis, which leads to a larger than expected overall drop in prices.

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